Explaining presupposition projection in (coordinations of) polar questions[*]

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Abstract

This article starts off the observation that in certain cases, presuppositions triggered by an element inside a question nucleus may fail to project. In fact, in what looks like coordinated structures involving polar questions, presupposition projection patterns are exactly parallel to what is observed when the corresponding assertions are coordinated. The article further shows that these facts do not fall out straightforwardly from existing theories of polar questions, (apparent) coordinations of questions, and presupposition projection. Finally, it proposes a trivalent extension of inquisitive semantics such that the observed pattern can be understood in terms of existing theories of presupposition projection. The proposal has the following properties: (a) apparent coordinations of questions actually are coordinations of questions and (b) the semantic denotation of polar questions is asymmetric with respect to the "yes" and "no" answers.

1 How presuppositions project from polar questions

1.1 Introduction: presupposition projection and coordination

The issue of presupposition projection in coordinated structures has been the subject of a significant amount of attention within the formal semantics and pragmatics literature. The basic pattern to be explained is known at least since Karttunen (1973) and goes as follows: the utterance (1) presupposes that Syldavia is a monarchy, which means among other things that (1) can be judged to be of degraded felicity in a context where the nature of the Syldavian political system is in doubt. The presupposition is due to the presence of a *trigger*, here the definite DP *the Syldavian monarch*. When the clause in (1) occurs in an embedded context, the presupposition may or may not "project" depending on certain factors. Of interest to us is the fact that the coordinated structures in (2) lack the presupposition that Syldavia is a monarchy.

(1)   The Syldavian monarch is a progressive.

(2) a. Syldavia is a monarchy and the Syldavian monarch is a progressive.
    b. Syldavia is a republic or the Syldavian monarch is a progressive.[1]

The lack of a presupposition for the examples in (2) is described and explained in various ways in the literature, going back in particular to the influential work of Kartunnen (1973; 1974) and Heim (1983). *A minima*, an analysis of this data should derive the felicity conditions of (1) and (2) from more general assumptions about the semantics of declarative sentences, the semantics of the connectives *and* and *or*, and the semantics/pragmatics interface. For instance, Karttunen (1974) makes the following assumption about the semantics/pragmatics interface: a clause containing a trigger can be uttered felicitous if and only if its *local context* supports the trigger's presupposition. He further gives rules to determine the local context of conjuncts and disjuncts, allowing us to derive that the local context of the second conjunct in (2a) will support that Syldavia is a monarchy even when the global context does not, explaining why the presupposition trigger *the Syldavian monarch* can be used felicitously in such a global context. Heim (1983) recasts Karttunen's (1974) analysis in a more general framework: she assumes that sentences denote *Context Change Potentials* (CCPs). Then, one only has to specify how *and* and *or* string CCPs together to be able to derive what the local context of every constituent clause is.[2]

1.2 The core data

What I am interested in here is a very similar pattern of presupposition projection that can be observed in what looks like coordinations of *polar questions*, rather than coordinations of declarative clauses. Consider to begin with the simple polar question in (3a). As is well-known, (3a) presupposes that Syldavia is a monarchy, in the sense that it can be infelicitous in a context where that fact is under doubt. A similar presupposition is observed when an embedded polar question occurs in a declarative sentence, as in (3b).[3]

(3) a. Is the Syldavian monarch a progressive?
    b. Mary wonders whether the Syldavian monarch is a progressive.

The more general pattern is that a polar question, matrix or embedded, presupposes all that the corresponding declarative does — this fact is in fact often used as a test to establish what declaratives presuppose.

English allows for what at least looks like conjunctions and disjunctions of polar questions.[4] We can investigate presupposition projection in such structures in much the same way as we do for declaratives. The result of this investigation is that presuppositions project from the second member in a coordination of questions following non-trivial patterns, and that the patterns in question are strikingly similar to what is observed for declaratives.

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[1] (2b) does presuppose that Syldavia is either a republic or a monarchy. I ask the reader to assume that this is a well-known fact about countries in the part of the world where Syldavia is. The issue will not substantially affect the discussion.

[2] There is no explicit semantics for *and* in Heim 1983, and *or* is not discussed, but it is straightforward to apply Heim's approach to the discussion of *and* and *or* in Karttunen 1974 to reconstruct the full system.

[3] What (3b) presupposes is arguably "Mary believes that Syldavia is a monarchy" rather than "Syldavia is a monarchy", or possibly two readings are possible. Whether the inferences I describe are attributed to the speaker or the attitude holder when they differ does not affect my discussion of them.

[4] Most other connectives than *and* and *or*, such as for instance *but*, do not seem to ever occur inbetween two questions. Complex disjunctions such as "either... or..." cannot embed matrix questions either.

An example of a what we will call a conjunctive question is given in (4). The apparent structure of (4) is $?p \wedge ?q$, where ? is a question-forming operator, responsible for the auxiliary-fronting, p is *Syldavia is a monarchy* and q is *the Syldavian monarch is a progressive*.[5] In spite of the fact that $?q$, the second conjunct of (4), appears to be the question in (3a), the speaker who asks (4) is not understood to be presupposing that Syldavia is a monarchy. In fact, in a context where it is established that Syldavia is a monarchy, it is infelicitous to ask (4), because (4) presupposes that whether Syldavia is a monarchy is still not known.[6]

(4)   Is Syldavia a monarchy, and is the Syldavian monarch a progressive?

Thus, in (4), the presupposition triggered by the definite DP fails to project. In this respect, (4) is just like the conjunction of assertions (2a). As in (2a), the failure to project is due in some way or other to the presence of the first conjunct; if we replace it with something unrelated, as in (5), the presupposition does project. Again as for assertions, the utterance becomes infelicitous if the order is reversed to make $?q \wedge ?p$, as in (6). That (6) is infelicitous in any context can be explained as follows: the presupposition projects, but the second conjunct somehow triggers an ignorance inference about p, and the two inferences are incompatible. This explanation is very similar to how one can explain why (7), $q \wedge p$, is odd: the presupposition projects, but then the second conjunct is necessarily trivial.

(5)   Is Syldavia rich in mineral resources, and is the Syldavian monarch a progressive?

(6)   # Is the Syldavian monarch a progressive, and is Syldavia a monarchy?

(7)   # The Syldavian monarch is a progressive and Syldavia is a monarchy.

