On “zero” and semantic plurality

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1 Background

The numeric symbol “0” has two distinct guises. First of all, it is a place holder in the representation of large numbers. For example, in “103”, it indicates that there is nothing in the second column, helping us to distinguish “103” from “13”, “1030”, etc. The other use of “0” is as a number itself, the number between “-1” and “1”. Although quite a few ancient civilisations (but famously not the Ancient Egyptian, Greek and Roman ones) had positional number systems with punctuation marks that act like place-holder “0”, the concept of “0” as a number itself only first arose in the seventh century CE, in India. This concept of a “0” number then very slowly spread worldwide. For instance, only by the 17th century, the concept had become common in Europe (e.g. Kaplan, 1999).

Compared to other numbers, “0” is thus a relative latecomer. Naturally, this means that the *word* for the number “0” is similarly a relatively recent addition to the language families of the world. Given its late arrival, we can obviously do very well with a language that lacks a word like “zero”. One would expect that the only thing that changes once such a word is included, is that we can then express the newly formed numerical concept. It allows us to state scientific generalisations dependent on that concept. Newton’s first law of motion (*the law of inertia*), for instance, could now be stated as in (1).

(1) The acceleration of an object is zero if and only if the resultant force on the object is zero too.

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In non-mathematical contexts, it may seem that “zero” brings nothing new to the language. It amounts to simply another way of saying “no”. Take (2):

(2) There are zero emails in my inbox.

Here, “zero” indicates the number of emails in the speaker’s inbox. But, of course, saying that this number is 0 is no different from saying that there are “no” such emails. In non-scientific natural language, it may seem, there is little use for a word for “zero”, given the linguistic possibility of combining existential claims with negation to express the absence of stuff. Perhaps then, ‘zero’ in examples like (2) is a synonymous – maybe, more emphatic – alternative to ‘no’. In this paper, we will argue that such a characterisation is wrong. “Zero” and “no” are semantically and pragmatically distinct. In particular, we will argue that words for “0” are proper *numerals*, just like “one” and “thirteen”. Our conclusion will be that the meaning of “zero” is, quite simply, “0”, even in cases like (2). Additionally, we will show that the existence of a zero numeral has profound consequences for linguistic semantics. We will ultimately conclude that the fact that languages allow ascription of zero quantity to an entity provides evidence that linguistic semantics has access to what at first sight may seem like an ontological oddity: an entity with zero quantity. In other words, we will show that studying “zero” can inform us about the underlying semantic ontology of natural language.[1]

This work is structured as follows. In section 2, we will provide arguments against a quantifier analysis of “zero”. We will conclude that “zero” is a numeral and provide a detailed semantic analysis in sections 3 and 4. In particular, we will give an analysis of the limited ability of “zero” to license negative polarity items. We will end this article with a discussion of the wider semantic consequences of our analysis, in particular of our proposal to

[1] In this article, we will only discuss English “zero”. The word for 0 seems to behave as a member of the numeral class consistently across languages. That said, we know of two languages for which it has been reported that the zero numeral is unavailable in prenominal position. These languages are Western Armenian (as reported in Bale and Khanjian 2014) and Hungarian (as reported in Csirmaz and Szabolcsi 2012).

We believe this rare ban to be purely syntactic. In these languages, “zero” is perfectly acceptable in arithmetic contexts such as “Zero plus two is two”, for which, it has been argued, an atomic type d semantics is required of numerals (Rothstein, 2016). If “zero” can have a meaning of type d or $\langle e, t\rangle$, as we will argue below, then, compositionally, nothing prevents “zero” from combining with a noun (with or without a mediating head like MANY, introduced below). However, an argument position in an arithmetic statement and a modificational position inside a DP are clearly different syntactic environments, for which a syntactically different version of the numeral might have to be available – and in different languages “zero” may be subject to different syntactic requirements of the modificational position.

allow for the existence of a zero quantity entity.

2 "Zero" is not a quantifier

While it is clear that "zero" at times refers to the numerical concept "0", as in "division by zero", it is tempting to think that there is a distinct second use of "zero" which is purely quantificational. That is "zero emails" is simply synonymous to "no emails", albeit perhaps a bit more emphatic due to the choice of the more marked (less frequent) "zero" over "no".

It would be relatively straightforward to account for "zero" in this way if we think of it as a generalised quantifier: a relation between sets. Whereas "no A B" expresses that the intersection between A and B is empty, "zero A B" equivalently expresses that the intersection between A and B has cardinality 0. Given such an account, we would expect not to find any non-pragmatic differences. As we will show next, however, those predictions are wrong.

2.1 Distributional differences

If "no" and "zero" are equivalent generalised quantifiers, then we would expect them to have a similar distribution. It turns out, however, that in many respects, "zero" behaves syntactically like a numeral, not like a quantifier. A first indication of this is that, like other numerals, it allows NP ellipsis. In contrast, "no" disallows such ellipsis. Only the full DP "none" can be anaphoric to cars in (3).

(3) a. John owns four cars. Bill owns zero (*ones).
    b. John owns four cars. Bill owns thirteen (*ones).
    c. John owns four cars. Bill owns *no / none.

Measure nouns like "litre" and "metre" combine with numerals, and also with "zero". Yet, they are infelicitous with "no":

(4) There are { zero / thirteen / ??no } litres of milk in the fridge.

Ratio expressions like "DP per N" only allow a very specific class of DPs, especially numeral ones: "they sold { a hundred / *all } tickets per week". "Zero" pairs with numerals, not with "no". For instance:

(5) This drink contains { zero / thirteen / *no } grams of sugar per bottle.

Another environment that is suitable for numerals, also related to ratios, are *ratio comparatives*. Here, once more "zero" behaves like other numerals and not like “no”. The same goes for multiplicatives, as in (7).

(6) This table is { zero / thirteen / ??no } times longer than that one.

(7) John visited his grandmother { zero / thirteen / ??no } times.

In terms of distribution, then, “zero” appears to behave like a normal numeral. Although we take this to be suggestive of a non-quantificational nature to “zero”, it could of course in principle be that, although “zero” and “no” are semantically equivalent, they differ starkly in their syntactic requirements. That is, in order to show that a quantificational analysis is on the wrong track, we will have to show that “zero” and “no” differ in important *semantic* respects. As we will show next, they do.

2.2 Polarity and NPI licensing

“Zero” and “no” are obviously negative expressions. They differ, however, with respect to the nature of their negativity. In subject position, negative quantifiers trigger positive tag questions, just like sentential negation does, as shown in the following contrasts.

(8) John doesn’t love her, does he/*doesn’t he?

(9) John loves her, doesn’t he/*does he?

(10) No students love her, do they/*don’t they?

(11) Most students love her, *do they/don’t they?

“Zero”, however, pairs with positive quantifiers like “most”, not with “no”, as was previously noticed by De Clercq (2011).[2]

(12) Zero people love her, *do they/dont they?

