Number in NPI Licensing

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The acceptability of *any*-DPs in existential modal sentences presents a challenge for the theories of NPI licensing: existential modal sentences appear to differ substantially from other environments in which *any*-DPs are acceptable (in particular, they lack a downward-entailing operator). One approach to this challenge has been to, first, take *any*-DPs to be subject to an environment-based downward-entailingness condition – they have to occur in an environment that is Strawson downward-entailing with respect to their domain (cf. Kadmon & Landman 1993) – and, second, to derive such an environment in existential modal sentences by means of exhaustification (e.g., Fox 2007). This note presents new evidence for such a two-layered approach (cf. Crnič 2017, 2019). The evidence comes from a striking contrast in the behavior of singular vs. plural *any*-DPs in existential modal sentences.

Contents

1 The Observation

Any-DPs have a restricted distribution. Our understanding of it advanced significantly with Fauconnier's (1975) and Ladusaw's (1979) descriptive generalization that *any*-DPs must be c-commanded by a downward-entailing operator at LF (see von Fintel 1999 for a refinement). This generalization, however, fails to capture the famous distribution of *any*-DPs in existential modal sentences. Although these sentences lack a downward-entailing operator, an *any*-DP may nonetheless occur in them, as exemplified in (1). Such occurrences of *any*-DPs have been dubbed 'free choice occurrences' due to the universal-like 'free choice' inferences that they give rise to (Vendler 1967): for illustration, (1) conveys that every book is such that Mary is allowed to read it (that is, Mary is 'free to choose' which book to read).

(1) Mary is allowed to read any book.

Various proposals have been put forward on how the acceptability of (1) may be accounted for while maintaining that *any*-DPs denote existential quantifiers across the board (see Kadmon & Landman 1993, Lahiri 1998, Aloni 2007, Menéndez-Benito 2010, Chierchia 2013, Dayal 2013 for a selection). However, no attention has been paid to another striking fact about the distribution of *any*-DPs in existential modal sentences: while singular *any*-DPs are acceptable in them, plural *any*-DPs are not, as exemplified in (2).[1]

(2) #Mary is allowed to read any books.

The goal of this note is to show that the contrast between (1) and (2) is captured straightforwardly on an approach that combines the following two assumptions (cf. Crnič 2017, 2019):

First: The distribution of *any*-DPs is subject to the environment-based condition in (3) (cf. Kadmon & Landman 1993).[2] (Something closely akin to the condition in (3) falls out as a consequence of the approach of Crnič 2017, 2019. See Appendix C for a brief review.)

Second: The condition in (3) can be satisfied in existential modal sentences by applying exhaustification over the alternatives induced by the *any*-DP (cf., e.g., Fox 2007, Chierchia 2013).

[1] These data were brought to my attention by Naomi Francis and Elise Newman in a seminar (Crnič 2018).

[2] The definition of cross-categorial Strawson entailment is in (i) (cf. Gajewski 2011, Sect. 3). The notion of a constituent being Strawson downward-entailing with respect to the domain of an *any*-DP is defined in (ii).

(i) a. If α and β are of type t, then $\alpha \Rightarrow_s \beta$ iff $\alpha = \mathsf{F}$ or $\beta = \mathsf{T}$.
b. If α and β are of type $\sigma\tau$, then $\alpha \Rightarrow_s \beta$ iff $\alpha(x) \Rightarrow \beta(x)$ for all x of type σ s.t. $\beta(x)$ is defined.

(ii) A consitutent S is Strawson downward-entailing with respect to the domain D of an *any*-DP it dominates iff every D' such that $[\![D']\!] \Rightarrow [\![D]\!]$, $[\![S]\!] \Rightarrow_s [\![S[D/D']]\!]$ (where $S[D/D']$ is identical to S except that every occurrences of D in S is replaced with an occurrence of D').

(3) **The *Any* Condition:** A DP headed by *any* is acceptable only if its domain is dominated by a constituent that is Strawson downward-entailing with respect to it.

On such an approach, the contrast between singular vs. plural *any*-DPs in (1) and (2) is a consequence of the fact that while exhaustification can create an environment that is Strawson downward-entailing with respect to the domain of *any* in the case of singular *any*-DPs in existential modal sentences (Sect. 2.1), it crucially cannot do so in the case of plural *any*-DPs like *any books*, resulting in them violating the *Any* Condition (Sect. 2.2). Moreover, the approach gives rise to a prediction that the status of plural *any*-DPs in existential modal sentences can be improved with certain manipulations – a prediction that is borne out (Sect. 2.3).

The combination of the *Any* Condition and exhaustification is critical for capturing this contrast between singular vs. plural *any*-DPs. If either the *Any* Condition is dropped or not derived (Sect. 3.1; cf. Chierchia 2013) or certain mechanisms other than exhaustification are used to strengthen the import of *any*-DPs in modal sentences (Sect. 3.2; cf. Aloni 2007, Menéndez-Benito 2010), the contrast between sentences (1) and (2) remains unexplained, all else equal.

2 The *Any* Condition and Exhaustification

If the distribution of *any*-DPs is taken to be subject to the *Any* Condition, and one subscribes to the uniformity of *any*-DPs across all their occurrences, an additional mechanism must be assumed to allow them to satisfy the *Any* Condition in existential modal sentences. One candidate for this mechanism is exhaustification in grammar (e.g., Fox 2007). We show that an approach that adopts the *Any* Condition and exhaustification explains straightforwardly the contrast between the acceptability of singular vs. plural *any*-DPs in existential modal sentences.

2.1 Singular *Any*-DPs

We rehearse the derivation of the acceptability of singular *any*-DPs in existential modal sentences. (The presentation is kept concise for readability. See Appendix A for more details.)

The *Any* Condition. The condition in (3) subsumes acceptable occurences of *any*-DPs in existential modal sentences. This is supported by the intuitive validity of the Strawson entailment pattern in (4), where the substituted weaker domain and its stronger substitute are underlined.

(4) Mary is allowed to read any <u>book.</u> $\Rightarrow_{\text{S}}$ Mary is allowed to read any <u>long book.</u>

The pattern in (4) is unsurprising once we take the free choice inferences that *any*-DPs induce in existential modal sentences into account: if every <u>book</u> is such that Mary is allowed to read it, then every <u>long book</u> is such as well (if there are long books in the domain). The pertinent question now is how these free choice inferences accompanying *any*-DPs are derived.

Exhaustification. One approach to free choice inferences relies on a covert exhaustification operator in grammar, *exh* (e.g., Fox 2007, Chierchia 2013, Bar-Lev & Fox 2019). One formulation of *exh*, due to Bar-Lev & Fox (2019), is provided in (5) (the choice of the formulation is not crucial for our purposes): it negates all the relevant excludable alternatives, which are characterized in (6-a), and asserts all the includable alternatives, which are characterized in (6-b). (The set ALT(S) consists of all the sentences S' that can be derived from S by a substitution of constituents in S with their subconstituents or with lexical elements, see Katzir 2007.)

(5) $[\![\text{exh}_R\ S]\!](w) = 1$ iff

a. $\forall S' \in \text{Excl}(S) \cap R: \neg[\![S']\!](w)$, and

b. $\forall S' \in \text{Incl}(S): [\![S']\!](w)$.

(6) a. $\text{Excl}(S) = \bigcap\{M \mid M \text{ is a maximal subset of ALT(S)}$
such that $\{\neg[\![S']\!] \mid S' \in M\} \cup \{[\![S]\!]\}$ is consistent$\}$

b. $\text{Incl}(S) = \bigcap\{M \mid M \text{ is a maximal subset of ALT(S)}$
such that $\{[\![S']\!] \mid S' \in M\} \cup \{\neg[\![S']\!] \mid S' \in \text{Excl}(S)\}$ is consistent$\}$

Derivation. The sentence in (1) has the LF in (7-b), where *exh* takes matrix scope and the *any*-DP occurs in the scope of the modal. (Again, see Appendix A for some missing details.)

(7) a. Mary is allowed to read any book.

b. $[\text{exh}_R\ [\Diamond\ [\text{any}_D\ \text{book}_x\ [\text{Mary read x}]]]]$

The sister of *exh* in (7-b) induces the alternatives in (8). Specifically, following Chierchia (2013), we assume that *any*-DPs have so-called subdomain and universal quantifier alternatives.

(8) $\text{ALT}([\Diamond\ [\text{any}_D\ \text{book}_x\ [\text{Mary read x}]]]) =$
$\{[\Diamond\ [\text{any}_{D'}\ \text{book}_x\ [\text{Mary read x}]]], [\Diamond\ [\text{every}_{D'}\ \text{book}_x\ [\text{Mary read x}]]] \mid [\![D']\!] \subseteq [\![D]\!]\}$

The set of excludable alternatives in (8) is provided in (9) – all the maximal subsets of the alternatives in (8) that can be jointly negated with the sentence being true have these alternatives in common and nothing else.[3] The includable alternatives are provided in (10) – they, and no other alternatives, can all be asserted consistently if all the excludable alternatives are negated. (This is witnessed by the inference in (11) below being consistent.)

[3]The maximal sets of alternatives that can be jointly negated with the sentence being true have the form in (i). They differ from each other in what book is subtracted from the subdomains. See Appendix A for more details.

(i) $\{[\text{any}_{D'}\ \text{book}_x\ [\text{Mary read x}]], [\text{every}_{D''}\ \text{book}_x\ [\text{Mary read x}]] \mid$
$[\![D']\!], [\![D'']\!] \subseteq [\![D]\!] \wedge b \notin [\![D']\!] \cap [\![\text{book}]\!] \wedge [\![D'']\!] \cap [\![\text{book}]\!] \neq \{b\}\}$, for some $b \in [\![D]\!] \cap [\![\text{book}]\!]$.