More interestingly, we can try to replace the nucleus p of the first conjunct by a proposition p' that is (at least contextually) equivalent to $\neg p$, or to replace the first conjunct by an *or not* alternative question bearing upon the same proposition p. The reason these replacements are natural things to test is that (8b) and (8c) intuitively raise the same issue as (8a), in the sense that the knowledge one needs to answer any of these three questions is the same. They turn out, however, to not be interchangeable when it comes to presupposition projection: both replacement attempts, (9a) and (9b), yield sentences that are just infelicitous.

(8)   a.   Is Syldavia a monarchy?
        b.   Is Syldavia a republic?
        c.   Is Syldavia a monarchy or not?

(9)   a. # Is Syldavia a republic, and is the Syldavian monarch a progressive?
        b. # Is Syldavia a monarchy or not, and is the Syldavian monarch a progressive?

The generalization that emerges is most easily phrased in terms of local contexts:

(10)   **Generalization about conjunctive questions:** in a conjunction of polar questions $?p \wedge ?q$ (but not when *or not* questions are involved), while the local context

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[5]We are going to discuss in time whether (4) should indeed be analysed as $?p \wedge ?q$.

[6]In general, matrix questions pragmatically presuppose that their answer has not yet been established in the discourse. Thus, the first conjunct in (4), uttered on its own, presupposes that Syldavia might or might not be a monarchy as far as the Common Ground allows. How this presupposition arises exactly in the case of (4) is an interesting question, but I will not attempt to address it here.

of p is the global context, in at least some cases the local context of q is the global context enriched with p.

This generalization is entirely parallel to what Karttunen (1974) and Heim (1983) offer for the case of $p \wedge q$. It explains the felicity of (4), the infelicity of (6), the fact that the presupposition projects in (5),[7] and the infelicity of (9a) (the second conjunct's presupposition is necessarily not met).

Let us now turn to the case of disjunctive questions. An example of what looks like a disjunction of polar questions is given in (11); we can schematize it as $?p\vee?q$, without committing to the fact it should be analysed that way. An immediate complication is that disjunctive questions of this sort are known to be systematically ambiguous between at least two readings. Using the terminology of Roelofsen and Farkas (2015), (11) has a *closed* reading where it presupposes (a) that one of John or Mary is here (exhaustiveness) and (b) that John and Mary are not both here (exclusivity). This reading is also known as the *alternative question* reading. The most natural way of bringing it out is to pronounce (11) with a falling intonation on the second disjunct. There is also an *open* reading, that is most easily made salient by having a rising intonation on the second disjunct. The open reading of (11) does not presuppose either (a) or (b), and is amenable to a negative answer ("neither of them is here").

(11)   Is John here or is Mary here?

With this distinction in mind, consider the crucial presuppositional example (12). The question in (12) can be uttered felicitously in a context where the political system of Syldavia is in doubt; in fact, as in the conjunctive case, (12) would be infelicitous otherwise.

(12)   Is Syldavia a republic, or is the Syldavian monarch a progressive?

These facts obtain both with the "open" and "closed" intonation, and both under the open and closed reading. Note that we do not have to rely on the intonation to disambiguate readings: specifying a richer context lets us select one or the other reading and establish the presuppositional facts as well, e.g. the discourse in (13) brings about the closed reading while the discourse in (14) brings about the open reading. In what follows, it will be helpful for the reader to keep these two discourses in mind to verify that all examples are compatible with both.

(13)   My conviction is that you never see a conservative monarch enacting progressive policies. With what you told me about Syldavia's progressive laws, tell me, is Syldavia a republic, or is the Syldavian monarch a progressive? *(closed reading)*

(14)   I always thought Syldavia was a very conservative monarchy, but what I learned about their policies made me less certain about it. Tell me, is Syldavia a republic, or is the Syldavian monarch a progressive? *(open reading)*

Thus, in (12), as in the disjunction of assertions (2b), the presupposition triggered by the definite DP fails to project. We can in fact replicate all of the tests we applied to the conjunctive case to see how the failure to project depends on the first disjunct. An

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[7]To be precise, the fact that we infer from (5) that Syldavia is a monarchy and not that if Syldavia is rich in mineral resources, it is a monarchy is an instance of the *proviso problem*. The proviso problem occurs in exactly the same cases for questions as for assertions as far as I can tell.

unrelated first disjunct as in (15) fails to block projection. Reversing the order as in (16) leads to degraded felicity (the judgement is less sharp than for the conjunctive case, and in this respect questions do not differ from assertions; cf. (17)). Replacing the first disjunct by its "opposite" or adding *or not*, as in (18a) and (18b) respectively, leads to infelicity.

(15)   Is Syldavia rich in mineral resources, or is the Syldavian monarch a progressive?

(16)   ?? Is the Syldavian monarch a progressive, or is Syldavia a republic?

(17)   ?? The Syldavian monarch is a progressive or Syldavia is a republic.

(18)   a. # Is Syldavia a monarchy, or is the Syldavian monarch a progressive?
        b. # Is Syldavia a republic or not, or is the Syldavian monarch a progressive?

All of these observations lead us to the following generalization, which is strikingly similar to Karttunen's (1974) generalization about disjunction:

(19)   **Generalization about disjunctive questions:** in a disjunction of polar questions $?p \lor ?q$ (but not when *or not* questions are involved), under either an open or a closed reading, while the local context of p is the global context, in at least some cases the local context of q is the global context enriched with $\neg p$.

Thus, to conclude, presupposition projection in both (apparent) conjunctions and (apparent) disjunctions of polar questions obeys laws that are strikingly similar to those seen in conjunctions and disjunctions of declarative clauses. Before we move on to see why this is puzzling from a theoretical point of view, there is a final thing to remark about the data.

1.3   Some additional empirical points

A few other remarks can be made before we move on to the theoretical part of the paper. First, while we have been using matrix questions as examples, everything works exactly the same when looking at embedded questions; this is demonstrated below. In the disjunctive cases, the same ambiguity between open and closed reading is found in embedded examples, and as before it does not affect the projection facts we are interested in. In the rest of this article, we will switch between matrix and embedded examples depending on what makes the point clearer, but as far as I can ascertain there is nothing in the discussion that does not apply equally to both.

(20)   Mary wonders whether Syldavia is a monarchy and whether the Syldavian monarch is a progressive.
        $\not\leadsto$ (Mary believes that) Syldavia is a monarchy.

(21)   Mary wonders whether Syldavia is rich in mineral resources, and whether the Syldavian monarch is a progressive.
        $\leadsto$ (Mary believes that) Syldavia is a monarchy.

