Conceptual alternatives: Competition in language and beyond*

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Abstract

Things we can say, and the ways in which we can say them, compete with one another. And this has consequences: words we decide *not* to pronounce have critical effects on the messages we end up conveying. For instance, in saying *Chris is a good teacher*, we may convey that Chris is not an *amazing* teacher. How this happens is an unsolvable problem, unless a theory of *alternatives* indicates what counts, among all the things that have not been pronounced. It is sometimes assumed, explicitly or implicitly, that any word counts, as long as that word could have replaced one that was actually pronounced. We review arguments against this powerful idea. In doing so, we argue that the level of words is not the right level of analysis for alternatives. Instead, we capitalize on recent conceptual and associated methodological advances within the study of the so-called "language of thought" to reopen the problem from a new perspective. Specifically, we provide theoretical and experimental arguments that the relation between alternatives and words may be indirect, and that alternatives are not linguistic objects in the traditional sense. Rather, we propose that competition in language is better seen as primarily determined by general reasoning preferences, or thought preferences (preferences which may have forged the lexicons of modern languages in the first place, as argued

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elsewhere). We propose that such non-linguistic preferences can be measured and that these measures can be used to explain linguistic competition, non-linguistically, and more in depth.

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1 The phenomenon associated with alternatives

When it comes to semantic interpretation in natural language, words that have *not* been pronounced matter. In fact, language even makes available specific expressions whose interpretation depends on unpronounced, alternative utterances, e.g. *only* and *even*, to mention just two. For instance, *Only Ann passed the exam* conveys not only that *Ann passed the exam* is true, but also that all alternatives of the form *x passed the exam* are false, where *Ann* is replaced by some relevant individual *x*: Bill, Chris, etc. (Horn, 1972; Rooth, 1985). Similarly, *Even Ann passed the exam* conveys that *Ann passed the exam* is not only true, but was less likely to be true than all relevant alternatives of the form *x passed the exam*.

Inferences based on what was said versus what could've been said but wasn't (i.e. what was unsaid; see Horn, 1992) arise even in the absence of particles such as *only* and *even*, and they are known in linguistics as *implicatures*. Perhaps the most widely studied kind are *scalar implicatures*, so called because they involve scales of some kind. For instance, the sentence *Ann or Bill passed the exam* is readily understood as conveying that Ann and Bill didn't both pass. A standard explanation of how this inference arises, pioneered by the philosopher H.P. Grice (Grice, 1975) and refined further by Horn, 1972; Gazdar, 1979; Gamut, 1991 (among others), goes as follows: *or* and *and* form a scale, ⟨*or*, *and*⟩; the speaker could've used the alternative utterance made up from the other member of that scale, *Ann <u>and</u> Bill passed the exam*, and presumably would've (and should've) done so, if this alternative were true; since the speaker didn't use this alternative, it must be because it's false. Thus, it's true that Ann or Bill passed, but false that they both did.

The same disjunctive sentence also typically conveys that the speaker of the sentence is uncertain which of the two passed, and this ignorance inference can be explained along similar lines: the speaker could've used one of the alternative sentences *Ann passed the exam* or *Bill passed the exam* if she had known which of them was true; since she didn't use either one, it must be because she wasn't certain which one of them was true.

One role that alternatives play, then, is that they generate inferences, either plain (sometimes called exhaustive) inferences or ignorance inferences. As the flip side of this, alternatives also help us understand why certain sentences are judged to be odd. For example, the sentence *Italians come from Italy or France* is odd because, even if it is strictly speaking true, it may irresistibly trigger a false suggestion that Italians may not come from Italy, but rather from France. Thus, alternatives are responsible for routinely derived inferences, and these inferences are so robustly anchored in the interpretive process of language that alternatives may not be ignored, even when they result in plain oddness (Magri, 2009).

2 A theory of competition and constraints on alternatives

A complete theory of scalar implicature involves two components. First, one needs to describe a system predicting which inferences are generated by the competition between a given sentence and a given set of alternatives. This has been the subject of much research (Horn, 1972; Gazdar, 1979; Gamut, 1991; Chierchia et al., 2012). But a predictive theory also needs to specify, second, what the set of alternatives is for any given sentence (see Katzir, 2007 for a seminal proposal, on which we will rely heavily). Indeed, the phenomenon we are interested in is not restricted to the $\langle or, and \rangle$ alternation, but arises similarly with other scales, such as $\langle some, all \rangle$, $\langle possible, certain \rangle$, $\langle allowed, required \rangle$, and $\langle warm, hot \rangle$. It is arguably for this reason that each of the sentences in [1] typically licenses an exhaustivity implicature, viz. that the corresponding stronger alternative is false (Horn, 1992).

[1] a. Dev ate some of the cookies. ($\leadsto$ not all)
b. Emily read *Hamlet* or *Macbeth*. ($\leadsto$ not both)
c. It's possible that it's raining. ($\leadsto$ not certain)
d. You're allowed to stay home. ($\leadsto$ not required to)
e. The soup is warm. ($\leadsto$ not hot)

We can now highlight one of the most pressing and immediate challenges for a theory of alternatives, known as *the symmetry problem* (Fox, 2007; Katzir, 2007; Fox and Katzir, 2011), which arises if we do not constrain what scales are made of. Abstractly first, if α typically implicates 'not β', then a theory of alternatives needs to explain why β is the alternative to α, rather than *not* β. If the latter but not the former were an alternative, then α would actually implicate β (= not[not β]); and if both β and *not* β were alternatives, then α would implicate speaker uncertainty about β. In either case, the implicature 'not β' would remain unexplained. The symmetry problem is so called because, without constraints on what counts as an alternative, every potential alternative β has a symmetric partner, *not* β (α *and not* β could play the same role), whose existence preempts the derivation of the scalar implicature actually observed.

To illustrate, an unconstrained theory fails to capture the fact that *some* is routinely understood as 'some but not all' rather than as 'all' (or as ignorance between 'some but not all' and 'all'). A better theory would explain why *all* is an alternative to *some*, while *not all* or *some but not all* aren't. An immediately obvious difference between *all* and *some but not all* is that the former, like *some* itself, is a single word, whereas the latter is a string of multiple words. That is, *some* and *all* are, in some intuitive sense, equally complex, while *some but not all* is more complex than *some*. Building on this observation, the following recipe for generating alternatives is often explicitly or implicitly endorsed; it states that alternatives are derived on just a word-by-word basis.

[2] **Recipe for alternatives.** Take the string of words, and replace one word with another word.

