

Number in NPI Licensing*

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The acceptability of *any*-DPs in existential modal sentences presents a challenge for the theories of NPI licensing: existential modal sentences appear to differ substantially from other environments in which *any*-DPs are acceptable (in particular, they lack a downward-entailing operator). One approach to this challenge has been to, first, take *any*-DPs to be subject to an environment-based downward-entailingness condition – they have to occur in an environment that is Strawson downward-entailing with respect to their domain (cf. Kadmon & Landman 1993) – and, second, to derive such an environment in existential modal sentences by means of exhaustification (e.g., Fox 2007). This note presents new evidence for such a two-layered approach (cf. Crnić 2017, 2019). The evidence comes from a striking contrast in the behavior of singular vs. plural *any*-DPs in existential modal sentences. The paper concludes by charting some relations between *any*-DPs and other polarity items.

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1 The Observation

Any-DPs have a restricted distribution. Our understanding of it advanced significantly with Fauconnier’s (1975) and Ladusaw’s (1979) descriptive generalization that *any*-DPs must be c-commanded by a downward-entailing operator at LF (see von Stechow 1999 for a refinement). This generalization, however, fails to capture the famous distribution of *any*-DPs in existential modal sentences. Although these sentences lack a downward-entailing operator, an *any*-DP may nonetheless occur in them, as exemplified in (1). Such occurrences of *any*-DPs have been dubbed ‘free choice occurrences’ due to the universal-like ‘free choice’ inferences that they give rise to (Vendler 1967): for illustration, (1) conveys that every book is such that Gal is allowed to read it (that is, Gal is ‘free to choose’ which book to read).

- (1) Gal is allowed to read any book.

Modal environments provide an important arena for the study of *any*-DPs and other polarity items. On the one hand, they illuminate potential features of their interpretation that are concealed in other environments (see Crnič 2014b for a similar point about non-monotone environments). On the other hand, the assumptions of the different approaches to *any*-DPs and other polarity items are brought into a particularly sharp relief when dealing with these environments (see, e.g., Kadmon & Landman 1993, Lahiri 1998, Aloni 2007b, Menéndez-Benito 2010, Chierchia 2013 for a selection of approaches that adopt a uniform treatment of *any*-DPs as an existential quantifier). Accordingly, the behavior of *any*-DPs and other polarity items in modal environments has been discussed extensively in the literature in recent years (e.g., Dayal 1998, 2004, 2009, 2013, Kratzer & Shimoyama 2002, Kratzer 2005, Fălăuş 2009, 2014, Alonso-Ovalle & Menéndez-Benito 2010, 2018, Aloni & Port 2010, 2015, Buccola & Haida

2017, among others; see Chierchia 2013 for a comprehensive review).

However, no attention has been paid to a striking fact about the distribution of *any*-DPs in existential modal sentences: while singular *any*-DPs are acceptable in them, as exemplified in (1) above, plural *any*-DPs are not, as exemplified in (2).¹

(2) #Gal is allowed to read any books.

The goal of this note is to show that the contrast between (1) and (2) is captured straightforwardly on an approach that combines the following two assumptions (cf. Crnič 2017, 2019):

First: The distribution of *any*-DPs is subject to the environment-based condition in (3) (cf. Kadmon & Landman 1993).² (Something closely akin to the condition in (3) falls out as a consequence of the approach of Crnič 2017, 2019. See Appendix C for a brief review.)

Second: The condition in (3) can be satisfied in existential modal sentences by applying exhaustification to derive free choice inferences (cf., e.g., Fox 2007, Chierchia 2013).

(3) **The Any Condition:** A DP headed by *any* is acceptable only if its domain is dominated by a constituent that is Strawson downward-entailing with respect to it.

On such an approach, the contrast between singular vs. plural *any*-DPs in (1) and (2) is a consequence of the fact that while exhaustification can create an environment that is Strawson downward-entailing with respect to the domain of *any* in the case of singular *any*-DPs in existential modal sentences (Sect. 2.1), it crucially cannot do so in the case of plural *any*-DPs like *any books*, resulting in them violating the *Any Condition* (Sect. 2.2). The combination of the *Any Condition* and exhaustification is critical for capturing this contrast between singular vs. plural *any*-DPs: if either the *Any Condition* is dropped or not derived (cf. Chierchia 2013) or certain mechanisms other than exhaustification are used to strengthen the import of *any*-DPs in modal sentences (cf. Aloni 2007b, Menéndez-Benito 2010), the contrast between the sentences in (1) and (2) remains unexplained, all else equal (Sect. 2.4). Furthermore, the approach described above gives rise to several intricate predictions about how the status of plural *any*-DPs in existential modal sentences may be improved, all of which seem to be borne out (Sect. 3).

The study of the interaction of number and the availability of free choice readings with *any*-

¹These data were brought to my attention by Naomi Francis and Elise Newman in a seminar (Crnič 2018).

²The definition of cross-categorical Strawson entailment is in (i) (cf. Gajewski 2011, Sect. 3). The notion of a constituent being Strawson downward-entailing with respect to the domain of an *any*-DP is defined in (ii).

- (i) a. If α and β are of type t , then $\alpha \Rightarrow_s \beta$ iff $\alpha = F$ or $\beta = T$.
b. If α and β are of type $\sigma\tau$, then $\alpha \Rightarrow_s \beta$ iff $\alpha(x) \Rightarrow \beta(x)$ for all x of type σ s.t. $\beta(x)$ is defined.
- (ii) A constituent S is Strawson downward-entailing with respect to the domain D of an *any*-DP it dominates iff every D' such that $\llbracket D' \rrbracket \Rightarrow \llbracket D \rrbracket$, $\llbracket S \rrbracket \Rightarrow_s \llbracket S[D/D'] \rrbracket$ (where $S[D/D']$ is identical to S except that every occurrences of D in S is replaced with an occurrence of D').

DPs has repercussions that go well beyond the behavior of count *any*-DPs. On the one hand, it leads to a new perspective on a poorly understood variation among polarity items with respect to their acceptability in existential modal sentences (Sect. 4.1). On the other hand, it brings to light some more general features of an approach that adopts (or derives) the *Any* Condition and its relation to alternative approaches. This, in turn, allows for a new way of typologizing polarity items, one that takes the observed interaction of number and the availability of free choice readings as a key factor in assorting the polarity items (Sect. 4.2).

In light of all this, we conclude that the impact of number on the behavior of polarity items has substantial, previously underappreciated diagnostic value in determining what mechanisms govern their distribution. Although we cannot adequately survey, let alone engage with, all the issues brought up in the rich literature on *any*-DPs and other polarity items here, we flag some of the most critical ones to our study in the conclusion (Sect. 5).

2 The *Any* Condition and Exhaustification

If the distribution of *any*-DPs is taken to be subject to the *Any* Condition, and if one subscribes to the uniform analysis of *any*-DPs as existential quantifiers across all their occurrences, an additional mechanism must be invoked to allow them to satisfy the *Any* Condition in existential modal sentences. One candidate for this mechanism is exhaustification in grammar (e.g., Fox 2007). We show that an approach that adopts the *Any* Condition and exhaustification explains straightforwardly the contrast between the acceptability of singular vs. plural *any*-DPs in existential modal sentences.

2.1 Singular *Any*-DPs

We rehearse the derivation of the acceptability of singular *any*-DPs in existential modal sentences. (The presentation is kept concise for readability. See Appendix A for more details.)

The *Any* Condition. The condition in (3) subsumes acceptable occurrences of *any*-DPs in existential modal sentences. This is supported by the intuitive validity of the Strawson entailment pattern in (4), where the substituted weaker domain and its stronger substitute are underlined.

(4) Gal is allowed to read any book. \Rightarrow_s Gal is allowed to read any long book.

The pattern in (4) is unsurprising once we take the free choice inferences that *any*-DPs induce in existential modal sentences into account: if every book is such that Gal is allowed to read it, then every long book is such as well (if there are long books in the domain). The pertinent question now is how these free choice inferences accompanying *any*-DPs are derived.

Exhaustification. One approach to free choice inferences relies on a covert exhaustification operator in grammar, *exh* (e.g., Fox 2007, Chierchia 2013, Bar-Lev & Fox 2020). One formulation of *exh*, due to Bar-Lev & Fox (2020), is provided in (5) (the choice of the formulation is not crucial for our purposes): it negates all the relevant excludable alternatives, which are characterized in (6-a), and asserts all the includable alternatives, which are characterized in (6-b). (The set $\text{ALT}(S)$ consists of all the sentences S' that can be derived from S by a substitution of constituents in S with their subconstituents or with lexical elements, see Katzir 2007.)

- (5) $\llbracket \text{exh}_R S \rrbracket(w) = 1$ iff
- a. $\forall S' \in \text{Excl}(S) \cap R: \neg \llbracket S' \rrbracket(w)$, and
 - b. $\forall S' \in \text{Incl}(S): \llbracket S' \rrbracket(w)$.
- (6) a. $\text{Excl}(S) = \bigcap \{M \mid M \text{ is a maximal subset of } \text{ALT}(S) \text{ such that } \{\neg \llbracket S' \rrbracket \mid S' \in M\} \cup \{\llbracket S \rrbracket\} \text{ is consistent}\}$
- b. $\text{Incl}(S) = \bigcap \{M \mid M \text{ is a maximal subset of } \text{ALT}(S) \text{ such that } \{\llbracket S' \rrbracket \mid S' \in M\} \cup \{\neg \llbracket S' \rrbracket \mid S' \in \text{Excl}(S)\} \text{ is consistent}\}$

Derivation. The sentence in (1) has the LF in (7-b), where *exh* takes matrix scope and the *any*-DP occurs in the scope of the modal. (Again, see Appendix A for some missing details.)

- (7) a. Gal is allowed to read any book.
- b. $\llbracket \text{exh}_R [\diamond [\text{any}_D \text{book}_x [\text{Gal read } x]]] \rrbracket$

The sister of *exh* in (7-b) induces the alternatives in (8). Specifically, following Chierchia (2013), we assume that *any*-DPs have so-called subdomain and universal quantifier alternatives.

- (8) $\text{ALT}([\diamond [\text{any}_D \text{book}_x [\text{Gal read } x]]]) =$
 $\{[\diamond [\text{any}_{D'} \text{book}_x [\text{Gal read } x]]], [\diamond [\text{every}_{D'} \text{book}_x [\text{Gal read } x]]] \mid \llbracket D' \rrbracket \subseteq \llbracket D \rrbracket\}$

The set of excludable alternatives in (8) is provided in (9) – all the maximal subsets of the alternatives in (8) that can be jointly negated with the sentence being true have these alternatives in common and nothing else.³ The includable alternatives are provided in (10) – they, and no other alternatives, can all be asserted consistently if all the excludable alternatives are negated. (This is witnessed by the meaning in (11) below being consistent.)

³The maximal sets of alternatives that can be jointly negated with the sentence being true have the form in (i). They differ from each other in what book is subtracted from the subdomains. See Appendix A for more details.

- (i) $\{[\text{any}_{D'} \text{book}_x [\text{Gal read } x]], [\text{every}_{D''} \text{book}_x [\text{Gal read } x]] \mid$
 $\llbracket D' \rrbracket, \llbracket D'' \rrbracket \subseteq \llbracket D \rrbracket \wedge b \notin \llbracket D' \rrbracket \cap \llbracket \text{book} \rrbracket \wedge \llbracket D'' \rrbracket \cap \llbracket \text{book} \rrbracket \neq \{b\}, \text{ for some } b \in \llbracket D \rrbracket \cap \llbracket \text{book} \rrbracket\}.$

$$(9) \quad \text{Excl}([\diamond [\text{any}_D \text{ book}_x [\text{Gal read } x]]]) = \\ \{[\diamond [\text{every}_{D'} \text{ book}_x [\text{Gal read } x]] \mid \llbracket D' \rrbracket \subseteq \llbracket D \rrbracket \wedge \text{card}(\llbracket D' \rrbracket \cap \llbracket \text{book} \rrbracket) \geq 2\}$$

$$(10) \quad \text{Incl}([\diamond [\text{any}_D \text{ book}_x [\text{Gal read } x]]]) = \\ \{[\diamond [\text{any}_{D'} \text{ book}_x [\text{Gal read } x]] \mid \llbracket D' \rrbracket \subseteq \llbracket D \rrbracket \wedge \llbracket D' \rrbracket \cap \llbracket \text{book} \rrbracket \neq \emptyset\}$$

The output of the exhaustification in (7-b) is computed in (11): the assertion of the includable alternatives corresponds to the free choice inferences accompanying the *any*-DP, provided in the first line, while the negation of the excludable alternatives, provided in the second line, depends on the context (since the excludable alternatives need not be relevant, they need not get negated, as indicated by the parantheses; see, e.g., Fox & Katzir 2011, Katzir 2014, Crnić et al. 2015, Bar-Lev 2018 on some constraints on the relevance of alternatives).

$$(11) \quad \forall D': D' \neq \emptyset \wedge D' \subseteq \llbracket D \rrbracket \cap \llbracket \text{book} \rrbracket \rightarrow \diamond(\text{Gal read a book in } D') \\ (\wedge \forall D': D' \subseteq \llbracket D \rrbracket \wedge \text{card}(D' \cap \llbracket \text{book} \rrbracket) \geq 2 \rightarrow \neg \diamond(\text{Gal read every book in } D'))$$

This meaning corresponds to every book being such that Gal is allowed to read it (a consequence of the assertion of the includable alternatives) and Gal not being allowed to read two books (a consequence of the negation of the excludable alternatives):

$$(12) \quad \forall x: x \in \llbracket D \rrbracket \cap \llbracket \text{book} \rrbracket \rightarrow \diamond(\text{Gal read } x) \\ (\wedge \forall D': D' \subseteq \llbracket D \rrbracket \wedge \text{card}(D' \cap \llbracket \text{book} \rrbracket) \geq 2 \rightarrow \neg \diamond(\text{Gal read every book in } D'))$$

Satisfaction of the Condition. Sentence (1) is Strawson downward-entailing with respect to the domain of *any* on the construal in (7-b), meaning that the *any*-DP is correctly predicted to be acceptable. Namely, if you replace domain *D* in (7-b) with a stronger subdomain *D**, you obtain a Strawson weaker meaning: if every book in *D* is such that Gal is allowed to read it (and Gal is not allowed to read two books in *D*), then every book in *D** is such that Gal is allowed to read it if there are such books⁴ (and Gal is not allowed to read two books in *D**).

⁴The presupposition that there are books in the domain of *any* (indicated by the conditional *if there are such books* in the main text) is crucial for the sentence to be Strawson downward-entailing with respect to it: if there are no books in a stronger domain *D** that replaces the domain of *any* in evaluating Strawson downward-entailingness, existential quantification over it will yield a false meaning, and thus the *Any* Condition will be violated. This is different from what we observe in typical downward-entailing environments, where an empty domain leads to a tautologous meaning, which is trivially entailed by any sentence. Some support for the occurrences of *any*-DPs in existential modal sentences indeed being construed as presuppositional indefinites comes from, for example, their inability to occur in the pivot position of a *there is* construction (cf. Milsark 1974), (i), and from the standard presupposition projection tests, (ii) (sequence (ii) is infelicitous because the conditional sentence inherits the presupposition that there are mistakes in this manuscript, which is triggered on the free choice construal of the *any*-DP in the antecedent of the conditional sentence; cf. von Stechow 1998).

- (i) a. #There may be any boy in the garden.
b. There may be a boy in any garden.

(13) [exh_R [◇ [any_D book_x [Gal read x]]]]

is Strawson downward-entailing with respect to the domain D.

Some other environments. Before proceeding to plural *any*-DPs, it is instructive to discuss two further occurrences of *any*-DPs: its acceptable occurrence in the scope of negation, provided in (14), and its unacceptable occurrence in the scope of a universal modal, provided in (15). In the former environment, the *any*-DP satisfies the *Any* Condition without requiring an application of exhaustification, underscoring that exhaustification can be seen as a rescue mechanism in existential modal sentences.⁵ In the latter environment, exhaustification does not enable the *any*-DP to satisfy the *Any* Condition, accounting for its unacceptability (Crnič 2017, 2019).