(22)   a. #Mary wonders whether the Syldavian monarch is a progressive and whether Syldavia is a monarchy.
        b. #Mary wonders whether Syldavia is a republic and whether the Syldavian monarch is a progressive.
        c. #Mary wonders whether Syldavia is a monarchy or not and whether the Syldavian monarch is a progressive.

(23)   Mary wonders whether Syldavia is a republic or whether the Syldavian monarch is a progressive.

     ↛ (Mary believes that) Syldavia is a monarchy.

(24)   Mary wonders whether Syldavia is rich in mineral resources or whether the Syldavian monarch is a progressive.

     ⤳ (Mary believes that) Syldavia is a monarchy.

(25)   a. ??Mary wonders whether the Syldavian monarch is a progressive or whether Syldavia is a republic.

     b. #Mary wonders whether Syldavia is a monarchy or whether the Syldavian monarch is a progressive.

     c. #Mary wonders whether Syldavia is a republic or not or whether the Syldavian monarch is a progressive.

Second, alternative descriptions of the phenomenon that do not refer to local contexts would be possible. However, as pointed out by Schlenker (2009) (cf. also Singh 2007), when it comes to assertions, the generalization in terms of local contexts lets us predict certain cases of infelicity due to triviality effects, even in the absence of presuppositions. Such effects are also observed in polar questions. For instance, in (26a), the clause "Ann is in France" should always be in a local context where Ann is known to be in Paris, and therefore locally trivially true. Similarly in (26b), the clause "Ann is in London" should always be locally trivially false. We can therefore understand the infelicity of (26a) and (26b) in terms of our generalization.[8]

Similar data can be observed in the case of disjunction, as seen in (27a) and (27b). For now, we are going to focus on the presupposition projection data, and we will return to examples like (26a) and (26b) in Section 4.2.[9]

(26)   a. #Is Ann in Paris and is she in France?
     b. #Is Ann in Paris and is she in London?

(27)   a. #Is Ann away from Paris or is she in France?
     b. #Is Ann away from Paris or is she in London?

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[8]An anonymous reviewer points out that under certain intonations, these two examples can be felicitous. Discourse markers like *actually*, *in fact* etc. can help bring out such readings.

(i)   Is Ann in Paris? And (actually) is she (even) in France?

Intuitively, in these cases, the speaker changes discourse strategy in-between the two conjuncts, and would have asked the second question first, had they thought it through. The fact that the second conjunct is a correction is signalled through discourse markers and emphatic intonation. There is also a clear sentence break in-between the conjuncts.

The judgements reported here are based on a different intonation pattern, where there is no full sentence break in-between the clauses (which I have tried to indicate through punctuation), and the second conjunct does not bear any emphasis, and where there is no perception that the speaker changed their mind. Attempting an analysis of discourse-level uses of question coordination is beyond the scope of this paper.

[9]In particular, we will see that under the theory we are going to propose, it will be possible to explain the infelicity of (26a) and (26b) in terms of global redundancy or similar pragmatic constraints, without necessarily referring to local contexts.

2 Theoretical consequences and challenges

The fact that presupposition projection in coordinations of polar questions is very reminiscent of presupposition projection in coordinations of declarative clauses might seem expected. In this section, I am going to argue that it is actually puzzling given established theories on polar questions, coordination of questions and presupposition projection.

2.1 The need for a yes/no asymmetry in polar questions

Some remarks can be made before we even attempt a formal analysis. To begin with, if we pursue an analysis where the following two properties hold:

(i) in a conjunctive question like (4) ($?p \wedge ?q$), there is actually a constituent identical or equivalent to the simple polar question (8a) ($?p$), as the syntax suggests;

(ii) whether a presupposition triggered in a certain context projects or not is a function of the semantics of the sentence and of its subconstituents (a property true of all analyses of presupposition projection we will discuss);

then, we absolutely need our analysis of the simple polar question $?p$ to be *asymmetric*. What I mean by "asymmetric" here is that the semantic denotation of a polar question $?p$ should be such that the "yes" answer (p) and the "no" answer ($\neg p$) do not play interchangeable roles in it. In particular, $?p$ and $?(\neg p)$ should be different objects, and that all three questions in (28) should be semantically distinct. The reason this is a necessity is that without a difference between these questions, there is no hope to account for the contrast between (4) on the one hand, and (9a) and (9b) on the other hand (a similar point can be made with the disjunctive examples).

(28)   a.   Is Syldavia a monarchy?
       b.   Is Syldavia a republic?
       c.   Is Syldavia a monarchy or not?

The significance of this point comes from the fact that polar questions are frequently assigned denotations that are "yes/no-symmetric", e.g.:

(29)   a.   In Hamblin semantics: $\{p, \neg p\}$.
       b.   In partition semantics: $\lambda w. \lambda w'. p(w) = p(w')$.
       c.   In inquisitive semantics: $\{s \mid s \vdash p \vee s \vdash \neg p\}$

Such theories make (28c) indistinguishable from (28a) and (28b) contextually equivalent to the other two, which is problematic given that these are not interchangeable as far as the phenomenon we are looking at is concerned. We therefore need a more fine-grained view of polar questions.

An important caveat here is that the fact that there exists yes/no-asymmetries around polar questions is well-known, at least since Bolinger (1978). Some specific asymmetric phenomena that have been studied include discourse patterns, both in terms of felicity conditions (e.g. Biezma and Rawlins 2012) and future discourse potential (e.g. Roelofsen and Farkas 2015), as well as epistemic and evidential biases (e.g. Büring and Gunlogson 2000; Sudo 2013). These authors acknowledge the need for asymmetric representations of questions in order to account for the phenomena at hand. What is generally assumed, however, is that symmetric denotations of the kind shown above are adequate to account

for the *resolution conditions* of polar questions, even if they might not be an adequate representation of question meaning in general. Under this view, what our data tells us is that presupposition projection in questions and/or the mechanisms of question coordination cannot be derived purely from resolution conditions.[10]

2.2   The failure of a direct reduction to the declarative case

A second remark has to do with the way we have to analyse the connectives. As already noted, the presupposition projection patterns we observe are very similar to those observed in the declarative case. Additionally, the basic facts we started from about how presupposition project from declarative conjuncts and disjuncts are widely agreed-upon and a large and diverse array of theories have been devised to derive them. If we can reduce our cases of "conjunctive questions" and "disjunctive questions" to conjunctions or disjunctions of declaratives, then said theories are going to make predictions about presupposition projection, which we might hope will match the observed pattern.