This theory offers (at least) two immediate successes.

Success 1: Solving the symmetry problem. Given the string *Dev ate some of the cookies*, we can replace *some* with *all* to yield the alternative *Dev ate all of the cookies* (whose negation correctly delivers the observed implicature), but we can't replace *some* with *some but not all* (since the latter is not a single word) to yield the alternative *Dev ate some but not all of the cookies* (whose negation would deliver the unobserved inference that Dev ate all of the cookies). As such, we correctly predict that *some* implicates 'not all', rather than 'all'. Similarly, we correctly predict that *Ann or Bill passed the exam* has as an alternative *Ann and Bill passed the exam* (by replacing *or* with *and*), but not *Ann or Bill but not both passed the exam*, hence that *or* implicates 'not both', rather than 'both'. And similarly for all other scalar expressions.

Success 2: Explaining lexical constraints. Since the recipe makes reference to words, i.e. to single lexical items of the language, we predict that scalar inferences are constrained by the lexicon of the language under consideration. For instance, the English sentence *Fred is Gale's brother* doesn't imply one way or the other about whether (the speaker believes that, or is uncertain whether) Fred is older or younger than Gale. By contrast, Japanese has single-word sibling terms that semantically encode seniority (and, like English, gender), e.g. *ani* 'older brother' and *otooto* 'younger brother', as well as non-specific sibling terms that don't encode seniority, *kyoodai* 'brother'. As a result, the use of the non-specific term *kyoodai*, as in *Kochira wa Takashi-kun no kyoodai no Michio-kun desu* 'This is Takashi's brother Michio', implicates that the speaker was not in a position to use either of the more specific terms, *ani* or *otooto*, and as such implicates speaker uncertainty about whether Michio is Takashi's older or younger brother (Matsumoto, 1995).

Interim summary. Under a common view, alternatives are determined by the recipe in [2] and the lexicon of the language under consideration. Despite initial successes, however, careful attention reveals a number of challenges for such a view.

Challenge 1: Hierarchy of logical/content words. Consider the sentence in [3a], where *cards* refers to standard playing cards that can only be red (diamonds and hearts) or black (clubs and spades). This sentence can naturally be understood as conveying that not all of Mary's cards are red. On first glance, the recipe in [2] appears to explain this inference: [3b], but not [3c], is an alternative; thus, the sentence implicates the negation of [3b], rather than the negation of [3c].

[3] a. Some of Mary's cards are red.
 b. All of Mary's cards are red.
 c. Some but not all of Mary's cards are red.
 d. Some of Mary's cards are black.

However, the recipe in [2] also allows us to replace *red* with *black*, yielding the alternative [3d], whose negation means that none of Mary's cards are black. If [3d], and not [3b], were the alternative of [3a], then [3a] would actually implicate that all of Mary's cards are red; and if both [3b] and [3d] were alternatives, then [3a] would implicate speaker ignorance about whether some of Mary's cards are black. Empirically, however, while [3a] can convey either that not all of Mary's cards are red, or that the speaker is ignorant about whether that is true, it simply cannot convey that all of Mary's cards are red. In theoretical terms, this means that [3b] can be an active alternative without [3d] being active (which yields the implicature that not all of Mary's cards are red), or they can both be active (which yields ignorance), but it cannot be the case that [3d] is active without [3b] (since that would implicate that all of Mary's cards are red). Thus, there is an observable asymmetry between [3b] and [3d] that the recipe in [2] is not sensitive to.

Challenge 2: Alternatives beyond the lexicon. The English sentence *John broke all of his arms* is odd: it implies that John has more than two arms. The oddity, and this inference, can be explained by competition with the alternative *John broke both of his arms*, which is a more appropriate description of a situation in which John broke his left arm and his right arm. That the English lexicon contains the terms *all* and *both* is crucial to this explanation, and the prediction is that in a language with only a lexical term meaning 'all', and no lexical term meaning 'both', the sentence corresponding to *John broke all of his arms* ought to not be odd. French is a (not so frequent) case in point; however, the French sentence *Jean s'est cassé tous les bras* 'John broke all of his arms' is just as odd as its English counterpart (Chemla, 2007).

This data point thus indicates that competition goes beyond just the lexicon of a given language.

Challenge 3: Some lexical properties are invisible. Consider the sentence in [4a]. By replacing *all* with *some*, we derive the alternative in [4b], which (to the extent that it's interpretable) has the reading 'there are some books x such that no one read x'. The negation of [4b] is thus equivalent to 'there are no books x such that no one read x', i.e. 'for all books x, someone read x'. As such, we expect [4a] to implicate that each book was read by someone or other; however, this implicature is intuitively unavailable. Rather, the implicature we typically draw from [4a] is that someone (at least one person) read some book (at least one book). We can derive this implicature by instead replacing *all* with *any*, yielding the alternative in [4c], which has the reading 'there is no person x such that x read some book'. The negation of [4c] is thus equivalent to 'there is some person x such that x read some book', precisely the implicature observed.

[4] a. No one read <u>all</u> of the books.
b. No one read <u>some</u> of the books. ✗
c. No one read <u>any</u> of the books. ✓

The problem, however, is that the recipe in [2], which simply makes reference to word-by-word replacement in strings, cannot distinguish [4c] (which we want to be an alternative of [4a]) from [4b] (which we don't). More generally, when *all* occurs in the syntactic scope of a negative quantifier like *no one*, it should be replaceable by *any*, but not by *some*. One possible solution is to revise the recipe in [2] to take syntactic structure into account, and to assume that *all* is replaceable by either *some* or *any*, depending on structure. Another possible solution, which we will explore, is to assume that *some* and *any* have roughly the same meanings — they're existential quantifiers (see Kadmon and Landman, 1993; Chierchia, 2013) — and that universal quantification is replaceable by existential quantification.

Challenge 4: The quest for a deeper explanation. Even if all of the above challenges could be successfully met by a suitably revised lexical recipe, one may raise a more general, conceptual worry about linguistic approaches to alternatives. Why is the meaning 'all', but not 'some but not all', lexicalized in the first place, not just in English, but in language after language? A natural type of answer that has been sought is that 'all' is somehow more *natural*, or *primitive*, than 'some but not all'. Maybe this kind of answer can also provide an answer to the question of why *all*, but not *some but not all*, is an alternative to *some*. In other words, the observation that 'all' is lexicalized makes it a sensible candidate for being an alternative, but it leaves us with the question of why it's lexicalized in the first place. Once we understand why it's lexicalized, perhaps we won't need to explain why it's an alternative. It's an alternative for the same reason: because it's more natural/primitive.