(14) Gal did not read any book.

(15) #Gal must read any book.

Ad (14). If *any book* takes scope below negation in (14), as provided in (16), it occurs in a constituent that is (Strawson) downward-entailing with respect to it – and thus its domain: if there is no book in D that Gal read, there is no book in any subset of D that Gal read.

(16) [not [any_D book_x [Gal read x]]]

is Strawson downward-entailing with respect to the domain D.

Dropping the negation in (14)/(16) leaves us with an environment that is not Strawson downward-entailing with respect to the domain of the *any*-DP: there are subsets D* of D such that Gal reading a book in D may be true without Gal reading a book in D* being true. Thus, we get a violation of the *Any* Condition, which explains the unacceptability of the *any*-DP in #Gal read any book (see Fox 2007, Chierchia 2013 for a discussion of the unavailability of free choice readings for unembedded disjunction and existential quantifiers).

Ad (15). Similarly to existential modal sentences, universal modal sentences do not on their own garner a constituent that would be Strawson downward-entailing with respect to the domain of *any*. Unlike in existential modal sentences, however, introducing exhaustification does not

(ii) #I am not sure whether there are any mistakes in this manuscript, but if I am allowed to fix any mistake, it doesn't matter.

See Crnič 2019, Sect. 2 of Part 1, for a further discussion of, and support for, *any*-DPs in existential modal sentences obligatorily triggering existence presuppositions. We do not represent the presuppositional construals for reasons of simplicity (see, e.g., Diesing 1992, Büring 1998 for two possible approaches).

⁵In line with the level of abstraction adopted in this note, we remain agnostic about whether some alternative-sensitive operator must associate with every occurrence of an *any*-DP. We merely assume that, as with other expressions in language (plain indefinites, disjunction, etc.), *any*-DPs have formal alternatives that can feature in exhaustification. A derivation of the *Any* Condition in (3) may well require an assumption of an alternative-sensitive operator that associates with *any*-DPs more generally, an assumption shared by many recent theories of *any*-DPs and other polarity items (esp., Krifka 1995, Lahiri 1998, Chierchia 2013, Crnič 2017, 2019).

provide a constituent that would satisfy the *Any* Condition in universal modal sentences (Crnič 2017, 2019). Sentence (15) may be assigned the structure in (17), whose meaning is in (18) (see Bar-Lev & Fox 2020, Sect. 5.5, on recursive exhaustification and free choice under universal operators).⁶

(17) $[\text{exh}_{R'} [\text{exh}_R [\Box [\text{any}_D \text{book}_x [\text{Gal read } x]]]]]$

(18) $\Box(\text{Gal read a book in } D) \wedge \forall x: x \in D \cap [\text{book}] \rightarrow \Diamond(\text{Gal read } x)$
 $(\wedge \forall D': \text{card}(D' \cap [\text{book}]) \geq 2 \wedge D' \subseteq D \rightarrow \neg \Diamond(\text{Gal read every book in } D'))$

The structure in (17) is not Strawson downward-entailing with respect to the domain of *any*: while replacing the domain D with a subdomain in (17) may yield a subset of the free choice inferences (and a subset of negated inferences; see the second and third conjunct in (18)), it also yields a stronger meaning of the universal modal component of the exhaustified meaning (see the first conjunct in (18)), which suspends the entailment. *Any*-DPs are accordingly predicted to be unacceptable in universal modal sentences (though see Sect. 5.3).

(19) $[\text{exh}_R [\Box [\text{any}_D \text{book}_x [\text{Gal read } x]]]]$

is not Strawson downward-entailing with respect to the domain D .

Summary. The combination of the assumption of the *Any* Condition (or some mechanism that has it as a consequence) and the assumption that free choice inferences are generated in grammar by means of exhaustification yields the correct prediction that *any*-DPs with singular NP complements should be acceptable in existential modal environments, in downward-entailing environments, but not in universal modal environments or episodic upward-entailing environments.⁷ (How the distribution of *any*-DPs in environments that are not obviously subsumed by

⁶We obtain the meaning in (18) if we assume that the universal modal has an existential modal alternative. In this case, no universal modal alternative built on *any* is excludable (see Bar-Lev & Fox 2020, though they make a different assumption about modals). If we do not admit existential modal sentences to our alternatives, the conclusion about Strawson downward-entailingness remains the same, though the free choice inferences are derived from the negations of the subdomain alternatives, which are of the form *Gal is required to read a book in D'* , and a single exhaustification suffices to derive these (Bar-Lev & Fox 2020, Sect. 5.5). Although the choice between the two derivations is unimportant at this juncture, it may be important elsewhere, see Sect. 5.3.

⁷This is an oversimplification. Exhaustification is in principle predicted to be able to rescue occurrences of *any*-DPs in various non-modal environments in which they are c-commanded by an existential quantifier. The prediction is not borne out in general, which is often attributed to an intervention effect induced by the non-modal existential quantifier (esp., Chierchia 2013; see also Bar-Lev & Fox 2020). However, if *any*-DPs were admitted in the scope of some non-modal existential quantifiers, that is, if exhaustification could rescue them in those environments, one would expect them to exhibit a distribution that parallels that of *any*-DPs in existential modal sentences. One candidate for such an environment was brought up by a reviewer and is exemplified in (i): comparatives. *Any*-DPs exhibit sensitivity to number therein in a way that parallels existential modal sentences.

- (i) a. Gal is taller than any boy in her class.
b. #Gal is taller than any boys in her class.

A possible clausal structure of (i) that would account for this state of affairs is provided in (ii) (cf. Seuren 1973,

our classification here, such as generics and imperatives, can be accounted for is discussed in Crnić 2019.⁸ See Sect. 5.3 for some further discussion of universal modal sentences.)

We now turn to occurrences of plural *any*-DPs in existential modal sentences.

2.2 Plural *Any*-DPs

Plurality. The only difference between the sentences in (1) and (2) is in the number marking on the *any*-DP. What is the semantic import of this difference? While there are several different proposals about the semantics of number (e.g., Sauerland 2003, Spector 2007, Zweig 2009, Ivlieva 2013, Križ 2017), the choice between them does not affect the predictions of the approaches discussed in this paper. For concreteness, we will assume that the plural morpheme simply denotes a cumulation operator (*): it takes a set of atoms as an argument, and returns back a set of atoms and all their sums.⁹

- (20) $[[\text{books}]] = *[[\text{book}]] = \lambda x. x \text{ is a book or a sum of books}$
 $(\approx \{ \text{Anna Karenina, War and Peace, Anna Karenina+War and Peace, ...} \})$

For clarity of exposition, we assume that the set of elements constituting the domain of quantification of a plural *any*-DP is also closed under sum formation. More specifically, we will assume in the following that the domain D of *any* contains book atoms and all their sums (and no other sums of books) (see Appendix B for a more general derivation).

Derivation. The sentence in (2) has the LF in (21-b). The alternatives to the sister of *exh* in (21-b) are in (22): they differ from the sister of *exh* in that the determiner *any* is replaced by *all*

Schwarzschild 2008). Whether such an analysis is plausible, or whether some alternative analysis is to be preferred, as well as what the precise distribution of this contrast is, requires an extensive engagement with the syntax/semantics of comparatives (cf. Aloni & Roelofsen 2014). We hope to pursue this engagement in the future.

- (ii) $[\text{exh}_R [\exists_d [\text{than} [\text{any}_D \text{ boy/s}]_x [\text{NEG} [x \text{ is tall}_d]]] [\text{Gal is tall}_d]]$

⁸*Any*-DPs are acceptable in generic and imperative sentences. While the intuitive entailment patterns that those sentences give rise to appear to respect the *Any* Condition (for example, *any dog barks* intuitively entails *any large dog barks*), the derivation of these is controversial. According to the approach discussed in the main text, the acceptability of *any*-DPs would be expected in these environments if they had an underlying existential semantics, which may subsequently be strengthened to a universal one (cf. Bassi & Bar-Lev 2016 for one way of deriving such strengthening). This is in line with Nickel 2010, Menéndez-Benito 2010, Menéndez-Benito 2012 on generics, and Kaufmann 2011 on imperatives. See Crnić 2018, 2019 for some more details.

⁹Following Link 1983, we assume that the domain of individuals consists both of atoms and sums of individuals (proper pluralities), is partially ordered by a part-of relation (\sqsubseteq), and is closed under sum formation (+). A definition of an atom is provided in (i-a) (which could be further relativized to include sortal information) and that of the *-operator is provided in (i-b) (which closes a set under sum formation).

- (i) a. An individual $x \in D_e$ is an atom iff $\forall z (z \sqsubseteq x \rightarrow z = x)$.
 b. For any $P \in D_{(et)}$, $*P = \lambda x. P(x) \vee \exists y, z (x = y+z \wedge *P(y) \wedge *P(z))$.

or the domain of *any*, D , is replaced by a subdomain. (See Appendix B for more details.)

- (21) a. #Gal is allowed to read any books.
 b. $[\text{exh}_R [\diamond [\text{any}_D \text{books}_x [\text{Gal read } x]]]]$

(22) $\text{ALT}([\diamond [\text{any}_D \text{books}_x [\text{Gal read } x]]]) =$
 $\{[\diamond [\text{any}_{D'} \text{books}_x [\text{Gal read } x]]], [\diamond [\text{all}_{D''} \text{books}_x [\text{Gal read } x]]] \mid [[D']] \subseteq [[D]]\}$

What are the excludable alternatives in (22)? Maximal subsets of (22) that can be jointly negated with the sentence being true are of the form given in (23): their joint negation is consistent with Gal being allowed to read some book or books (namely, Gal may still be allowed to read book b) and expanding the set by a further alternative leads to a contradiction (e.g., negating that Gal is allowed to read some book or books in $\{b\}$ is incompatible with the sentence being true and all other alternatives in the expanded set being false). (We paraphrase the import of *any books* with ‘some book or books’ since *books* has a number-neutral meaning.)

(23) $\{[\diamond [\text{any}_{D'} \text{books}_x [\text{Gal read } x]]], [\diamond [\text{all}_{D''} \text{books}_x [\text{Gal read } x]]] \mid$
 $[[D']], [[D'']] \subseteq [[D]] \wedge b \notin [[D']] \cap [[\text{books}]] \wedge [[D'']] \cap [[\text{books}]] \neq \{b\},$
 for some $b \in [[D]] \cap [[\text{book}]]$.

The intersection of all such sets – the set of excludable alternatives – is provided in (24): it consists of all the alternatives that convey that Gal is allowed to read two or more books. (We omit some alternatives from the set, say, those based on *any* with the domain of *any* containing only pluralities consisting of two books, etc., for reasons of brevity.)

(24) $\text{Excl}([\diamond [\text{any}_D \text{books}_x [\text{Gal read } x]]]) =$
 $\{[\diamond [\text{all}_{D'} \text{books}_x [\text{Gal read } x]]] \mid [[D']] \subseteq [[D]] \wedge \text{card}([[D']] \cap [[\text{books}]]) \geq 2\}$

The includable alternatives are all the subdomain alternatives that contain at least one atom book in the domain of *any*, as given in (25): these and only these alternatives can all be jointly asserted when all the alternatives in (24) are negated.

(25) $\text{Incl}([\diamond [\text{any}_D \text{books}_x [\text{Gal read } x]]]) =$
 $\{[\diamond [\text{any}_{D'} \text{books}_x [\text{Gal read } x]]] \mid [[D']] \subseteq [[D]] \wedge [[D']] \cap [[\text{book}]] \neq \emptyset\}$

Accordingly, the meaning of the structure in (21-b) is provided in (26): every alternative in which the domain of *any* contains at least one atom book is such that it is true (and every alternative in which a universal quantifier has two or more books in its domain is false).

(26) $\forall D': D' \subseteq [[D]] \wedge \exists x (x \in D' \cap [[\text{book}]]) \rightarrow \diamond(\text{Gal read some book or books in } D')$

$$(\wedge \forall D': D' \subseteq \llbracket D \rrbracket \wedge \text{card}(D' \cap \llbracket \text{book} \rrbracket) \geq 2 \rightarrow \neg \diamond(\text{Gal read all books in } D'))$$

This meaning is equivalent to every book being such Gal is allowed to read it (and her not being allowed to read two or more books), as given in (27): namely, (26) entails that every singleton domain containing an atom book is such that Gal is allowed to read the book in it (which in turn entails that every domain containing at least one atom book is such that Gal is allowed to read some book or books in it).

$$(27) \quad \forall x: x \in D \cap \llbracket \text{book} \rrbracket \rightarrow \diamond(\text{Gal read } x)$$

$$(\wedge \forall D': D' \subseteq \llbracket D \rrbracket \wedge \text{card}(D' \cap \llbracket \text{book} \rrbracket) \geq 2 \rightarrow \neg \diamond(\text{Gal read all books in } D'))$$

The exhausted meanings of the sentences with singular and plural *any*-DPs in (1) and (2) are thus equivalent (compare plural (26)-(27) with their singular counterparts in (11)-(12) above).¹⁰ How can they, then, differ with respect to the licensing of singular vs. plural *any*-DPs? We first show that they do in fact differ in this respect, and then diagnose why.

Violation of the Condition. To show that the structure in (21-b) is not Strawson downward-entailing with respect to the domain of *any*, and so the occurrence of the *any*-DP in it is correctly ruled out as unacceptable, it suffices to come up with one replacement of D in (21-b) with a subdomain that will not result in a Strawson weaker meaning. Consider domain D^* in (28), which consists only of proper plural individuals, and is a proper subdomain of D .

$$(28) \quad D^* = \{x \mid x \in D \wedge x \notin \llbracket \text{book} \rrbracket\} (\subset D)$$

The meaning of the counterpart of (21-b) in which domain D is replaced by domain D^* is provided in (29): since there are no atom books in domain D^* , both the includable and excludable alternatives involve non-atoms only: the includable alternatives are all the alternatives based on *any* whose domain contains at least one plurality of exactly two books; the excludable alternatives are all the alternatives based on *all* whose domain contains at least two pluralities. This means that the free choice inferences that are generated only involve non-atoms as well. This is represented in (29) by the domains quantified over having an empty intersection with the set of atom books (that is, $\llbracket \text{book} \rrbracket$); the exclusion inferences correspond to Gal not being allowed to read three books (if the excludable alternatives are relevant).

¹⁰A reviewer observes that bare plurals appear to license stronger inferences than singular indefinites in existential modal sentence like (i), specifically, they appear to license free choice inferences involving all pluralities. We suggest that this follows from a generic construal of bare plurals (that is, generic quantification over all pluralities of books). Why generic quantification is unavailable for *any*-DPs is a vexing issue (see Dayal 2004, 2009, Chierchia 2013 on the absence of quantificational variability effects with *any*-DPs).