(9) $\text{Excl}([\Diamond\ [\text{any}_{\text{D}}\ \text{book}_{\text{x}}\ [\text{Mary read x}]]]) =$
$\{[\Diamond\ [\text{every}_{\text{D}'}\ \text{book}_{\text{x}}\ [\text{Mary read x}]] \mid \text{card}([\![\text{D}']\!]\cap[\![\text{book}]\!])\geq 2 \wedge [\![\text{D}']\!] \subseteq [\![\text{D}]\!]\}$

(10) $\text{Incl}([\Diamond\ [\text{any}_{\text{D}}\ \text{book}_{\text{x}}\ [\text{Mary read x}]]]) =$
$\{[\Diamond\ [\text{any}_{\text{D}'}\ \text{book}_{\text{x}}\ [\text{Mary read x}]] \mid [\![\text{D}']\!] \neq \emptyset \wedge [\![\text{D}']\!] \subseteq [\![\text{D}]\!]\cap[\![\text{book}]\!]\}$

The output of the exhaustification in (7-b) is computed in (11): the assertion of the includable alternatives corresponds to the free choice inferences accompanying the *any*-DP, provided in the first line, while the negation of the excludable alternatives, provided in the second line, depends on the context (since the excludable alternatives can be pruned, they need not end up being negated, as indicated by the parantheses; see, e.g., Fox & Katzir 2011, Katzir 2014, Crnič et al. 2015, Bar-Lev 2018 on some constraints on the pruning of alternatives).

(11) $\forall \text{D}'\text{: } \text{D}' \neq \emptyset \wedge \text{D}' \subseteq [\![\text{D}]\!]\cap[\![\text{book}]\!] \rightarrow \Diamond(\text{Mary read a book in D}')$
$\big(\wedge\ \forall \text{D}'\text{: } \text{card}(\text{D}'\cap[\![\text{book}]\!])\geq 2 \wedge \text{D}' \subseteq [\![\text{D}]\!] \rightarrow \neg\Diamond(\text{Mary read every book in D}')\big)$

This meaning corresponds to every book being such that Mary is allowed to read it (a consequence of the assertion of the includable alternatives) and Mary not being allowed to read two books (a consequence of the negation of the excludable alternatives):

(12) $\forall \text{x}\text{: } \text{x} \in [\![\text{D}]\!]\cap[\![\text{book}]\!] \rightarrow \Diamond(\text{Mary read x})$
$\big(\wedge\ \forall \text{D}'\text{: } \text{card}(\text{D}'\cap[\![\text{book}]\!])\geq 2 \wedge \text{D}' \subseteq [\![\text{D}]\!] \rightarrow \neg\Diamond(\text{Mary read every book in D}')\big)$

Satisfaction of the Condition. Sentence (1) is Strawson downward-entailing with respect to the domain of *any* on the construal in (7-b), meaning that the *any*-DP is correctly predicted to be acceptable. Namely, if you replace domain D in (7-b) with a stronger subdomain D^*, you obtain a Strawson weaker meaning: if every book in D is such that Mary is allowed to read it (and Mary is not allowed to read two books in D), then every book in D^* is such that Mary is allowed to read it if there are such books[4] (and Mary is not allowed to read two books in D^*).

(13) $[\text{exh}_{\text{R}}\ [\Diamond\ [\text{any}_{\text{D}}\ \text{book}_{\text{x}}\ [\text{Mary read x}]]]]$
is Strawson downward-entailing with respect to the domain D.

We now turn to occurrences of plural *any*-DPs in existential modal sentences.

[4]The presupposition that there are books in the domain of *any* is crucial for the sentence to be Strawson downward-entailing with respect to it: if there are no books in a stronger domain D^* that replaces the domain of *any*, existential quantification over it will yield a false meaning, and thus the *Any* Condition will be violated. This is different from what we observe in typical downward-entailing environments, where an empty domain leads to a tautologous meaning, which is trivially entailed by any sentence. See Crnič 2019, Sect. 2, for further discussion of, and support for, *any*-DPs in existential modal sentences obligatorily triggering existence presuppositions.

2.2 Plural *Any*-DPs

Plurality. The only difference between the sentences in (1) and (2) is in the number marking on the *any*-DP. What is the semantic import of this difference? While there are several different proposals about the semantics of number (e.g., Sauerland 2003, Spector 2007, Zweig 2009, Ivlieva 2013, Križ 2017), the choice between them does not affect the predictions of the current approach (or those of the approaches discussed in the following section). For concreteness, we will assume that the plural morpheme simply denotes a cumulation operator ($*$): it takes a set of atoms as an argument, and returns back a set of atoms and all their sums.[5] (See Appendix B for a discussion of the assumption that plural NPs denote sets of proper plural individuals. The same conclusions are reached on that assumption as in the main text.)

(14) $[\![\text{books}]\!] = *[\![\text{book}]\!] = \lambda x.$ x is a book or a sum of books
$\big(= \{\text{Anna Karenina, War and Peace, Anna Karenina+War and Peace, ...}\}\big)$

Derivation. The sentence in (2) has the LF in (15-b). The alternatives to the sister of *exh* in (15-b) are in (16): they differ from the sister of *exh* in that the determiner *any* is replaced by *all* or the domain of *any*, D, is replaced by a subdomain. (See Appendix B for more details.)

(15) a. #Mary is allowed to read any books.
b. $[\text{exh}_R\ [\Diamond\ [\text{any}_D\ \text{books}_x\ [\text{Mary read x}]]]]$

(16) $\text{ALT}([\Diamond\ [\text{any}_D\ \text{books}_x\ [\text{Mary read x}]]]) =$
$\{[\Diamond\ [\text{any}_{D'}\ \text{books}_x\ [\text{Mary read x}]]], [\Diamond\ [\text{all}_{D'}\ \text{books}_x\ [\text{Mary read x}]]] \mid [\![D']\!] \subseteq [\![D]\!]\}$

What are the excludable alternatives in (16)? Maximal subsets of (16) that can be jointly negated with the sentence being true are of the form given in (17): their joint negation is consistent with Mary being allowed to read some book or books (namely, Mary may still be allowed to read book *b*) and expanding the set by a further alternative leads to a contradiction (e.g., negating that Mary is allowed to read some book or books in $\{b\}$ is incompatible with the sentence being true and all other alternatives in the expanded set being false). (We paraphrase the import of *any books* with 'some book or books' since the *books* has a number-neutral meaning.)

(17) $\{[\Diamond\ [\text{any}_{D'}\ \text{books}_x\ [\text{Mary read x}]]], [\Diamond\ [\text{all}_{D''}\ \text{books}_x\ [\text{Mary read x}]]] \mid$

[5] Following Link 1983, we assume that the domain of individuals consists both of atoms and sums of individuals (proper pluralities), is partially ordered by a part-of relation ($\sqsubseteq$), and is closed under sum formation (+). A definition of an atom is provided in (i-a) (which could be further relativized to include sortal information) and that of the $*$-operator is provided in (i-b) (which closes a set under sum formation).

(i) a. An individual $x \in D_e$ is an atom iff $\forall z\ (z \sqsubseteq x \rightarrow z = x)$.
b. For any $P \in D_{(et)}$, $*P = \lambda x.\ P(x) \vee \exists y,z\ (x = y+z \wedge *P(y) \wedge *P(z))$.

$[\![D']\!], [\![D'']\!] \subseteq [\![D]\!] \wedge b \notin [\![D']\!] \cap [\![books]\!] \wedge [\![D'']\!] \cap [\![books]\!] \not\subseteq \{b\}\}$

for some $b \in [\![D]\!] \cap [\![book]\!]$.

The intersection of all such sets – the set of excludable alternatives – is provided in (18): it consists of all the alternatives that convey that Mary is allowed to read two or more books. (We omit equivalent or stronger alternatives from the set, say, those with the domain of *any* consisting of only pluralities consisting of two books, etc., for reasons of brevity.)

(18) $\text{Excl}([\Diamond\ [\text{any}_D\ \text{books}_x\ [\text{Mary read x}]]]) =$

$\{[\Diamond\ [\text{all}_{D'}\ \text{books}_x\ [\text{Mary read x}]]] \mid [\![D']\!] \subseteq [\![D]\!] \wedge \text{card}([\![D']\!] \cap [\![books]\!]) \geq 2\}$

The includable alternatives are all the subdomain alternatives that contain at least one atom book in the domain of *any*, as given in (19): these and only these alternatives can all be jointly asserted when all the alternatives in (18) are negated.

(19) $\text{Incl}([\Diamond\ [\text{any}_D\ \text{books}_x\ [\text{Mary read x}]]]) =$

$\{[\Diamond\ [\text{any}_{D'}\ \text{books}_x\ [\text{Mary read x}]]] \mid [\![D']\!] \subseteq [\![D]\!] \wedge \exists x\ (x \in [\![D']\!] \cap [\![book]\!])\}$

Accordingly, the meaning of the structure in (15-b) is provided in (20): every alternative in which the domain of *any* contains at least one atom book is such that it is true (and every alternative in which a universal quantifier has two or more books in its domain is false).

(20) $\forall D'\text{: } D' \subseteq [\![D]\!] \wedge \exists x\ (x \in D' \cap [\![book]\!]) \rightarrow \Diamond(\text{Mary read some book or books in } D')$

$\big(\wedge\ \forall D'\text{: } \text{card}(D' \cap [\![book]\!]) \geq 2 \wedge D' \subseteq [\![D]\!] \rightarrow \neg\Diamond(\text{M. read all books in } D')\big)$

This meaning is equivalent to every book being such Mary is allowed to read it (and her not being allowed to read two or more books), as given in (21): namely, (20) conveys that every singleton domain containing an atom book is such that Mary is allowed to read the book in it (which in turn entails that every domain containing at least one atom book is such that Mary is allowed to some book or books in it).