3 Logical primitives and the language of thought

The challenges above arise only if competition is assumed to occur at the linguistic level, i.e. between properly linguistic objects (utterances). If, instead, competition is between 'primitives' at some more conceptual level, then the problems vanish. To be more concrete, we revise the recipe as in [5].

[5] **Recipe (revised).** Take the conceptual representation of the sentence, and replace one primitive element with another primitive element.

This recipe may be seen as a 'neo-Katzirian' view of alternatives. We elaborate on this view in much greater detail in §5, but for now we want to discuss some initial consequences of this view and how to make it concrete and testable. The main difference with Katzir, 2007 is that replacements are (preferably) drawn from a cognitive set of primitive elements, rather than from a lexicon. The full Katzirian set of alternatives, including those involving replacements made available by context, can be recovered as well by assuming that some primitive elements can anaphorically refer to previously mentioned material (it seems easy to mentally point at or refer to thoughts or linguistic expressions).

Now, reconsider challenge 3, for instance (*No one read all of the books*). If we assume that existential ($\exists$) and universal ($\forall$) quantification are primitives, and are the conceptual counterparts of *some* and *all*, respectively, then substitution of one for the other becomes possible regardless of syntactic environment. More precisely, $\forall$ can be replaced by $\exists$, yielding the attested implicature, despite the fact that replacing the word *all* by the word *some* (and keeping the structure the same) would yield an ungrammatical structure, due to the polarity sensitivity of *some*.

Importantly, moving from a linguistic to a conceptual level provides new insight into the symmetry problem and into lexicalization (challenge 4). Suppose that $\exists$ and $\forall$ are primitive, while the conceptual counterpart of *some but not all* is not; rather, it's composed of other primitives like $\exists$, $\wedge$ (conjunction), $\neg$ (negation), and $\forall$.

[6] **Hypothesis.** The conceptual counterparts of *some* and *all* are primitive, while that of *some but not all* is not.

Combined with the natural hypothesis that lexical items, across languages, are more likely to be primitive elements than non-primitive elements, this explains the cross-linguistic stability of lexicalization (Strickland, 2017). Moreover, if primitive elements are also privileged candidates for being alternatives, then this also addresses the symmetry problem. (In §5, we provide a more complete algorithm to derive alternatives in the language of thought.) Our experiment is intended to be a proof-of-concept in support of hypothesis [6].

3.1 What is a primitive element?

Here we try to give an intuitive sense of what we mean by a primitive element. The word *birds* describes an intuitively well-formed natural class (the set of all birds), and so does the word *red* (the set of all red things). Moreover, the combination *red (and) birds* is an intuitively well-formed natural class (the set that includes cardinals but not strawberries or blue jays). However, *red or birds* (the set that includes cardinals, strawberries, and blue jays, but not blue berries) is a rather unnatural class, and *red xor birds* (where *xor* means exclusive *or*) even less so (it includes strawberries and blue jays, but not cardinals).

These clear facts could be used to argue that (the concept) 'and' is, in some sense, more primitive, or preferred, than 'or', and both of those more so than 'xor'. The specific question we aim to address is whether it can be shown that (the concept) 'all' is more primitive, or preferred, than 'some but not all'.

3.2 The logical primitives of thought

In the domain of content words, such as nouns, Gärdenfors, 2014 has most forcefully discussed constraints on lexicalized meanings, presumably originating from conceptual constraints on natural classes. These notions have been operationalized in psychology experiments since then (e.g. Xu and Tenenbaum, 2007), showing that specific properties of some word meanings were more readily accessible than others, in both children and adults. In the domain of logical words, which concerns us the most here, comparable investigations have been proposed from various traditions as well, e.g. Piantadosi et al., 2016; Katzir and Singh, 2013; Horn, 1972; Seuren and Jaspers, 2014 (see Chemla et al., 2018 for a unification of content and logical words).

4 Experiment

The goal of this experiment is to establish preferences between expressions, or rather between their counterparts at the level of thought (i.e. in a hypothesized language of thought). Using again the example above, we would like to know whether one can find an intrinsic preference for 'all' over 'some but not all', which would help solve the symmetry problem exposed above (as well as the lexicalization facts, potentially). To do so, we used an implicit rule discovery task, very much inspired by Piantadosi et al., 2016 (as well as predecessors in the 1960s, e.g. Haygood and Bourne, 1965; King, 1966; Bourne, 1970). We adjusted the task, however, so as to be in a position to draw conclusions about pairwise differences between potential alternatives (while Piantadosi et al., 2016 had the wider-reaching ambition to evaluate as a whole the functional lexicon of the language of thought). The first two pairwise comparisons involve the 'some'/'all' scale of alternatives. Specifically, we will investigate whether 'all' is preferred over 'some but not all', thus explaining the paradigm in [7] (slightly modified from [3]). We will also explore whether 'no' is preferred over 'some but not all', thus explaining the parallel paradigm in [8] (a variant of [4]).[1]

[7] a. Some of the shapes are red.
  b. Not all of the shapes are red. (observed inference)
  c. All of the shapes are red. (actual alternative)
  d. Some but not all of the shapes are red. (missing alternative)

[8] a. Not all of the shapes are red.
  b. Some of the shapes are red. (observed inference)
  c. None of the shapes are red. (actual alternative)
  d. Some but not all of the shapes are red. (missing alternative)

4.1 Participants

A group of 45 participants were recruited on Mechanical Turk and paid $3 each for their participation. IPs were restricted to the United States, and all participants

1 (Many thanks to Jacopo Romoli for raising the following issue clearly and forcefully.) In this experiment, we only address part of the problem, having to do with 'some but not all' not being an alternative neither in the direct case, (7), nor in the indirect case, (8). To be more complete, we would need to explain more, namely that:

(i) Direct case: 'all' is an active alternative, while neither 'some but not all' nor 'not all' are.

(ii) Indirect case: 'none' is an active alternative, while neither 'some but not all' nor 'some' are.

The direct case doesn't seem to add much difficulty: the absence of 'not all' as an active alternative could simply be because 'not all' is intrinsically complex. This would need to be proven independently, but the ingredients seem to be there (on this view, 'not all' would not be an elementary primitive; it would have to be constructed from a negation and 'all'). Note that this would not be so easy to test because of the logical relations between 'all' and 'not all' and the constraints on the task (there needs to be an overlap between the items to be compared). The indirect case may be more challenging, however, because 'some' is likely to be a good primitive (given, e.g., that it is typically lexicalized), and so we need to explain why it does not play an active role in blocking the effect of the 'none' alternative. The experiment will not speak to this, but we mention here two possible solutions. First, even if both are primitives, it could be that 'some' is *less* primitive than 'none' (see §5.3). Second, if 'some' is a good primitive, then 'not some' is a very strong candidate in the context of 'not all'; that is, if single word replacements are favored in the first place, then this in fact makes 'not some' = 'none' a more salient candidate for being an alternative than 'some' on its own.