(i) Gal can read books. (\Rightarrow Every plurality of books is such that Gal can read them.)

$$(29) \quad \forall D': D' \subseteq \llbracket D \rrbracket \wedge D' \cap \llbracket \text{two books} \rrbracket \neq \emptyset \wedge D' \cap \llbracket \text{book} \rrbracket = \emptyset \rightarrow \\ \diamond(\text{Gal read some book or books in } D') \\ (\wedge \forall D': D' \subseteq \llbracket D \rrbracket \wedge \text{card}(D' \cap \llbracket \text{book} \rrbracket) \geq 3 \rightarrow \neg \diamond(\text{Gal read all books in } D'))$$

The meaning in (29) is equivalent to every two books being such Gal is allowed to read them (and her not being allowed to read three or more books): namely, every plurality consisting of exactly two books is according to (29) such that Gal is allowed to read some book or books in the singleton set containing that plurality (which then entails that Gal is allowed to read some book or books in a set that contains that and other pluralities).

$$(30) \quad \forall x: x \in \llbracket D \rrbracket \cap \llbracket \text{books} \rrbracket \wedge \text{card}(x)=2 \rightarrow \diamond(\text{Gal read } x) \\ (\wedge \forall D': D' \subseteq \llbracket D \rrbracket \wedge \text{card}(D' \cap \llbracket \text{book} \rrbracket) \geq 3 \rightarrow \neg \diamond(\text{Gal read all books in } D'))$$

The *Any Condition* dictates that the entailment relation in (31) obtains: namely, for the condition to be satisfied, it has to hold that there is a constituent in (21-b) such that replacing domain D with the stronger D^* results in a Strawson weaker meaning of the thus modified constituent; the only plausible candidate for this constituent is (21-b) itself. (The first line in (31) corresponds to (26)-(27), while the second line corresponds to (29)-(30).)

(31) **Requirement of the *Any Condition*:**

$$\llbracket [\text{exh}_R [\diamond [\text{any}_D \text{ books}_x [\text{Gal read } x]]]] \rrbracket \Rightarrow_s \\ \llbracket [\text{exh}_R [\diamond [\text{any}_{D^*} \text{ books}_x [\text{Gal read } x]]]] \rrbracket$$

The required Strawson entailment does not obtain, however: not only does the meaning in (26)-(27) fail to entail any inferences about pluralities consisting of two books, which are entailed by (29)-(30), it actually excludes them from being true (unless the excludable alternatives, parenthesized in (26)-(27), are taken not to be relevant). This means that the *Any Condition* cannot be satisfied in the structure in (21-b).

$$(32) \quad \llbracket [\text{exh}_R [\diamond [\text{any}_D \text{ books}_x [\text{Gal read } x]]]] \rrbracket$$

is not Strawson downward-entailing with respect to the domain D .

An approach that adopts the *Any Condition* and exhaustification thus correctly captures the contrast in acceptability between the singular and plural *any*-DPs in existential modal sentences.

Diagnosis. Why, more generally, does Strawson downward-entailingness obtain with singular but not plural *any*-DPs? This difference follows from what replacements of the domain of *any* with its subdomains are available on the singular vs. plural number marking (which are needed to check the *Any Condition*) and how this interacts with exhaustification. Specifically, the free choice inferences generated in the sentences under discussion crucially involve only the

minimal elements in the domain of quantification of *any*¹¹ (in both the singular and plural case these are atoms, given the number-neutral meaning of plural NPs, see also Appendix B). This can be seen most clearly in the representations in (12) and (27), both of which involve universal quantification over atoms.

(33) **Exhaustification and free choice:**

The free choice inferences generated by exhaustification over the alternatives to an *any-DP* involve only the minimal elements in the domain of the *any-DP*.

Singular *any-DPs*: Exhaustification enables the satisfaction of the *Any Condition* in the case of singular *any-DPs*, where the domain of quantification of *any* is restricted to the minimal elements (atoms) by the singular morphology on the NP: if we replace the domain of *any* in (7-b) with a subdomain, any subdomain, we will obtain free choice inferences involving a subset of the atoms that the free choice inferences of the initial sentence involved, that is, we obtain a Strawson weaker meaning. This corresponds to Strawson downward-entailingness.

Plural *any-DPs*: In contrast, the restriction of free choice inferences to minimal elements guarantees a violation of the *Any Condition* in the case of plural *any-DPs*, where *any* does not quantify only over atoms: if we replace the domain of *any* in (21-b) with a subdomain that consists only of non-atoms (say, only of pluralities consisting of two books, etc.), the free choice inferences that get generated will involve non-atoms, while the free choice inferences of the initial sentence involved only atoms. Since no free choice inferences involving non-atoms are entailed by the initial sentence, we obtain a Strawson independent meaning at best.

The crucial difference between the sentences with singular vs. plural *any-DPs* is summarized in (34) in terms of the relations between the sets of includable alternatives, whose assertion corresponds to the free choice inferences. The disjointness or only partial overlap of the sets of includable alternatives of a sentence and its subdomain variant implies at least independence of their exhaustified meanings (esp., of their free choice inferences), and thus absence of Strawson downward-entailingness with respect to the domain of *any*.

(34) **Facts about subdomains and free choice:**

Sg: For every domain $D' \subseteq D$, the set of includable alternatives in $ALT([\diamond [any_D \text{ book}_x \text{ Gal read } x]])$ is a superset of that in $ALT([\diamond [any_{D'} \text{ book}_x \text{ Gal read } x]])$.

Pl: For some domains $D' \subseteq D$, the set of includable alternatives in $ALT([\diamond [any_D \text{ books}_x \text{ G. read } x]])$ is not a superset of that in $ALT([\diamond [any_{D'} \text{ books}_x \text{ G. read } x]])$.

¹¹A minimal element in a set is one that has no other element in the set as a part.

2.3 Intermediate Summary

We showed that the asymmetry in the acceptability of singular vs. plural *any*-DPs in existential modal sentences, exemplified in (1) and (2), follows naturally on an approach that makes the following two assumptions (the first one stems originally from Kadmon & Landman 1993, the second one is found in Fox 2007, Chierchia 2013, among others; see Crnič 2017, 2019):

- *Any*-DPs must satisfy the *Any* Condition.
- The import of *any*-DPs (just like that of other existential quantifiers and disjunction) can be strengthened in modal sentences by means of exhaustification.

Exhaustification over the alternatives induced by singular *any*-DPs in existential modal sentences creates an environment that is Strawson downward-entailing with respect to the domain of *any*, as rehearsed in Sect. 2.1. In contrast, exhaustification over the alternatives induced by plural *any*-DPs does not have this effect, as shown in Sect. 2.2. The crucial difference between the two cases was that in the case of singular *any*-DPs free choice inferences are generated for every element in the domain of quantification of *any* (namely, all the atoms), while in the case of plural *any*-DPs these were generated only for a proper subset of this domain (namely, only the minimal elements – the atoms on a number-neutral analysis of plurality). When it comes to other environments, number should not play an important role: exhaustification is either not needed (downward-entailing environments) or it fails to deliver an environment that would be Strawson downward-entailing with respect to the domain of an *any*-DP on either choice of number (universal modal and episodic upward-entailing environments, though see fn. 7), as summarized in Table 1. (Again, how the distribution of *any*-DPs in environments that are not obviously subsumed by our classification here, such as generics and imperatives, can be accounted for is discussed in Crnič 2019. See fn. 8 and Sect. 5.3 for some further discussion.)

	DE	\diamond	\square / UE
Singular <i>any</i> -DPs	✓	✓	✗
Plural <i>any</i> -DPs	✓	✗	✗

Table 1: The predicted distribution of *any*-DPs in different environments given the *Any* Condition and the availability of *exh*. (‘UE’ stands for episodic upward-entailing environments.)

In the following section, we emphasize the critical importance of both components of the approach (namely, the *Any* Condition and exhaustification) in the explanation of the contrast between singular and plural *any*-DPs in existential modal sentences.

2.4 The Significance of the Two Components

We describe some issues raised by the contrast between (1) and (2) for, first, an approach to *any*-DPs that relies on exhaustification but neither adopts the *Any* Condition as a primitive nor

derives it (Chierchia 2013) and, second, for two approaches that employ a mechanism other than exhaustification to generate free choice inferences (Aloni 2007b, Menéndez-Benito 2010).

2.4.1 Dropping the *Any* Condition

One approach that neither assumes nor derives the *Any* Condition is that of Chierchia (2013), on which the acceptability of *any*-DPs depends simply on whether exhaustification over the alternatives induced by the sentences with *any*-DPs yields a consistent interpretation.

No contrast. On this exhaustification-only approach, both singular and plural *any*-DPs are *prima facie* predicted to be acceptable in existential modal sentences. This holds because the exhaustified meanings of both (1) and (2), computed in (11) and (26), respectively, are consistent and stronger than the simple sentence, and thus admitted as grammatical, all else equal, as stated in (35). (Chierchia 2013 adopts slightly different LFs, a special definition of *exh*, and additional auxiliary assumptions to deal with *any*-DPs in modal sentences than those adopted here, but this does not affect the prediction of acceptability, see also fns. ?? and 21.)

$$(35) \quad [[[\text{exh}_R [\diamond [\text{any}_D \text{book}_x [\text{Gal read } x]]]]]] = \\ \quad [[[\text{exh}_R [\diamond [\text{any}_D \text{books}_x [\text{Gal read } x]]]]]] \quad (\text{consistent, with } \textit{exh} \text{ non-vacuous})$$

Intervention. The incorrect prediction of acceptability may be avoided by identifying adequate auxiliary assumptions. These might be found in the independently needed theory of intervention, which Chierchia (2013) provides. Its underlying idea is that the set of alternatives over which *exh* quantifies may be larger than what we assumed above, leading in some cases to a failure of exhaustification rather than what would otherwise be a licit interpretation.¹² The hope could be that the problematic occurrences of plural *any*-DPs are ruled out by this machinery. The plural number would thereby have to be classified as an intervener that expands the set of alternatives over which *exh* quantifies. Now, the additional alternatives that we would obtain for plural *any*-DPs on this assumption would differ from the other alternatives merely in the number marking on the DP (singular, instead of plural). However, unlike in other cases of intervention, these additional alternatives would not obviously yield a pathology in exhaustification since they would be equivalent to their plural counterparts, as stated in (36). Note, furthermore, that the equivalence is maintained also after exhaustification, as stated in (35) above. (These facts parallel those involving the equivalence of sentences in which singular and plural *any*-DPs occur in downward-entailing environments.)

¹²The failure of exhaustification due to an intervention effect can arise in Chierchia's system because of a specific definition of *exh* that he employs. The definition employed in the main text, taken from Bar-Lev & Fox 2020, does not admit contradictions, and thus additional conditions would need to be introduced to account not only for the intervention effects but even for the basic unacceptability of NPIs in episodic upward-entailing environments. See Crnić 2021 for some further discussion of these issues.

$$(36) \quad \llbracket [\diamond [\text{any}_D \text{books}_x [\text{Gal read } x]]] \rrbracket = \llbracket [\diamond [\text{any}_D \text{book}_x [\text{Gal read } x]]] \rrbracket$$

A similar state of affairs would obtain even if the meanings of plural NPs were strengthened prior to the application of the exhaustification over the alternatives induced by *any*-DPs (cf. Spector 2007, Zweig 2009, Ivlieva 2013). In this case, the sister of *exh* in (21) would have the meaning in (37), and the alternatives with singular morphology would be properly weaker than the alternatives with plural morphology, as stated in (38), meaning that they should not obviously have an adverse effect on exhaustification.

$$(37) \quad \llbracket [\diamond [\text{any}_D \text{books}_x [\text{Gal read } x]]] \rrbracket = 1 \text{ iff } \diamond(\text{Gal read at least 2 books in } D)$$

$$(38) \quad \llbracket [\diamond [\text{any}_D \text{books}_x^{\text{str}} [\text{Gal read } x]]] \rrbracket \Rightarrow_s \llbracket [\diamond [\text{any}_D \text{book}_x [\text{Gal read } x]]] \rrbracket$$

Summary. We conclude that an exhaustification approach that does not adopt the *Any* Condition (or some mechanism that would derive it) fails to predict the pattern in (2) absent further assumptions, assumptions that have yet to be provided. A more detailed study of the application of the theory of Chierchia (2013) to the data under discussion is thus mandated.

2.4.2 Replacing Exhaustification

There are mechanisms other than exhaustification that have been put forward for strengthening the import of existential quantifiers and disjunction in existential modal sentences. We discuss two such mechanisms in the following (Aloni 2007b, Menéndez-Benito 2010).

Universal closure. Aloni (2007b) and Menéndez-Benito (2010) treat indefinites, incl. *any*-DPs, as inducing sets of alternatives in which they are effectively replaced with their potential verifiers (cf. also Kratzer 2005). Aloni assumes that this set is quantified over by the embedding modal, while Menéndez-Benito assumes that this is done by two covert operators. The structure assigned to (2) in the system of Aloni is provided in (39); its interpretation corresponds to universal quantification over the potential verifiers of the existential quantification, as in (40).

$$(39) \quad [\diamond [\text{Gal read any}_D \text{books}]]$$

$$(40) \quad \forall x: x \in \llbracket [D] \rrbracket \cap \llbracket [\text{books}] \rrbracket \rightarrow \diamond(\text{Gal read } x)$$

The structure assigned to (2) in the system of Menéndez-Benito is given in (41), in which the universal quantifier over alternatives, \forall , takes matrix scope and the exhaustive operator, *Excl*, takes scope below the modal. The meaning of the structure is provided in (42): every proposition of the form ‘Gal is allowed to read *x* and no other book’, where *x* is a book or a sum of books in *D*, is true (the universal quantification is due to \forall , and the exhaustive ‘no other book’ is due to *Excl*); this is equivalent to every book or sum of books being such that

Gal is allowed to read just them (see Menéndez-Benito 2010 for arguments for the obligatory exhaustification below the modal, and Chierchia 2013, Ch. 6, for a reply).

(41) $[\forall [\diamond [\text{Excl} [\text{Gal read any}_D \text{ books}]]]]$

(42) $\forall x: x \in [[D]] \cap [[\text{books}]] \rightarrow \diamond(\text{Gal read } x \text{ and no other books})$

Importantly, both structures in (39) and (41) have consistent interpretations and are, moreover, Strawson downward-entailing with respect to the domain of *any*: if every plurality of books in D is such that Gal is allowed to read (only) them, then for any subset D' of D , it holds that every plurality of books in D' is such that Gal is allowed to read (only) them. This means that the plural *any-DPs* in (39) and (41) are incorrectly predicted to be acceptable.

Summary. The difference between exhaustification and the strengthening mechanisms employed in (39) and (41) is that exhaustification affirms only the alternatives whose restrictors of *any* contain at least one atom, meaning that no information is conveyed about the alternatives whose restrictors do not contain any atoms (potentially these alternatives are even negated due to exclusion). This is not the case for the mechanisms presented in this section: they induce universal quantification over all the pluralities in the restrictor of *any*. While this quantification could be restricted to only minimal potential verifiers, there seems to be no independent motivation for this (it may even have negative global repercussions for the respective systems).

In conclusion, we discussed the consequences of dropping the *Any Condition* and relying merely on exhaustification to account for the distribution of *any-DPs* (Sect. 2.4.1), as well as the consequences of replacing exhaustification with an alternative strengthening mechanism (Sect. 2.4.2). Overgeneration obtains in both cases: the occurrences of plural *any-DPs* in existential modal sentences are incorrectly predicted to be acceptable, all else equal. Further study of how these systems could and should be constrained is required.

3 Predictions

Two types of manipulations are predicted to affect the acceptability of plural *any-DPs* in existential modal sentences. The first type involves a restriction of free choice inferences, so that they involve the same individuals in both the sentences under discussion and in their variants used to check Strawson downward-entailingness. This is achieved by means of numeral modification in Sect. 3.1. The second type involves an expansion of free choice inferences, so that they involve all the elements in the domain of *any*, not just the minimal ones. This is achieved by means of collective predication in Sect. 3.2.

3.1 Numeral *Any*-DPs

We observed that the unacceptability of plural *any*-DPs in existential modal sentences arises due to free choice inferences being generated solely with respect to the minimal elements in the domain of *any*. Accordingly, one strategy for rescuing plural *any*-DPs in existential modal sentences is by restricting the minimal elements in the domain of *any* to be of the same size across all the replacements of the domain with its subdomains. In this case, Strawson downward-entailingness with respect to the domain of *any* may obtain just like in the singular *any*-DP case, where such a restriction is in place due to the singular morphology. One way of achieving such a restriction is by using a prenominal numeral modifier (see Dayal 2004, 2013, Chierchia 2013, as well as Sect. 5.3 below for further discussion). The prediction of the approach is borne out, as exemplified in (43). We derive it in the following.