(21) $\forall x\text{: } x \in D \cap [\![book]\!] \rightarrow \Diamond(\text{Mary read x})$

$\big(\wedge\ \forall D'\text{: } \text{card}(D' \cap [\![book]\!]) \geq 2 \wedge D' \subseteq [\![D]\!] \rightarrow \neg\Diamond(\text{M. read all books in } D')\big)$

The exhaustified meanings of the sentences with singular and plural *any*-DPs in (1) and (2) are thus equivalent (compare plural (20)-(21) with their singular counterparts in (11)-(12) above). How can they, then, differ with respect to the licensing of singular vs. plural *any*-DPs? We first show that they do in fact differ in this respect, and then elaborate on why.

Violation of the Condition. The structure in (15-b) is not Strawson downward-entailing with respect to the domain of *any*, and so the occurrence of the *any*-DP in it is correctly ruled out as unacceptable. In order to show this, it suffices to come up with one replacement of D in (15-b) with a stronger subdomain that will not result in a Strawson weaker meaning. Consider domain D^* in (22), which consists only of proper plural individuals, and is a proper subdomain of D.

(22) $D^* = \{x \mid x \in D \wedge x \notin [\![\text{book}]\!]\} \left(\subset D\right)$

The meaning of the counterpart of (15-b) in which domain D is replaced by domain D^* is provided in (23): since there are no atom books in domain D^*, the includable (and excludable) alternatives involve non-atoms only; this means that the free choice inferences that are generated can only involve non-atoms as well, which is represented in (23) by the domains quantified over not intersectiong with the set of atom books.

(23) $\forall D'$: $D' \neq \emptyset \wedge D' \subseteq [\![D]\!] \cap [\![\text{books}]\!] \wedge D' \cap [\![\text{book}]\!] = \emptyset \rightarrow \Diamond$(M. read some book or books in D') $\left(\wedge \forall D'\text{: } \text{card}(D' \cap [\![\text{book}]\!]) \geq 3 \wedge D' \subseteq [\![D]\!] \rightarrow \neg\Diamond\text{(M. read all books in } D')\right)$

The meaning in (23) is equivalent to every two books being such Mary is allowed to read them (and her not being allowed to read three or more books): namely, every plurality consisting of exactly two books is according to (23) such that Mary is allowed to read some book or books in the singleton set containing that plurality (which then entails that Mary is allowed to read some book or books in a set that contains that and other pluralities).

(24) $\forall x\text{: } x \in D \cap [\![\text{books}]\!] \wedge \text{card}(x)=2 \rightarrow \Diamond$(Mary read x)
$\left(\wedge \forall D'\text{: } \text{card}(D' \cap [\![\text{book}]\!]) \geq 3 \wedge D' \subseteq [\![D]\!] \rightarrow \neg\Diamond\text{(M. read all books in } D')\right)$

The *Any* Condition dictates that the entailment relation in (25) obtains: namely, for the condition to be satisfied, it has to hold that there is a constituent in (15-b) such that replacing domain D with the stronger D^* results in a Strawson weaker meaning of the thus modified constituent; the only plausible candidate for this constituent is (15-b) itself. (The first line in (25) corresponds to (20)-(21), while the second line corresponds to (23)-(24).)

(25) **Consequence of the *Any* Condition:**

$[\![[\text{exh}_R\ [\Diamond\ [\text{any}_D\ \text{books}_x\ [\text{Mary read x}]]]]]\!] \Rightarrow_s$
$[\![[\text{exh}_R\ [\Diamond\ [\text{any}_{D^*}\ \text{books}_x\ [\text{Mary read x}]]]]]\!]$

The required Strawson entailment does not obtain, however: not only does the meaning in (20)-(21) fail to license any inferences about pluralities consisting of two books, it actually excludes them from being true (unless the excludable alternatives, parenthesized in (20)-(21), are pruned). This means that the *Any* Condition cannot be satisfied in the structure in (15-b). An approach that adopts the *Any* Condition and exhaustification thus correctly captures the contrast

in acceptability between the singular and plural *any*-DPs in existential modal sentences.

Diagnosis. Why, more generally, does Strawson downward-entailingness obtain with singular but not plural *any*-DPs? The source of the difference lies in how number marking interacts with the replacement of the domain of *any*. In particular, free choice inferences crucially involve only the minimal elements in the domain of quantification of *any*[6] (in both singular and plural cases these are atoms, given our choice of number-neutral meaning of plural NPs, see also Appendix B). This can be seen most clearly in the representations in (12) and (21), both of which involve universal quantification over atoms.

(26) **Exhaustification and free choice:**
Free choice inferences generated by exhaustification over the alternatives to an *any-DP* involve only the minimal elements on which the *any*-DP lives.

Singular *any*-DPs: Exhaustification enables a satisfaction of the Condition in the case of singular *any*-DPs, where the domain of quantification of *any* is restricted to the minimal elements (atoms) by the singular morphology on the NP – if we replace the domain of *any* in (7-b) with a subdomain, any subdomain, we will obtain free choice inferences involving a subset of the atoms that the free choice inferences of the initial sentence involved, that is, we obtain a Strawson weaker meaning. This corresponds to Strawson downward-entailingness.

Plural *any*-DPs: On the other hand, the restriction of free choice inferences to minimal elements guarantees a violation of the Condition in the case of plural *any*-DPs, where *any* does not quantify only over atoms – if we replace the domain of *any* in (15-b) with a subdomain that consists only of non-atoms (say, only of pluralities consisting of two books, etc.), the free choice inferences that get generated will involve non-atoms, while the free choice inferences of the initial sentence involved only atoms. Since no free choice inferences involving non-atoms are licensed by the initial sentence, we obtain a Strawson independent meaning at best.

The crucial difference between the sentences with singular vs. plural *any*-DPs is summarized in (27) in terms of the relations between the sets of includable alternatives (the assertion of includable alternatives corresponds to the free choice inferences). The disjointness of the sets of includable alternatives for sentences built on stronger and weaker domains of *any* implies independence of their exhaustified meanings (esp., of their free choice inferences), and thus absence of Strawson downward-entailingness with respect to the domain of *any*.

(27) **Facts about subdomains and free choice:**

Sg: For every domain $D' \subseteq D$, the set of includable alternatives in ALT([$\Diamond$ [any_{D} book_{x} Mary read x]]) is a superset of that in ALT([$\Diamond$ [$\text{any}_{D'}$ book_{x} Mary read x]]).

[6] A minimal element in a set is one that has no other element in the set as a part.

Pl: For some domains $D' \subseteq D$, the set of includable alternatives in ALT([$\Diamond$ [any$_D$ books$_x$ M. read x]]) is disjoint from that in ALT([$\Diamond$ [any$_{D'}$ books$_x$ M. read x]]).

2.3 Prediction

In light of the above diagnosis, we can devise a configuration in which a plural *any*-DP is predicted to satisfy the *Any* Condition in an existential modal sentence: if one forces the minimal elements in the domain of *any* to remain of the same size across all the replacements of the domain with a subdomain, Strawson downward-entailingness with respect to the domain of *any* is predicted to obtain, just like in the singular *any*-DP case. One way of achieving this is by using a prenominal numeral modifier (cf. Dayal 2004, Chierchia 2013, Dayal 2013). The prediction of the approach is borne out, as exemplified in (28).

(28) Mary is allowed to read any three books.

The interpretation of the sentence in (28) is provided in (29). This meaning is equivalent to every plurality of three books being such that you are allowed to read the three books (and that you are not allowed to read more than three books), as given in (30). (Recall that the free choice inferences involve only the minimal elements in the domain of quantification of *any*; the presence of *three* forces these to consist of three atoms of books.)[7]

(29) $\forall D'$: $D' \neq \emptyset \wedge D' \subseteq [\![D]\!] \cap [\![\text{three books}]\!] \rightarrow \Diamond$(Mary read three books in D')
$\big(\wedge \forall D'$: card$(D' \cap [\![\text{book}]\!]) \geq 4 \wedge D' \subseteq [\![D]\!] \rightarrow \neg\Diamond$(Mary read all books in $D')\big)$

(30) $\forall x$: $x \in D \cap [\![\text{books}]\!] \wedge$ card$(x)=3 \rightarrow \Diamond$(Mary read x)
$\big(\wedge \forall D'$: card$(D' \cap [\![\text{book}]\!]) \geq 4 \wedge D' \subseteq [\![D]\!] \rightarrow \neg\Diamond$(Mary read all books in $D')\big)$

Since the prenominal numeral imposes a restriction to pluralities of three books, this restriction obviously obtains in all the sentences derived from (28) by replacing the domain of *any* with a subdomain: the resulting sentences necessarily induce a subset of the free choice inferences induced by the initial sentence (that there are such inferences is warranted by the existential presupposition construal of the *any*-DP, see footnote 4). Consequently, since the resulting sentences are Strawson entailed by the initial sentence, the *Any* Condition is satisfied.

[7] The expression *three books* picks out the set of pluralities consisting of exactly three books (cf., e.g., Kennedy 2015, Buccola & Spector 2016). The meaning of *any three books* is provided in (i).

(i) $[\![\text{any}_D \text{ three books}]\!](P) = 1$ iff $\exists x \in D$ (books$(x) \wedge P(x) \wedge$ card$(x) = 3)$

2.4 Summary

We showed that the asymmetry in the acceptability of singular vs. plural *any*-DPs in existential modal sentences, exemplified in (1) and (2), follows naturally on an approach that makes the following two assumptions (cf. Crnič 2017, 2019):

- *Any*-DPs must satisfy the *Any* Condition.
- The import of *any*-DPs (just like that of other existential quantifiers and disjunction) can be strengthened in existential modal sentences by exhaustification.