This datum suggests that, in subject position, “zero” contributes a different *semantics of negation* than “no”. This is confirmed by the inability of “zero” to license NPIs. In (14), we show this for the strong NPI “in years”.

(13) No student has visited me in years.

[2]This is perhaps reminiscent of pairs like “not quite all” and “almost all”, which although similar in meaning differ in terms of their negativity (Horn, 2002; Nouwen, 2010).

(i) Not quite all students love her, do they/*don’t they?

(ii) Almost all students love her, *do they/don’t they?

(14) *Zero students have visited me in years.

The judgements reported in the literature regarding weak NPIs are not totally consistent. However, what *is* consistently reported is that there is a clear contrast between 'no' and 'zero' in licensing weak NPIs. For instance, Gajewski (2011) reports the contrast between (15) and (16), giving (15) a question mark.

(15) No student ever said anything.

(16) ?Zero students said anything.

Zeijlstra (2007) thinks the contrast is clearer and gives (17), a totally parallel example, a star:

(17) *Zero students bought any car.

If both "no" and "zero" are equivalent negative quantifiers, it is hard to see how their licensing of tag questions and NPIs can be so different. We take this fact alone to be sufficient to dismiss an analysis of "zero" as a negative quantifier. In the next subsection, we will show one further semantic difference: the scope of negation for "zero" is more flexible than that of "no".[3]

2.3 Split scope

In some languages, negative indefinites give rise to split scope (Jacobs, 1980; Rullmann, 1995; de Swart, 2000; Penka and Zeijlstra, 2005; Abels and Martí, 2010; Penka, 2011). For instance, Dutch "geen" appears to be able to split into negation and existential quantification, stranding a third operator in the middle.

(18) Je hoeft **geen** stropdas te dragen.
you must-NPI GEEN tie to wear
'You do <u>not</u> have to wear <u>a</u> tie.'

[3]There are other polarity-related phenomena that distinguish 'zero' from 'no' – such as negative inversion and licensing, (i) and (ii), respectively, as reported in Gajewski (2011).

(i) On no/*zero occasion(s) did he mention my help. (Deprez, 1999)

(ii) No/*Zero students but Bill came. (Moltmann, 1995)

These observations buttress our point that 'zero' differs from 'no' in terms of polarity. In the rest of the paper we will focus on NPI licensing as a representative case for polarity phenomena.

In English, a similar effect with "no" can be observed in examples like (19) (Potts, 2000).

(19) The company need fire **no** employees.
'It is <u>not</u> the case that the co. is obligated to fire <u>an</u> employee.'

However, as observed by Potts (2000), in English the phenomenon is much more limited than in other Germanic languages. When the negative polar modal "need" in (19) is replaced by the neutral "to have to", no split scope arises:

(20) The company has to fire **no** employees.
#'It's <u>not</u> the case that the company has to fire <u>an</u> employee.'

Strikingly, however, the version with "zero" *does* have a split reading.

(21) The company has to fire **zero** employees.
'It's <u>not</u> the case that the company has to fire <u>an</u> employee.'

In summary, the negative force of "zero" differs from that of "no" in two important respects: (i) it is weaker in the sense that it does not license sentence negation phenomena like positive tag questions and NPIs; (ii) it is more flexible in that it can split and scopally interact with other operators. In the remainder of this article, we will show that these properties follow once we assume that the semantics of "zero" is parallel to that of (other) numerals.

3 Numeral semantics

Numerals are not quantificational determiners in the classical generalised quantifier sense. In particular, it is often remarked that numerals lack quantificational force, as evidenced by minimal pairs like (22) and (23), from Link (1987).

(22) Three men lifted the piano.

(23) Three men can lift the piano.

While (22) has existential force, (23) is a generic statement about the lifting capacities of groups of three men. The semantics for the numeral "three", then, should be void of existential force, since that existential force must come from the particular environment that is present in (22) and absent in (23).

Most contemporary semantic analyses of numerals assume that numerals only have indirect quantificational force. We will ultimately propose a semantics of "zero" that is entirely parallel to such existing proposals for other numerals. Before we can do this, we will need to be very clear about the underlying assumptions that exist in the literature.

3.1 Non-quantificational views on numeral semantics

We will discuss three prominent views from the literature that are more or less closely related in the sense that the semantics they propose for the numeral lacks quantificational force.

3.1.1 The modificational view

On the first analysis, which could be called *the modificational view*, numerals are like intersective adjectives, they denote properties (for instance, Rothstein, 2016). On this view, the semantics of (22) and (23) first of all concerns the intersection between three properties: being men, lifting or being able to lift the table and *being 3*. This means that at their core they involve the open propositions in (24) and (25), respectively,

(24) $\#x = 3 \wedge {}^*\mathbf{man}(x) \wedge {}^*\mathbf{lift\text{-}the\text{-}piano}(x)$

(25) $\#x = 3 \wedge {}^*\mathbf{man}(x) \wedge \Diamond{}^*\mathbf{lift\text{-}the\text{-}piano}(x)$

Here, the cardinality information is given as a property of the variable x: $\#x = 3$ says that x is a group consisting of three atomic entities. If x has this property of being 3-*many*, then x must obviously be an complex entity, a so-called *plural individual*. This is why the predicates **man** and **lift-the-piano** are lifted to properties of plurals, using the * operator. (See below).

For the respective existential and generic force, external quantifiers are introduced to yield (26) and (27), respectively.

(26) $\exists x[\#x = 3 \wedge {}^*\mathbf{man}(x) \wedge {}^*\mathbf{lift\text{-}the\text{-}piano}(x)]$

(27) $\textsc{Gen}x[\#x = 3 \wedge {}^*\mathbf{man}(x) \wedge \Diamond{}^*\mathbf{lift\text{-}the\text{-}piano}(x)]$

(We will not have much to say about how this quantificational force comes about, and in what follows, we ignore generic uses completely.) The modificational view derives these meanings by analysing numerals as denoting sets of equally sized individuals. For instance:

(28) $[\![\text{three}]\!] = \lambda x.\#x = 3$

This says that “three” denotes the set of all complex entities that consist of exactly 3 atoms. Just like the combination of a noun and an (intersective) adjective like “yellow” is interpreted as the intersection between their two extensions, numeral-noun combinations are interpreted via intersection, too.

(29) $[\![\text{three men}]\!] = \lambda x.\#x = 3 \wedge {}^{*}\mathbf{man}$

According to (29), “three men” is a set denoting indefinite. It is subject to whatever future compositional operations other indefinites are subject too, like gaining existential force from elsewhere.