(43) Gal is allowed to read any three books.

Derivation. The structure of the sentence in (43) is provided in (44), and its interpretation is in (45). The alternatives to the sister of *exh* parallel those discussed above and encompass the subdomain alternatives, the universal quantifier alternatives, and their recombinations. The meaning in (45) is equivalent to every plurality of three books in the domain of *any* being such that Gal is allowed to read those three books (and that Gal is not allowed to read more than three books, due to the negation of the excludable universal quantifier alternatives in which the quantifier has four books in its domain), as provided in (46). The free choice inferences involve only the minimal elements in the domain of quantification of *any*, all of which importantly consist of exactly three books here.¹³

(44) $[\text{exh}_R [\diamond [\text{any}_D \text{ three books}]_x [\text{Gal read } x]]]$

(45) $\forall D': D' \neq \emptyset \wedge D' \subseteq [[D]] \cap [[\text{three books}]] \rightarrow \diamond(\text{Gal read three books in } D')$
 $(\wedge \forall D': D' \subseteq [[D]] \wedge \text{card}(D' \cap [[\text{book}]]) \geq 4 \rightarrow \neg \diamond(\text{Gal read all books in } D'))$

(46) $\forall x: x \in D \cap [[\text{books}]] \wedge \text{card}(x) = 3 \rightarrow \diamond(\text{Gal read } x)$
 $(\wedge \neg \diamond(\text{Gal read four books in } D))$

Satisfaction of the Condition. Since the prenominal numeral imposes a restriction to pluralities of three books, this restriction obviously obtains in all the sentences derived from (43) by replacing the domain of *any* with a subdomain (as required by the *Any* Condition): the sentences resulting from this replacement necessarily induce a subset of the free choice inferences

¹³The expression *three books* picks out the set of pluralities consisting of exactly three books (cf., e.g., Kennedy 2015, Buccola & Spector 2016). The meaning of *any three books* is provided in (i).

(i) $[[\text{any}_D \text{ three books}]](P) = 1$ iff $\exists x \in D (\text{books}(x) \wedge P(x) \wedge \text{card}(x) = 3)$

induced by the initial sentence. Consequently, since the resulting sentences are Strawson entailed by the initial sentence, the *Any* Condition is satisfied, and the plural *any*-DP is acceptable.

(47) $[\text{exh}_R [\diamond [\text{any}_D \text{ three books}]_x [\text{Gal read } x]]]$

is Strawson downward-entailing with respect to the domain D.

3.2 Collective Predication

The unacceptability of plural *any*-DPs in the sentences we discussed was conditioned by exhaustification generating free choice inferences involving only the minimal elements in the domain of *any*. If the sentences could be manipulated so that this restriction to the minimal elements would be lifted – that is, so that free choice inferences involving all the elements in the domain of *any* would be generated – plural *any*-DPs would be predicted to be acceptable in existential modal sentences. One way of achieving this is by using collective predication.

Collective predicates. A distributive predicate holds of a plurality if and only if it holds of all the atoms (subpluralities) constituting it. This is not the case for collective predicates. Consider the sentence in (48): on its collective/cumulative construal, it is compatible with it being false that Gal and Tal danced with Niv (or that they danced with Ziv). Similarly, the sentence in (49) is compatible with it being false that a proper subset of the girls formed a PEN club together.

(48) Gal and Tal danced with Niv and Ziv.

\nRightarrow Gal and Tal danced with Niv.

(49) Those girls formed a PEN club together.

\nRightarrow Two of those girls formed a PEN club together.

***Any*-DPs and collective predication.** When one turns to free choice occurrences of plural *any*-DPs, this property of collective predication effects an expansion of the set of includable alternatives, one that will encompass all subdomain alternatives. Consider the sentences in (50)-(51). They are significantly more acceptable than their distributive counterparts discussed above. We elaborate on why this is predicted in the following, where we focus on (50).

(50) Gal and Tal are allowed to dance with any boys.

(51) Any poets may form a PEN club together.

The sentence in (50) can be assigned the structure in (52). The alternatives to the sister of *exh* are those provided in (53): as assumed previously, they comprise of the subdomain alternatives, the universal quantifier alternatives, and their various recombinations.

$$(52) \quad [\text{exh}_R [\diamond [\text{any}_D \text{ boys}_x [\text{Gal and Tal danced with } x]]]]$$

$$(53) \quad \text{ALT}([\diamond [\text{any}_D \text{ boys}_x [\text{Gal and Tal danced with } x]]]) = \\ \{[\diamond [\text{any}_{D'} \text{ boys}_x [\text{Gal and Tal danced with } x]]], \\ [\diamond [\text{all}_{D'} \text{ boys}_x [\text{Gal and Tal danced with } x]]] \mid [D'] \subseteq [D]\}$$

Which of these alternatives are excludable? Since their verifications by different pluralities are mutually independent, we have many more maximal sets of jointly negatable alternatives than in the distributive case. In particular, Gal and Tal (collectively) dancing with a specific plurality of boys is compatible with them not (collectively) dancing with any plurality of boys distinct from it, including its proper subpluralities, and them not being allowed to collectively dance with more than one plurality of boys (that is, we assume that the universal quantifier scopes over event quantification; cf., e.g., Link 1987, Roberts 1987 on plural quantification). The maximal sets of alternatives of jointly negatable alternatives thus have the form in (54):

$$(54) \quad \{[\diamond [\text{any}_{D'} \text{ boys}_x [\text{Gal and Tal danced with } x]]], \\ [\diamond [\text{all}_{D''} \text{ boys}_x [\text{Gal and Tal danced with } x]]] \mid \\ [D'], [D''] \subseteq [D] \wedge x \notin [D'] \wedge \text{card}([D''] \cap [\text{boys}]) \geq 2 \text{ or } [D''] \neq \{x\}\}, \\ \text{for some plurality of boys } x \in D.$$

The excludable alternatives, arrived at by intersecting all the sets of the form in (54), are provided in (55): they consist of all the universal quantifier alternatives in which the domain of the universal quantifier consists of at least two elements. Consequently, all the existential quantifier alternatives can be asserted consistently with the negation of all the excludable alternatives, making all of them includable, as provided in (56).

$$(55) \quad \text{Excl}([\diamond [\text{any}_D \text{ boys}_x [\text{Gal and Tal danced with } x]]]) = \\ \{[\diamond [\text{all}_{D'} \text{ boys}_x [\text{Gal and Tal danced with } x]]] \mid [D'] \subseteq [D] \wedge \text{card}([D'] \cap [\text{boys}]) \geq 2\}$$

$$(56) \quad \text{Incl}([\diamond [\text{any}_D \text{ boys}_x [\text{Gal and Tal danced with } x]]]) = \\ \{[\diamond [\text{any}_{D'} \text{ boys}_x [\text{Gal and Tal danced with } x]]] \mid [D'] \subseteq [D] \wedge [D'] \cap [\text{boys}] \neq \emptyset\}$$

The resulting meaning of the sentence is provided in (57): we obtain free choice inferences involving all pluralities of boys (as well as the negation of all the relevant excludable alternatives). Note that this is different from what we obtained in the distributive cases discussed previously, where free choice inferences involved only all minimal pluralities of objects.

$$(57) \quad \forall x: x \in [D] \cap [\text{books}] \rightarrow \diamond(\text{Gal and Tal danced with } x) \\ (\wedge \forall D': D' \subseteq [D] \wedge \text{card}(D' \cap [\text{books}]) \geq 2 \rightarrow \\ \neg \diamond(\text{Gal and Tal danced with all pluralities in } D'))$$

Satisfaction of the Condition. The structure in (52) is Strawson downward-entailing with respect to the domain of the *any*-DP: since we obtain free choice inferences involving every element in the domain of the *any*-DP (and negation of every universal quantifier alternative whose domain of *any* contains at least two elements from the domain), replacing the domain with a subdomain will yield a subset of the free choice inferences (and a subset of the negation inferences). Consequently, the occurrence of the *any*-DP is correctly predicted to be acceptable.

(58) $[\text{exh}_R [\diamond [\text{any}_D \text{boys}_x [\text{Gal and Tal danced with } x]]]]$

is Strawson downward-entailing with respect to the domain D.

In conclusion, we discussed two manipulations of existential modal sentences with plural *any*-DPs. While the manipulation involving prenominal numeral modifiers brought about a restriction of free choice inferences to pluralities of only a certain size (Sect. 3.1), the manipulation involving collective predication brought about an expansion of free choice inferences to all pluralities (Sect. 3.2). Both manipulations were predicted to rescue the occurrences of plural *any*-DPs. And both predictions were shown to be borne out.

4 Variation

How are *any*-DPs related to other polarity items? We first discuss expressions that, we hypothesize, are subject to a variant of the *Any* Condition and whose domains of quantification are structurally similar to those of plural *any*-DPs. Their distribution is predicted to parallel that of plural *any*-DPs (Sect. 4.1). We then turn to expressions that, we hypothesize, are subject to obligatory exhaustification, either instead of or in addition to being subject to a variant of the *Any* Condition. Their distribution is predicted to differ from that of *any*-DPs (Sect. 4.2).

4.1 Cumulativity More Generally

We begin by outlining some predictions involving expressions that resemble plural *any*-DPs in that they (i) denote existential quantifiers, (ii) are subject to the condition that their domain must be dominated by a constituent that is Strawson downward-entailing with respect to it, and (iii) have a cumulative domain of quantification, as defined in (59).

(59) **Cumulative domains:**

A domain D is cumulative iff all x, y in D are such that $x+y$ is in D as well.

If there are expressions of this kind, they should be unacceptable in modal environments, as stated in (60) (since the condition on their distribution cannot be satisfied in them). And they should be acceptable in downward-entailing environments (since the condition on their

distribution can be satisfied in them). Are there such expressions other than plural *any*-DPs?

- (60) **Prediction:** If an existential quantifier must be dominated by a constituent that is Strawson downward-entailing with respect to its domain, and if this domain is cumulative, it should be unacceptable in (existential) modal sentences.

In the following, we describe some assumptions on which two sets of expressions in English would qualify as instantiations of this prediction.

Mass nouns. While the semantics of mass nouns is controversial (see, e.g., Lasersohn 2011, Lima 2018, Chierchia 2019 for recent reviews), there is consensus that it is at least cumulative. For instance, if two grains of rice are in the denotation of *rice*, or two pieces of furniture are in the denotation of *furniture*, their sums are as well.

- (61) **Hypothesis:** The meanings of mass nouns are cumulative.

How does this affect the distribution of mass *any*-DPs? Since their domain is cumulative, their behavior in modal environments should, all else equal, parallel that of plural *any*-DPs: even if exhaustification over the alternatives induced by mass *any*-DPs were to yield free choice inferences, the *Any* Condition could not be satisfied for reasons discussed in Sect. 2.2. The prediction appears to be borne out: mass *any*-DPs are unacceptable in existential modal sentences (see Chemla et al. 2011, fn.4, and Crnič & Haida 2020).

- (62) a. #Gal can donate any blood.
b. #We are allowed to buy any furniture.

More concretely, the sentence in (62-b) can be assigned a structure like (63). If we assume that the domain of *any* in (62-b) consists of all the pieces of furniture and their sums (e.g., Gillon 1992, Chierchia 1998a,b), the result of exhaustification should correspond to (64) (the exclusion inferences are left out for brevity).¹⁴ Replacing the domain of *any* in (63) with a subdomain need not yield a weaker meaning, however: for example, if a subdomain would contain a sum of tables without containing the separate tables from the initial domain making up this sum, the exhaustification of the initial sentence would not entail a free choice inference

¹⁴If the denotations of mass nouns were divisive (e.g., Bunt 1979, Link 1983), exhaustification over the alternatives induced by mass *any*-DPs in existential modal sentences may yield contradictory meanings. Specifically, Crnič & Haida 2020 show that on the assumption of divisiveness, one would not be able to identify a maximal set of alternatives to an existential modal sentence with a mass *any*-DP that could be jointly negated with the sentence being true. Accordingly, all alternatives would be excludable, and their exclusion would yield a contradiction (unless it were appropriately restricted, but see Fox & Hackl 2006 on the limits of contextual restriction; see also Gajewski 2009). Thus, no matter what assumptions one makes about the structure of mass noun denotations, mass *any*-DPs are predicted to be unacceptable in existential modal sentences. Similar considerations may extend to *ever* in the main text since the domain of events/times may be divisive as well (cf. Humberstone 1979). See Crnič & Haida 2020 for a more extensive discussion.

with respect to that sum, though that of the second sentence would.¹⁵

(63) $[\text{exh}_R [\diamond [[\text{any}_D \text{ furniture}]_x [\text{Gal buys } x]]]]$

is not Strawson downward-entailing with respect to the domain D.

(64) $\diamond(\text{Gal buys furniture in } D) \wedge \forall x \in \{x \mid x \text{ is a piece of furniture in } D\}: \diamond(\text{Gal buys } x)$

In contrast, mass *any*-DPs are acceptable in sentences that are downward-entailing with respect to them (and thus their domains), as exemplified in (65), since the *Any* Condition may be satisfied in those cases without a recourse to exhaustification, as represented in (66).

(65) a. Gal didn't donate any blood.

b. No one bought any furniture.

(66) $[\text{neg} [\text{any}_D \text{ blood}_x [\text{Gal donate } x]]]$

is Strawson downward-entailing with respect to the domain D.

Ever. An expression that is often categorized together with *any*-DPs is *ever* (the two expressions are in the class of so-called 'weak NPIs' in English, cf. Zwarts 1998). We can analyze it as an adverbial existential quantifier over events, and take its domain to be cumulative and unrestricted, as in (67)-(68) (cf. Krifka 1995; see, e.g., Landman 1991 on event structures).

(67) $[[\text{ever}_D]] = \lambda P_{(vt)}. \exists e \in D: P(e)$

(68) **Hypothesis:** The domain of *ever* is cumulative.

In order to qualify as an instantiation of the above prediction, we must further assume that it is subject to the condition in (69), a generalization of the *Any* Condition (hence the parentheses around *any*).

(69) **The (*Any*) Condition:** An occurrence of a designated existential quantifier expression is acceptable only if the domain of the quantifier is dominated by a constituent that is Strawson downward-entailing with respect to it.

Given these assumptions, the behavior of *ever* in modal environments should parallel that of plural and mass *any*-DPs: even if exhaustification over the alternatives induced by *ever* would yield free choice inferences, the (*Any*) Condition could not be satisfied (see also fn. 14). The prediction is famously borne out:¹⁶

¹⁵The acceptability of mass *any*-DPs in existential modal sentences improves if we coerce a container/subkind construal of the mass expressions. The coercion in such readings arguably makes the domain of *any* non-cumulative (corresponding to the singular case), allowing for the satisfaction of the *Any* Condition.

¹⁶An alternative account of the data is conceivable, one on which the condition on the distribution of *ever* is substantively different from the (*Any*) Condition: it pertains not to the domain of *ever* but to *ever* itself. Consequently, the exhaustification could not rescue *ever* in modal environments (recall that exhaustification yields a Strawson

(70) #Gal is allowed to ever read ‘War and Peace’.

More concretely, the sentence in (70) can be assigned the structure in (71). If we assume that the domain of *ever* in (70) consists of all events, including plural events (sums of events), the result of exhaustification should correspond to (72) (the exclusion inferences are left out and the transworld identity of events is assumed for brevity). Replacing the domain of *ever* with a subdomain need not yield a weaker meaning, however: for example, if the subdomain contains a sum of events without containing its parts from the initial domain, the exhaustification of the initial sentence would not entail a free choice inference with respect to that sum, though that of the second sentence would (say, that Gal is allowed to read ‘War and Peace’ in January and March).

(71) $[\text{exh}_R [\diamond [\text{ever}_D [\text{Gal read } x]]]]$

is not Strawson downward-entailing with respect to D.