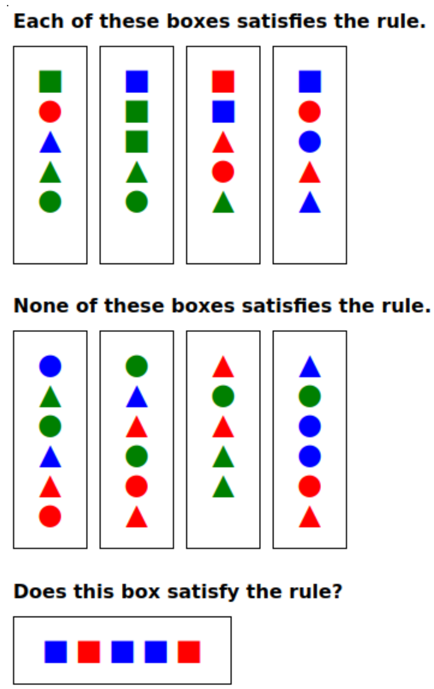


Figure 1: Example stimulus of the condition QUANTIFIERS=(some, SBNA), TYPE=TARGET, where the relevant property is 'square'. The positive boxes all satisfy the two rules 'some (or all) of the objects in the box are square' and 'some but not all of the objects in the box are square', while the negative boxes satisfy neither of those rules. The participants' task was to infer a rule and then to indicate whether the additional box at the bottom satisfies that rule. In this TARGET condition, the final box satisfies one but not the other of the two relevant rules.

reported to be native speakers of English. We precommitted to excluding participants with a median response time below 500ms on the assumption that they would not have been doing the task, but we found that this applied to none.

4.2 Design and task

Figure 1 presents an example of a stimulus that was shown to participants. Their task was to infer a rule that certain boxes of objects satisfy. They were presented with a group of boxes that they were told satisfy the rule (henceforth, *positive* boxes), and another group of boxes that they were told do not satisfy the rule (henceforth, *negative* boxes). At the bottom of the screen was another box, and the participants' task was to indicate whether they thought that this final box satisfied the rule, by clicking either "Satisfies the rule" or "Doesn't satisfy the rule".

The experimental design involved two fully crossed factors: QUANTIFIERS and

TYPE. The levels of the QUANTIFIERS factor are based on pairs of quantifiers, one of which is logically weaker than (entailed by) the other. In association with a particular property (random factor), a given quantifier determines a rule. For instance, in association with the property 'red' (see below for specific details about the stimuli), the quantifier 'all' determines a rule that can be stated as 'All the objects in the box are red'. Two quantifiers (in association with a given property) determine two rules which can be used to characterize the positive and negative boxes of items: positive boxes satisfied both rules, while negative boxes satisfied neither rule. The 8 levels of the QUANTIFIERS factor were: (some, all), (some, SBNA), (not all, no), (not all, SBNA), (at least 3, exactly 3), (at most 3, exactly 3), (at least 4, exactly 4), (at most 4, exactly 4). For the sake of the analysis, the last four were collapsed into two of the form (at least n, exactly n) and (at most n, exactly n). These numerical cases will allow us to check that the method can detect rather uncontroversial cases: the 'at least' rules should be easier than the 'at most' rules. This would align well with the known preference for reasoning with upward-monotonic over downward-monotonic quantifiers (Geurts and Slik, 2005) and with the fact that, in the absence of explicit modifier phrases, bare numerals more naturally acquire an 'at least' reading than an 'at most' reading (see Spector, 2013 for discussion).

The TYPE factor, together with the two rules determined by the QUANTIFIERS factor, determined the final test box. It varied according to whether the final test box satisfied both of the rules determined by the QUANTIFIERS level (in the YES type), neither of them (in the NO type), or exactly one of them (in the TARGET type). As we will explain in further detail in the analysis, the YES and NO types serve as baselines to decide whether participants inferred a rule coherent with our goal, while the TARGET type helped us evaluate which of these two rules they favor: a positive response would correspond to the weaker rule (e.g. 'some'), a negative response to the stronger rule (e.g. 'SBNA').

The quantifiers in a pair had to allow for us to create boxes that satisfy both rules associated with them, none, or only one of them. Concretely, they always stood in an entailment relation. But we are also able to compare two quantifiers which do not stand in a logical entailment relation by comparing each of them to a third quantifier that they both entail and obtaining a measure of how strongly each of them is preferred (or dispreferred) with respect to that common reference point. For example, if we learn that 'all' is strongly preferred to 'some', while 'SBNA' is only weakly preferred to 'some', then this indirectly indicates that 'all' is preferred to 'SBNA' (precisely the preference that would solve the symmetry problem).

4.3 Stimuli

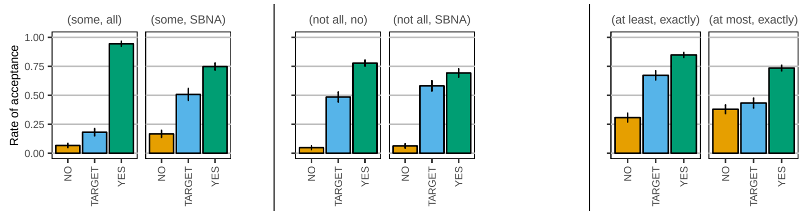
Figure 1 provides a full example of an item, with its characteristics described in the caption. Each item of a given condition was constructed by first randomly selecting the following superficial characteristics: the property required to determine the rules ('red', 'blue', 'green', 'triangle', 'square', or 'circle'), the number of positive boxes (3 or 4), the number of negative boxes (3 or 4), and for each box, the number of objects in that box (5 or 6). The shape and color of the objects in each box were then chosen so that each box would satisfy both, neither, or exactly one of the rules, depending on whether that box was a positive, negative, or test box. To simplify the task of the participants, and in particular the extraction of the relevant property, objects that satisfied the selected property were put together at the beginning (top or left) of their box.