There is in fact an independent reason we might want to reduce our examples to coordinations of assertions. The point would be to avoid committing to the fact that questions can be conjoined or disjoined at all. While the basic truth-conditional effect of *and* and *or* when acting upon declaratives is relatively well-understood, there is much less of a consensus on what they do to questions. In various existing theories of questions, the meaning that should be assigned to *and*, to *or* or to both to get non-degenerate results is unclear, or it does not clearly relate to the declarative meaning, or there is an lack of uniformity between what has to be assumed for *and* and for *or*. Because of these pervasive issues, it has been proposed that questions can be conjoined but not disjoined, or not directly (e.g. Szabolcsi 1997; though see Szabolcsi 2016 and Ciardelli, Groenendijk, et al. 2018, sec. 9.2.2 for counterpoints), or that they can be disjoined but not directly conjoined (e.g. Hirsch 2017), or that they can neither be directly conjoined nor directly disjoined (e.g. Krifka 2001).

In these accounts of question coordination, which I am going to refer to collectively as the *reductionist view*, an apparent coordination of questions can always be paraphrased as a coordination of declaratives. For instance, one would analyse (30a) in such a way that it is equivalent to either (30b) or (30c), and (31a) in such a way that it is equivalent to (31b) or (31c).[11]

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[10] I thank a reviewer for stressing the importance of this distinction.

[11] Two strategies are possible. One option consists in assuming that the connective takes higher or lower scope than the surface syntax would suggest, with ellipsis or semantically inert elements potentially involved. For instance, Hirsch (2017) proposes that apparent conjunctive questions actually involve "and" taking high scope. The second option, proposed among others by Krifka (2001), consists in *lifting* the question from whatever type α our theory of simple questions would assign to it to the higher-order type $(\alpha \rightarrow t) \rightarrow t$, as shown in (i), and then assume that conjunction and disjunction apply (classically) to this higher-order denotation. The second strategy only delivers "high" readings, as in (30b) and (31b).

(i)   LIFT$(Q) := \lambda P.\, P(Q)$ (type $(\alpha \rightarrow t) \rightarrow t$)
       "Q and Q'" $\approx$ LIFT$(Q) \wedge$ LIFT$(Q') = \lambda P.\, P(Q) \wedge P(Q')$
       "Q or Q'" $\approx$ LIFT$(Q) \vee$ LIFT$(Q') = \lambda P.\, P(Q) \vee P(Q')$

To apply either strategy to the case of matrix questions, we need to assume that there are silent truth-conditional speech-act operators above them, such as a silent *I wonder* or a silent *you should tell me*. Alternatively, along with the second strategy, we might assume that the higher-order type is the "normal" type of questions, and that there are no linguistic constituents with semantic type α. I use embedded examples in this section to avoid dealing with the issue.

(30) a. Mary wonders whether John is here and whether it's raining.
      b. Mary wonders whether John is here, and she wonders whether it's raining.
      c. Mary wonders whether it is the case that John is here and it's raining.

(31) a. Mary wonders whether John is here or whether it's raining.
      b. Mary wonders whether John is here, or she wonders whether it's raining.
      c. Mary wonders whether it is the case that John is here or it's raining.

Unfortunately, such approaches do not lead to a satisfactory account. The low scope strategy spelt out by (30c) and (31c) would make the presupposition projection facts unremarkable: the question nucleus would be a conjunctive or disjunctive proposition that is well-known to lack a presupposition, and making a question out of it should not change anything to that. However, the problem is that our examples are not actually polar questions, or at the very least, some of the readings they allow for are not polar, and yet the presuppositions still fail to project. If (4) were a polar question, then the asker should be satisfied upon being told by a fully knowledgeable answerer that Syldavia's leader is a conservative, without being told whether we are talking about a monarch or a president. This is wrong: someone who asks (4) wants to actually know whether Syldavia is a monarchy or a republic. Similarly for the disjunctive case (12): we expect the asker to be satisfied upon being told that Syldavia cannot possibly have a conservative monarch. With the open intonation, such a answer is actually judged to be incomplete, and only felicitous if it is all the answerer knows. Under the closed reading, the answer is completely infelicitous, as we fully expect given that the asker is already presupposing the fact in question. Thus, if either of our examples has a polar reading at all, it is not very salient and the lack of presupposition projection that we observe is not dependent on it.[12]

The high scope strategy for conjunctive questions, as spelt out by (30b), has been argued to predict adequate truth conditions in general (Krifka 2001; Hirsch 2017). For disjunctive questions, it has been proposed as an analysis of certain cases of disjunctive constituent questions (e.g. Xiang 2020). However, as already pointed out by Hoeks and Roelofsen (2019), the resulting truth conditions are clearly wrong in the case of disjunctive polar questions: on its most natural reading, (31a) is not a wide-scope disjunction (it does not suggest speaker ignorance) and it does not indicate a disjunctive desire on Mary's part.[13] Regardless of the truth conditions, in both the conjunctive and disjunctive cases, the high-scope strategy makes incorrect predictions about presupposition projection. Indeed, various theories of presupposition projection agree that "ϕ and ψ", where ϕ and ψ are declarative sentences, is felicitous whenever the presuppositions of ϕ are satisfied, and it is sufficient to grant ϕ to make the presuppositions of ψ satisfied (I'm ignoring here any issues related to the so-called proviso problem, as they do not affect the discussion as far as I can tell). Applying this to our conjunctive example (20), we predict that it should be

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[12]Another reason to think that our questions are not polar questions is that "yes" and "no" are not good answers to them. My impression from an informal survey is that in the conjunctive case (4), speakers' initial intuition is to accept "yes" and "no" as answers, but they are not sure as to how to interpret either. In the disjunctive case (12), with the "open" intonation, "no" is a felicitous answer, meaning that both disjuncts are false, but "yes" is of unclear interpretation. With the alternative question intonation, both "yes" and "no" are unacceptable. More generally, "yes" and "no" as an answer to an alternative question can sometimes be interpreted as "both are true" and "neither is true" respectively, but they tend to come across as joke answers.