3.1.2 The degree type view

On a closely related view, numerals do not express properties, but rather directly express cardinalities (Hackl, 2000). For instance:

(30) $[\![\text{three}]\!] = 3$

The connection between the noun phrase and the numeral is mediated by a silent counting quantifier “MANY”. We will follow Hackl (2000) and many others in assuming that MANY simultaneously allows the numeral to express the cardinality of something and introduces existential force.[4]

(31) $[\![\text{MANY}]\!] = \lambda n.\lambda A.\lambda B.\exists x[\#x = n \wedge {}^{*}A(x) \wedge {}^{*}B(x)]$

This says that silent MANY expresses a determiner that takes a number and two properties and returns the proposition that there exists a group of that number that has both properties. For instance, “three men lifted the piano” is analysed as [[[three MANY] men] lifted the piano], resulting in:

(32) $\exists x[\#x = n \wedge {}^{*}\mathbf{man}(x) \wedge {}^{*}\mathbf{lift\text{-}the\text{-}piano}(x)]$

This result is exactly the same as what we arrived at via the modificational account. Importantly, the result represents an *at least* reading for the numeral: (32) says that for some group of three it is the case that this group consists of piano-lifting men, but it does not exclude the possibility that more men lifted the piano. This is a welcome prediction, since “three men lifted the piano” is consistent with more men doing so, as evidenced by the contrast between (33) and (34):

(33) Three men lifted the piano, if not more.

[4] This means that special arrangements are in order to account for the generic readings. While it is easy to sever the quantificational force from the cardinality function, the proper account of generic readings is far from straightforward. See Buccola (2017) for discussion.

(34) Exactly three men lifted the piano, #if not more.

This is not to say that "three" is not regularly interpreted to mean *exactly three*. One would have to posit some additional mechanism, like that of a scalar implicature (see below), to strengthen (32) into such a reading.

3.1.3 The de-Fregean view

The third account of numeral semantics we discuss goes in the opposite direction. It takes the exactly reading as basic and derives the at least reading evidenced in (33) via an extra mechanism. Kennedy's so-called de-Fregean account (2015) also makes use of the counting quantifier *many*, but rather than having numerals denote numbers directly, they denote quantifiers over numbers. If we take cardinalities to be of the type of degrees, d, then numerals are not simply of that type, but rather of type $\langle\langle d,t\rangle,t\rangle$. On this account, "three" expresses a property of sets of numbers: it is true only of those sets of numbers that have 3 as their maximum.

(35) $[\![\text{three}]\!] = \lambda P.max(P) = 3$

Since MANY takes a number as its argument and numerals provide a degree quantifier, the numeral will have to raise, leaving behind a trace of type d. For our running example, "Three men carried the piano", we get the structure in (36) and the interpretation in (37).

(36) [three [λ_i [[[t_i MANY] men] carried the piano]]]

(37) $max(\lambda i.\exists x[\#x = i \wedge {}^*\mathbf{men}(x) \wedge {}^*\textbf{lift-the-piano}(x)]) = 3$

Here, the maximality operator looks at a set that collects those numbers i such that there are at least i men lifting the piano. In a world in which exactly three men (each) carried the piano, this set is going to be {1,2,3}. The maximum will then be 3 and the sentence is correctly predicted to be true. In case there are more men lifting the piano, the maximum will be higher and the sentence is predicted to be false. This means that (37) is the exactly reading of the numeral. Kennedy's route to derive the at least reading involves type shifting the degree quantifier $\lambda P.max(P) = 3$ to the degree denoting 3. From there, the at least reading is derived straightforwardly in combination with MANY, just like it was in the account that takes the degree meaning as basic. Note that this account, like the two accounts described above, does not encode quantification over individuals in the semantics of a numeral – it is part of the meaning of MANY, like in the degree type view. Two differences between this analysis and the other two are that, 1) the semantics of statements with numerals under this account is a statement about a set of degree intervals – in particular, a set of intervals that share a particular maximum; 2) the exactly meaning is basic and not derived from the at-least reading. These two differences will be important for 'zero'.

3.2 Numerals and plurality

Because in all these views the cardinality information provided by a numeral is thought to be the property of an entity (as in $\#x = n$), the semantics of numerals is intrinsically related to semantic plurality. Clearly, singular entities cannot have the property "thirteen", only *plural* entities, i.e. so-called *pluralities*, can. Because our argument will be that "zero" is important to semantic plurality, it is worthwhile to be a bit more specific about the assumptions on semantic plurality.[5]

Pluralities are made of atoms. In particular, if a and b are two atomic entities, then there exists a plural entity $a \sqcup b$, the plurality that consists of nothing but a and b, or, the *sum* of a and b. In general, for any set of entities X, there exists an entity $\sqcup X$ whose parts are the elements of X as well as *their* parts, while nothing else is part of that individual. So, $\sqcup\{\text{john}, \text{mary}\}$ is $\text{john}\sqcup\text{mary}$ and $\sqcup\{\text{john} \sqcup \text{mary}, \text{sue}, \text{ann}\}$ is $\text{john} \sqcup \text{mary} \sqcup \text{sue} \sqcup \text{ann}$.

Following Link (1983), we write * for the operator that allows us to map any set to the set of all pluralities that can be built from the elements of this set.

(38) $\quad {}^*Z \;=\; \{\sqcup X \mid X \subseteq Z \;\&\; X \neq \emptyset\}.$

[5] Note that what we are interested in is the relation between "zero" and *semantic* plurality. There are some very interesting questions regarding morphosyntactic plurality that are outside the scope of this article. One such issue is the cross-linguistic variation in number marking on a noun that "zero" combines with. In languages that have different number marking on the noun depending on the numeric value on the numeral (singular with 1, plural with the rest), "zero" patterns with numerals higher than 1. An example of such language is English, and the majority of Indo-European (we don't have complete data). Why does number marking with "zero" in these languages pattern with higher numerals rather than with "one"? We believe that this is not a semantic issue. Semantically speaking, both the nominal and the VP predicate in sentences with numerals are plural, as seen from our denotations throughout the paper. The inevitability of semantic pluralisation for statements about cardinality makes morphological marking of plurality logical – but also excessive. In English-like languages, numerals more often than not require the noun to be marked for plural, which makes numeral "one" an odd exception that requires explanation. In other languages, however – one example being Turkish – nouns are systematically morphologically singular in combination with numerals. In such languages, "zero" also combines with a singular noun. For a defence of a syntactic agreement view on number marking on nouns with numerals, see, for example, Krifka (2003).