We used a Python script (`https://osf.io/6qarq/`) and a Bash script (`https://osf.io/7dskm/`) to randomly construct the set of items that all participants saw, including 6 repetitions of each of the 24 combinations of the levels of TYPE (3 levels) and QUANTIFIERS (8 levels). The full set of 144 items was randomized for each participant. This randomized set was preceded by 4 practice items (the same ones, in the same order, for each participant). Their nature as practice items was not apparent to the participants, and they all involved a non-numerical QUANTIFIERS factor and a non-target TYPE factor. Our motivation for these choices was to ensure that participants did not hit a target item at the very beginning, and to ensure that participants did not see any discontinuity between the practice trials and subsequent trials. Once the list of items was generated offline, we used the online experimental platform created by Alex Drummond (`http://spellout.net/ibexfarm/`) to implement the task, presenting the (non-practice) items in a fresh random order to each participant.

4.4 Results

All data and an analysis script are available at `https://osf.io/uq3qp/`. Mean results are presented in Figure 2. First, note that the YES type (green bars) and NO type (orange bars) generate appropriate high and low acceptance rates, respectively (grand means of 79% and 17%), while the TARGET type (blue bars) generates intermediate response rates (average 50%). This shows that participants were paying attention and were able to satisfactorily perform what might seem like a difficult task (of a similar nature to the pattern recognition tasks in IQ tests, in that participants have to identify both the dimensions/properties to which the rule pertains and the rule itself).

Furthermore, observe the secondary but interesting difference between the QUANTIFIERS (some, all) and (not all, no). These two pairs are duals of each other



(a) some paradigm. 'all' vs. 'SBNA'

(b) not.all paradigm. 'no' vs. 'SBNA'

(c) num paradigm. 'at least' vs. 'at most'

Figure 2: Proportion of 'yes' responses for each pair of quantifiers and each type of item. no cases made both rules attached to the quantifiers false; yes cases made them both true; and target cases made one of them true and the other false. Error bars represent the standard error of the within-subject mean. Results are grouped by 'paradigm', i.e. pairs of quantifiers which meaningfully share one member ('some', 'not all', or 'exactly *n*') and are used for the comparison between their other members.

(the first member of one is the negation of the second member of the other). Concretely, this means that the constraints on the positive and negative boxes were simply reversed from one to the other: what served as a potential yes test box for one was a no test box for the other. Interestingly, we observe an asymmetry in response rates: the yes bar for (not all, no) is not as high as the no bar for (some, all) is low. This suggests that participants did not perform the task via judgments of similarity (which of the two groups does the test box resemble the most?) — in which case the asymmetry would be unexpected — but rather engaged in a rule discovery task in which the positive exemplars played a different role from the negative exemplars. Overall, these first considerations suggest that participants solved the task as we were hoping.

Let us now move on to the question of how to interpret these data to shed light on the main point of interest. Imagine an ideal world in which participants always infer one of the two rules based on the quantifiers in the pair, and never any other rule. Then the yes items (where both rules are satisfied) and no items (where neither rule is satisfied) should be at ceiling and floor, respectively. In this ideal world, the responses to target items give us a direct measure of preference between the two rules. In actual fact, various biases, errors, and additional rules can play into the task, and hence yes and no items are not at ceiling/floor. Still, the positioning of target items within the range between the two extremes provided by yes and no items provides a measure of preference: the closer target is to yes, the more the weaker quantifier is preferred to the stronger. This rescaling of the range for target items to the span between yes and no eliminates noise created by general yes/no biases and rules that are weaker or stronger than both quantifiers in the pair (for more details, see the discussion in the appendix, §A).

	ALL VS SBNA	NO VS SBNA	AT LEAST VS AT MOST
Δ_{elpd}	–32.0	–3.8	–11.7
$se(\Delta_{elpd})$	7.9	3.1	7.2

Table 1: Differences in estimated log pointwise predictive likelihood and their standard errors for models with one or two γ parameters for sets of quantifier pairs sharing a weaker member. Negative values of Δ_{elpd} indicate evidence in favor of the two-γ model.

For each pair p of quantifiers, we thus fitted models with three parameters: α_p determined the rate of acceptance of NO items, β_p determined the rate of acceptance of YES items, and γ_p determined where in the range between YES and NO we find the TARGET items. For details on the analysis to follow, see §A.

Recall, however, that what we are really interested in is comparing two quantifiers that are not ordered in a given pair. This we can now do by comparing the degree to which they are preferred or dispreferred relative to a third quantifier. We can compare two quantifiers y_1 and y_2 by fitting models for the data for the pairs (x, y_1) and (x, y_2), for some x, and seeing whether we obtain the same γ-parameter for both pairs. If there is a difference between the γ-parameters, this tells us that one of y_1 and y_2 is preferred to the other, in that it is more strongly preferred to x than the other is. The results of our Bayesian model comparison, done by leave-one-out cross-validation, are shown in Table 1.

We find evidence for a difference in all cases. Given the direction of the effect, preferences for 'at least' over 'at most' (our baseline case) and for 'all' over 'SNBA' are strongly supported, while there is more modest evidence for a preference of 'no' over 'SBNA'. (See details in §A, and Burnham and Anderson, 2002, §2.6 for heuristics about differences on the deviance scale; our Δ_{elpd} values are converted to that scale by multiplying them by –2.)

4.5 Discussion

The evidence we found for the preference of 'all' over 'some but not all' (and of 'no' over 'some but not all') provides a new solution to the symmetry problem, casts new light on the scalar inferences from 'some' to 'not all' (and the dual inference from 'not all' to 'some'), and gives new insight into the underlying source of cross-linguistic lexicalization facts. More generally, our work establishes a point of contact between linguistics and psychology, where the two domains seem to coincide: robust facts about language show the footprints of more general properties of cognition.

We took inspiration from work in psychology to design a measure of 'primitivity' of logical elements, one that would apply both to non-linguistic rules and that could impact linguistic competition and linguistic lexicons. One may wonder whether the paradigm we used really taps into non-linguistic abilities, or whether its results merely reproduce the complexity facts at the linguistic (lexico-syntactic) level. We note that it is a complex issue. In particular, such an issue does not quite arise under a view of language as the ability to manipulate abstract representations, whether linguistic representations, non-linguistic representations, abstract rules, pronounced utterances, or unpronounced alternatives, etc. Concretely, it would be interesting to replicate our results under standard dual-tasks, which are supposed to block linguistic abilities (Norman and Bobrow, 1975), but failure to do so would not be very informative since this may simply show that the dual-tasks and the (linguistic) ability they block may already be at play for manipulating abstract rules in the first place. Investigating similar results with pre-linguistic infants or non-human animals could prove interesting too, allowing us to compare the logical repertoires of different animals, at various linguistic stages — that is, independently of the development of a pronounced language, at least as we know (hear/see) it.