Exhaustification over the alternatives induced by singular *any*-DPs in existential modal sentences creates an environment that is Strawson downward-entailing with respect to the domain of *any*, as rehearsed in Sect. 2.1. In contrast, exhaustification over the alternatives induced by plural *any*-DPs does not have this effect (unless the DPs are appropriately restricted by a prenominal numeral), as shown in Sect. 2.2. The crucial difference between the two cases was that in the case of singular *any*-DPs free choice inferences are generated for every element in the domain of quantification of *any* (namely, all the atoms), while in the case of plural *any*-DPs these were generated only for a proper subset of this domain (namely, only the atoms). In the following section, we emphasize the importance of both components of the approach (namely, the *Any* Condition and exhaustification) in the explanation of the contrast between (1) and (2).

3 The Significance of the Two Components

We describe some issues raised by the contrast between (1) and (2) for, first, an approach to *any*-DPs that relies on exhaustification but neither adopts the *Any* Condition as a primitive nor derives it (Chierchia 2013) and, second, for two approaches that employ a mechanism other than exhaustification to generate free choice inferences (Aloni 2007, Menéndez-Benito 2010).

3.1 Dropping the *Any* Condition

One approach that neither assumes nor derives the *Any* Condition is that of Chierchia (2013), on which the acceptability of NPIs depends simply on whether exhaustification over the alternatives induced by sentences with NPIs yields a consistent interpretation.

No contrast. On this exhaustification-only approach, both singular and plural *any*-DPs are *prima facie* predicted to be acceptable in existential modal sentences. This holds because the exhaustified meanings of both (1) and (2), computed in (11) and (20), respectively, are consistent, and thus admitted as grammatical, all else equal, as stated in (31). (Chierchia 2013 adopts slightly different LFs and auxiliary assumptions to deal with *any*-DPs in modal sentences than those adopted here, but this does not affect the prediction of consistency.)

(31) $[\![[\text{exh}_R\ [\Diamond\ [\text{any}_D\ \text{book}_x\ [\text{Mary read x}]]]]]\!] =$

$[\![[\text{exh}_R\ [\Diamond\ [\text{any}_D\ \text{books}_x\ [\text{Mary read x}]]]]]\!] \neq \bot$

Intervention. There might be a way of avoiding this conclusion by making some appropriate auxiliary assumptions. Since Chierchia's account imposes one condition less on *any*-DPs than the type of approach discussed above, it must be enriched elsewhere in order to avoid overgeneration. Accordingly, Chierchia recruits, and further develops, an independently needed theory of intervention: the set of alternatives over which *exh* quantifies may be larger than what we assumed above, leading in some cases to a failure of exhaustification rather than what would otherwise be a consistent interpretation. The hope could be that the problematic occurrences of plural *any*-DPs are also ruled out by this machinery. The plural number would thereby have to be classified as an intervener that expands the set of alternatives over which *exh* quantifies. The additional alternatives we would obtain on this assumption would differ from the other alternatives merely in the number marking on the DP (singular, instead of plural). However, unlike in other cases of intervention, these additional alternatives would not obviously yield a pathology in exhaustification since they would be equivalent to their plural counterparts, as stated in (32). Note, furthermore, that the equivalence is maintained also after exhaustification, as stated in (33) and demonstrated above. (These facts parallel the properties of sentences in which singular and plural *any*-DPs occur in the scope of negation.)

(32) $[\![[\Diamond\ [\text{any}_D\ \text{books}_x\ [\text{Mary read x}]]]]\!] \Leftrightarrow_s [\![[\Diamond\ [\text{any}_D\ \text{book}_x\ [\text{Mary read x}]]]]\!]$

(33) $[\![[\text{exh}_R\ [\Diamond\ [\text{any}_D\ \text{books}_x\ [\text{Mary read x}]]]]]\!] \Leftrightarrow_s [\![[\text{exh}_R\ [\Diamond\ [\text{any}_D\ \text{book}_x\ [\text{Mary read x}]]]]]\!]$

A similar state of affairs would obtain even if the meanings of plural NPs were strengthened prior to the application of the exhaustification over the alternatives induced by *any*-DPs (cf. Spector 2007, Zweig 2009, Ivlieva 2013). In this case, the sister of *exh* in (15) would have the meaning in (34), and the alternatives with singular morphology would be properly weaker than the alternatives with plural morphology, as stated in (35), meaning that they should not obviously have an adverse effect on exhaustification.

(34) $[\![[\Diamond\ [\text{any}_D\ \text{books}_x\ [\text{Mary read x}]]]]\!] = \Diamond$(Mary read at least 2 books in D)

(35) $[\![[\Diamond\ [\text{any}_D\ \text{books}_x\ [\text{Mary read x}]]]]\!] \Rightarrow_s [\![[\Diamond\ [\text{any}_D\ \text{book}_x\ [\text{Mary read x}]]]]\!]$

Summary. We conclude that on an exhaustification approach that does not adopt the *Any* Condition (or some mechanism that would derive it), the pattern in (2) is unexpected absent further assumptions, assumptions that have yet to be provided. A more detailed study of the application of the theory of Chierchia (2013) to the data under discussion is thus mandated.

3.2 Replacing Exhaustification

There are mechanisms other than exhaustification that have been put forward for strengthening the import of existential quantifiers and disjunction in existential modal sentences. We discuss two such mechanisms in the following (Aloni 2007, Menéndez-Benito 2010).

Universal closure. Aloni (2007) and Menéndez-Benito (2010) treat indefinites, incl. *any*-DPs, as inducing sets of alternatives in which they are effectively replaced with their potential verifiers (cf. also Kratzer 2005). Aloni assumes that this set is quantified over by the embedding modal, while Menéndez-Benito assumes that this is done by two covert operators. The structure assigned to (2) in the system of Aloni is provided in (36); its interpretation corresponds to universal quantification over the potential verifiers of the existential quantificiation, as in (37).

(36) [$\Diamond$ [Mary read any$_{\mathrm{D}}$ books]]

(37) $\forall$x: x $\in$ [[D]]$\cap$[[books]] $\rightarrow$ $\Diamond$(Mary read x)

The structure assigned to (2) in the system of Menéndez-Benito is given in (38), in which the universal quantifier over alternatives, $\forall$, takes matrix scope and the exhaustive operator, *Excl*, takes scope below the modal. The meaning of the structure is provided in (39): every proposition of the form 'Mary is allowed to read x and no other book', where x is a book in D, is true (the universal quantification is due to $\forall$, and the exhaustive 'no other book' is due to *Excl*); this is equivalent to every book being such that Mary is allowed to read just it (see Menéndez-Benito 2010 for arguments for this reading, and Chierchia 2013, Ch. 6, for a reply).

(38) [$\forall$ [$\Diamond$ [Excl [Mary read any$_{\mathrm{D}}$ books]]]]

(39) $\forall$x: x $\in$ [[D]]$\cap$[[books]] $\rightarrow$ $\Diamond$(Mary read x and no other books)

Importantly, both structures in (36) and (38) have consistent interpretations and are, moreover, Strawson downward-entailing with respect to the domain of *any*: if every plurality of books in D is such that Mary is allowed to read (only) them, then for any subset D$'$ of D, it holds that every plurality of books in D$'$ is such that Mary is allowed to read (only) them. This means that the plural *any-DPs* in (36) and (38) are incorrectly predicted to be acceptable.

Diagnosis. The difference between exhaustification and the strengthening mechanisms employed in (36) and (38) is that exhaustification affirms only the alternatives whose restrictors of *any* contain at least one atom, meaning that no information is conveyed about the alternatives whose restrictors do not contain any atoms (potentially these alternatives are negated). This is not the case for the mechanisms presented in this section: they induce universal quantification over all the pluralities in the restrictor of *any*. While this quantification could be restricted to

only minimal potential verifiers, there seems to be no independent motivation for this (it may even have negative global repercussions for the respective systems).

3.3 Summary

We discussed the consequences of, on the one hand, dropping the *Any* Condition and relying merely on exhaustification to account for the distribution of *any*-DPs and, on the other hand, replacing exhaustification with an alternative strengthening mechanism (universal quantification over the alternatives induced by indefinites). Overgeneration obtains in both cases: the occurrences of plural *any*-DPs in existential modal sentences are incorrectly predicted to be acceptable. Further study of how these systems could and should be constrained is required.

	Cond+Exh	**NoCond+Exh**	**Cond+NoExh**
Approaches	Crnič 2019	Chierchia 2013	Aloni 2007
Predicted unacceptability of free choice plural *any*-DPs	✓	✗	✗

Table 1: Summary of the approaches to NPI licensing discussed and whether they correctly capture the behavior of plural *any*-DPs in existential modal sentences, all else equal.

4 Outlook

We showed that an approach that adopts the *Any* Condition and exhaustification adequately and straightforwardly predicts the distribution of both singular and plural *any*-DPs in existential modal sentences. We conclude the paper by pointing to three areas left for future research.

Variation among NPIs. A distribution similar to that of plural *any*-DPs is found with *any*-DPs that have a mass noun complement (say, *furniture*) and with *ever*, an expression that has been classified as a so-called weak NPI, just like *any*-DPs (esp., Zwarts 1998): namely, they are all unacceptable in existential modal sentences, as exemplified in (40).

(40) a. #Mary is allowed to buy any furniture.
b. #Mary is allowed to ever read a book.