(72) $\diamond(\text{Gal read ‘War and Peace’ at an event or events in } D) \wedge$

$\forall e \in \{e \mid e \text{ is an event in } D\}: \diamond(\text{Gal read ‘War and Peace’ at } e)$

In contrast, *ever* is acceptable in sentences that are downward-entailing with respect to it (and thus its domain), as exemplified in (73), since the *Any* Condition may be satisfied in those cases without a recourse to exhaustification, as represented in (74).

(73) No student ever ‘War and Peace’.

(74) $[\text{no student}_x [\text{ever}_D [x \text{ read ‘War and Peace’}]]]$

is Strawson downward-entailing with respect to the domain D.

Summary. If we assume that an existential quantifier expression is subject to the (*Any*) Condition, and if its domain of quantification is cumulative, we expect it to be unacceptable in modal environments, as summarized in Table 2. We presented two candidates for such expressions (mass *any*-DPs and *ever*). On this view, the variation among certain NPIs can be seen as falling out wholly from the differences in their semantic properties while their so-called licensing conditions (that is, the mechanisms involved in governing their distribution) are kept constant. The assumptions that were adopted in our discussion surely require further study, as does comparison with alternative approaches (esp., Chierchia 2013, Crnič & Haida 2020).

downward-entailing environment with respect to the domain of an existential quantifier, not the existential quantifier itself). A fully fledged out theory of this may well give rise to, however, the expectation that the distribution of *ever* should parallel that of minimizer NPIs, contrary to fact (cf. Heim 1984, Crnič 2014a).

	Non-cumulative domains	Cumulative domains
DE environments	✓	✓
◇ environments	✓	✗
	(count <i>any</i> -DPs)	(mass <i>any</i> -DPs, <i>ever</i>)

Table 2: Predicted acceptability of existential quantifier expressions subject to the (*Any*) Condition as dependent on the (non-)cumulativity of their domains of quantification.

4.2 Obligatory Exhaustification

There is a myriad of expressions across languages whose restricted distributions and interpretations in modal environments resemble those of *any*-DPs, and which are often classed under the heading of ‘polarity items’, ‘free choice items’, and ‘modal indefinites’ (e.g., Kratzer & Shimoyama 2002, Kratzer 2005, Fălăuş 2009, 2014, Alonso-Ovalle & Menéndez-Benito 2010, 2018, Aloni & Port 2010, Chierchia 2013, among many others). One question that arises in light of our discussion is whether these expressions could/should be treated as being subject to the (*Any*) Condition in (69). For many of the expressions, the answer is ‘no’. Importantly, however, the existence of such expressions is neither surprising nor antipathetic to an approach that takes *any*-DPs to be subject to the *Any* Condition – rather, it further buttresses it.

We chart some predictions about the distribution of existential quantifiers akin to *any*-DPs that are, we hypothesize, not subject to the (*Any*) Condition but are obligatorily exhaustified (e.g., Chierchia 2013) as well as of those that are, we hypothesize, both subject to the (*Any*) Condition and are obligatorily exhaustified. The two factors yield the following typology:

(75) **Potential typology:**

- (C) Subject to the (*Any*) Condition ✓ | Obligatory exhaustification ✗
- (E) Subject to the (*Any*) Condition ✗ | Obligatory exhaustification ✓
- (CE) Subject to the (*Any*) Condition ✓ | Obligatory exhaustification ✓

We derive the distributional profiles in (75) and point to some expressions that realize them. In doing this, we report some results and insights from the extensive literature on exhaustification-requiring polarity items (esp., Kratzer & Shimoyama 2002, Alonso-Ovalle & Menéndez-Benito 2010, Chierchia 2013). (In addition to the (*Any*) Condition and the simple-minded obligatory exhaustification, further factors have been recognized as determining the behavior of polarity items; see, e.g., Fălăuş 2009, 2014, Alonso-Ovalle & Menéndez-Benito 2010, 2018, Chierchia 2013, Buccola & Haida 2017 for a sampling.)

Modal environments. Ad (C) and (CE). If an expression denotes an existential quantifier that is subject to the (*Any*) Condition, we expect its distribution to parallel that of *any*-DPs in modal environments: in particular, they should be unacceptable in universal modal sentences, and their free choice occurrences should resist plural complements. In addition to *any*-DPs,

qualsiasi NPs in Italian (Chierchia 2013), *cualquiera NPs* in Spanish (Menéndez-Benito 2010), and *wh-bhii* expressions in Hindi (Lahiri 1998) seem to fit the bill. The distribution of *qualsiasi NPs* in modal environments is exemplified in (76) (Gennaro Chierchia, p.c.) (see Sect. 5.1 for some further parallels with *any-DPs* involving partitivity). (Note that since exhaustification is necessary for the *Any* Condition to be satisfied in existential modal sentences, expressions in (C) and (CE) are predicted to have the same distribution in them. A difference between them emerges in downward-entailing environments, however.)

- (76) a. Puoi leggere qualsiasi libro.
 can read qualsiasi book
 ‘You can read any book’
 b. #Puoi leggere qualsiasi libri.
 can read qualsiasi books
 ‘#You can read any books’
- (77) #Devi leggere qualsiasi libro/i.
 must read qualsiasi book/s
 ‘#You must read any book(s).’

Ad (E). If we have an existential quantifier expression that is not subject to the (*Any*) Condition, and whose alternatives match those of *any-DPs* but must be exhaustified, two expectations arise, all else equal: (i) when it comes to modal environments, such expressions need not be restricted to existential modal sentences (recall that exhaustification of *any-DPs* yields consistent meanings in all modal sentences, Sect. 2.1); and (ii) they should not exhibit sensitivity to number (since this is determined by the (*Any*) Condition). Several expressions seem to behave like this (see Kratzer 2005, Martí 2008, Alonso-Ovalle & Menéndez-Benito 2010, Chierchia 2013): *algun(os) NPs* in Spanish, *irgend* indefinites in German, (*un*) *NP qualsiasi* in Italian, and *wh* indefinites in Slovenian. This is illustrated for German *irgend* indefinites in (78)-(79). Such sentences tend to give rise to free choice inferences, which can be derived along the lines discussed in Sect. 2.1 (see below for another possible construal).¹⁷

- (78) a. Gal darf irgendein Buch lesen.
 Gal can irgend-a book read
 b. Gal darf irgendwelche Bücher lesen.
 Gal can irgend-which books read
- (79) a. Gal muss irgendein Buch lesen.
 Gal must iregnd-a book read
 b. Gal muss irgendwelche Bücher lesen.
 Gal must irgend-which books read

¹⁷Alonso-Ovalle & Menéndez-Benito 2010 point to intricate differences between *algunos* and *irgend* indefinites (for example, involving non-uniqueness inferences). The additional restrictions discussed in the literature that these expressions may be subject to can be adopted wholesale without affecting our classification.

The coarse predictions about the distribution of polarity items in modal environments are summarized in Table 3. Polarity items that can take plural NP complements in existential modal sentences are predicted to be able to occur in universal modal sentences and *vice versa*. At a more fine-grained level of description, further distinctions may be necessary, but cannot be surveyed here (see the papers cited above and references therein).

	(C) and (CE)	(E)
◇ environments	✓(sg)	✓(sg,pl)
□ environments	✗	✓(sg,pl)

(any-DPs, qualsiasi NPs, cualquiera NPs, bhii-wh indefs.) *(irgend, algun, wh indefs. in Slo.)*

Table 3: Predicted acceptability in modal environments of existential quantifier expressions from the three hypothesized typological classes in (75).

Non-modal environments. Ad (C). An expression subject to the (*Any*) Condition should be acceptable in downward-entailing environments, and unacceptable in episodic upward-entailing environments (see, e.g., Kadmon & Landman 1993, Lahiri 1998, Crnič 2019 for discussion). In addition to *any*-DPs, *bhii-wh* indefinites in Hindi (Lahiri 1998) exhibit this behavior.

(80) koi bhii nahiiN aayaa
 anyone even not came
 ‘No one came.’

(81) #koi bhii aayaa
 anyone even came
 ‘#Anyone came.’

Ad (E). The distribution of exhaustification-requiring expressions in non-modal environments depends on the properties of exhaustification and the nature of its obligatoriness, as discussed extensively by Chierchia (2013). We begin by looking at downward-entailing environments. A critical fact about these is that exhaustification is averse to them, which can be attributed to (82) (see Chierchia et al. 2011, Spector 2014, Fox & Spector 2018 for details).

(82) **Non-weakening Condition:**

An occurrence of *exh* is not licensed in sentence S if deleting it does not alter or strengthens the meaning of S (that is, if *exh* is globally weakening or vacuous in S).

Given this, two general kinds of distributions are conceivable. On the one hand, if the distribution of *exh* when accompanying the polarity items is subject to the Non-weakening Condition, and the obligatoriness of exhaustification is strict, the items should be unacceptable in downward-entailing environments: namely, given that the hypothesized polarity items are the weakest among their alternatives, their exhaustification will either be vacuous or

lead to global weakening if they occur in downward-entailing environments, no matter where exhaustification would apply, in violation of (82). On the other hand, if exhaustification is merely preferred (necessary if possible, cf. Crnič 2014a for a similar pattern with minimizer NPIs), or if it were not subject to the Non-weakening Condition (perhaps because it satisfies some other requirement, cf. Chierchia 2013), then the expressions should be acceptable in downward-entailing environments, and have import identical to that of plain indefinites therein. Both patterns can be observed: *un NP qualsiasi* is marked in downward-entailing environments, as exemplified in (83), while *irgend* indefinites in German are acceptable, as exemplified in (84).

(83) #Non tutti i ragazzi hanno letto un libro qualsiasi.
not all the boys have read a book qualsiasi

(84) Nicht alle Jungs haben irgendein Buch gelesen.
not all boys have irgendein book read

Let us now turn to episodic upward-entailing environments. Almost all of the expressions we listed as falling under (E) are acceptable in such environments, and are accompanied therein by certain modal inferences (ignorance or agent indifference inferences), as illustrated in (85).

(85) Gal hat irgendein Buch gelesen.
⇒ The speaker is ignorant of what book Gal read, or
⇒ Gal is indifferent with respect to what book she read.

These obligatory modal inferences have been attributed to the exhaustification applying above an appropriate covert modal operator (say, an assertive K_{\square} operator), as presented in (86) (e.g., Alonso-Ovalle & Menéndez-Benito 2010, Chierchia 2013). The idea that underlies the derivation of ignorance inferences is that each sentence comes with an appended universal epistemic modal relativized to the speaker that exhaustification can take scope over in order to generate the observed ignorance inferences.^{18,19}

(86) a. $[\text{exh}_R [K_{\square} [\text{irgend-which}_D \text{ books}_x [\text{Gal read } x]]]]$
b. $\square_{\text{sp}}(\text{Gal read some books in } D) \wedge \forall x \in [[\text{book}]] \cap D: \diamond_{\text{sp}}(\text{Gal read } x)$

¹⁸Alonso-Ovalle & Menéndez-Benito 2010 present data that suggest that the behavior of plural *irgend* and *algunos* indefinites come apart with respect to the obligatoriness of exhaustification: with the latter, it seems to be optional, as witnessed by the optionality of the ignorance inferences.

¹⁹Not all expressions discussed under (E) above are acceptable in non-modalized upward-entailing environments. For example, *wh* indefinites in Slovenian are not, as shown in (i). Chierchia 2013 ascribes this potential variation to a parameter pertaining to where the *exh* accompanying the pertinent expressions may attach. This is not easily captured on the assumptions adopted in this section, on which there is a single exhaustification mechanism accompanying all ‘modal indefinites’. Accordingly, this assumption may need to be given up.

(i) #(Prepričan sem, da) Marija bere kako knjigo.
(certain be that) Marija read what book

Ad (CE). Expressions that are subject both to the (*Any*) Condition and obligatory exhaustification should be unacceptable in downward-entailing environments. This holds for *cualquiera NPs* and *qualsiasi NPs* (Chierchia 2013). (Note that if exhaustification were only conditionally obligatory, the distribution of the expressions would be indistinguishable from that of the expressions that are only subject to the (*Any*) Condition, that is, that are in class (C).)

(87) ??Non tutti i ragazzi hanno letto qualsiasi libro.
not all the boys have read qualsiasi book

Unlike expressions that are subject only to obligatory exhaustification, expressions that are also subject to the (*Any*) Condition cannot be rescued in episodic environments since these may only be construed as involving covert universal modality (see also footnote 19 for a potential additional reason, and Sect. 5.2 for apparent obviation).

(88) #Gal ha letto qualsiasi libro.
Gal has read qualsiasi book

The coarse predictions about the distribution of polarity items in non-modal environments are summarized in Table 4 (see Chierchia 2013 for some more fine-grained distinctions, as noted also in footnote 19). (The split in the middle right-most cell indicates that the acceptability depends on the nature of obligatoriness of exhaustification.)

	(C)	(CE)	(E)
DE environments	✓	✗	✓/✗
UE environments	✗	✗	✓

(*any*-DPs, *bhii-wh*, *wh-koli* indefs.) (*qualsiasi NPs*, *cualquiera NPs*) (*irgend* indefs., *un NP qualsiasi*)

Table 4: Predicted acceptability in downward-entailing and episodic upward-entailing environments of existential quantifier expressions from the hypothesized typological classes in (75).

Summary. The goal of this section was to bring into sharper focus some consequences of adopting the (*Any*) Condition by comparing the predictions arising from it to those that follow from the assumption of obligatory exhaustification. We showed that dropping the (*Any*) Condition leads to an expectation of the acceptability of the polarity items in universal modal sentences and the acceptability of them taking plural complements in modal environments. The expressions discussed bear out this expectation, though we left many nuances of their distribution aside (see the abovementioned authors for more comprehensive discussions of additional factors). We can conclude that a theory that adopts the (*Any*) Condition and one that adopts obligatory exhaustification are not only compatible with each other, but even dovetail once one looks beyond *any*-DPs. Their joint study remains promising.

5 Outlook

An approach that adopts the Any Condition and admits exhaustification adequately predicts the restricted distribution of singular and plural *any*-DPs in existential modal sentences (Sect. 2), including the mitigating effects of numeral modification and collective predication in the case of plural *any*-DPs (Sect. 3). Furthermore, it offers a straightforward way of capturing variation among weak NPIs in modal contexts, and dovetails with approaches that assume obligatory exhaustification when a broader set of polarity items is considered (Sect. 4). It is not without challenges, however. We conclude the paper by pointing out three left for future research.

5.1 Partitivity

In addition to *any*-DPs with prenominal numeral modifiers, partitive plural *any*-DPs are also acceptable in existential modal sentences, as exemplified in (89). A similar improvement can be found also with mass *any*-DPs, which behave like plural *any*-DPs, as discussed in Sect. 4.1.

- (89) a. #Gal is allowed to read any books.
b. Gal is allowed to read any of the books.
- (90) a. #Gal is allowed to drink any water.
b. Gal is allowed to drink any of the water.

We speculate that the acceptability of the partitive plural and mass *any*-DPs may have an explanation similar to the one presented for prenominal numeral modifiers above: the partitive may impose an additional restriction on the domain of quantification of *any*, via a covert partitive head (Chierchia 1998a, Sauerland 2004, Longenbaugh 2019, among others), which may sanction Strawson downward-entailingness (just like prenominal numerals do). For example, if the unpronounced partitive head were singular *book* in (89-b), as provided in (91) (see Longenbaugh 2019 on this type of number mismatch), we would effectively obtain a configuration that mirrors the one with the singular *any*-DP.

- (91) [exh_R [\diamond [any_D ~~book~~ of the books]_x [Gal read x]]]

We hope to pursue this direction of explanation in the future. Tentative support for it comes from Spanish: *cualquiera* NPs with plural NPs are acceptable in existential modal sentences as partitives, but only if the determiner is singular. If the determiner is plural, the partitive becomes unacceptable, as provided in (92) (this support is only tentative due to the archaic nature of *cualesquiera*, Paula Menéndez-Benito, p.c.)