5 A program for alternatives

5.1 Katzirian view

Two lessons we learn from the Katzirian view of alternatives (Katzir, 2007) are that structure matters (and not just strings of words) and that different sorts of operations over structures exist. To be more precise, the alternatives of a given structure are best conceived as the result of applying operations to that structure. This view allows one to measure the relative complexity of a structure and its alternatives, by tracing the operations needed to obtain those alternatives. What operations, then, may transform a structure? Katzir proposes the following three:

(i) a part of the structure may be deleted;

(ii) a part of the structure may be replaced by a piece of structure made available in the current discourse (in virtue of its presence in the structure of the current sentence or of a previous sentence in the discourse); and

(iii) a part of the structure may be replaced by an individual lexical item.

5.2 Neo-Katzirian view

We propose an updated version of Katzir's algorithm, in which alternatives are located in the language of thought, and replacements occur only on the basis of single lexical items, albeit lexical items of the language of thought. Assume that a sentence and its structure can always be translated into the language of thought, a language which would thus be structurally similar to that of classical Logical Forms, but with potentially more lexical items. We propose that there is single operation available (recursively) to transform a structure: replacement of part of the structure by a lexical item of the language of thought. The lexical items are diverse, however, and may be:

(i′) a special empty element;

(ii′) a special pronoun capable of pointing at structures; and

(iii′) other elements that may or may not be lexicalized in the actual language.

There is a clear correspondence between (i′) and (i), and between (ii′) and (ii). We note, by the way, that (ii) is not very Katzirian in the first place, in that it may lead to more complex alternatives. However, its updated formulation (ii′) may be seen as Katzirian again (in the sense of not adding complexity), given that the resulting structure would definitely be simpler.[2]

The main innovation, then, is in (iii′), where replacements can occur even on the basis of elements (of the language of thought) that are not lexicalized in the actual language. This immediately gives us a handle on the challenge from French: the oddity of *tous les bras* 'all the arms' is due to the existence of an alternative 'both arms' at the level of thought, even though 'both' is not lexicalized in French.

However, our innovation in (iii′) also raises the important question of what else the lexicon of the language of thought contains (besides the two special elements mentioned in (i′) and (ii′)), and we risk losing the Katzirian solution to the symmetry problem: if, for example, both 'all' and 'some but not all' were lexical items of the language of thought, then we would expect symmetric alternatives and hence fail to explain the observed implicatures. One straightforward way to interpret our experimental results, then, is as follows: they establish that 'all' is (more likely to be) a lexical item of the language of thought, while 'some but not all' is not (likely to be one), hence that 'all', but not 'some but not all', may be recruited during replacements by the structural rule in (iii′), thus solving the symmetry problem.

2 This may look like a technical trick, but the actual existence of a special pronoun of this form, capable of anaphorically referring to a piece of structure made available in the discourse, may actually be a sensible idea. For what it's worth, it seems appealing to say that we can refer to thoughts, at least for ourselves (as in, "*This* and *that* are the reasons why…"). This may even happen in a rather visual manner, and the analogy with pointing pronouns in sign language may be of relevance (Schlenker, 2017). Such considerations are not of central importance, but it is useful to note that even if quite abstract, they could lead to actual predictions: under the view that a pronoun is responsible for picking up the piece of structure from the context, then no further replacement should be possible, even if we assume that replacements can occur recursively.

The resulting theory is a rather direct translation of the Katzirian view from natural language to the language of thought, but in order to fit with our graded experimental results, it would require there to be either a graded conception of membership in the lexicon of the language of thought, or genuine epistemic uncertainty about what belongs to the lexicon of the language of thought. Both of these appear rather questionable, which suggests that something is wrong with the above formulation of (iii'). Instead, we may prefer a more graded notion of *alternativehood*, where rule (iii') involves replacement by potentially *any* element of the language of thought, but replacements are associated with costs that vary, among other things, according to the 'primitiveness' of the element being used.

5.3 A more nuanced view: Costs

In various current approaches to implicatures, alternatives come with relative costs, so that more costly alternatives are less likely to influence the inferential processes (Bergen et al., 2016). As a result, we may escape the symmetry between two alternatives A_1 and A_2 not by justifying that one is actually not an alternative, but simply from the fact that they come at different costs. In a Katzirian view, it is natural to try to obtain these costs from the way the alternative was derived, potentially by assigning costs to the transformations in (i), (ii), and (iii) above. For instance, alternatives obtained by type (i) may by design be simpler, and therefore produce a low cost.

From our neo-Katzirian perspective, we may simply say that the cost of an alternative varies negatively with the size of the structure being replaced, and positively with the complexity of the replacement material. The complexity of replacement material may then actually be the same thing as what we call primitiveness. Surely, (i'), for instance, may incur a very low cost, but the cost of a replacement of the other types may depend on how primitive these elements are.

Of course, we have no way to give precise numbers, but we may still sketch the general idea. If 'both' is relatively primitive, then, first of all, it is likely to be lexicalized in a variety of languages; and, second, even if it is not in a given language, replacing 'all' with 'both' (at the level of thought) is relatively cheap, hence could explain the challenge raised by French *tous les bras* 'all the arms'. Similarly, although our experimental results don't (and could never) definitively establish that 'all' is a lexical item of the language of thought, while 'some but not all' is not, they do support the hypothesis that the former is more primitive than the latter. As such, deriving alternatives via replacement with 'some but not all' is more costly than with 'all', which in turn gives us a considerable handle on solving the symmetry problem.

5.4 Connection with non-graded view of alternatives

The cost-based approach is compatible with a potentially more common, non-graded view of alternatives, according to which the inferential mechanism works from a set of alternatives which do not come with graded costs; a structure simply is or is not an alternative. Formally, one may still use the costs derived from a recipe as sketched above, and require that for a sentence to be in the set of (effective) alternatives, its cost has to pass some threshold. The threshold may also be variable from one occasion to the next, and the relative costs associated with different types of alternatives (i′)–(iii′) may then lead to generalizations of the form: when one disjunct is an alternative to a disjunction, then so is the other disjunction (because they come at the same cost). It would be interesting to see whether the well-described generalizations from Fox and Katzir, 2011 can be derived with appropriate parameters in such a model.