Could this parallel behavior of NPIs have the same source? On the assumption that mass nouns denote sets that contain both atoms and proper plural individuals (cf. Chierchia 1998), and that the domain of *ever* consists of both (singleton sets of) moments of time and their intervals – note that both of these assumptions may be controversial – this state of affairs might

be naturally explained on the *Any* Condition and exhaustification approach. However, at least one alternative derivation of the facts exemplified in (40) is available, one that relies on slightly different assumptions about mass noun and tense semantics (namely, that they are inherently divisive, Crnič & Haida 2019). A more careful study of these issues is beyond the scope of this note. We hope to attend to it in the future, not least since it promises to shed new light on some fundamental questions about the analysis of mass nouns and tense.

Partitive *any*-DPs. In addition to *any*-DPs with prenominal numeral modifiers, partitive plural *any*-DPs are also acceptable in existential modal sentences, as exemplified in (41). (Note that a similar improvement in acceptability is found also with mass *any*-DPs.)

(41) Mary is allowed to read any of the books.

We speculate that the acceptability of the plural *any*-DP in (41) may have an explanation similar to the one presented for prenominal numeral modifiers above: the partitive may impose an additional restriction on the domain of quantification of *any*, via a covert partitive head (Chierchia 1997, Sauerland 2004, Longenbaugh 2019, among others), which may sanction Strawson downward-entailingness (just like prenominal numerals do). For example, if the unpronounced partitive head were singular *book* (see Longenbaugh 2019 on this type of number mismatch), we would effectively obtain a configuration that mirrors the one with the singular *any*-DP. We hope to pursue this direction of explanation in the future.

(42) $[\text{exh}_{\text{R}}\ [\Diamond\ [\text{any}_{\text{D}}\ \sout{\text{book}}\ \text{of the books}]_{\text{x}}\ [\text{Mary read x}]]$

More on *any* with prenominal numerals. Dayal (2004) observed a puzzling fact about the distribution of plural *any*-DPs with prenominal numeral modifiers: they are acceptable even in universal modal sentences. This is exemplified in (43), whose meaning is paraphrased in (44). (The distribution of *any*-DPs with prenominal numeral modifiers comes apart from that of plain *any*-DPs in several other environments. See Chierchia 2013, Ch. 6, for a detailed discussion).

(43) Mary is required to read any_{D} two books.

(44) Mary is required to read two books in D,
and for each pair of books in D, she may read them.

In view of (44), the sentence intuitively violates the *Any* Condition: while the conjunct in the second line in (44) does license Strawson entailment to the subdomains of D, the conjunct in the first line does not (cf. Crnič 2017, 2019 on singular *any*-DPs in universal modal sentences). We lack an adequate account of this puzzle, but we hope to be able to address it in the future.

References

Aloni, Maria. 2007. Free choice, modals, and imperatives. *Natural Language Semantics* 15(1). 65–94.

Bar-Lev, Moshe. 2018. *Free choice, homogeneity, and innocent inclusion*: The Hebrew University of Jerusalem dissertation.

Bar-Lev, Moshe E & Danny Fox. 2019. Free choice, simplification, and innocent inclusion. MS, Hebrew University of Jerusalem & MIT.

Buccola, Brian & Andreas Haida. 2017. Obligatory irrelevance and the computation of ignorance inferences. Manuscript, The Hebrew University of Jerusalem.

Buccola, Brian & Benjamin Spector. 2016. Modified numerals and maximality. *Linguistics and philosophy* 39(3). 151–199.

Chierchia, Gennaro. 1997. Partitives, reference to kinds and semantic variation. In *Semantics and linguistic theory*, vol. 7, 73–98.

Chierchia, Gennaro. 1998. Plurality of mass nouns and the notion of "semantic parameter". In *Events and grammar*, 53–103. Springer.

Chierchia, Gennaro. 2013. *Logic in grammar*. Oxford: Oxford University Press.

Crnič, Luka. 2014. Non-monotonicity in NPI licensing. *Natural Language Semantics* 22(2). 169–217.

Crnič, Luka. 2017. Free choice under ellipsis. *The Linguistic Review* 34(2). 249–294.

Crnič, Luka. 2018. Topics in Semantics: Negative Polarity Item (Fall 2018). Massachusetts Institute of Technology: MIT OpenCourseWare. `https://ocw.mit.edu/`.

Crnič, Luka. 2019. *Any*: Logic, likelihood, and context (Parts I, II). *Language and Linguistics Compass* 13(11). doi: 10.1111/lnc3.12353, 10.1111/lnc3.12354.

Crnič, Luka, Emmanuel Chemla & Danny Fox. 2015. Scalar implicatures of embedded disjunction. *Natural Language Semantics* 23(4). 271–305.

Crnič, Luka & Andreas Haida. 2019. Free choice and divisiveness. Manuscript, The Hebrew University of Jerusalem.

Dayal, Veneeta. 2004. The universal force of free choice any. *Linguistic variation yearbook* 4(1). 5–40.

Dayal, Veneeta. 2013. A viability constraint on alternatives for free choice. In Anamaria Falaus (ed.), *Alternatives in semantics*, Basingstoke: Palgrave Macmillan.

Fauconnier, G. 1975. Pragmatic scales and logical structure. *Linguistic Inquiry* 6(3). 353–375.

von Fintel, Kai. 1999. NPI licensing, Strawson entailment, and context dependency. *Journal of Semantics* 16(2). 97–148.

Fox, Danny. 2007. Free choice and the theory of scalar implicatures. In Uli Sauerland & Penka Stateva (eds.), *Presupposition and Implicature in Compositional Semantics*, 71–120. Palgrave Macmillan.

Fox, Danny & Roni Katzir. 2011. On the characterization of alternatives. *Natural Language Semantics* 19(1). 87–107.

Francis, Naomi. 2018. *Even* in presupposition denials. Manuscript, MIT.

Gajewski, Jon. 2011. Licensing strong NPIs. *Natural Language Semantics* 1–40.

Guerzoni, Elena. 2003. *Why even ask? On the pragmatics of questions and the semantics of answers*: Massachusetts Institute of Technology dissertation.

Ivlieva, Natasha. 2013. *Scalar implicatures and the grammar of plurality and disjunction*: MIT dissertation.

Kadmon, Nirit & Fred Landman. 1993. *Any*. *Linguistics and Philosophy* 16(4). 353–422.

Karttunen, Lauri & Stanley Peters. 1979. Conventional implicature. In C.-K. Oh & D. A. Dinneen (eds.), *Syntax and semantics*, vol. 11, 1–56. Academic Press.

Katzir, Roni. 2007. Structurally defined alternatives. *Linguistics and Philosophy* 30. 669–690.

Katzir, Roni. 2014. On the roles of markedness and contradiction in the use of alternatives. In Salvatore Pistoa-Reda (ed.), *Pragmatics, semantics and the case of scalar implicatures*, 40–71. Springer.

Kennedy, Christopher. 2015. A" de-fregean" semantics (and neo-gricean pragmatics) for modified and unmodified numerals. *Semantics and Pragmatics* 8. 10–1.

Kratzer, Angelika. 2005. Indefinites and the operators they depend on: From japanese to salish, vol. 173, 113–142. CSLI Publications Stanford, CA.

Krifka, Manfred. 1995. The semantics and pragmatics of weak and strong polarity items. *Linguistic Analysis* 25. 209–257.

Križ, Manuel. 2017. Bare plurals, multiplicity, and homogeneity. Manuscript, ENS.

Ladusaw, William. 1979. *Polarity sensitivity as inherent scope relations*: University of Texas, Austin, PhD dissertation.

Lahiri, Utpal. 1998. Focus and negative polarity in Hindi. *Natural Language Semantics* 6(1). 57–123.

Link, Godehard. 1983. The logical analysis of plurals and mass terms: A lattice-theoretical approach. *Formal semantics: The essential readings* 127–146.

Longenbaugh, Nicholas. 2019. Agreement mismatch in partitive relatives. *Linguistic Inquiry* 50(4). 847–861.

Menéndez-Benito, Paula. 2010. On universal free choice items. *Natural Language Semantics* 18(1). 33–64.

Sauerland, Uli. 2003. A new semantics for number. In *Semantics and linguistic theory*, vol. 13, 258–275.

Sauerland, Uli. 2004. Scalar implicatures in complex sentences. *Linguistics and Philosophy* 27(3). 367–391.

Spector, Benjamin. 2007. Aspects of the pragmatics of plural morphology: On higher-order implicatures. In *Presupposition and implicature in compositional semantics*, 243–281. Springer.

Vendler, Zeno. 1967. *Linguistics in philosophy*. Cornell University Press.

Zwarts, F. 1998. Three types of polarity. In F. Hamm & E. Hinrichs (eds.), *Plurality and quantification*, vol. 69, 177–238. Kluwer.

Zweig, Eytan. 2009. Number-neutral bare plurals and the multiplicity implicature. *Linguistics and philosophy* 32(4). 353–407.

A Singular *any*-DPs in existential modal sentences, in detail

This section elaborates on the presentation in Sect. 2.1 (see Crnič 2017, 2019 for further details). In the following, we take the resource domain D of *any* to consist of three books, {a, b, c}, and we represent alternatives with their translations into predicate logic formulas. Moreover, existential quantification is represented with disjunction, while universal quantification (and distributive predication over plurals) with conjunction in the standard manner.

The sentence in (45-a) has the LF in (45-b).