- (92) Puedes leer cualquiera de los libros.
can read cualquiera-sg of the books

- (93) #Puedes leer cualesquiera de los libros.
can read cualesquiera-pl of the books

5.2 Subtriggering

The status of plural *any*-DPs in existential modal sentences improves also in the presence of post-nominal modifiers, as illustrated with a relative clause in (94).

- (94) Gal is allowed to read any books that she finds.

This mitigating effect of the relative clause is unexpected given the *Any* Condition and the structures we have been assuming so far: the presence of a post-nominal modifier does not obviously affect the output of exhaustification in a way that would obviate the problem described in Sect. 2.2. There is, however, another puzzling property of post-nominal modification in this domain: it can rescue occurrences of *any*-DPs more generally (LeGrand 1975). This is exemplified in (95), where the *any*-DP occurs unembedded, in an apparently episodic sentence. Such occurrences of *any*-DPs are called ‘subtriggered *any*-DPs’.

- (95) Gal read any book that she found.

Now, although the facts concerning subtriggered *any*-DPs may be *prima facie* unexpected, the sentences hosting them seem to intuitively satisfy the *Any* Condition, as hinted at in (96).

- (96) Gal read any book that she found.
⇒ Gal read any long book that she found.

What mechanics underlies the pattern in (96) is controversial, however (see, e.g., von Stechow 1996, Aloni 2007a, Menéndez-Benito 2010, Chierchia 2013, Crnič 2019 for discussion). Importantly, though, to the extent that the standard paraphrase of (95) with a universal quantifier – that every book that Gal (would have) found is such that she (would have) read it – is adequate and can be derived, it does entail that every plurality of books that Gal (would have) found is such that she (would have) read them. Accordingly, the pursuit of the acceptability of (94) as an instance of subtriggering looks promising.

5.3 Numeral *Any*-DPs Again

Numeral *any*-DPs were used in Sect. 3.1 to instantiate a prediction of the approach that adopts the *Any* Condition. The distribution of numeral *any*-DPs is more complex than our discussion may have suggested, however. In particular, Dayal (2004, Sect. 3) observed that, unlike other occurrences of *any*-DPs, they are acceptable in universal modal sentences, as in (97).²⁰

²⁰Dayal 2004 makes another pertinent observation, which is not at odds with the *Any* Condition, however: numeral *any*-DPs are marked in downward-entailing environments, (i-a). This seems to be part of a more general

- (97) a. #Gal is required to read any book(s).
 b. Gal is required to read any three books.

This state of affairs is challenging for, and thus requires workarounds on, any approach that rules out occurrences of *any*-DPs in universal modal sentences.²¹ This holds also for an approach that adopts the *Any* Condition: we saw in Sect. 2.1 that *any*-DPs do not satisfy the *Any* Condition in universal modal sentences, at least not on parses that parallel those of existential modal sentences with *any*-DPs, provided in (98). The main culprit for this is the assertion of the meaning of the sister of the exhaustification operator, that is, the first conjunct in (98): substituting the domain of the indefinite with a subdomain results in a stronger conjunct, blocking Strawson downward-entailingness (the exclusion inferences are left out for brevity.)

$$(98) \quad [[[\text{exh}_{R'} [\text{exh}_R [\Box [\text{any}_D \text{ three books}]_x \text{ Gal read } x]]]] = 1 \text{ iff} \\
\Box(\text{Gal read three books in } D) \wedge \forall x \in D \cap [[\text{three books}]]: \Diamond(\text{Gal read } x)$$

In the following, we outline one possible way of meeting this challenge. We rely on an independent mechanism that can ‘neutralize’ the first conjunct in (98) with respect to Strawson downward-entailingness. We begin by taking a short detour through how discourse is structured by questions, which underpins and constrains this mechanism (cf. Beaver & Clark 2009).

Questions and answers. Each sentence in discourse can be analyzed as an answer to a (potentially implicit) question (Roberts 2012). A sentence signals what question it is answering by means of its focus structure. Now, since questions presuppose that they have a maximally informative true answer (Dayal 1996), a sentence can indirectly give rise to such a presupposition (via the potentially implicit question it invokes). We take this to be the source of the presuppositions (optionally) accompanying focus (cf. Beaver & Clark 2009; see, e.g., Geurts & van der Sandt 2004, Abusch 2010 for some alternative treatments of focus presuppositions).

For illustration, the sentence in (99), in which *Gal* is focused, can signal that it is providing an answer to question *Who read a book?* We assume that this happens through a mediation of

pattern: negative quantifiers are also marked when they contain a numeral, (i-b). An explanation of these facts is beyond the scope of this paper. See, e.g., Chierchia 2013, Ch. 5, for a direction involving intervention.

- (i) a. #Gal didn’t read any two books.
 b. #No two students read a book.

²¹Chierchia 2013, Ch. 6, provides a detailed treatment of this phenomenon. In his system, *any*-DPs are admitted in existential, and ruled out in universal modal sentences, due to an interplay of two assumptions: obligatory wide-scope of *any*-DPs in modal contexts (Wide-Scope Constraint) and the alternatives in the domain of *exh* obligatorily differing in a specific way in the conversational backgrounds of the modals occurring in them (Modal Containment). Numeral *any*-DPs are, then, admitted in universal modal sentences due to the Wide-Scope Constraint being a violable default: it is ranked below a constraint that prohibits sentences with polarity items to have equivalent alternatives in which the items are replaced with designated alternatives (Scale Economy). In the case at hand, these would be the alternatives based on *any*-DPs with other numerals: all of them yield contradictory meanings if scoping above the modal, which allows them to exceptionally scope below the modal. See also Dayal 2013.

a focus-sensitive operator in grammar, which interpolates the question from the focus structure of the sentence. If the question was not used in the preceding discourse, the conversational participants have to accommodate it (that is, they have to adjust their beliefs so that they entail that the question is currently under discussion). Importantly for our purposes, this means taking on board the presupposition of the question, namely, that someone read a book, as in (100).

(99) Gal_F read a book.

(100) **Accommodated question:**

Who read a book? ($\Rightarrow \exists x: x \text{ is a person} \wedge x \text{ read a book}$)

Now, some sentences can invoke different types of questions. Particularly relevant for us are the sentences that invoke modalized questions like (101). Spector (2008) observes that these questions are ambiguous, as hinted at by the two answers in (101). On one reading, the answers to them involve pluralities: they are of the form *Gal is required to read x*, where *x* picks out a plurality of books. On the other reading, the so-called higher-order reading, the answers to them involve upward-entailing (UE) quantifiers: they are of the form *Gal is required to read QP*, where *QP* picks out an UE quantifier over books. One such quantifier is paraphrased in (101-b): an existential quantifier whose domain consists of the sum of ‘War & Peace’ and ‘Anna Karenina’ and the sum of ‘The Brothers Karamazov’ and ‘Demons’.

(101) What books is Gal required to read?

a. ‘War & Peace’ and ‘Anna Karenina’.

b. ‘War & Peace’ and ‘Anna Karenina’ or ‘The Brothers Karamazov’ and ‘Demons’.

The two readings differ in their presuppositions, as provided in (102). On the simple reading, the presupposition involves pluralities; on the higher-order reading, the presupposition involves quantifiers over which the *wh* indefinite ranges (see Spector 2008 for details).

(102) a. $\exists x: x \text{ are books} \wedge \Box(\text{Gal read } x)$

b. $\exists Q: Q \text{ is an UE quantifier over books} \wedge \Box(Q(\lambda x. \text{Gal read } x))$

Finally, the distribution of higher-order readings of questions is restricted in specific ways. Specifically, they require the presence of certain embedding quantificational operators in the sentence, such as universal modals, and of plural marking on the *wh*-phrases (esp., Elliott et al. 2018, Fox 2018 on the latter, but see Xiang 2020; see also Gentile & Schwarz 2018, Hirsch & Schwarz 2020 for some related discussion). With this in hand, we can now return to sentences with numeral *any*-DPs.

Derivation. Consider the sentence in (103), which may answer *What three books is Gal required to read?* on its higher-order construal, provided in (104). This question is interpolated

from the alternatives induced by the *any*-DP (in particular, its subdomain alternatives).

(103) Gal is required to read any_D three books.

(104) **Accommodated question:**

For what UE quantifier Q over $D \cap \text{[[three books]]}$: $\Box(Q(\lambda x. \text{Gal read } x))$?

Sentence (103) provides a full answer to the question invoked without requiring the *any*-DP to take semantic scope above the modal \Box . It triggers the presupposition in (105).

(105) $\exists Q$: Q is an UE quantifier over triples of books in $D \wedge \Box(Q(\lambda x. \text{Gal read } x))$

The assertive meaning of sentence (103) is in (106). It parallels the meaning we derived in Sect. 2.1: it comprises of the meaning of the sister of *exh* and the free choice inferences.

(106) $\Box(\text{Gal read three books in } D) \wedge \forall x \in D \cap \text{[[three books]]}: \Diamond(\text{Gal read } x)$

Now, the *Any* Condition is satisfied on the construal of sentence (103) that invokes the presupposition in (105). Namely, the sentence entails any sentence in which the domain of *any*, D , is replaced with a subdomain, D^* , because the presupposition of the resulting sentence entails that Gal is required to read three books in D^* , and the free choice inferences of the resulting sentence are entailed by those of the initial one, provided in (106). The *any*-DP in (103) is thus predicted to be acceptable, at least to the extent that the question in (104) can be accommodated in the context.

Avoiding overgeneration. But is undergeneration avoided at the cost of overgeneration? We review three overgeneration suspects and why they are ruled out on the assumptions above.

First: Dropping the numeral from the *any*-DP in (107) yields the same issue for Strawson downward-entailingness that we observed for plural *any*-DPs in Sect. 2.2.

(107) $\#$ Gal is required to read any_D books.

Specifically, the free choice inferences that would obtain in (107) pertain to the minimal elements in the domain of *any*. For example, if we assume that these are atomic individuals, the assertive meaning of the structure in (107) is the one provided in (108). Now, if we replace D with a domain that contains only proper pluralities, the free choice inferences that we obtain pertain to those pluralities, and are thus not entailed by (108). Thus, sentence (107) cannot satisfy the *Any* Condition, and is correctly predicted to be unacceptable.

(108) $\Box(\text{Gal read a book or books in } D) \wedge \forall x \in D \cap \text{[[book]]}: \Diamond(\text{Gal read } x)$

Second: Sentences with singular *any*-DPs like (109) cannot invoke higher-order questions

(cf. Elliott et al. 2018, Fox 2018, but see Xiang 2020). And the questions that they might invoke, they do not provide a full answer to: *What book is Gal required to read?* is not fully answered by a sentence conveying that a book is such that Gal is required to read it. The mechanism we employed to deal with numeral *any*-DPs is thus inconsequential for singular *any*-DPs.²²

(109) #Gal is required to read any book.

Third: Relatedly, an episodic sentence with an *any*-DP (with or without a numeral) runs into the issue that the question it would need to invoke in order to have a chance at satisfying the *Any* Condition cannot be fully answered by it. (In fact, even if this condition of answerhood could be obviated, we would face the problem that replacing the domain of *any* with a subdomain would yield a Strawson-equivalent meaning, which should be ruled out on the full version of the *Any* Condition; see Lahiri 1998, Gajewski & Hsieh 2014, Crnič 2019, Part II, among others.)

(110) #Gal read any book.

While the proposal outlined does not obviously lead to overgeneration, further study of it is necessary, as is its comparison with alternative derivations (esp., Dayal 2004, 2013, Chierchia 2013). Furthermore, more foundational questions about the connection between questions and the *Any* Condition are raised by our discussion. In particular, we relied in our derivation on questions that require one to identify the domain of *any* that leads to a more informative true proposition than its alternatives do, in (104) – this resembles the requirement imposed by the *Any* Condition (namely, the domain of *any* is supposed to lead to a more informative meaning of a constituent than its subdomain alternatives do). Whether this suggestive resemblance reflects a deeper connection between questions and certain polarity items remains to be explored.

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²²If the plural marking requirement for higher-order readings of questions would be obviated, singular *any*-DPs would, all else equal, be admitted in universal modal sentences (if the requisite questions could be accommodated).

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A Singular *any*-DPs in existential modal sentences, in detail

This section elaborates on the presentation in Sect. 2.1 (see Crnič 2017, 2019 for further details). In the following, we take the resource domain D of *any* to consist of three books, $\{a,$

b, c}, and we represent alternatives with their translations into propositional logic formulas. Moreover, existential quantification is represented with disjunction, while universal quantification (and distributive predication over plurals) is represented with conjunction in the standard manner.

The sentence in (111-a) has the LF in (111-b).

- (111) a. Gal is allowed to read any book.
 b. $[\text{exh}_R [\diamond [\text{any}_D \text{book}_x [\text{Gal read } x]]]]$

Formal alternatives. The alternatives to the sister of *exh* in (111-b) are provided in (112) (following Krifka 1995 and, esp., Chierchia 2013): they are derived by replacing the domain of *any* with its subdomains, and *any* with *every*. In the following, we rely on the characterization in the bottom row of (112), where ‘ $\diamond(a \vee b)$ ’ stands for the meaning corresponding to Gal being allowed to read a book in $\{a, b\}$, etc. (The sentence has other alternatives, say, those in which *Gal* is replaced with *John*, but these are irrelevant for the purpose at hand.)

- (112) $\text{ALT}([\diamond [\text{any}_D \text{book}_x [\text{Gal read } x]]]) =$
 $\{[\diamond [\text{any}_{D'} \text{book}_x [\text{G. read } x]]], [\diamond [\text{every}_{D'} \text{book}_x [\text{G. read } x]]] \mid [D'] \subseteq \{a, b, c\}\} \approx$
 $\{\diamond a, \diamond b, \diamond c, \diamond(a \vee b), \diamond(a \vee c), \diamond(b \vee c), \diamond(a \vee b \vee c), \diamond(a \wedge b), \diamond(b \wedge c), \diamond(a \wedge c), \diamond(a \wedge b \wedge c)\}$

Excludable alternatives. Recall the definition of excludable alternatives:

- (113) $\text{Excl}(S) = \bigcap \{M \mid M \text{ is a maximal subset of } \text{ALT}(S)$
 such that $\{\neg[S'] \mid S' \in M\} \cup \{[S]\}$ is consistent}

What are the excludable alternatives in (112)? One maximal set of alternatives in (113) that can be jointly negated with the sister of *exh* being true is provided in (114). This is witnessed by (115) being consistent, and by the fact that if the set of the alternatives that get negated is expanded by any of the subtracted alternatives, we obtain a contradiction.

- (114) $\{\diamond b, \diamond c, \diamond(b \vee c), \diamond(a \wedge b), \diamond(b \wedge c), \diamond(a \wedge c), \diamond(a \wedge b \wedge c)\}$
 Subtracted alternatives: $\diamond a, \diamond(a \vee b), \diamond(a \vee c), \diamond(a \vee b \vee c)$

- (115) $\diamond(a \vee b \vee c) \wedge \forall p \in (114): \neg p$ (consistent meaning)
 $= \diamond a \wedge \neg \diamond b \wedge \neg \diamond c$

All other maximal sets of alternatives in (112) that can be jointly negated with the sister of *exh* being true have a similar form and are provided in (116)-(117).