5.5 Influence of language on (the language of) thought

We have argued that at the level of thought, there is an inherent preference for the concept 'all' over the concept 'some but not all', and that such conceptual preferences help explain certain cross-linguistic lexicalization facts, such as the fact that language after language lexicalizes 'all', but not 'some but not all'. Thus, language proper (specifically, its lexicon) appears ready to host natural classes of concepts that fit whatever the constraints of the language of thought are.

However, if, as we have suggested, competition occurs at the level of thought, then nothing precludes the possibility that language proper (e.g. the lexicon) may also influence conceptual alternatives, either directly or indirectly. Directly, one could say that lexicalized alternatives get a cost reduction, simply because they are easily expressible both in the language of thought and presumably in language. Indirectly, we may expect a bidirectional influence between language and language of thought, in a weak version of the Sapir-Whorf hypothesis. Concretely, assume, as we have argued, that competition occurs primarily at the level of thought, and that the concepts 'older brother' and 'younger brother' may well be universal (they appear in some languages, so they should be available at the level of the language of thought in the first place). Yet the fact that Japanese, but not English, lexicalizes them matters: as we have seen, the use of the word corresponding to the concept 'brother' in Japanese (*kyoodai*) implicates speaker uncertainty about 'older brother' vs. 'younger brother', whereas in English (*brother*) this is not the case. A possible translation of this in the current framework is that the costs associated with these concepts vary: they are moderate for all by default in the language of thought, but for Japanese speakers, the alternatives involving these concepts may benefit from a lower cost, simply because of the lexicalization. It is hard to describe, let alone test, the details of such an approach, but it is also hard to see reasons to discard this possibility.

6 Conclusion

Alternatives are pervasive in natural language, and they lie at the heart of scalar inference, such as the inference from *some* to 'not all'. However, a complete theory of alternatives needs to explain how alternatives are constrained, e.g. why *all*, but not *some but not all*, is an alternative to *some*. That is, it needs to overcome the symmetry problem (Katzir, 2007; Fox and Katzir, 2011). We propose that alternatives are constrained both by the lexicon of a language, as standard wisdom has it, but also by more general restrictions on what counts as a primitive concept. Although the two sources remain necessary, we argued that the latter type of constraint may favorably replace the former, both for conceptual and practical reasons. Because this view relies on claims about the lexicon of an invisible language, it cannot be easily tested by standard means. But we propose that the relevant elements of this language can be inferred from regularities observed across languages (in language after language, we may find a word meaning *all*, but not necessarily one meaning *some but not all*), or from general preferences in *a priori* non-linguistic tasks. Overall, a complete evaluation of the proposed approach will only be possible through a better integration of facts and methods that may typically come from psychology and formal linguistics independently of one another.

A Appendix: Statistical analysis

A.1 General description and notations

Semi-formally, our measure is based on three parameters for each quantifier pair p. First, $\overline{\alpha_p}$ represents our best estimate of the true rate of 'yes' responses in the NO cases, and $\overline{\beta_p}$ represents our best estimate of the true rate of 'yes' responses in the YES cases. In an ideal world, we should find $\overline{\alpha_p} = 0$ and $\overline{\beta_p} = 1$.[3] Critically, $\overline{\gamma_p}$ represents the preference between the weak and the strong quantifier, estimated as the position of the responses to the TARGET types not as an absolute rate, but as a proportion with $\overline{\alpha_p}$ and $\overline{\beta_p}$ as extreme points. In pseudo-formula, this means that the actual response in the TARGET type is predicted by $\overline{\alpha_p} + \overline{\gamma_p} \cdot (\overline{\beta_p} - \overline{\alpha_p})$. Concretely, a maximal preference for the weaker quantifier in p would correspond

3 In the upcoming implementation of this idea using logit models, the parameters will not directly represent rates of 'yes' responses, and they will not range between 0 and 1 but rather between $-\infty$ and $+\infty$. To convey the spirit of the analysis more simply, however, we ignore this technicality in this paragraph. To keep track of it, however, we mark the parameters in this informal description with a line above them, as in $\overline{\alpha_p}$.

to $\overline{\gamma_p} = 100\%$, i.e. an expected response rate in the TARGET case as high as in the YES cases: $\overline{\alpha_p} + 100\% \cdot (\overline{\beta_p} - \overline{\alpha_p}) = \overline{\beta_p}$. Conversely, an extreme preference for the stronger quantifier would correspond to $\overline{\gamma_p} = 0\%$, i.e. an expected response rate in the TARGET case as low as in the NO cases: $\overline{\alpha_p} + 0\% \cdot (\overline{\beta_p} - \overline{\alpha_p}) = \overline{\alpha_p}$. Finally, a neutral preference corresponds to $\overline{\gamma_p} = 50\%$ and an expected response rate exactly in between $\overline{\alpha_p}$ and $\overline{\beta_p}$: $\overline{\alpha_p} + 50\% \cdot (\overline{\beta_p} - \overline{\alpha_p}) = (\overline{\alpha_p} + \overline{\beta_p})/2$.

To explain, imagine an ideal world in which participants always infer one of the two rules based on the quantifiers in the pair, and never any other rule. Then the YES items (where both rules are satisfied) and NO items (where neither rule is satisfied) should be at ceiling and floor, respectively. In this ideal world, the responses to TARGET items give us a direct measure of preference between the two rules. In actual fact, various biases, errors, and additional rules can play into the task, and hence YES and NO items are not at ceiling/bottom. Still, the positioning of TARGET items within the range between the two extremes provided by YES and NO items provides a measure of preference: the closer TARGET is to YES, the more the weaker quantifier is preferred to the stronger. This rescaling of the range for TARGET items to the span between YES and NO eliminates noise created by general yes/no biases and rules that are weaker or stronger than both quantifiers in the pair.

One source of simplification here is that we ignore the possibility that participants might infer a rule that is 'intermediate' between the two members of the pair; for example, 'at least 3' and 'more than half' are intermediate between 'some' and 'all'. Depending on the specific rule participants infer and the particular test box showing, this could alter the rate of 'yes' responses, but not because participants opt for quantifiers *from our pair*. We thus work on the assumption that the two rules we compare are the most salient in their range. However, we also note that our main interpretations are based not only on pairs, but on the comparison between two pairs of quantifiers. We submit that the intermediate rules relevant for these two pairs are similar across the pairs we use, so that their role should be cancelled out in our analysis. For instance, when we compare 'all' and 'SBNA' through the pairs (some, all) and (some, SBNA), we assume that the relevant intermediate rules, which are compatible with the positive and negative examples being what they are, and which could make the test box true, are of the form 'at least x' in both cases.