[13]An analysis following Xiang (2020), where *wonder* is decomposed into *want* and *know*, and disjunction takes scope in-between the two, would predict such a disjunctive desire: Mary wants it to be the case that either she knows whether John is here, or she knows whether it is raining.

felicitous in a context where, if we come to know that Mary wonders whether Syldavia is a monarchy, then we can grant that Syldavia is a monarchy. Such a context is extremely odd and we clearly do not need to accommodate it to accept (20). Similarly, in the disjunctive example, we predict the presupposition no to project as long as if Mary does not wonder whether Syldavia is a republic, then she believes that Syldavia is a monarchy, which again is not a context that one needs to accomodate to accept (23).[14]

To conclude, the low-scope reductionist view delivers inadequate polar interpretations for the questions we are looking at. The high-scope reductionist view makes specific predictions about presupposition projection, as long as we accept well-established facts about presupposition projection in declaratives, and these predictions do not bring us any closer to an explanation of what we observe. All this suggests that our data is problematic for reductionist accounts of question coordination, and that explaining it will require that we actually engage with question semantics and with the thorny issue of question coordination.[15]

2.3   Taking a broader look: our desiderata

In the previous two sections, I developed an argument that an analysis of our data should have the following two properties: it should feature a yes/no-asymmetric representation to polar questions, and it should not adopt a reductionist view of question coordination.

Some other properties are arguably desired in any account of question coordination. The effect of *and* and *or* when acting upon questions should relate in a natural way to their effect when acting upon propositions. Ideally, said relation should be the same for conjunction and for disjunction. The syntax we assume should be what we observe: two polar questions, each with insertion or in embedded cases with *whether*, connected by *and* or *or*; in other words, $?p \wedge ?q$ and $?p \vee ?q$. Deviating from these requirements means that we

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[14]Under the want $\gg$ or $\gg$ know analysis, the condition for filtering would instead be something like: "according to Mary's beliefs, if she does not know whether Syldavia is a republic, then Syldavia is a monarchy", which is nonsensical.

[15]A potential reductionist counter-argument worth commenting upon is that if we attempt to explicitly spell out the semantics that a reductionist account would give to our examples, it is not entirely clear whether the presupposition projects in the resulting sentence. This is especially true in the conjunctive case (i), which is only somewhat degraded in my judgement. In the disjunctive case, the sentence we obtain (ii) is somewhat involved and hard to interpret.

(i)   ? Mary wonders / wants to know whether Syldavia is a monarchy and she wonders / wants to know whether the Syldavian monarch is a progressive.

(ii)   ?? Mary wants to know whether Syldavia is a republic or to know whether the Syldavian monarch is a progressive.

A counter-counter-argument is that using complex conjunctions and disjunctions to make the scope explicit, even though it does not affect presupposition projection in the general case, makes both our attempts more clearly contradictory:

(iii)   a.   Not only is Syldavia a monarchy, but the Syldavian monarch is a progressive.
         b.   #Not only does Mary wonder whether Syldavia is a monarchy, but she wonders whether the Syldavian monarch is a progressive.

(iv)   a.   Either Syldavia is a republic or the Syldavian monarch is a progressive.
        b.   #Mary wants either to know whether Syldavia is a republic or to know whether the Syldavian monarch is a progressive.

need to assume a more complicated syntax/semantics interface. It might of course turn out that we need to violate some of them, but such a move requires good motivation.

Additionally, we want to account for the actual meaning of conjunctive and disjunctive questions (both open and closed), in terms of answerhood conditions, behaviour under embedding, and so on. This goes both as far as our specific examples are concerned and in the general case. The effect of *or not* should be explained as well.

The desiderata we listed so far bear entirely on the analysis of questions and of the question connectives that we adopt, and do not relate to the matter of presupposition projection directly. As I already mentioned, presupposition projection in declaratives is the subject of a large number of theories in the literature. These accounts have the following structure: they provide a system where the presuppositions of a complex declarative sentence can be derived from those of simpler sentences based on a few (ideally motivated) assumptions about the semantics/pragmatics interface and the denotation of sentences. Our goal here is to apply this methodology to questions. As we have seen above, we cannot get away with a reduction of our data to the sort of data that these theories have been designed to explain. However, we can still attempt to extend them in the most natural way. Concretely, what we want is to adopt the assumptions of an established theory of presupposition projection, apply them to an account of polar questions and coordinated questions, and derive the observed projection patterns.

How to do this, for a particular theory of presupposition projection and a particular theory of question semantics, is not necessarily obvious. In particular, it is difficult to extend a dynamic theory of presupposition projection, such as that of Heim (1983), to a static account of questions. In what follows, I am instead going to focus on theories of presupposition projection that are based on a static system, such as Schlenker's (2008) Transparency Theory (and the related derivation of local contexts he offers in Schlenker 2009), and what I am going to refer to as the *trivalent theory* (Beaver and Krahmer 2001; George 2014). The fact that these theories are based on a static and truth-conditional view of propositions will make it easier to extend them to various theories of questions.

My purpose in the rest of this section is to highlight the pervasive issues that one encounters when applying the methodology I described to various established accounts of questions. The end goal is to motivate a new approach to question semantics, which will be detailed in the next section. We will proceed in the following way: first, in Section 2.4, we will review in detail how one could try to understand the phenomenon at hand within the framework of Hamblin/Karttunen semantics. Then, in Section 2.5, I will briefly show how parallel issues to those that arise in the case of H/K semantics arise in other major theories as well. This will motivate the presentation of a new theory of questions in Section 3.

2.4   Case study: an analytical attempt based on answer set semantics and the Transparency Theory

Probably the most common view in question semantics is to analyse them as denoting a set of answers. This view is known as the *answer set theory* or as *Hamblin-Karttunen semantics*, after Hamblin (1976) and Karttunen (1977). I will now attempt an analysis of our data within H/K semantics, in order to show the many issues that arise.

The basic idea of H/K semantics is that a question is at some level a set of propositions, and these propositions are construed as the possible answers to the question. Asking a question means prompting the participants in the conversation to identify one of the proposition as true. There are various implementations of the idea. In a thread of work

based on Hamblin's (1976) system, a question's denotation is simply the set of all its possible answers, regardless of the present situation. In contrast, under the approach of Karttunen (1977), questions have an intension and an extension, and the extension is the set of true answers at the current world. We are going to adopt Hamblin's view here for the sake of concreteness, but the choice does not really matter: both systems map onto each other in a way that preserves the properties of interest to us (this is illustrated in the formal companion to this section, Appendix A, where both Hamblin's and Karttunen's systems are considered).