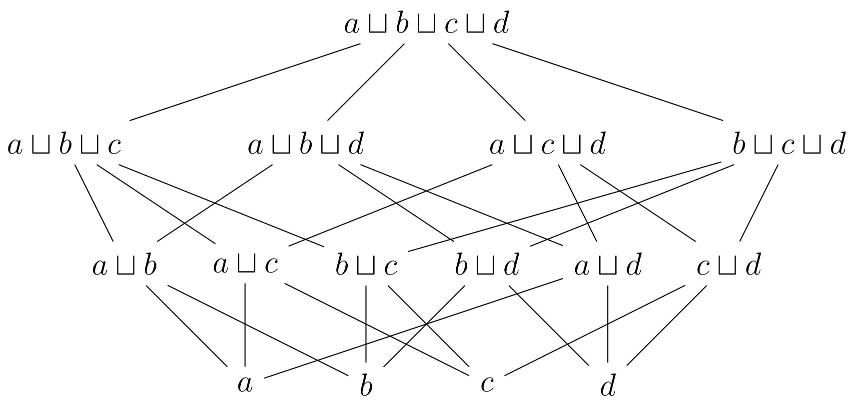

Figure 1: The set of pluralities $^*\{a, b, c, d\}$

This operation is illustrated in figure 1. The arcs between the nodes correspond to *inclusion*, $\sqsubset$, when read from bottom to top. So, $a \sqsubset a \sqcup b$, and $a \sqcup b \sqsubset a \sqcup b \sqcup c$.[6]

The operation * is essential to explain how predicates that are incompatible with group action can nevertheless combine with plural arguments. For instance, *breathing* is a property of atoms only and so, one would expect that its extension only contains atoms, this in contrast to collective predicates like "to meet". However, if "to breathe" is true only of atoms, how come we can truthfully state that "John and Mary are breathing"? The answer is that in this case the predicate is pluralised using *. If B is the set of breathing atoms, and if that set includes both John and Mary, then $^*B$ will include john $\sqcup$ mary. "John and Mary are breathing" is true if and only if the plurality john $\sqcup$ mary is indeed a member of $^*B$.

Once we have plural individuals in this way, we can also express numerical properties. In figure 1, there are four layers. The bottom layer is the layer of atoms, entities of cardinality 1. The layer above that has the pluralities of cardinality 2, and so forth.

The definition we provided for * is the one commonly assumed in the literature. It explicitly excludes the sum of the empty set. Formally, this makes the resulting structure a so-called *semi-lattice.*

The sum of the empty set is an individual that has no proper parts. It is the bottom element in a *full* lattice.[7] We will write this element as $\perp$ and,

[6] Moreover, not shows in the diagram, $a \sqsubset a \sqcup b \sqcup c$. That is to say, the $\sqcup$ relation is *transitive.*

[7] The crucial property of semi-lattices is that for any two individuals in the semi-lattice,

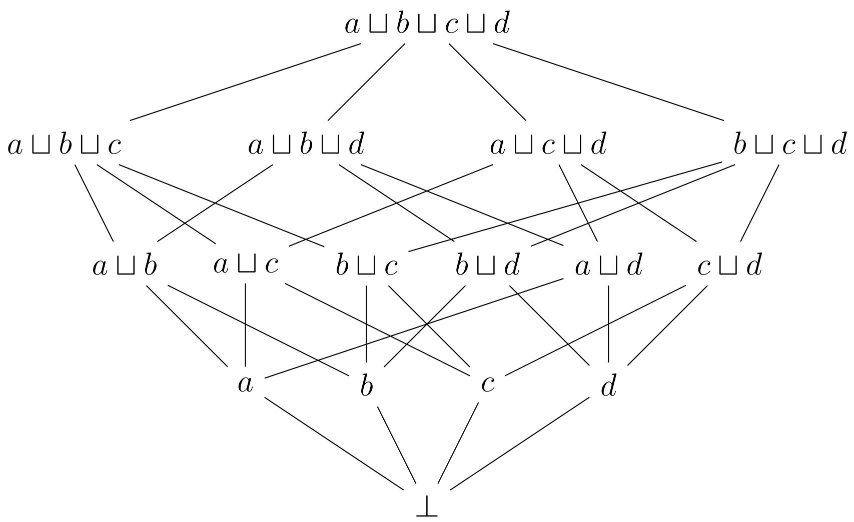

Figure 2: The set of pluralities ${}^{\times}\{a, b, c, d\}$

so, $\sqcup\emptyset = \perp$. Correspondingly, we could have suggested a way of forming pluralities from a set Z that includes the sum of the empty subset of Z. We will write this operation as ${}^{\times}$:

(39) $\quad {}^{\times}Z \;=\; \{\sqcup X \mid X \subseteq Z\}$

This operation yields the full lattice in figure 2, which is exactly the same as the semi-lattice in figure 1 except that it has this extra element, which turns out to be a proper part of any other entity in the structure. Crucially for our argument, while the semi-lattice in figure 1 only allows us to express cardinalities of 1 or more, the full lattice of figure 2 contains an entity of cardinality 0 as well.

For most scholars, the choice between a semi-lattice domain and one in the shape of a full lattice is purely cosmetic: since the bottom element has no obvious use, it is easier to do without it. As Landman (1991) explains:

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the sum of these individuals is also in the lattice. To qualify as a *full* rather than a *semi*-lattice, the mirror image operation to summation, $\sqcap$ or *meet*, should also always result in individuals in the lattice. The meet operation returns what two individuals have in common: it is the biggest entity that is part of both individuals. For instance, $(a \sqcup b) \sqcap b = b$. For a set of atoms Z, ${}^{*}Z$ is not a full lattice, though, because the meet of two distinct atoms is not in ${}^{*}Z$. The meet of two distinct atoms, $a \sqcap b$, is precisely $\perp$, since this is the biggest element that is a part of both a and b. In other words, to go from a semi-lattice structure to a full lattice structure, one has to assume the existence of a bottom element.

"In a full [lattice] we have this 0 element. In all the important definitions, we want to apply our concepts to singular or plural individuals, excluding 0. This means that we have to add exclusion clauses ($x \neq 0$) to all of them. Assuming from the start that 0 is not there, will make the definitions simpler and more readable." (p. 302).

A straightforward illustration of the consequences of including $\perp$ comes from bare plurals. Intuitively, a sentence like (40) should receive an analysis along the lines of (41).

(40) There are typos in the text.

(41) $\exists x[{}^{*}\mathbf{typo}(x) \wedge {}^{*}\mathbf{in\text{-}the\text{-}text}(x)]$

This simply says that the text is not without typos. But what would happen if we applied ${}^{\times}$ instead of *, as in (42)?

(42) $\exists x[{}^{\times}\mathbf{typo}(x) \wedge {}^{\times}\mathbf{in\text{-}the\text{-}text}(x)]$

It turns out that this is a tautology. The reason for this is that for any predicate P, it is true that ${}^{\times}P(\perp)$. This is because the empty set is a subset of the extension of P no matter what that extension is. Consequently, $\perp$, the sum of $\emptyset$, is a member of the extension of ${}^{\times}P$, no matter what P denotes. It follows that there will always be an x such that ${}^{\times}\mathbf{typo}(x) \wedge {}^{\times}\mathbf{in\text{-}the\text{-}text}(x)$, just take $x = \perp$.