A.2 Models and analyses

The logit models we fit thus had a slightly different structure from the generalized linear models commonly used in the analysis of binary response data. Their basic form was as follows:

$$Y_{pi} \sim \mathsf{bernoulli}(\mathsf{logit}^{-1}(\pi_{pi}))$$
$$\text{with: } \pi_{pi} = \alpha_p \cdot \textsc{no}_i + \beta_p \cdot \textsc{yes}_i + (\alpha_p + \gamma_p \cdot (\beta_p - \alpha_p)) \cdot \textsc{target}_i.$$

Given a pair of quantifiers $p = (x, y)$, the parameter $\gamma_{(x,y)}$ thus represents the preference for the weaker member over the stronger member. Given two pairs of quantifiers (x, y_1) and (x, y_2) that share a common member x, we can then compare y_1 and y_2 by fitting a model to the data from both pairs and seeing whether there is evidence for a difference between $\gamma_{(x,y_1)}$ and $\gamma_{(x,y_2)}$. If there is one, then one of the y's is preferred to the other, in that it is more distinct from x than the other is.

We partitioned our data into three sets based on the quantifier pairs that shared a member: (i) a set comprising data from (some, all) and (some, SBNA); (ii) a set comprising data from (not all, no) and (not all, SBNA); and (iii) a set comprising data from (at least n, exactly n) and (at most n, exactly n). On each of these three data sets, we fitted models that included either one or two γ parameters (allowing, or not allowing, for a relative preference between the two non-shared quantifiers in the two pairs, as discussed in the main text),[4] and different varying-effects structures: there was (or was not) a subject intercept u_{0s}, and either no modification of the parameters, parameter modifiers differing by subject, or parameter modifiers differing by both subject and pair. All models were fitted with MCMC methods using STAN through the `rstan` package in R, with STAN's default uniform (improper) prior over α, β, and γ (in the latter case restricted to $[0, 1]$) as well as the standard deviation hyperparameters for the varying subject effects (which themselves were assumed to be normally distributed around 0). Models were evaluated by leave-one-out cross-validation, approximated by Pareto-smoothed importance sampling with the `loo` package (Vehtari et al., 2016) on the basis of 5,000 samples of the likelihood for each data point, drawn after 5,000 burn-in iterations.

4 α and β were always allowed to be distinct for the two pairs.

	ALL VS SBNA	NO VS SBNA	AT LEAST VS AT MOST
Δ_{elpd}	−32.0	−3.8	−11.7
$se(\Delta_{elpd})$	7.9	3.1	7.2

Table 2: Differences in estimated log pointwise predictive likelihood and their standard errors for models with one or two γ parameters for sets of quantifier pairs sharing a weaker member. Negative values of Δ_{elpd} indicate evidence in favor of the two-γ model.

	$\gamma_{(\text{some,all})} - \gamma_{(\text{some,SBNA})}$	$\gamma_{(\text{not all,no})} - \gamma_{(\text{not all,SBNA}))}$	$\gamma_{(\text{at least,exactly})} - \gamma_{(\text{at most,exactly})}$
mean	−0.58	−0.19	0.52
95% CI	(−0.75, −0.41)	(−0.32, −0.07)	(0.34, 0.70)

Table 3: Posterior means and 95% credible intervals for differences between γ parameters, reflecting relative preference between quantifiers not shared between the pairs.

A.3 Results

For all three data sets, the best model turned out to be the two-γ model with a subject intercept and parameter modifiers varying by subject, but not by pair:[5]

$$
\begin{aligned}
Y_{spi} &\sim \text{bernoulli}(\text{logit}^{-1}(\pi_{spi})) \\
\text{with: } \pi_{spi} &= (\alpha_p + u_{\alpha s}) \cdot \text{NO}_i + (\beta_q + u_{\beta s}) \cdot \text{YES}_i \\
&\quad + (\alpha_p + u_{\alpha s} + (\gamma_p + u_{\gamma s})((\beta_p + u_{\beta s}) - (\alpha_p + u_{\alpha s}))) \cdot \text{TARGET}_i + u_{0s}.
\end{aligned}
$$

In order to see whether the preference for two γ-parameters was meaningful, we compared the optimal models to the variants with identical varying effects structure, but only one (pair-independent) γ-parameter. Differences in estimated log pointwise predictive likelihood and their standard errors for models with one and two γ-parameters are given in Table 2.

In addition, posterior means and 95% credible intervals for the difference between γ-parameters are shown in Table 3:

With a Δ_{elpd} of −32.0, there is strong evidence for a preference for 'all' over

5 Models where parameter modifiers differed by subject and pair had this form:

$$
\begin{aligned}
Y_{spi} &\sim \text{bernoulli}(\text{logit}^{-1}(\pi_{spi})) \\
\text{with: } \pi_{spi} &= (\alpha_p + u_{\alpha sp}) \cdot \text{NO}_i + (\beta_q + u_{\beta sp}) \cdot \text{YES}_i \\
&\quad + (\alpha_p + u_{\alpha sp} + (\gamma_p + u_{\gamma sp})((\beta_p + u_{\beta sp}) - (\alpha_p + u_{\alpha sp}))) \cdot \text{TARGET}_i + u_{0s}.
\end{aligned}
$$

'SBNA'.[6] The direction of this preference is given by the negative difference in γ parameters. If x is stronger than y in a pair (x,y), then a low $\gamma_{(x,y)}$ indicates a stronger preference for x, and since $\gamma_{(\text{some,all})}$ is smaller than $\gamma_{(\text{some,SBNA})}$, we know that 'all' is preferred to 'some' more strongly than 'SBNA' is preferred to 'some', hence that 'all' is preferred to 'SBNA'.

The same reasoning applies to the comparison of 'no' with 'SBNA' via comparison of each with 'not all'. In this instance, however, the effect is smaller and the strength of the evidence, with a Δ_{elpd} of −3.8, much more modest.

For 'at least' and 'at most', a greater γ value indicates a stronger preference over 'exactly', because now the quantifiers of interest are the weaker members of the pair. Since $\gamma_{(\text{at least,exactly})}$ is greater than $\gamma_{(\text{at most,exactly})}$, we can infer a preference for 'at least' over 'at most'. The effect size here is in the same ballpark as with 'all' vs. 'SBNA', and the evidence quite strong.

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6 See Burnham and Anderson, 2002, §2.6 for heuristics about differences on the deviance scale. Our Δ_{elpd} values can be converted to that scale by multiplying them by −2.

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