Presupposition projection   In what follows, I will assume that presupposition projection is governed by the Transparency Theory for the sake of concreteness. My claim is that this choice is not crucial. The discussion can easily be recast in terms of local contexts or in trivalent terms, and in Appendix A I derive several formal results that illustrate the points made in the discussion in both the Transparency Theory and George's (2014) trivalent theory. Moreover, at least certain dynamic accounts of presupposition projection would face very similar issues when being extended to questions.[16]

I will not describe the Transparency Theory in full detail here, and only says how it applies to the examples: Appendix A contains proofs of some of the relevant formal results, as well as parallel results for trivalent theories of presupposition projection. The Transparency Theory's predictions derive from a notion of contextual equivalence over sentence denotations. Thus, to apply it to questions, we need a notion of contextual equivalence over them. The simplest possible one goes like this: two questions are equivalent if the sets that they denote are equal, after each member proposition is contextually restricted. We can use this simple notion to derive the results discussed here; cf. Appendix A for a formal definition (under the name H-equivalence) as well as potential alternatives.

General considerations   In theories based on Hamblin 1976, a polar question like (32a) (repeated from (8a)) is traditionally analysed as denoting the two-element set in (32b). This analysis is based on the fact that the two propositions in (32b) are intuitively perceived to be the two ways one may felicitously answer (32a). More generally, one will analyse the polar question we schematize as $?p$ as denoting $\{p, \neg p\}$.

(32) a. Is Syldavia a monarchy?
      b. {That Syldavia is a monarchy, That Syldavia is not a monarchy}

Sets being unordered objects, $?p = \{p, \neg p\}$ is indistinguishable from $?(\neg p)$ (at least under a classical view of negation where it is involutive). Thus at first look, the traditional view in H/K semantics fails to meet one of our desiderata: it does not analyse polar questions in a yes/no-asymmetric way. However, in some accounts of the internal syntax of disjunctive and polar questions, there is a yes/no-asymmetric intermediate constituent with the type of a question which is what is assumed to be engaging in disjunction. In particular, Biezma and Rawlins (2012) propose that a polar question $?p$ really denotes the singleton set $\{p\}$ rather than a two-element set, and that an additional "coercion operation" is responsible for adding $\neg p$ to the set. In a very similar way, Karttunen (1977) offers a syntax involving a "proto-question" constituent whose denotation is the moral

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[16]In particular, an account that combines the answer-set theory with Heim's (1983) theories of propositions, as in Li's (2019) "dynamicised Hamblin sets", would suffer the same problems as static answer set approaches.

equivalent of $\{p\}$ (translating from his system to Hamblin's). The negative proposition is added by a special semantic rule. In both theories, while the denotation of a plain polar question is yes/no-symmetric, question coordination can involve yes/no-asymmetric (proto-)questions and our desideratum is met.

For purposes of illustration I am going to give the sketch of an analysis in this vein. Proto-questions are formed by a question operator ? such that $?p = \{p\}$. Then, a closure operator C, defined in (33), can apply. This operator generalizes the coercion operation of Biezma and Rawlins (2012) and the semantic rule of Karttunen (1977). What it does is add a "catch-all" answer to a question denotation, that is true when all pre-existing answers are false. In particular, applied to $\{p\}$, it will give $\{p, \neg p\}$. The definition given here is sufficiently general that the operator C could in principle apply to non-singleton sets as well. Neither Karttunen (1977) nor Biezma and Rawlins (2012) allow for this possibility, but it will prove worth considering for our purposes.

(33) $\text{C} = \lambda Q.\, Q \cup \left\{ \neg \left(\bigvee_{p \in Q} p \right) \right\}$

Following Karttunen (1977) as well as Biezma and Rawlins (2012), we assume that C is optional in at least some cases, and in particular while a plain polar question is essentially $\text{C}[?p]$, a disjunctive question might be just $?p \vee ?q$.[17]

It would be most natural to assume that *and* and *or* denote set intersection and set union respectively when applied to questions. For disjunction, this is indeed the assumption that is generally made in H/K semantics. However, for conjunction, such an assumption leads to degenerate results. In most cases, the two questions being conjoined are disjoint sets; there is no proposition that is an answer to both, and therefore their intersection is an empty set. We cannot hope to derive the felicity and answerhood conditions of conjunctive questions from the empty set. Because of that, one either needs to adopt a reductionist view of conjunction, or to assume that conjunction of questions applies *pointwise* inside the set. All this leaves us with the following definitions of question conjunction and disjunction:

(34)   a. $Q \wedge Q' = \{p \wedge p' \mid p \in Q, p' \in Q'\}$
         b. $Q \vee Q' = Q \cup Q'$

H/K semantics thus fails to fulfil another of our desiderata: the treatment of question conjunction and question disjunction is not uniform. This is disappointing in itself, in as much as it makes the syntax-semantics interface less transparent.[18] Additionally, it entails that we cannot hope to derive the presupposition projection patterns similarly for conjunction and disjunction, and therefore that the similarity between what is observed

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[17]The presentation here is not true to Karttunen (1977) and Biezma and Rawlins (2012) in the sense that neither theory actually features such an operator, and the corresponding device is more like an interpretation rule.

Additionally, Biezma and Rawlins (2012) assume that the corresponding coercion operation only occurs in embedded questions. Since the phenomenon we are interested in is not affected by the distinction between matrix and embedded questions it will be easier to follow Karttunen (1977) and assume that both kinds are composed uniformly.

[18]Here there are several things to say to defend H/K semantics. First, pointwise composition is independently motivated, not only to analyse the internal composition of questions (Hamblin 1976) but also for other phenomena, such as focus (Rooth 1992). Second, given that non-pointwise conjunction leads to degenerate results, it seems unproblematic to assume that the syntax allows for it just as it allows for non-pointwise disjunction, but that it is banned on semantic grounds.

in questions and in declaratives will turn out to be a coincidence. This is an instance of a more general problem where the lack of parallelism makes the system not well-behaved from a formal perspective, leading to the impossibility of deriving formal properties and to strange predictions in edge cases (see Ciardelli, Roelofsen, et al. 2017 for a discussion of this point).

Conjunctive questions   These concerns aside, let us move to an account of presupposition projection in coordinated questions. We have the following building blocks: polar questions denoting singleton sets, conjunction and disjunction as defined above, and the closure operator C.