3.3 Numeral "zero"

If we want to maintain that "zero" is numeral, a significant dilemma emerges. On the one hand, if we maintain the dominant choice in the literature of adopting *, we have no hope of doing justice to "zero", since it expresses 0 cardinality, a concept that is not defined in the semi-lattice obtained by applying *. On the other hand, if we go against the grain and adopt ${}^{\times}$, then it turns out that sentences with "zero" inherit the problem we just observed for bare plurals: we wrongly predict them to be tautological (and, as we will see below, other problematic cases of the same kind can be found).

We first illustrate this dilemma using the degree denoting account of numerals. On such an account, we would interpret "zero" as in (43).

(43) $[\![\text{zero}]\!] = 0$

In combination with MANY, it yields the determiner meaning in (44):

(44) $\lambda A.\lambda B.\exists x[\#x = 0 \wedge {}^{*}A(x) \wedge {}^{*}B(x)]$

Given this, "zero students passed the test" is assigned the truth-conditions in (45).

(45) $\exists x[\#x = 0 \wedge {}^{*}\mathbf{student}(x) \wedge {}^{*}\mathbf{pass\text{-}the\text{-}test}(x)]$

This is a contradiction. Since on the assumption that there is no $\bot$, there is no entity with zero quantity, and any proposition of the form $\exists x[\#x = 0 \wedge \varphi]$ is contradictory. This is, of course, an unwelcome result. We can very well imagine what it is like for it to be true that zero students passed.

Using a full lattice, the result is not much better, however.

(46) $\exists x[\#x = 0 \wedge {}^{\times}\mathbf{student}(x) \wedge {}^{\times}\mathbf{pass\text{-}the\text{-}test}(x)]$

No matter what the extensions of *student* and *pass-the-test* is, the full lattices formed by applying $^{\times}$ to them is always going to contain $\bot$. So, (46) is always true: just take $x = \bot$.

Note that the modificational view, on which "zero" would denote the property of having 0 cardinality ($\lambda x.\#x = 0$) fares just as badly, simply because it would also assign (45) or (46) to "zero students passed the test".[8]

"Zero" creates a problem for the degree quantifier account (Kennedy, 2015) as well, although this problem comes about in a somewhat different way. On that view, the sentence 'Zero students passed the test' would yield (47) or (48).

(47) $max(\lambda i.\exists x[\#x = i \wedge {}^{*}\mathbf{student} \wedge {}^{*}\mathbf{pass\text{-}the\text{-}test}]) = 0$

(48) $max(\lambda i.\exists x[\#x = i \wedge {}^{\times}\mathbf{student} \wedge {}^{\times}\mathbf{pass\text{-}the\text{-}test}]) = 0$

In a world with no students passing the test, the set operated on by the maximality operator in (47) differs from that in (48). The former is the empty set. Since there is no student passing the test, there is also no number such that at least so many students passed the test. Assuming maximality is undefined for the empty set, the sentence is wrongly predicted to be semantically ill-formed.

In contrast, (48) yields the right truth-conditions.[9] In a world without

[8] On this account, failing to include $\bot$ means that "zero" denotes the empty set. If we do include it, it denotes $\{\bot\}$.

[9] One could suggest that instead of changing * for $^{\times}$ one could change the max function for a function that is defined on the empty set and returns 0 in such case. Such a function indeed exists: this would be the supremum function sup. Changing max for sup could give us 0 without introducing $\bot$, because of the way it is defined: $sup(\{\}) = 0$. However, introducing sup as part of meaning of numerals in the place of max will cause problems elsewhere. Consider the following scenario: You are allowed to take any amount of luggage on the flight as long as it's under 20kg, in other words, the allowance is an open interval

passing students, there is exactly one entity that is a student and passed the test, namely $\perp$. That entity has cardinality zero, and, so, the maximality operator will be applied to the set $\{0\}$, the maximum of which is 0. In any other kind of world, there will be entities that are students who passed the test of higher cardinality, and so the sentence is correctly predicted to be false in such worlds.

As they stand, things are not promising: While in section 2, we presented arguments that "zero" is not a quantifier, we now have seen that giving "zero" a numeral semantics creates a myriad of problems. On the modifier and degree approaches, only non-informative readings are generated. Under an *exactly* analysis using the degree quantifier meaning for numerals, semantic ill-formedness is wrongly predicted, unless we assume the existence of the bottom element. It is not clear what consequences such an assumption has. As we will show next, however, once we take *exactly* readings of numerals into account and derive them from *at least* readings, things start falling into place.

4 Proposal for "zero"

As we will argue now, the dilemma posed by numerical accounts can be resolved once we take into account the fact that numerals alternate between at least and exactly readings, and once we allow the exactly reading to be derived from the at least one. In fact, in this section we will show that the numeral analysis precisely predicts the way "zero" is of different polarity than "no".

4.1 Tautological semantics and exhaustification

Recall that on an *at least* reading account of numerals (both on the modificational and the degree-denoting account), (49) can be given the semantics in (50).

(49) Zero MANY students passed the test.

(50) $\exists x[\#x = 0 \;\&\; {}^{\times}\mathbf{student}(x) \;\&\; {}^{\times}\mathbf{pass\text{-}the\text{-}test}(x)]$

[0,20). The claim 'You are allowed to bring 20kg of luggage' would still be false, even though $sup[0, 20) = 20$. This is clearly an unwelcome result.

Another possibility would be to stipulate that $max(\emptyset) = 0$ and leave the max function otherwise unaltered. Using such a stipulation, (47) then expresses the correct truth-conditions, just like (48) does. However, it also inherits the issues we discuss below.

As we explained, as it stands this is a trivial statement: it irrespective of how many students passed, since the bottom element is true of any predication ${}^{\times}P$. This means it is an at-least meaning, compatible with there being a y s.t. $\#y > 0$ & ${}^{\times}$**student**(y) & ${}^{\times}$**pass-the-test**(y). And, so, in other words, (50) says that zero or more students passed the test.

Like with other numerals, however, an exactly meaning can be derived from the at-least meaning by exhaustification. Here, and in what follows, we will assume that this meaning comes about via a silent operator EXH. Nothing much depends on this assumption, however. We could have equally assumed that the exhaustification effect is not represented syntactically and is a pragmatic quantity implicature.

Since at sentence-level "zero" means "zero or more", other numerals offer stronger statements ("one or more", "two or more" etc.). Exhaustification denies all such stronger statements (the meaning component added by exhaustification is underlined):

(51) $[\![$EXH Zero student passed$]\!]$ =

$$\exists x_e[\ \#x = 0\ \&\ {}^{\times}\mathbf{student}(x)\ \&\ {}^{\times}\mathbf{pass\text{-}the\text{-}test}(x)\ \&\ \underline{\neg\exists y_e\ [\#y > 0\ \&\ {}^{\times}\mathbf{student}(x)\ \&\ {}^{\times}\mathbf{pass\text{-}the\text{-}test}(x)]}\ ]$$

This now states that there are zero or more students that passed the test, but that there are not more than 0. In other words, the number of passing students is exactly 0: everyone failed.