- (116) $\{\diamond a, \diamond c, \diamond(a \vee c), \diamond(a \wedge b), \diamond(a \wedge c), \diamond(b \wedge c), \diamond(a \wedge b \wedge c)\}$

Subtracted alternatives: $\diamond b, \diamond(a \vee b), \diamond(b \vee c), \diamond(a \vee b \vee c)$

$$(117) \quad \{\diamond a, \diamond b, \diamond(a \vee b), \diamond(a \wedge b), \diamond(a \wedge c), \diamond(b \wedge c), \diamond(a \wedge b \wedge c)\}$$

Subtracted alternatives: $\diamond c, \diamond(a \vee c), \diamond(b \vee c), \diamond(a \vee b \vee c)$

The intersection of these maximal sets – that is, the set of excludable alternatives in (112) – is provided in (118): it consists of all the alternatives that have a (non-trivial) conjunctive meaning (if a proposition is conjoined with itself, we obtain a trivial conjunction).

$$(118) \quad \text{Excl}([\diamond [\text{any}_D \text{ book}_x [\text{Gal read } x]]]) = \{\diamond(a \wedge b), \diamond(a \wedge c), \diamond(b \wedge c), \diamond(a \wedge b \wedge c)\}$$

More generally, the sets of the alternatives that can be jointly negated with the sister of *exh* being true have the form in (119), while their intersection has the form in (120).

$$(119) \quad \{[\diamond [\text{any}_{D'} \text{ book}_x [\text{Gal read } x]]], [\diamond [\text{every}_{D''} \text{ book}_x [\text{Gal read } x]]] \mid \\ \llbracket D' \rrbracket, \llbracket D'' \rrbracket \subseteq \llbracket D \rrbracket \wedge z \notin \llbracket D' \rrbracket \wedge \llbracket D'' \rrbracket \cap \llbracket \text{book} \rrbracket \neq \{z\}\}, \\ \text{for some } z \in \llbracket D \rrbracket \cap \llbracket \text{book} \rrbracket$$

$$(120) \quad \text{Excl}([\diamond [\text{any}_D \text{ book}_x [\text{Gal read } x]]]) = \\ \{\diamond [\text{every}_{D'} \text{ book}_x [\text{Gal read } x]] \mid \llbracket D' \rrbracket \subseteq \llbracket D \rrbracket \wedge \text{card}(\llbracket D' \rrbracket \cap \llbracket \text{book} \rrbracket) \geq 2\}$$

Includable alternatives. Recall Bar-Lev & Fox's (2020) definition of includable alternatives:

$$(121) \quad \text{Incl}(S) = \bigcap \{M \mid M \text{ is a maximal subset of } \text{ALT}(S) \\ \text{such that } \{\llbracket S' \rrbracket \mid S' \in M\} \cup \{\neg \llbracket S' \rrbracket \mid S' \in \text{Excl}(S)\} \text{ is consistent}\}$$

What are the includable alternatives in (112)? They are all the disjunction alternatives (incl. the trivial disjunction alternatives), as given in (122). This is witnessed by the consistency of (123), which is a conjunction of all the alternatives in (122) with the negations of all the excludable alternatives; moreover, asserting a non-trivial conjunction alternative (which is the only type of alternatives that is not in (122)) would obviously contradict the negation of one of the excludable conjunctive alternatives, which shows that the set in (122) is indeed maximal.

$$(122) \quad \text{Incl}([\diamond [\text{any}_D \text{ book}_x [\text{Gal read } x]]]) = \\ \{\diamond a, \diamond b, \diamond c, \diamond(a \vee b), \diamond(a \vee c), \diamond(b \vee c), \diamond(a \vee b \vee c)\}$$

$$(123) \quad \diamond a \wedge \diamond b \wedge \diamond c \wedge \diamond(a \vee b) \wedge \diamond(a \vee c) \wedge \diamond(b \vee c) \wedge \diamond(a \vee b \vee c) \wedge \quad (\text{inclusion}) \\ \neg \diamond(a \wedge b) \wedge \neg \diamond(a \wedge c) \wedge \neg \diamond(b \wedge c) \wedge \neg \diamond(a \wedge b \wedge c) \quad (\text{exclusion})$$

Exhaustification and obligatory pruning. Recall Bar-Lev & Fox's (2020) definition of *exh*:

- (124) $[[\text{exh}_R S]](w) = 1$ iff
- $\forall S' \in \text{Excl}(S) \cap R: \neg[[S']](w)$, and
 - $\forall S' \in \text{Incl}(S): [[S']](w)$.

What is the meaning of the structure (111-b)? This depends on what alternatives are relevant, that is, on the resolution of R . If all the formal alternatives to the sister of *exh* are relevant, $\text{ALT}([\diamond [\text{any}_D \text{book}_x [\text{Gal read } x]]]) \subseteq [[R]]$, the meaning we obtain is the one provided in (125) (the same result is obtained for any R that denotes a superset of the set consisting of all the excludable alternatives, $\text{Excl}([\diamond [\text{any}_D \text{book}_x [\text{Gal read } x]]]) \subseteq [[R]]$). On the other hand, if none of the alternatives are relevant, the meaning that we obtain is the one provided in (126). There are further conceivable resolutions of R (but see Katzir 2014, Crnič et al. 2015, Bar-Lev 2018 on some constraints on the pruning of alternatives).

(125) If $[[R]] \supseteq \text{Excl}([\diamond [\text{any}_D \text{book}_x [\text{Gal read } x]]])$,

$$[[[\text{exh}_R [\diamond [\text{any}_{D'} \text{book}_x [\text{Gal read } x]]]]]] =$$

$$\diamond a \wedge \diamond b \wedge \diamond c \wedge \neg \diamond(a \wedge b) \wedge \neg \diamond(a \wedge c) \wedge \diamond(b \wedge c)$$

(126) If $[[R]] \cap \text{Excl}([\diamond [\text{any}_D \text{book}_x [\text{Gal read } x]]]) = \emptyset$,

$$[[[\text{exh}_R [\diamond [\text{any}_{D'} \text{book}_x [\text{Gal read } x]]]]]] = \diamond a \wedge \diamond b \wedge \diamond c$$

How do we choose between the potential resolutions of R ? It turns out that the choice is not completely free – it is partly dictated by the *Any* Condition. In order to obtain a Strawson downward-entailing environment with respect to the domain of *any*, all the disjunctive (incl. the single disjunct alternatives) must count as irrelevant (unless they are equivalent to the sister of *exh*). In other words, from the set in (112), only the excludable alternatives in (118) may be in $[[R]]$ (together with the alternatives that are equivalent to the sister of *exh*) (cf. Buccola & Haida 2017 on obligatorily irrelevant alternatives):

(127) $[[R]] \cap \text{ALT}([\diamond [\text{any}_D \text{book}_x [\text{Gal read } x]]]) \subseteq \text{Excl}([\diamond [\text{any}_D \text{book}_x [\text{Gal read } x]]])$

Let us first show that we indeed get Strawson downward-entailingness with respect to the domain of *any* if (127) obtains. The interpretation of the structure in (111-b) is provided in (128) (the parentheses indicate that the exclusion of the alternatives depends on R).

(128) $[[[\text{exh}_R [\diamond [\text{any}_D \text{book}_x [\text{Gal read } x]]]]]] =$

$$\diamond a \wedge \diamond b \wedge \diamond c \wedge (\neg \diamond(a \wedge b) \wedge \neg \diamond(a \wedge c) \wedge \neg \diamond(b \wedge c))$$

If we replace D with a proper (non-empty) subdomain of D (and keep R fixed), we obtain a Strawson weaker meaning. Two such replacements are provided in (129), where each of the meanings is entailed by the meaning in (128): there are fewer asserted includable alternatives,

while the negated excludable alternatives are constant across all the substitutions of D.

- (129) a. If $[[D']] = \{a,b\}$, then $[[[\text{exh}_R [\diamond [\text{any}_{D'} \text{book}_x [\text{Gal read } x]]]]]] = \diamond a \wedge \diamond b (\wedge \neg \diamond(a \wedge b) \wedge \neg \diamond(a \wedge c) \wedge \neg \diamond(b \wedge c))$
- b. If $[[D']] = \{a\}$, then $[[[\text{exh}_R [\diamond [\text{any}_{D'} \text{book}_x [\text{Gal read } x]]]]]] = \diamond a (\wedge \neg \diamond(a \wedge b) \wedge \neg \diamond(a \wedge c) \wedge \neg \diamond(b \wedge c))$

What happens, however, if (127) does not obtain? In this case, at least one replacement of domain D will not result in a Strawson weaker meaning. For illustration, assume that $\diamond(a \vee b) \in [[R]]$ (falsifying (127), since $\diamond(a \vee b)$ is not an excludable alternative). Now, the meaning of the sentence with domain D is provided in (130), where the content of φ depends on what alternatives are in $[[R]]$ besides $\diamond(a \vee b)$. The meaning of the corresponding structure in which D is replaced by a stronger domain D' , namely $[[D']] = \{c\}$, is provided in (131), where ψ depends on what alternatives are in $[[R]]$ besides $\diamond(a \vee b)$.

(130) If $\diamond(a \vee b) \in [[R]]$, then $[[[\text{exh}_R [\diamond [\text{any}_D \text{book}_x [\text{Gal read } x]]]]]] = \diamond a \wedge \diamond b \wedge \diamond c \wedge \varphi$

(131) If $[[D']] = \{c\}$ and $\diamond(a \vee b) \in R$, then

$$[[[\text{exh}_R [\diamond [\text{any}_{D'} \text{book}_x [\text{Gal read } x]]]]]] = \diamond c \wedge \neg \diamond(a \vee b) \wedge \psi$$

The meaning in (130) clearly does not entail the meaning in (131), no matter what the content of φ and ψ is – in fact, (130) entails the negation of (131). The same conclusion is reached for every other R that fails to satisfy the condition in (127). This is summarized in (132), where the alternatives equivalent to the sister of *exh* are omitted for brevity.

(132) **Generalization about the obligatory pruning of alternatives:**

An otherwise well-formed LF $[\text{exh}_R [\diamond [\text{any}_D \text{NP}_x \text{XP}]]]$ is Strawson downward-entailing with respect to D iff

$$[[R]] \cap \text{ALT}([\diamond [\text{any}_D \text{NP}_x \text{XP}]]) \subseteq \text{Excl}([\diamond [\text{any}_D \text{NP}_x \text{XP}]]).$$

Summary. Exhaustification over the alternatives introduced by singular *any*-DPs in existential modal sentences, as in (111-b), creates a Strawson downward-entailing environment with respect to the domain of *any* – which results in the *any*-DP satisfying the *Any* Condition – if none of the includable alternatives are in the resource domain of *exh*, R (besides the sister of *exh* and any alternatives equivalent with it).

B Plural *any*-DPs in existential modal sentences, in detail

This section elaborates on the presentation in Section 2.2. We assume in the following that the domain D corresponds to the closure of the set of three books under sum formation, $\ast\{a,b,c\}$,

that is, $\{a, b, c, a+b, a+c, b+c, a+b+c\}$. We adopt the notation of the preceding section, with existential quantification (distributive predication) over $\{a+b\}$ being represented as $a \wedge b$, etc.

The sentence in (133-a) may have the LF in (133-b).

- (133) a. #Gal is allowed to read any books.
 b. $[\text{exh}_R [\diamond [\text{any}_D \text{books}_x [\text{Gal read } x]]]]$

Formal alternatives. The alternatives to the sister of *exh* in (133-b) are provided in (134): they are the alternatives derived by replacing the domain of *any* with its subdomains, and *any* with *all*. (An element is in the disjunctive closure of a set P iff it is in P or if it is a disjunction of two elements each of which is in the disjunctive closure of P.)

- (134) $\text{ALT}([\diamond [\text{any}_D \text{books}_x [\text{Gal read } x]]]) =$
 $\{[\diamond [\text{any}_{D'} \text{books}_x [\text{G. read } x]]], [\diamond [\text{all}_{D'} \text{books}_x [\text{G. read } x]]] \mid [[D']] \subseteq^* \{a, b, c\}\} \approx$
 $\{\diamond\varphi \mid \varphi \in \text{DISJCLOSURE}(\{a, b, c, a \wedge b, a \wedge c, b \wedge c, a \wedge b \wedge c\})\} =$
 $\{\diamond a, \diamond b, \diamond c, \diamond(a \vee b), \dots, \diamond(a \vee b \vee c \vee \dots \vee (a \wedge b \wedge c)), \diamond(a \wedge b), \diamond(b \wedge c), \diamond(a \wedge c), \diamond(a \wedge b \wedge c)\}$

Excludable alternatives. What are the excludable alternatives in (134)? One maximal set of alternatives in (134) that can be jointly negated with the sister of *exh* being true is provided in (135). This is witnessed by (136) being consistent, and by the fact that if the set of alternatives that get negated is expanded by any of the subtracted alternatives, we obtain a contradiction.

- (135) $\{\diamond\varphi \mid \varphi \in \text{DISJCLOS}(\{b, c, a \wedge b, a \wedge c, b \wedge c, a \wedge b \wedge c\})\} =$
 $\{\diamond b, \diamond c, \diamond(b \vee c), \diamond((a \wedge b) \vee c), \diamond((a \wedge c) \vee b), \dots, \diamond(a \wedge b), \diamond(b \wedge c), \diamond(a \wedge c), \diamond(a \wedge b \wedge c)\}$
 Subtracted alternatives: $\{\diamond(a \vee \psi) \mid \psi \in \text{DISJCLOS}(\{a, b, c, a \wedge b, a \wedge c, b \wedge c, a \wedge b \wedge c\})\}$

- (136) $\diamond(a \vee b \vee c \vee \dots \vee (a \wedge b \wedge c)) \wedge \forall p \in (135): \neg p$ (consistent meaning)
 $= \diamond a \wedge \neg \diamond b \wedge \neg \diamond c$

All other maximal sets of alternatives in (134) that can be jointly negated with the sister of *exh* being true have a similar form and are provided in (137)-(138).

- (137) $\{\diamond\varphi \mid \varphi \in \text{DISJCLOS}(\{a, c, a \wedge b, a \wedge c, b \wedge c, a \wedge b \wedge c\})\}$

- (138) $\{\diamond\varphi \mid \varphi \in \text{DISJCLOS}(\{a, b, a \wedge b, a \wedge c, b \wedge c, a \wedge b \wedge c\})\}$

The intersection of these maximal sets – that is, the set of excludable alternatives in (134) – is provided in (139): it consists of all the conjunctive alternatives and all the disjunctive alternatives in which all the disjuncts are (non-trivial) conjunctions.