The most common analysis of a conjunctive question with the apparent structure of $?p \wedge ?q$ in H/K semantics is that it should denote a 4-element set: $\{p \wedge q, p \wedge \neg q, \neg p \wedge q, \neg p \wedge \neg q\}$. I will call this set the quadripartition. The quadripartition leads to the prediction that the answerer to a conjunctive question has to specify the truth values of both propositions, resulting in four mutually-exclusive answers. The received view is that this is indeed what conjunctive questions generally mean.

Recall that in the theory sketched above, $?p = \{p\}$, $?q = \{q\}$, and $\wedge$ applies pointwise. Further on, C may apply on top of question-type constituents. We can therefore generate the quadripartition if we assume that C applies to both conjuncts separately, resulting in a structure like $C[?p] \wedge C[?q]$.

Yet, the quadripartition is problematic for us. To predict the filtering observed in (35) (repeated from (4)), we would want the proposition expressed by p (that Syldavia is a monarchy) to be transparent in the position of q (the second conjunct's nucleus). However, it is easy to verify that it is not; in fact, nothing non-trivial is transparent in the position of q in $C[?p] \wedge C[?q]$ in any context (cf. Appendix A for a proof). Regardless of the presupposition projection theory we adopt, we know at any rate that the quadripartition will not let us derive the observed pattern, because it is not yes/no-asymmetric: substituting $\neg p$ for p in the quadripartition does not change the set.

(35)   Is Syldavia a monarchy, and is the Syldavian monarch a progressive?

Then, our data is problematic for the view that conjunctive questions always denote the quadripartition. It is also problematic for answer set theories not based on proto-questions, and where polar questions always denote $\{p, \neg p\}$, as in these theories the quadripartition is the only thing that a conjunctive question could denote. In our system, however, it is possible to generate other denotations, as long as we give ourselves as much freedom as we want and allow for C to apply to any question-type constituent. This allows us to generate six alternative denotations for (35), which are listed in (36).[19] This wide use of C is of course not at all in the spirit of Karttunen (1977) and Biezma and Rawlins (2012), for whom C is a last-resort coercion rule.

(36)   a. $?p \wedge ?q = \{p \wedge q\}$
        b. $?p \wedge C[?q] = \{p \wedge q, p \wedge \neg q\}$
        c. $C[?p] \wedge ?q = \{p \wedge q, \neg p \wedge q\}$
        d. $C[?p \wedge ?q] = \{p \wedge q, \neg(p \wedge q)\}$
        e. $C[?p \wedge C[?q]] = \{p \wedge q, p \wedge \neg q, \neg p\}$
        f. $C[C[?p] \wedge ?q] = \{p \wedge q, \neg p \wedge q, \neg q\}$

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[19]Adding extra C's would only add logical contradictions to the sets, as the reader can verify. We are going to assume that it would make the questions unacceptable.

Options (a), (b) and (c) are uninteresting. (a) leads to a question with only one possible answer; such a question is presumably not well-formed. (b) and (c) should correspond to readings where the answer to one of the conjuncts is presupposed, which is presumably banned on pragmatic grounds.

Option (d) leads to a yes/no question. We have already seen when discussing the reductionist view that (35) is not or at least does not have to be a yes/no question.

Option (f) is yes/no-symmetric as well (with respect to the first conjunct), leaving us with option (e). Option (e) is in fact what we want. First, p is transparent in the position of q. Intuitively, this is because in the denotation, "q" always appears somewhere after "$p\wedge$".[20] We therefore correctly predict (4) to lack a presupposition that Syldavia is a monarchy.[21] Furthermore, the three mutually-exclusive propositions in the set (that I will refer to as the tripartition) correspond exactly to the possible answers to (35):

(37) a. Syldavia is not a monarchy. ($\neg p$)
      b. Syldavia is a monarchy and/but the Syldavian monarch is not a progressive. ($p \wedge \neg q$)
      c. Syldavia is a monarchy and the Syldavian monarch is a progressive. ($p \wedge q$)

Then, what we now have is a new analysis for conjunctive questions within H/K semantics, schematized by (e), such that both the presupposition projection facts and the answerhood conditions of (35) are easily understood.[22]

In my view, the analysis in question is not entirely satisfactory. Other than the general problems with H/K semantics and pointwise conjunction I mentioned above, the main issue is with the asymmetric structure of (e). This asymmetry raises a number of challenges if we make the natural assumption that C is in the syntax and that it is some sort of complementizer. First, assuming that coordination can only occur between constituents of the same category, then we need to explain why a constituent like $?p \wedge \text{C}[?q]$ is at all possible. Second, we need to explain why (e) is allowed, while the alternative tripartition in (f), which is not attested, is not. Even the polar reading predicted by (d) might have to be ruled out, as the evidence for such a reading is scarce.[23] Finally, it is puzzling that nothing in the surface syntax of (35) suggests there is left/right asymmetry in the structure. This is probably why, to my knowledge, the possibility of something like (e) (or (f)) has in fact never been considered worth discussing before.[24] None of these problems is insurmontable, but they will require to complicate the syntax, or to find another analysis of what C is where they become easier to deal with.

To conclude, while H/K semantics provides the necessary ingredients to understand our cases of conjunctive questions, doing so requires non-standard assumptions about how

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[20]Since all theories of presupposition projection are designed so that a presupposition that p is true does not project after "$p\wedge$", here we can see that the choice of the Transparency Theory was not crucial. See also the derivations in Appendix A.

[21]Our generalization in terms of local contexts given in (10) also follows if we derive local contexts in the spirit of the Transparency Theory along the lines of Schlenker (2009).

[22]We will discuss in Section 3.5 the implications of the resolution conditions predicted by (e) in non-presuppositional examples.

[23]Hoeks and Roelofsen (2019) note that, like the theory we are sketching here, inquisitive semantics predicts conjunctive questions to have polar readings, and they claim that this prediction is correct. My own judgement is that the reading they characterize as polar is the one corresponding to the tripartition, that is, option (e). In my view, the issue remains to be investigated more thoroughly.

[24]In particular, the extensive discussion of potential structures for conjunctive questions offered by Hoeks and Roelofsen (2019) does not mention the possibility of (e) or (f).

conjunctive questions' internal composition (with unconstrained, asymmetric application of a closure operation), so as to allow them to be interpreted as a tri-partition, and possibly suffers from an overgeneration of readings.

Disjunctive questions We can apply the same methodology to our disjunctive example (38) (repeated from (12)). The most common analysis of closed disjunctive questions (alternative questions) within H/K semantics is that a question with the apparent structure $?p \lor ?q$ denotes $\{p, q\}$, based on the intuition that the two potential answers to the questions are the two disjuncts (cf. discussion by Roelofsen and Farkas (2015)). Both Karttunen (1977) and Biezma and Rawlins (2012) essentially adopt this view.