(45) a. Mary is allowed to read any book.

b. $[\text{exh}_{\text{R}}\ [\Diamond\ [\text{any}_{\text{D}}\ \text{book}_{\text{x}}\ [\text{Mary read x}]]]]$

Formal alternatives. The alternatives to the sister of *exh* in (45-b) are provided in (46) (following Krifka 1995 and, esp., Chierchia 2013): they are derived by replacing the domain of *any* with its subdomains, and *any* with *every*. In the following, we rely on the characterization in the bottom row of (46), where '◇(a∨b)' stands for the meaning corresponding to Mary being allowed to read a book in {a, b}, etc. (The sentence has other alternatives, say, those in which *Mary* is replaced with *John*, but these are irrelevant for the purpose at hand.)

(46) ALT([◇ [any_D book_x [Mary read x]]]) =

{[◇ [$\text{any}_{D'}$ book_x [M. read x]]], [◇ [$\text{every}_{D'}$ book_x [M. read x]]] | $[\![D']\!] \subseteq \{a,b,c\}\}$ ≈

{◇a, ◇b, ◇c, ◇(a∨b), ◇(a∨c), ◇(b∨c), ◇(a∨b∨c), ◇(a∧b), ◇(b∧c), ◇(a∧c), ◇(a∧b∧c)}

Excludable alternatives. Recall the definition of excludable alternatives:

(47) Excl(S) = ⋂{M | M is a maximal subset of ALT(S)
such that $\{\neg[\![S']\!] \mid S' \in M\} \cup \{[\![S]\!]\}$ is consistent}

What are the excludable alternatives in (46)? One maximal set of alternatives in (47) that can be jointly negated with the sister of *exh* being true is provided in (48). This is witnessed by (49) being consistent, and by the fact that if the set of the alternatives that get negated is expanded by any of the subtracted alternatives, we obtain a contradiction.

(48) {◇b, ◇c, ◇(b∨c), ◇(a∧b), ◇(b∧c), ◇(a∧c), ◇(a∧b∧c)}
Subtracted alternatives: ◇a, ◇(a∨b), ◇(a∨c), ◇(a∨b∨c)

(49) ◇(a∨b∨c) ∧ ∀p ∈ (48): ¬p (consistent meaning)
= ◇a ∧ ¬◇b ∧ ¬◇c

All other maximal sets of alternatives in (46) that can be jointly negated with the sister of *exh* being true have a similar form and are provided in (50)-(51).

(50) {◇a, ◇c, ◇(a∨c), ◇(a∧b), ◇(a∧c), ◇(b∧c), ◇(a∧b∧c)}
Subtracted alternatives: ◇b, ◇(a∨b), ◇(b∨c), ◇(a∨b∨c)

(51) {◇a, ◇b, ◇(a∨b), ◇(a∧b), ◇(a∧c), ◇(b∧c), ◇(a∧b∧c)}
Subtracted alternatives: ◇c, ◇(a∨c), ◇(b∨c), ◇(a∨b∨c)

The intersection of these maximal sets – that is, the set of excludable alternatives in (46) – is provided in (52): it consists of all the alternatives that have a (non-trivial) conjunctive meaning (if a proposition is conjoined with itself, we obtain a trivial conjunction).

(52) Excl([◇ [any_D book_x [Mary read x]]]) = {◇(a∧b), ◇(a∧c), ◇(b∧c), ◇(a∧b∧c)}

More generally, the sets of the alternatives that can be jointly negated with the sister of *exh* being true have the form in (53), while their instersection has the form in (54).

(53) $\{[\Diamond\ [\text{any}_{D'}\ \text{book}_x\ [\text{Mary read x}]]], [\Diamond\ [\text{every}_{D''}\ \text{book}_x\ [\text{Mary read x}]]] \mid$
$[\![D']\!], [\![D'']\!] \subseteq [\![D]\!] \wedge z \notin [\![D']\!] \wedge [\![D'']\!] \cap [\![\text{book}]\!] \neq \{z\}\}$, for some $z \in [\![D]\!] \cap [\![\text{book}]\!]$

(54) $\text{Excl}([\Diamond\ [\text{any}_D\ \text{book}_x\ [\text{Mary read x}]]]) =$
$\{[\Diamond\ [\text{every}_{D'}\ \text{book}_x\ [\text{Mary read x}]]] \mid [\![D']\!] \subseteq [\![D]\!] \wedge \text{card}([\![D']\!] \cap [\![\text{book}]\!]) \geq 2\}$

Includable alternatives. Recall Bar-Lev & Fox's (2019) definition of includable alternatives:

(55) $\text{Incl}(S) = \bigcap\{M \mid M$ is a maximal subset of $\text{ALT}(S)$
such that $\{[\![S']\!] \mid S' \in M\} \cup \{\neg[\![S']\!] \mid S' \in \text{Excl}(S)\}$ is consistent$\}$

What are the includable alternatives in (46)? They are all the disjunction alternatives (incl. the trivial disjunction alternatives), as given in (56). This is witnessed by the consistency of (57), which is a conjunction of all the alternatives in (56) with the negations of all the excludable alternatives; moreover, asserting a non-trivial conjunction alternative (which is the only type of alternatives that is not in (56)) would obviously contradict the negation of one of the excludable conjunctive alternatives, which shows that the set in (56) is indeed maximal.

(56) $\text{Incl}([\Diamond\ [\text{any}_D\ \text{book}_x\ [\text{Mary read x}]]]) =$
$\{\Diamond a, \Diamond b, \Diamond c, \Diamond(a \vee b), \Diamond(a \vee c), \Diamond(b \vee c), \Diamond(a \vee b \vee c)\}$

(57) $\Diamond a \wedge \Diamond b \wedge \Diamond c \wedge \Diamond(a \vee b) \wedge \Diamond(a \vee c) \wedge \Diamond(b \vee c) \wedge \Diamond(a \vee b \vee c) \wedge$ (includable)
$\neg\Diamond(a \wedge b) \wedge \neg\Diamond(a \wedge c) \wedge \neg\Diamond(b \wedge c) \wedge \neg\Diamond(a \wedge b \wedge c)$ (excludable)

Exhaustification and obligatory pruning. Recall Bar-Lev & Fox's (2019) definition of *exh*:

(58) $[\![\text{exh}_R\ S]\!](w) = 1$ iff
a. $\forall S' \in \text{Excl}(S) \cap R$: $\neg[\![S']\!](w)$, and
b. $\forall S' \in \text{Incl}(S)$: $[\![S']\!](w)$.

What is the meaning of the structure (45-b)? This depends on what alternatives are relevant, that is, on the resolution of R. If all the formal alternatives to the sister of *exh* are relevant, $\text{ALT}([\Diamond\ [\text{any}_D\ \text{book}_x\ [\text{Mary read x}]]]) \subseteq [\![R]\!]$, the meaning we obtain is the one provided in (59) (the same result is obtained for any R that denotes a superset of the set consisting of all the excludable alternatives, $\text{Excl}([\Diamond\ [\text{any}_D\ \text{book}_x\ [\text{Mary read x}]]]) \subseteq [\![R]\!]$). On the other hand, if none of the alternatives are relevant, the meaning that we obtain is the one provided in (60).

There are further conceivable resolutions of R (but see Katzir 2014, Crnič et al. 2015, Bar-Lev 2018 on some constraints on the pruning of alternatives).

(59) If $[\![R]\!] \supseteq \text{Excl}([\Diamond\ [\text{any}_D\ \text{book}_x\ [\text{Mary read x}]]])$,

$$[\![[\text{exh}_R\ [\Diamond\ [\text{any}_{D'}\ \text{book}_x\ [\text{Mary read x}]]]]]\!] = \Diamond a \wedge \Diamond b \wedge \Diamond c \wedge \neg\Diamond(a\wedge b) \wedge \neg\Diamond(a\wedge c) \wedge \Diamond(b\wedge c)$$

(60) If $[\![R]\!] \cap \text{Excl}([\Diamond\ [\text{any}_D\ \text{book}_x\ [\text{Mary read x}]]]) = \emptyset$,

$$[\![[\text{exh}_R\ [\Diamond\ [\text{any}_{D'}\ \text{book}_x\ [\text{Mary read x}]]]]]\!] = \Diamond a \wedge \Diamond b \wedge \Diamond c$$

How do we choose between the potential resolutions of R? It turns out that the choice is not completely free – it is partly dictated by the *Any* Condition. In order to obtain a Strawson downward-entailing environment with respect to the domain of *any*, all the disjunctive (incl. the single disjunct alternatives) must count as irrelevant (unless they are equivalent to the sister of *exh*). In other words, from the set in (46), only the excludable alternatives in (52) may be in $[\![R]\!]$ (together with the alternatives that are equivalent to the sister of *exh*) (cf. Buccola & Haida 2017 on obligatorily irrelevant alternatives):

(61) $[\![R]\!] \cap \text{ALT}([\Diamond\ [\text{any}_D\ \text{book}_x\ [\text{Mary read x}]]]) \subseteq \text{Excl}([\Diamond\ [\text{any}_D\ \text{book}_x\ [\text{Mary read x}]]])$

Let us first show that we indeed get Strawson downward-entailingness with respect to the domain of *any* if (61) obtains. The interpretation of the structure in (45-b) is provided in (62) (the parentheses indicate that the exclusion of the alternatives depends on R).

(62) $$[\![[\text{exh}_R\ [\Diamond\ [\text{any}_D\ \text{book}_x\ [\text{Mary read x}]]]]]\!] = \Diamond a \wedge \Diamond b \wedge \Diamond c \wedge \big(\neg\Diamond(a\wedge b) \wedge \neg\Diamond(a\wedge c) \wedge \neg\Diamond(b\wedge c)\big)$$

If we replace D with a proper (non-empty) subdomain of D (and keep R fixed), we obtain a Strawson weaker meaning. Two such replacements are provided in (63), where each of the meanings is entailed by the meaning in (62): there are fewer asserted includable alternatives, while the negated excludable alternatives are constant across all the substitutions of D.