- (139) $\text{Excl}([\diamond [\text{any}_D \text{books}_x [\text{G. read } x]]]) = \{\diamond\varphi \mid \varphi \in \text{DISJCLOSURE}(\{a \wedge b, b \wedge c, a \wedge b \wedge c\})\}$

Excludable alternatives more generally. More generally, the maximal sets of alternatives that can be jointly negated with the sentence being true are those that have the form in (140): all the alternatives in (140) can be negated with the sentence being true (namely, it can hold that Mary is allowed to read book or books x).

$$(140) \quad \{[\diamond [\text{any}_{D'} \text{books}_x [\text{G. read } x]]], [\diamond [\text{all}_{D'} \text{books}_x [\text{G. read } x]]] \mid \emptyset \neq [D'] \subseteq [D] \cap [\text{books}] \setminus \{x\}\} \text{ for some plurality } x \text{ in, and with no proper parts in, } [D] \cap [\text{books}]$$

The set of excludable alternatives is, then, the set of alternatives based on universal quantifiers (and their equivalents based on *any*) whose meaning asymmetrically entails one of the alternatives that have a singleton domain containing a plurality of books from D none of whose proper parts are also in the domain D of *any*.

$$(141) \quad \text{Excl}([\diamond [\text{any}_D \text{books}_x [\text{G. read } x]]]) = \{[\diamond [\text{any}_{D'} \text{books}_x [\text{G. read } x]]], [\diamond [\text{all}_{D'} \text{books}_x [\text{G. read } x]]] \mid \emptyset \neq [D'] \subseteq [D] \cap [\text{books}] \setminus \{x \mid \neg \exists y \in [D] \cap [\text{books}] [y \sqsubset x]\}\}$$

Includable alternatives. What are the includable alternatives in (134)? They are all the disjunction alternatives in which at least one disjunct is not a (non-trivial) conjunction, as given in (142). This is witnessed by the consistency of (143), which is a conjunction of all the alternatives in (142) with the negations of all the excludable alternatives (it is verified in any situation in which $\diamond a \wedge \diamond b \wedge \diamond c$ is true and all excludable alternatives are false); moreover, asserting a conjunction or a disjunction of conjunctions (which is the only type of formal alternatives that is not in (142)) would obviously contradict a negation of one of the excludable conjunctive alternatives, which shows that the set in (142) is indeed maximal.

$$(142) \quad \text{Incl}([\diamond [\text{any}_D \text{books}_x [\text{Gal read } x]]]) = \{[\diamond (\varphi \vee \psi) \mid \varphi \in \{a, b, c\} \wedge \psi \in \text{DISJCLOSURE}(\{a, b, c, a \wedge b, a \wedge c, b \wedge c, a \wedge b \wedge c\})]\}$$

$$(143) \quad \forall p \in \{[\diamond (\varphi \vee \psi) \mid \varphi \in \{a, b, c\} \wedge \psi \in \text{DISJCLOSURE}(\{a, b, c, a \wedge b, a \wedge c, b \wedge c, a \wedge b \wedge c\})]\}: p \wedge \forall q \in \{[\diamond \varphi \mid \varphi \in \text{DISJCLOSURE}(\{a \wedge b, b \wedge c, a \wedge b \wedge c\})]\}: \neg q$$

(consistent)

Includable alternatives more generally. On the assumption of the more general formulation of excludable alternatives in (141), the includable alternatives are those based on *any* (and their equivalents based on *every*) such that the domain of *any* in them contains at least one plurality none of whose proper parts are in the domain D .

$$(144) \quad \text{Incl}([\diamond [\text{any}_D \text{books}_x [\text{Gal read } x]]]) = \{[\diamond [\text{any}_{D'} \text{books}_x [\text{Gal read } x]]] \mid \emptyset \neq [D'] \subseteq [D] \cap [\text{books}] \wedge \{x \mid \neg \exists y \in [D] \cap [\text{books}] [y \sqsubset x]\} \cap [D'] \cap [\text{books}] \neq \emptyset\}$$

Exhaustification. What is the meaning of the structure in (133-b)? This depends on what alternatives are relevant, that is, on the resolution of R. If we assume that all the excludable alternatives are relevant, we obtain the meaning in (145), which is identical to (144).

$$(145) \quad \text{If } D = * \{a, b, c\} \text{ and } \llbracket R \rrbracket \supseteq \text{Excl}([\diamond [\text{any}_D \text{ books}_x [\text{Gal read } x]]]),$$

$$\llbracket [\text{exh}_R [\diamond [\text{any}_D \text{ books}_x [\text{Gal read } x]]]] \rrbracket = \diamond a \wedge \diamond b \wedge \diamond c \wedge$$

$$\forall q \in \{\diamond \varphi \mid \varphi \in \text{DISJCLOSURE}(\{a \wedge b, b \wedge c, a \wedge b \wedge c\})\}: \neg q$$

Given this resolution of R, the structure in (133-b) is not Strawson downward-entailing with respect to the domain of *any*. Consider the counterpart of (133-b) with a stronger domain D^* that consists of all proper pluralities, that is, $D^* = \{a+b, a+c, b+c, a+b+c\}$. The sets of excludable and includable alternatives in $\text{ALT}([\diamond [\text{any}_{D^*} \text{ books}_x [\text{Gal read } x]]])$ are provided in (146)-(147).

$$(146) \quad \text{Excl}([\diamond [\text{any}_{D^*} \text{ books}_x [\text{Gal read } x]]]) = \{\diamond(a \wedge b \wedge c)\}$$

$$(147) \quad \text{Incl}([\diamond [\text{any}_{D^*} \text{ books}_x [\text{Gal read } x]]]) = \{\diamond(a \wedge b), \diamond(b \wedge c), \diamond(a \wedge c)\}$$

Consequently, this structure has the meaning in (148).

$$(148) \quad \llbracket [\text{exh}_R [\diamond [\text{any}_D \text{ books}_x [\text{Gal read } x]]]] \rrbracket = \diamond(a \wedge b) \wedge \diamond(b \wedge c) \wedge \diamond(a \wedge c) \wedge \neg \diamond(a \wedge b \wedge c)$$

This meaning is obviously not entailed by the meaning in (145), which demonstrates that the structure (133-b) on the proposed resolution of R is not Strawson downward-entailing with respect to the domain D. Now, adding to R alternatives that are not excludable cannot improve the situation (see the discussion in Appendix A) nor can pruning of excludable alternatives: the includable alternatives that are asserted when the domain of *any* is D^* are not entailed by the includable alternatives that are asserted when the domain of *any* is D.

Exhaustification more generally. Combining the more general formulations of the excludable and includable alternatives, (141) and (144), and the same assumptions about R as above, we obtain the meaning in (149). This corresponds to every plurality of books that does not have proper parts in D being such that Gal is allowed to read it, and every plurality of books that does have proper parts in D being such that Gal is not allowed to read it.

$$(149) \quad \llbracket [\text{exh}_R [\diamond [\text{any}_D \text{ books}_x [\text{Gal read } x]]]] \rrbracket =$$

$$\forall D': \emptyset \neq D' \subseteq \llbracket [D] \rrbracket \cap \llbracket [\text{books}] \rrbracket \wedge \{x \mid \neg \exists y \in \llbracket [D] \rrbracket \cap \llbracket [\text{books}] \rrbracket [y \sqsubset x]\} \cap D' \cap \llbracket [\text{books}] \rrbracket$$

$$\neq \emptyset: \diamond(\text{Gal read a book or books in } D') \wedge$$

$$\forall D': \emptyset \neq D' \subseteq \llbracket [D] \rrbracket \cap \llbracket [\text{books}] \rrbracket \setminus \{x \mid \neg \exists y \in \llbracket [D] \rrbracket \cap \llbracket [\text{books}] \rrbracket [y \sqsubset x]\}: \neg \diamond(\text{Gal read books in } D')$$

$$(150) \quad \llbracket [\text{exh}_R [\diamond [\text{any}_D \text{ books}_x [\text{Gal read } x]]]] \rrbracket =$$

$$\forall x: x \in \llbracket [D] \cap \llbracket \text{books} \rrbracket \wedge \neg \exists y \in D \cap \llbracket \text{books} \rrbracket [y \sqsubset x]: \diamond(\text{Gal read } x) \wedge$$

$$\forall x: x \in \llbracket [D] \cap \llbracket \text{books} \rrbracket \wedge \exists y \in D \cap \llbracket \text{books} \rrbracket [y \sqsubset x]: \neg \diamond(\text{Gal read } x)$$

The meaning in (149)-(150) need not entail one of the sentence in which the domain D is replaced with a subdomain, in particular, if the subdomain has distinct minimal elements than D . This means that the Strawson Downward-Entailingness required by the *Any* Condition does not obtain in (133-b). The generalization about free choice inferences pertaining to the minimal elements, provided in (33) in the main text, can be read off from the representations in (149)-(150).

Strong meanings of plural NPs. The state of the affairs does not change substantively if the plural NPs do not have number-neutral interpretations. For example, assume that $D = \{a+b, a+c, b+c, a+b+c\}$ and that $D^* = \{a+b+c\}$. The meanings of the pertinent structures with domains D and D^* are provided in (151) and (152), respectively (again, the exclusion depends on the choice of R , thus the parentheses). Given that (151) fails to entail (152), the requisite Strawson downward-entailingness does not obtain here either.

$$(151) \quad \llbracket [\text{exh}_R [\diamond [\text{any}_D \text{ books}_x [\text{Gal read } x]]]] \rrbracket = \diamond(a \wedge b) \wedge \diamond(b \wedge c) \wedge \diamond(a \wedge c) (\wedge \neg \diamond(a \wedge b \wedge c))$$

$$(152) \quad \llbracket [\text{exh}_R [\diamond [\text{any}_{D^*} \text{ books}_x [\text{Gal read } x]]]] \rrbracket = \diamond(a \wedge b \wedge c)$$

Summary. Exhaustification over the alternatives introduced by plural *any*-DPs in existential modal sentences does not create a Strawson downward-entailing environment with respect to the domain of *any* – resulting in the *any*-DP not being able to satisfy the *Any* Condition.

C The *even* approach to *any*-DPs

We describe how the results obtained in the main text can be derived in the approach of Crnič 2017, 2019 (who builds on Kadmon & Landman 1993, Lahiri 1998). The approach has as a consequence a condition on the distribution of *any*-DPs that differs from the *Any* Condition when it comes to non-monotone environments (as detailed in Crnič 2019).

The setup. On this approach, *any*-DPs are accompanied by a covert *even* operator that quantifies over the alternatives built on the alternatives to the domain of *any* (see Lahiri 1998 for a derivation in which *even* associates with the determiner). We follow Krifka 1995 and Chierchia 2013 in assuming that the relevant alternatives to a (focused) domain are its subdomains.

$$(153) \quad [\text{even} [\dots \text{any}_{D_F} \dots]]$$

Even primarily triggers the ordering presupposition in (154), where the relevant ordering relation ($<$) is filled in by the context (it could be logical entailment, Strawson entailment, contextual entailment, lower-likelihood, or some other relation that respects additivity).

(154) $[[\text{even } S]]$ is defined only if $\forall S' \in F(S): S \not\Leftarrow S' \rightarrow S < S'$.

The precise predictions of the theory depend on the resolution of $<$ in the context. If this is resolved to (contextual) Strawson entailment relation, we obtain straightforwardly that *any*-DPs are acceptable in Strawson downward-entailing environments that are not also Strawson upward-entailing, that singular *any*-DPs are acceptable in existential modal sentences, and that plural *any*-DPs are unacceptable in existential modal sentences (see Crnić 2019 for a detailed discussion of the first two cases).

Singular *any*-DPs. The sentence in (155-a) has the LF in (155-b).

- (155) a. Gal is allowed to read any book.
 b. $[[\text{even } [\text{exh}_R [\diamond [\text{any}_{D_F} \text{book}_x [\text{Gal read } x]]]]]]$

The presupposition of (155-b) is provided in (156). Since the exhaustified constituent is Strawson downward-entailing with respect to the domain of *any*, as seen in Sect. 2.1 and Appendix A, the presupposition is trivially satisfied, and thus the *any*-DP is predicted to be acceptable.

(156) $\forall D^*: [[D^*]] \subseteq [[D]] \wedge$
 $[[[\text{exh}_R [\diamond [\text{any}_D \text{book}_x [\text{Gal read } x]]]]]] \not\Leftarrow$
 $[[[\text{exh}_R [\diamond [\text{any}_{D^*} \text{book}_x [\text{Gal read } x]]]]]] \rightarrow$
 $[[[\text{exh}_R [\diamond [\text{any}_D \text{book}_x [\text{Gal read } x]]]]]] <$
 $[[[\text{exh}_R [\diamond [\text{any}_{D^*} \text{book}_x [\text{Gal read } x]]]]]]$

Plural *any*-DPs. In the case of plural *any*-DPs, the scalar presupposition triggered by *even* is inconsistent (at least on the assumption that $<$ picks out the contextual Strawson entailment relation). The sentence in (157-a) has the LF in (157-b).

- (157) a. #Gal is allowed to read any books.
 b. $[[\text{even } [\text{exh}_R [\diamond [\text{any}_{D_F} \text{books}_x [\text{Gal read } x]]]]]]$

Let us focus on the following subdomain of D : $D^* = \{a+b+c\}$ (recall our assumption from the preceding appendix section that $D = \{a, b, c\}$). The presupposition of *even* in (157) depends on how we resolve the resource domain of *exh*, R . There are several options available to us, but it suffices to consider just two. First: If $[[R]]$ contains no excludable alternatives, the meanings of the exhaustified sentences with domains D and D^* are the following:

$$(158) \quad \llbracket [\text{exh}_R [\diamond [\text{any}_D \text{ books}_x [\text{Gal read } x]]]] \rrbracket = \diamond a \wedge \diamond b \wedge \diamond c$$

$$(159) \quad \llbracket [\text{exh}_R [\diamond [\text{any}_{D^*} \text{ books}_x [\text{Gal read } x]]]] \rrbracket = \diamond(a \wedge b \wedge c)$$

Since the meaning in (159) is not equivalent to that in (158), *even* presupposes that it is (contextually) Strawson entailed by (158). This is not the case since the logical entailment goes the other way around. On this resolution of R, the *any*-DP is correctly predicted to be unacceptable. Second: If R contains an excludable alternatives, say, just $\diamond(a \wedge b)$, the meanings of the exhaustified sentences with domains D and D^* are the following:

$$(160) \quad \llbracket [\text{exh}_R [\diamond [\text{any}_D \text{ books}_x [\text{Gal read } x]]]] \rrbracket = \diamond a \wedge \diamond b \wedge \diamond c \wedge \neg \diamond(a \wedge b)$$

$$(161) \quad \llbracket [\text{exh}_R [\diamond [\text{any}_{D^*} \text{ books}_x [\text{Gal read } x]]]] \rrbracket = \diamond(a \wedge b \wedge c)$$

Since the meaning in (161) is not equivalent to that in (160), *even* presupposes that it is (contextually) Strawson entailed by (160). This is not the case since (160) actually entails the negation of (161). On this resolution of R, the *any*-DP is also correctly predicted to be unacceptable.

Other ordering relations. If the ordering used by *even* is resolved to a weaker relation than (contextual) Strawson entailment, say, lower-likelihood in the context (cf., e.g., Karttunen & Peters 1979, Lahiri 1998, Crnič 2014b), nothing changes in the analysis of the singular *any*-DP case (see Crnič 2019 for details). In the case of plural *any*-DPs, on the other hand, we obtain contradictory presuppositions on some resolutions of R, and contingent (potentially implausible) presuppositions on other resolutions. We discuss two cases here. First: If no alternatives to the sister of *exh* are in R, we obtain a contradictory presupposition, namely, the meaning of the structure in which D^* replaces D is stronger than the meaning of the structure with D, and thus at most as likely. On this resolution of R, the *any*-DP is correctly predicted to be unacceptable.

$$(162) \quad \text{If } \llbracket R \rrbracket \cap \text{ALT}([\diamond [\text{any}_D \text{ books}_x [\text{Gal read } x]]]) = \emptyset, \text{ then}$$

$$\diamond a \wedge \diamond b \wedge \diamond c <_{\text{pr},c} \diamond(a \wedge b \wedge c), \text{ etc.}$$

Second: If the excludable alternatives to the sister of *exh* are relevant, we obtain a contingent presupposition. Namely, while the meaning of the structure in which D^* replaces D contradicts the meaning of the structure with D, it may nonetheless be more likely than it.

$$(163) \quad \text{If } \llbracket R \rrbracket \subseteq \text{Excl}([\text{exh}_R [\diamond [\text{any}_D \text{ books}_x [\text{Gal read } x]]]]), \text{ then}$$

$$\diamond a \wedge \diamond b \wedge \diamond c \wedge \neg \diamond(a \wedge b) \wedge \neg \diamond(a \wedge c) \wedge \neg \diamond(b \wedge c) <_{\text{pr},c} \diamond(a \wedge b \wedge c), \text{ etc.}$$

Consequently, on this resolution of $<$ and R, the prediction of the approach described above is not that we have an unacceptable occurrence of an *any*-DP, but rather one whose felicity depends on the plausibility of the scalar presupposition in the context (if this is implausible, the sentence is marked). However, another property of *even* affects the acceptability of the

sentence on this resolution: *even* tends to be infelicitous when the alternatives it quantifies over are incompatible with its sister (see Guerzoni 2003, Francis 2018 for a discussion). Since this is necessarily the case if [[R]] contains excludable alternatives, the unacceptability of the plural *any*-DPs may be attributed to this property of *even* (see Crnič 2019 for some related discussion).