Unlike other numerals, "zero" invokes exhaustification obligatorily. This is for purely pragmatic reasons. Quite simply, statements with "zero" are semantically defective without exhaustification. On our syntactic view on exhaustification, interpretation of a sentence with "zero" always yields two meanings, a defective one derived without EXH and a non-defective one with EXH. That latter meaning will always be the only one to surface (provided it is not itself defective for independent reasons).

In contrast to other scalar implicatures, the "not more than 0" component of exhaustified "zero" does not disappear in embedded contexts. While (52) strongly implicates that John does not take both sugar or milk, (53) does not just mean that nobody takes both sugar and milk. Instead, it means the stronger *nobody takes either*. This is standard behaviour for scalar terms. Weak scalar terms, like disjunction, trigger implicatures to make them more informative. Yet, in certain embedded positions, like in the scope of negation, for instance, their weak semantics ends up being very strong because of the scale reversal introduced by the higher operator. Weak scalar terms are strong in downward entailing context and this is why they fail to trigger implicatures in such contexts.

(52) John takes sugar or milk.

(53) Nobody takes sugar or milk.

(54) Nobody read zero books.

The example in (54) differs from (53) in that the exactly implicature is still in place, even though “zero” is in the scope of a downward entailing operator. This is simply because in contrast to other scalar terms, the semantics of “zero” is not more informative in downward entailing contexts. For instance, the negation of a tautology is just as defective as the tautology itself.

Crucially, having exhaustification rescue the defective semantics of “zero” does not mean that “zero” effectively means the same as “exactly zero”. This is because the exhaustification effect will in some cases happen at a distance. In fact, we have already seen an example of this, when we discussed split scope in section 2. For (55), we observed it had a reading paraphrasable as *the company does not have to fire any employees.* We obtain this reading straightforwardly, by exhaustifying at the top level, above the modal verb, as in (56). Here, once more, the first conjunct (the non-exhaustified semantics) is tautological. It is the exhausification effect, the second conjunct, that shapes the meaning of this statement.

(55) The company has to fire zero employees.

(56) $$\Box\exists x[\#x = 0 \land {}^{\times}\mathbf{employee}(x) \land {}^{\times}\mathbf{fire}(\mathbf{theC}, x)] \land \neg\Box\exists x[\#x > 0 \land {}^{\times}\mathbf{employee}(x) \land {}^{\times}\mathbf{fire}(\mathbf{theC}, x)]$$

Our observation for (55) is parallel to any similar examples with any other numerals, like for instance, (57).

(57) The company has to fire three employees.

On its most salient reading, this means that the company needs to fire three or more employees, but that there is no requirement to fire more. Of course, here too, this is accounted for straightforwardly in terms of a sentence-level scalar implicature. In other words, “zero” yields split scope readings just like other bare numerals do.

As we reported in section 2, (58) differs from (55) in that is fails to yield a split scope reading.

(58) The company has to fire no employees.

This is to be expected given that “no” is the end point of a scale and, thus, no exhaustification effects can occur. The fact that in other languages (Dutch and German, for instance) parallel examples to (58) *do* give rise to split scope, means that there must be more than one split scope phenomenon.

Note that split scope readings can be captured under the degree quantifier analysis without our current assumption that the *exactly* meaning is derived from *at least* meaning – simply by quantifier raising of the numeral above the modal verb:

(59) $[\![\text{The company has to fire three MANY employees}]\!] =$
$[\![\text{three}]\!](\lambda n.\Box\exists x[\#x = n \wedge {}^{\times}\mathbf{employee}(x) \wedge {}^{\times}\mathbf{fire}(\mathbf{theC}, x)]) =$
$max(\lambda n.\Box\exists x[\#x = n \wedge {}^{\times}\mathbf{employee}(x) \wedge {}^{\times}\mathbf{fire}(\mathbf{theC}, x)]) = 3$

These truth-conditions capture the split-scope reading – the lack of obligation to fire more than three employees, a reading derived in this case without the combination of *at least* meaning and scalar implicature.

As such, all three approaches to the semantics of numerals now allow us to derive the right truth-conditions while at the same time offering an account for the split scope readings with "zero". However, as we will now see, there is a crucial difference between *at least* and *exactly*-semantics-based analyses. This difference concerns one of major differences between "zero" and "no": polarity.

4.2 "Zero" and negative polarity items

As we observed in section 2, "zero" appears not to be able to license negative polarity items, in contrast to "no". As we will explain in this section, with adopting an at least semantics for "zero" and other numerals provides a way to approach this contrast. Under that view we adopted above, "zero" differs semantically from "no" in that its negative effect comes about via exhaustification. As we will show, this difference can indeed account for the differences in NPI licensing – unlike the *exactly*-based degree quantifier semantics. Let us first introduce our assumptions.

Traditionally, NPIs are thought to be licensed in downward entailing (DE) environments (Ladusaw, 1979). We follow Gajewski (2011) in assuming that an NPI is licensed when the environment it occurs in is DE. Assuming an exhaustification operator as we have done above, this licensing condition has two parts:

(60) Two licensing conditions for NPIs (Gajewski, 2011)

Given a structure $[_\alpha$ EXH $[_\beta \ldots [_\gamma$ NPI $] \ldots]]$:
Condition 1: the environment γ is DE in β
Condition 2: the environment γ is DE in α

These two conditions are needed to account for differences between weak NPIs (for instance, "any") and strong NPIs (for instance, "either", "to lift a finger").

(61) a. Weak NPIs are subject to Condition 1.
b. Strong NPIs are subject to both Condition 1 and Condition 2.

To illustrate, consider the case of "few". "Few" licenses weak NPIs, as in (62), but not strong ones, (63).

(62) Few students read any books by Auster.

(63) *Few students visited me in years.

"Few" is downward entailing in its second argument. If it is true that "few" students read City of Glass, then it will also be true that "few" students read City of Glass twice. For any statement "few A B" there is a stronger alternative statement "no A B", though. This means that the exhaustified meaning of "few A B" will make it false that "no A B". In other words, after exhaustification "few" is no longer downward entailing. If nobody read City of Glass twice, then "few students read City of Glass" is true on the non-exhaustified and false on the exhaustified reading. In terms of Gajewski's licensing conditions this means the following (assuming no polarity reversal intervenes between "few" and the NPI:

(64) $[_\alpha$ EXH $[_\beta$ Few NP $\ldots [_\gamma$ NPI $] \ldots]]$
a. γ is DE in β
b. γ is not DE in α

Given the distinction in licensing conditions between weak and strong NPIs stated in (61), this setup now correctly predicts that "few" licenses weak, but not strong NPIs.