(63) a. If $[\![D']\!] = \{a,b\}$, then $[\![[\text{exh}_R\ [\Diamond\ [\text{any}_{D'}\ \text{book}_x\ [\text{Mary read x}]]]]]\!] = \Diamond a \wedge \Diamond b \big(\wedge \neg\Diamond(a\wedge b) \wedge \neg\Diamond(a\wedge c) \wedge \neg\Diamond(b\wedge c)\big)$

b. If $[\![D']\!] = \{a\}$, then $[\![[\text{exh}_R\ [\Diamond\ [\text{any}_{D'}\ \text{book}_x\ [\text{Mary read x}]]]]]\!] = \Diamond a \big(\wedge \neg\Diamond(a\wedge b) \wedge \neg\Diamond(a\wedge c) \wedge \neg\Diamond(b\wedge c)\big)$

What happens, however, if (61) does not obtain? In this case, at least one replacement of domain D will not result in a Strawson weaker meaning. For illustration, assume that $\Diamond(a\vee b) \in R$ (falsifying (61), since $\Diamond(a\vee b)$ is not an excludable alternative). Now, the meaning of the sentence with domain D is provided in (64), where the content of φ depends on what alternatives

are in R besides ◇(a∨b). The meaning of the corresponding structure in which D is replaced by a stronger domain D′, namely D′={c}, is provided in (65), where ψ depends on what alternatives are in R besides ◇(a∨b).

(64) If ◇(a∨b) ∈ R, then [[[exh$_R$ [◇ [any$_D$ book$_x$ [Mary read x]]]]]] = ◇a ∧ ◇b ∧ ◇c ∧ φ

(65) If [[D′]] = {c} and ◇(a∨b) ∈ R, then
[[[exh$_R$ [◇ [any$_{D'}$ book$_x$ [Mary read x]]]]]] = ◇c ∧ ¬◇(a∨b) ∧ ψ

The meaning in (64) clearly does not entail the meaning in (65), no matter what the content of φ and ψ is – in fact, (64) entails the negation of (65). The same conclusion is reached for every other R that fails to satisfy the condition in (61). This is summarized in (66), where the alternatives equivalent to the sister of *exh* are omitted for brevity.

(66) **Generalization about the obligatory pruning of alternatives:**
An otherwise well-formed LF [exh$_R$ [◇ [any$_D$ NP$_x$ XP]]] is Strawson downward-entailing with respect to D iff
[[R]] ∩ ALT([◇ [any$_D$ NP$_x$ XP]]) ⊆ Excl([◇ [any$_D$ NP$_x$ XP]]).

Summary. Exhaustification over the alternatives introduced by singular *any*-DPs in existential modal sentences, as in (45-b), creates a Strawson downward-entailing environment with respect to the domain of *any* – which results in the *any*-DP satisfying the *Any* Condition – if none of the includable alternatives are in the resource domain of *exh*, R (besides the sister of *exh* and any alternatives equivalent with it).

B Plural *any*-DPs in existential modal sentences, in detail

This section elaborates on the presentation in Section 2.2. We assume in the following that the domain D corresponds to the closure of the set of three books under sum formation, $*\{a,b,c\}$, that is, {*a, b, c, a+b, a+c, b+c, a+b+c*}. We adopt the notation of the preceding section, with existential quantification (distributive predication) over {*a+b*} being represented as a ∧ b, etc.

The sentence in (67-a) may have the LF in (67-b).

(67) a. #Mary is allowed to read any books.
b. [exh$_R$ [◇ [any$_D$ books$_x$ [Mary read x]]]]

Formal alternatives. The alternatives to the sister of *exh* in (67-b) are provided in (68): they are the alternatives derived by replacing the domain of *any* with its subdomains, and *any* with *all*. (An element is in the disjunctive closure of a set P iff it is in P or if it is a disjunction of two elements each of which is in the disjunctive closure of P.)

(68) ALT([◇ [any$_D$ books$_x$ [Mary read x]]]) =

{[◇ [any$_{D'}$ books$_x$ [M. read x]]], [◇ [all$_{D'}$ books$_x$ [M. read x]]] | [[D′]]⊆∗{a, b, c}} ≈

{◇φ | φ ∈ DISJCLOSURE({a, b, c, a∧b, a∧c, b∧c, a∧b∧c})} =

{◇a, ◇b, ◇c, ◇(a∨b), ..., ◇(a∨b∨c∨...∨(a∧b∧c)), ◇(a∧b), ◇(b∧c), ◇(a∧c), ◇(a∧b∧c)}

Excludable alternatives. What are the excludable alternatives in (68)? One maximal set of alternatives in (68) that can be jointly negated with the sister of *exh* being true is provided in (69). This is witnessed by (70) being consistent, and by the fact that if the set of alternatives that get negated is expanded by any of the subtracted alternatives, we obtain a contradiction.

(69) {◇φ | φ∈ DISJCLOS({b, c, a∧b, a∧c, b∧c, a∧b∧c})} =
{◇b, ◇c, ◇(b∨c), ◇((a∧b)∨c), ◇((a∧c)∨b), ..., ◇(a∧b), ◇(b∧c), ◇(a∧c), ◇(a∧b∧c)}
Subtracted alternatives: {◇(a ∨ ψ) | ψ∈DISJCLOS({a, b, c, a∧b, a∧c, b∧c, a∧b∧c})}

(70) ◇(a∨b∨c∨...∨(a∧b∧c)) ∧ ∀p ∈ (69): ¬p (consistent meaning)
= ◇a ∧ ¬◇b ∧ ¬◇c

All other maximal sets of alternatives in (68) that can be jointly negated with the sister of *exh* being true have a similar form and are provided in (71)-(72).

(71) {◇φ | φ∈ DISJCLOS({a, c, a∧b, a∧c, b∧c, a∧b∧c})}

(72) {◇φ | φ∈ DISJCLOS({a, b, a∧b, a∧c, b∧c, a∧b∧c})}

The intersection of these maximal sets – that is, the set of excludable alternatives in (68) – is provided in (73): it consists of all the conjunctive alternatives and all the disjunctive alternatives in which all the disjuncts are (non-trivial) conjunctions.

(73) Excl([◇ [any$_D$ books$_x$ [M. read x]]]) = {◇φ | φ ∈ DISJCLOSURE({a∧b, b∧c, a∧b∧c})}

Includable alternatives. What are the includable alternatives in (68)? They are all the disjunction alternatives in which at least one disjunct is not a (non-trivial) conjunction, as given in (74). This is witnessed by the consistency of (75), which is a conjunction of all the alternatives in (74) with the negations of all the excludable alternatives (it is verified in any situation in which ◇a∧◇b∧◇c is true and all excludable alternatives are false); moreover, asserting a conjunction or a disjunction of conjunctions (which is the only type of formal alternatives that is not in (74)) would obviously contradict a negation of one of the excludable conjunctive alternatives, which shows that the set in (74) is indeed maximal.

(74) Incl([◇ [any$_D$ books$_x$ [Mary read x]]]) =

$$\{\Diamond(\varphi \vee \psi) \mid \varphi \in \{a, b, c\} \wedge \psi \in \textsc{DisjClosure}(\{a, b, c, a\wedge b, a\wedge c, b\wedge c, a\wedge b\wedge c\})\}$$

(75) $\forall p \in \{\Diamond(\varphi \vee \psi) \mid \varphi \in \{a, b, c\} \wedge \psi \in \textsc{DisjClosure}(\{a, b, c, a\wedge b, a\wedge c, b\wedge c, a\wedge b\wedge c\})\}$:
$p \wedge \forall q \in \{\Diamond\varphi \mid \varphi \in \textsc{DisjClosure}(\{a\wedge b, b\wedge c, a\wedge b\wedge c\})\}$: $\neg q$ (consistent)

Exhaustification. What is the meaning of the structure in (67-b)? This depends on what alternatives are relevant, that is, on the resolution of R. If we assume that all the excludable alternatives are relevant, we obtain the meaning in (76), which is identical to (75).

(76) If $D = *\{a, b, c\}$ and $[\![R]\!] \supseteq \text{Excl}([\Diamond\ [\text{any}_D\ \text{books}_x\ [\text{Mary read x}]]])$,
$[\![[\text{exh}_R\ [\Diamond\ [\text{any}_D\ \text{books}_x\ [\text{Mary read x}]]]]]\!] = \Diamond a\wedge\Diamond b\wedge\Diamond c \wedge$
$\forall q \in \{\Diamond\varphi \mid \varphi \in \textsc{DisjClosure}(\{a\wedge b, b\wedge c, a\wedge b\wedge c\})\}$: $\neg q$

Given this resolution of R, the structure in (67-b) is not Strawson downward-entailing with respect to the domain of *any*. Consider the counterpart of (67-b) with a stronger domain D^* that consists of all proper pluralities, that is, $D^* = \{a+b, a+c, b+c, a+b+c\}$. The sets of excludable and includable alternatives in $\text{ALT}([\Diamond\ [\text{any}_{D^*}\ \text{books}_x\ [\text{Mary read x}]]])$ are provided in (77)-(78).

(77) $\text{Excl}([\Diamond\ [\text{any}_{D^*}\ \text{books}_x\ [\text{Mary read x}]]]) = \{\Diamond(a\wedge b\wedge c)\}$

(78) $\text{Incl}([\Diamond\ [\text{any}_{D^*}\ \text{books}_x\ [\text{Mary read x}]]]) = \{\Diamond(a\wedge b), \Diamond(b\wedge c), \Diamond(a\wedge c)\}$

Consequently, this structure has the meaning in (79).

(79) $[\![[\text{exh}_R\ [\Diamond\ [\text{any}_D\ \text{books}_x\ [\text{Mary read x}]]]]]\!] = \Diamond(a\wedge b)\wedge\Diamond(b\wedge c)\wedge\Diamond(a\wedge c)\wedge\neg\Diamond(a\wedge b\wedge c)$

This meaning is obviously not entailed by the meaning in (76), which demonstrates that the structure (67-b) on the proposed resolution of R is not Strawson downward-entailing with respect to the domain D. Now, adding to R alternatives that are not excludable cannot improve the situation (see the discussion in Appendix A) nor can pruning of excludable alternatives: the includable alternatives that are asserted when the domain of *any* is D^* are not entailed by the includable alternatives that are asserted when the domain of *any* is D.

Strong meanings of plural NPs. The state of the affairs does not change substantively if the plural NPs do not have number-neutral interpretations. For example, assume that $D = \{a+b, a+c, b+c, a+b+c\}$ and that $D^* = \{a+b+c\}$. The meanings of the pertinent structures with domains D and D^* are provided in (80) and (81), respectively (again, the exclusion depends on the choice of R, thus the parentheses). Given that (80) fails to entail (81), the requisite Strawson downward-entailingness does not obtain here either.

(80) $[\![[\text{exh}_R\ [\Diamond\ [\text{any}_D\ \text{books}_x\ [\text{Mary read x}]]]]]\!] = \Diamond(a\wedge b) \wedge \Diamond(b\wedge c)\ \Diamond(a\wedge c)\ \big(\wedge\neg\Diamond(a\wedge b\wedge c)\big)$

(81) $[\![[\text{exh}_R\ [\Diamond\ [\text{any}_{D^*}\ \text{books}_x\ [\text{Mary read x}]]]]]\!] = \Diamond(a\wedge b\wedge c)$

Summary. Exhaustification over the alternatives introduced by plural *any*-DPs in existential modal sentences does not create a Strawson downward-entailing environment with respect to the domain of *any* – resulting in the *any*-DP not being able to satisfy the *Any* Condition.

C The *even* approach to *any*-DPs

We describe how the results obtained in the main text can be derived in the approach of Crnič 2017, 2019 (who builds on Kadmon & Landman 1993, Lahiri 1998). The approach has as a consequence a condition on the distribution of *any*-DPs that differs from the *Any* Condition when it comes to non-monotone environments (as detailed in Crnič 2019).

The setup. On this approach, *any*-DPs are accompanied by a covert *even* operator that quantifies over the alternatives built on the alternatives to the domain of *any* (see Lahiri 1998 for a derivation in which *even* associates with the determiner). We follow Krifka 1995 and Chierchia 2013 in assuming that the relevant alternatives to a (focused) domain are its subdomains.

(82) [even [... any_{D_F} ...]]

Even primarily triggers the ordering presupposition in (83), where the relevant ordering relation ($<$) is filled in by the context (it could be logical entailment, Strawson entailment, contextual entailment, lower-likelihood, or some other relation that respects additivity).

(83) [[even S]] is defined only if $\forall S' \in F(S)$: $S \not\Leftrightarrow S' \rightarrow S < S'$.

The precise predictions of the theory depend on the resolution of $<$ in the context. If this is resolved to (contextual) Strawson entailment relation, we obtain straightforwardly that *any*-DPs are acceptable in Strawson downward-entailing environments that are not also Strawson upward-entailing, that singular *any*-DPs are acceptable in existential modal sentences, and that plural *any*-DPs are unacceptable in existential modal sentences (see Crnič 2019 for a detailed discussion of the first two cases).

Singular *any*-DPs. The sentence in (84-a) has the LF in (84-b).

(84) a. Mary is allowed to read any book.
b. [even [exh_R [$\Diamond$ [any_{D_F} book_x [Mary read x]]]]]

The presupposition of (84-b) is provided in (85). Since the exhaustified constituent is Strawson downward-entailing with respect to the domain of *any*, as seen in Sect. 2.1 and Appendix A, the presupposition is trivially satisifed, and thus the *any*-DP is correctly predicted to be acceptable.

(85) $\forall D^*$: $[\![D^*]\!] \subseteq [\![D]\!] \wedge$

$[\![[\text{exh}_R\ [\Diamond\ [\text{any}_D\ \text{book}_x\ [\text{Mary read x}]]]]]\!] \not\Leftrightarrow$
$[\![[\text{exh}_R\ [\Diamond\ [\text{any}_{D^*}\ \text{book}_x\ [\text{Mary read x}]]]]]\!] \rightarrow$

$[\![[\text{exh}_R\ [\Diamond\ [\text{any}_D\ \text{book}_x\ [\text{Mary read x}]]]]]\!] <$
$[\![[\text{exh}_R\ [\Diamond\ [\text{any}_{D^*}\ \text{book}_x\ [\text{Mary read x}]]]]]\!]$

Plural *any*-DPs. In the case of plural *any*-DPs, the scalar presupposition triggered by *even* is inconsistent (at least on the assumption that < picks out the contextual Strawson entailment relation). The sentence in (86-a) has the LF in (86-b).

(86) a. #Mary is allowed to read any books.
b. $[\text{even}\ [\text{exh}_R\ [\Diamond\ [\text{any}_{D_F}\ \text{books}_x\ [\text{Mary read x}]]]]]$

Let us focus on the following subdomain of D: $D^* = \{a+b+c\}$ (recall our assumption from the preceding appendix section that $D = *\{a, b, c\}$). The presupposition of *even* in (86) depends on how we resolve the resource domain of *exh*, R. There are several options available to us, but it suffices to consider just two. First: If $[\![R]\!]$ contains no excludable alternatives, the meanings of the exhaustified sentences with domains D and D^* are the following:

(87) $[\![[\text{exh}_R\ [\Diamond\ [\text{any}_D\ \text{books}_x\ [\text{Mary read x}]]]]]\!] = \Diamond a \wedge \Diamond b \wedge \Diamond c$

(88) $[\![[\text{exh}_R\ [\Diamond\ [\text{any}_{D^*}\ \text{books}_x\ [\text{Mary read x}]]]]]\!] = \Diamond(a \wedge b \wedge c)$

Since the meaning in (88) is not equivalent to that in (87), *even* presupposes that it is (contextually) Strawson entailed by (87). This is not the case since the logical entailment goes the other way around. On this resolution of R, the *any*-DP is correctly predicted to be unacceptable. Second: If R contains an excludable alternatives, say, just $\Diamond(a \wedge b)$, the meanings of the exhaustified sentences with domains D and D^* are the following:

(89) $[\![[\text{exh}_R\ [\Diamond\ [\text{any}_D\ \text{books}_x\ [\text{Mary read x}]]]]]\!] = \Diamond a \wedge \Diamond b \wedge \Diamond c \wedge \neg\Diamond(a \wedge b)$

(90) $[\![[\text{exh}_R\ [\Diamond\ [\text{any}_{D^*}\ \text{books}_x\ [\text{Mary read x}]]]]]\!] = \Diamond(a \wedge b \wedge c)$

Since the meaning in (90) is not equivalent to that in (89), *even* presupposes that it is (contextually) Strawson entailed by (89). This is not the case since (89) actually entails the negation of (90). On this resolution of R, the *any*-DP is also correctly predicted to be unacceptable.

Other ordering relations. If the ordering used by *even* is resolved to a weaker relation than (contextual) Strawson entailment, say, lower-likelihood in the context (cf., e.g., Karttunen & Peters 1979, Lahiri 1998, Crnič 2014), nothing changes in the analysis of the singular *any*-DP case (see Crnič 2019 for details). In the case of plural *any*-DPs, on the other hand, we obtain

contradictory presuppositions on some resolutions of R, and contingent (potentially implausible) presuppositions on other resolutions. We discuss two cases here. First: If no alternatives to the sister of *exh* are in R, we obtain a contradictory presupposition, namely, the meaning of the structure in which D^* replaces D is stronger than the meaning of the structure with D, and thus at most as likely. On this resolution of R, the *any*-DP is correctly predicted to be unacceptable.

(91) If $[\![R]\!] \cap ALT([\Diamond\ [any_D\ books_x\ [Mary\ read\ x]]]) = \emptyset$, then
$\Diamond a \wedge \Diamond b \wedge \Diamond c <_{pr,c} \Diamond(a \wedge b \wedge c)$, etc.

Second: If the excludable alternatives to the sister of *exh* are relevant, we obtain a contingent presupposition. Namely, while the meaning of the structure in which D^* replaces D contradicts the meaning of the structure with D, it may nonetheless be more likely than it.

(92) If $[\![R]\!] \subseteq Excl([exh_R\ [\Diamond\ [any_D\ books_x\ [Mary\ read\ x]]]])$, then
$\Diamond a \wedge \Diamond b \wedge \Diamond c \wedge \neg\Diamond(a \wedge b) \wedge \neg\Diamond(a \wedge c) \wedge \neg\Diamond(b \wedge c) <_{pr,c} \Diamond(a \wedge b \wedge c)$, etc.

Consequently, on this resolution of < and R, the prediction of the approach described above is not that we have an unacceptable occurrence of an *any*-DP, but rather one whose felicity depends on the plausibility of the scalar presupposition in the context (if this is implausible, the sentence is marked). However, another property of *even* affects the acceptability of the sentence on this resolution: *even* tends to be infelicitous when the alternatives it quantifies over are incompatible with its sister (see Guerzoni 2003, Francis 2018 for a discussion). Since this is necessarily the case if $[\![R]\!]$ contains excludable alternatives, the unacceptability of the plural *any*-DPs may be attributed to this property of *even* (see Crnič 2019 for some related discussion).