If we apply this approach to "zero", we are not immediately going to be successful. Given our *at least*+EXH semantics, "zero" creates the following entailment patterns. (Again we are assuming nothing affecting polarity intervenes between "zero" and the NPI.)

(65) $[_\alpha$ EXH $[_\beta$ Zero NP $\ldots [_\gamma$ NPI $] \ldots]]$
a. γ is DE in α
b. γ is DE in β

"Zero" is clearly downward entailing after exhaustification. If exactly 0 students read City of Glass, then exactly 0 students read it twice. The non-exhaustified version is a bit trickier. On our approach, "zero" has a defective semantics. Any statement of the form "zero A B" is – without exhaustification – a tautology. In other words, "zero A B" is a tautology, but "zero A C" is also a tautology, irrespective of our choices of A, B, and C. Let $\top$ be the tautological proposition, the proposition that is always true. Obviously, $\top$ entails $\top$. Now choose B and C such that the extension of C is proper subset of that of B. Since "zero A B" and "zero A C" both express $\top$, they entail each other. Since the extension of C is contained in that of B, "zero" must be both downward entailing *and* upward entailing.

Given the situation in (65), we wrongly predict "zero" to license both kinds of NPIs. It is well-known, however, that the requirement of being in a *downward entailing* environment is at times too lax. In quite a few places in the literature, it is suggested that not only should NPI environments be *downward* entailing, they should also *not be upward* entailing (Progovac, 1993; Lahiri, 1998; Gajewski and Hsieh, 2014; Barker, 2017). One illustration of this comes from singular definite descriptions. Since definite descriptions are presuppositional, the relevant notion of entailment we need is so-called *Strawson* entailment (Von Fintel, 1999). A premise Strawson-entails a conclusion, if the premise together with the presuppositions of the conclusion entail the conclusion. Given such a definition, the restrictor argument of a singular definite description is Strawson DE, as illustrated in (66).

(66)

The student read City of Glass.	*premise*
There is a unique and salient happy student.	*presupposition of conclusion*
The happy student read City of Glass.	*conclusion*

Given the above setup, it would now be predicted that definite descriptions license (at least weak) NPIs. They do not:

(67) *The student that attended any class read City of Glass.

However, the restrictor argument is not just downward but also upward entailing (Lahiri, 1998).

(68)

The happy student read City of Glass.	*premise*
There is a unique and salient student.	*presupposition of conclusion*
The student read City of Glass.	*conclusion*

Given such observations, we can revise our earlier statement of NPI licensing conditions, replacing downward entailingness by non-trivial downward entailingness (NTDE), i.e. being downward, and not upward entailing.

(69) Two licensing conditions for NPIs (revised)

Given a structure $[_\alpha \text{ EXH } [_\beta \ldots [_\gamma \text{ NPI }] \ldots]]$:

Condition 1: the environment γ is non-trivially DE in β

Condition 2: the environment γ is non-trivially DE in α

If we now return to "zero", we see the following:

(70) $[_\alpha \text{ EXH } [_\beta \text{ Zero NP } \ldots [_\gamma \text{ NPI }] \ldots]]$

a. γ is NTDE in α

b. γ is DE but not NTDE in β

In other words, the case of "zero" is exactly the opposite of the case of "few". Whereas the latter met condition 1 but not condition 2, "zero" meets condition 2 but not 1. Since both kinds of NPIs are subject to condition 1, we now correctly predict that "zero" licenses no NPIs of any kind.

Our account of the lack of NPI licensing by "zero" is dependent on our assumption that numerals comes with an *at least* semantics, which is turned into an *exactly* meaning by the additional operation of exhaustification. If we had assumed that the *exactly* meaning is basic, then we would have had no account of the NPI licensing contrast between 'zero' and 'no', since "zero" is non-trivially downward entailing on that reading.

In other words, two crucial ingredients have allowed us to explain the polarity profiles of "zero": (i) the *at least* semantics of numerals; (ii) the inclusion of $\perp$ in the domain of entities.[10] The first of these is relatively uncontroversial. The second, as we have hinted above, is potentially problematic. For that reason, we now turn to the consequences of that second assumption.[11]

[10] A relatively simple modification would turn the degree quantifier analysis into an *at least* analysis for numerals, by adopting the semantics $\lambda P.max(P) \geq n$ for numeral n instead of $\lambda P.max(P) = n$. However, such a move would go against the motivation behind *at least* approaches to numeral semantics. On the modificational and degree account that we introduced in sections 3.1.1 and 3.1.2 the lower bounded semantics is built from the combination of exactly cardinality function $\#x = n$ and existential quantification. This allows these approaches to account for exactly readings of numerals in predicative position, as in "We are three", where existential quantification is absent. The same holds for other contexts in other non-existential contexts, like for instance "the three boys". A lower-bounded degree quantifier analysis would leave these cases unaccounted for.

[11] Although we believe that the contrast between NPIs with 'zero' and 'no' is solid, occasional examples of strong NPIs with 'zero' can be found, usually with heavy stress on 'zero'. (We thank Daniel Lassiter for pointing out such examples to us.)

(i) Yes, you heard us right, ZERO payments until July 2016!

It could be that "zero" with heavy stress is at times reanalysed as "exactly zero", without the need of an additional exhaustification operator.

5 The bottom entity and triviality

With the assumption of the existence of a 0 quantity bottom entity, the *at least* semantics of "zero" becomes trivial. As we argued above, the observed polarity behaviour of "zero" follows from how this triviality is overcome. Even thought, semantically, statements with "zero" are tautological, the scalar inferences they generate are not.

However, whereas we argued that triviality is a core part of how "zero" works, the inclusion of $\perp$ leads to triviality much more generally. We need to now make the case that our proposal does not cause spurious triviality outside the domain of 0 numerals.

The first case to consider is that of bare plurals. If we simply interpret them as existential statements, using $\exists$, then triviality emerges. For instance, (71) when interpreted as (72) is predicted to be a tautology.

(71) There are typos in the manuscript.

(72) $\exists x[{}^{\times}\mathbf{typo}(x) \wedge {}^{\times}\mathbf{in\text{-}the\text{-}manuscript}(x)]$

Since $\perp$ is in the extension of any predicate pluralised with ${}^{\times}$, any statement of the form $\exists x[{}^{\times}P(x)]$ will be true in any model.

We do not think this is a particularly serious problem. Since ${}^{\times}$ renders $\exists$ no longer truly *existential*, we simply need a new, properly existential quantifier, combining $\exists$ with non-emptiness. Call this operator $\mathbf{E}$, defined as in (73). The form in (74) is now the proper analysis of (71)

(73) $\mathbf{E}x[\varphi] :\Leftrightarrow \exists x[\#x > 0 \wedge \varphi]$

(74) $\mathbf{E}x[{}^{\times}\mathbf{typo}(x) \wedge {}^{\times}\mathbf{in\text{-}the\text{-}manuscript}(x)]$

Importantly, whatever is responsible for introducing the existential entailment represented in (74), it will have to be different from the mechanism that introduces existential quantificational force for numerals. Obviously, if we were to use $\mathbf{E}$ as the existential force of "zero", statements with "zero" would be predicted to be unsalvageable contradictions. If we want to maintain the assumption that there is only one kind of existential closure for all indefinite-like DPs, including numeral-noun combinations and bare plurals, then we would have to assume that the latter come with an empty determiner that contributes the exclusion of the bottom element.

It is important to point out, however, that it is not the case that $\perp$ is semantically excluded from the interpretation from all expressions, except for numerals. A plural definite description "the X" will refer to $\perp$ in worlds in which the extension of X is empty. So, (75) is predicted to be true in a situation without Australian students.

(75) The Australian students left the room.

This surely seems like an odd prediction to make and it is therefore tempting to conclude that the definite article must somehow semantically exclude the empty entity. Landman (2011), however, argues that in these cases $\perp$ is excluded pragmatically via domain restriction. The definite description in a sentence like (75) is analysed along the lines of (76), where A is the set of Australians, S is the set of students and C is the contextual set of entities.

(76) that unique $x \in \sqcup A \cap \sqcup S \cap C$ such that there is no y in the same set such that x is a part of y

Landman now suggests that C may or may not include the bottom entity. The interpretation of (75) is only non-trivial in case it does not. A pragmatic principle may help steer clear of tautological readings. Landman proposes the following maxim:

(77) **Avoid Triviality**: A contingent statement is better than a trivial one. (Landman, 2011, p. 14)

If the avoidance of triviality is pragmatic in nature, then we should be able to observe trivial readings in certain context. Landman mentions a few cases like that. In (78), he describes the relevant context as "Suppose I stand trial for fraud, and I say [(78-a)] to the judge, but add sotto voce [(78-b)] to you:"[12]

(78) a. Your honor, the persons who have come to me during 2004 with a winning lottery ticket have gotten a prize.
b. Fortunately, I was on a polar expedition the whole year. (Landman, 2011, ex. 2)

Note in particular the contrast to (79), which is fully infelicitous, as is to be expected on the assumption that the singular restricts the domain to atoms and, thus, semantically excludes the bottom entity.

(79) a. Your honor, the person who has come to me during 2004 with a winning lottery ticket has gotten a prize.
b. Fortunately, I was on a polar expedition the whole year.

[12] Gajewski and Hsieh (2014) tentatively follow Landman's suggestion that plural definite descriptions include the bottom entity in their domain to account for the licensing of NPIs in the restrictor of plural definites.

(Landman, 2011, ex. 9)

Inclusion of the bottom element in the denotation of plural predicates has effect on one more class of expressions: downward monotone degree quantifiers – expressions like "fewer than ten". These expressions, quite like bare numerals under Kennedy's (2015) analysis, denote sets of degree intervals. For instance, ⟦fewer than ten⟧ takes a property of numbers as its argument and states that there is no number ≥ 10 that has this property:

(80) $[\![\text{fewer than ten}]\!] = \lambda P_{\langle d,t \rangle}.max(P) < 10$

(81) ⟦Fewer than ten MANY students passed the test⟧ =
$max(\lambda i.\exists x[\#x = i \wedge {}^{\times}\textbf{student} \wedge {}^{\times}\textbf{pass-the-test}]) < 10$

Something that hasn't been noticed before is that unless we assume the bottom element, "fewer than ten" will not be downward monotone – the sentence "Fewer than ten students passed the test" will fail to come out true in a situation where no students passed the test. In such a situation, the bottom element will be the only element in the intersection of ${}^{\times}$**student** and ${}^{\times}$**pass-the-test**. Then, the interval defined in (81) will contain one number – namely, 0. 0 is indeed smaller than 10, and thus the sentence will come out true under the $\perp$ assumption, as desired. Without $\perp$ (i.e., using * instead of ${}^{\times}$), the situation with no students passing the test will not make the sentence in (81) true and will make "fewer than ten" non-downward-monotone – not supporting downward scalar inferences. The same reasoning holds for all downward monotone degree quantifiers without existential entailment – with or without existential implicatures: "fewer than", "at most", "few", etc.

As we have argued, the inclusion of $\perp$ in the domain of entities, although unorthodox, is not as problematic as it looked at first sight. In fact, it may be that analyses of numerals and modified numerals cannot do without this assumption.

We have assumed that $\perp$ enters into the semantics via semantic pluralisation. All our examples were distributive, however, and, so, there is a clear alternative way of introducing the bottom element, not via pluralisation, but rather via some overt distributivity operator.[13] As (Buccola and Spector, 2016) show, the exact mechanism behind the creation of the full lattice is worthwhile future research. They observe that collective predicates, unlike distributive ones, come with an existential entailment – and therefore, there is a contrast between examples with modified numerals such (82-a), with distributive predicates, and those like (82-b), with collective predicates:

[13]On quite a few accounts, distributivity *is* pluralisation. See, however, the discussion in Scha and Winter (2014) and Nouwen (2016).

(82) a. Fewer than 100 students passed the test.
b. Fewer than 100 soldiers surrounded the castle.

According to them, only the latter, but not the former, entails that at least one (non-empty) individual satisfies the VP predicate. Distributive predicates thus seem to have $\perp$ in their denotation, while collective predicates seem to lack it.

If this is indeed the case the following dilemma emerges: either a bottom element is, as we suggested in this paper, an inherent part of the denotation of all plural predicates (formed by $^{\times}$) that is somehow removed from the denotation of collective ones – or plural predicates systematically lack $\perp$ in the denotation (and are thus formed by *), and then it is added by a distributivity operator that is distinct from a pluralisation operator.

To resolve this dilemma, one has to look for cases that require the presence of the bottom element – but do not involve distributive predicates. "Zero" can provide exactly the case needed. If sentences with "zero" and collective predicates are well-formed and can be judged true in a scenario where no (group) individual satisfies the collective predicate, this could be seen as argument against introducing $\perp$ by the distributivity operator:

(83) Zero soldiers surrounded the castle.

Unfortunately, the judgements concerning such cases are not clear to us, and, for that reason, we leave this dilemma for future work.

6 Conclusion

We have conducted the first in-depth study of the semantics of "zero". As we have hinted at in several places above, the semantic literature has occasionally touched upon the relevance of "zero" to matters of negation and polarity licensing. We have built on some of the observations already present in the literature, and have offered a predictive account of these that is fully conservative in the sense that we give "zero" a numeral semantics, just like other number words. In addition, we have shown that "zero" is not just relevant to matters of negation, but also to plurality and, in particular, to assumptions about semantic ontology.

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