(38)   Is Syldavia a republic, or is the Syldavian monarch a progressive?

Assuming that disjunction is set union, $\{p, q\}$ can easily be derived as the direct disjunction of the proto-questions $\{p\}$ and $\{q\}$, i.e. $?p \lor ?q$. This is in fact part of the motivation for proto-questions: if a polar question denoted $\{p, \neg p\}$ from the get-go, a disjunction of two such questions would be $\{p, \neg p, q, \neg q\}$. Yet there is no reading of disjunctive questions such that the set of felicitous answers is this 4-element set. For instance, a fully knowledgeable speaker cannot cooperatively answer "John is not here" ($\neg p$) to (39), whatever the intonation.

(39)   Is John here, or is Mary here?

The version of H/K semantics we are working with here partially solves the problem by allowing for $?p \lor ?q = \{p, q\}$. We do generate $\text{C}[?p] \lor \text{C}[?q] = \{p, \neg p, q, \neg q\}$, but we can rule it out by assuming a *unique answer* constraint: at every contextually permissible world, there should be exactly one answer that is true. It can be verified that there is no context where $\{p, \neg p, q, \neg q\}$ satisfies this constraint without making some of the answers trivially false in the context (see Hoeks and Roelofsen 2019 for extensive discussion of this issue, including a discussion of more refined constraints).

The open reading of disjunctive questions has been the subject of less theorizing, as it is often mischaracterized as a polar reading. As Roelofsen and Farkas (2015) argue, the open reading is not polar and in fact allows for three possible answers corresponding to the disjuncts (p, q) and to their joint negation ($\neg p \land \neg q$). The natural analysis is therefore $\{p, q, \neg p \land \neg q\}$. We can generate it as $\text{C}[?p \lor ?q]$, as Roelofsen and Farkas (2015) propose (Roelofsen and Farkas (2015) work within the framework of inquisitive semantics, but the variant of H/K semantics that we are using here is close enough that the analysis carries over).[25]

We therefore have two potential denotations, $\{p, q\}$ and $\{p, q, \neg p \land \neg q\}$. Unfortunately for us, neither of them leads to an immediate explanation of the projection facts. Recall that in the conjunctive case, the key property that we could capitalize on, under the Transparency theory but also under other theories of presupposition projection, is that q always appeared after "$p \land$" in the denotation. There is no similar property in either of our proposed denotations: q appears on its own.[26] This translates into a failure of transparency: for $\neg p$ to be transparent in the position of q in either case, a necessary

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[25]In contrast, Biezma and Rawlins (2012) analyse both open and closed questions as $\{p, q\}$, but with different presuppositions.

[26]As in the conjunctive case, there are in principle other possibilities, such as $?p \lor \text{C}[?q] = \{p, q, \neg q\}$ and $\text{C}[?p] \lor ?q = \{p, \neg p, q\}$. These options do not have the required property either, and I will ignore them here to keep the discussion contained.

condition is that $\{p, q\} = \{p, \neg p \wedge q\}$ for any arbitrary proposition q (cf. also Appendix A). There is no reason for this to be the case in any realistic example.

At this point, one might remark that the notion of question equivalence that we have been using so far is naïve and excessively fine-grained. To fix this, first, we would have to define equivalence relative to a context, as Schlenker (2008) in fact does. Additionally, we might want to take the unique answer presupposition into account. There are many natural approaches that one could think of to that effect — Schlenker's theory as exposed in Schlenker 2008 does not cover the case of multiple sources of presupposition in the same sentence — and in Appendix A I prove for a number of them that they do not let us derive the desired result. Thus (12)/(38) remain entirely mysterious from the perspective of H/K semantics and the Transparency theory.

In the end, H/K semantics lets us to explain presupposition projection in conjunctive questions under the Transparency Theory (among others), if we are ready to make certain *ad hoc* assumptions on question composition. In a more problematic way, while there is a natural analysis for open and closed disjunctive questions within H/K semantics, the observed presupposition filtering does not follow in a similar mostly theory-neutral way, and neither does engaging in detail with the Transparency Theory (as is done in the Appendix) let us derive it. At any rate, the fact that conjunctive and disjunctive questions receive a very different treatment and that considerations on one case do not extend straightforwardly to the other is unsatisfactory, given that we hope to explain the parallelism between questions and assertions. Thus, given the desiderata exposed above, our data is problematic for the H/K analysis of polar questions, even in its proto-question-based yes/no-asymmetric form.

2.5 Other theories in question semantics

We have seen that our data is puzzling from the perspective of the answer set theory of questions. We are naturally prompted to look for alternative views on question semantics that do not suffer from the same problems as H/K semantics. While I lack the space to review them with the same level of detail, I will discuss the most common approaches here and show that largely the same issues arise.[27]

Questions as partitions Groenendijk and Stokhof (1984) propose (as a component of a more complex theory) an analysis of questions based on the following intuition: what questions contribute to the conversation is that they raise an *issue*, and the issue is a partition of the logical space. Thus, the denotation of a question will be a partition, represented in the form of an equivalence relation over possible worlds (type $s \to s \to t$). Two worlds are related if and only if they resolve the issue in the same way. Concretely, this will be the denotation of a polar question $?p$:

(40) $\lambda u. \lambda v. p(u) = p(v)$

The partition theory fails to meet our desiderata in a more fundamental way than H/K semantics. First, the denotation of a polar question is yes/no-symmetric: substituting $\neg p$

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[27]All the theories discussed in what follows are static; while there exist also dynamic accounts of questions, I will set them aside for lack of space. In general, these accounts' handling of the connectives is derivative of a static theory and the issues that we are going to discuss carry over. This applies for instance to Dotlačil and Roelofsen 2019 (based on inquisitive semantics) or to Li 2019 (based on answer set theory).

for p in (40) would keep the object unchanged. Unlike in the answer set theory, there is no obvious fix without deeply changing the account. A related fact is that a conjunction of two questions will denote what I have called earlier the quadripartition, where the desired transparency property does not obtain, and there is no obvious way to derive the tripartition that we want. Finally, since a disjunction of equivalence relations is not in general an equivalence relation, the partition theory predicts that disjunctions of questions will be ill-formed. It therefore needs to resort to reductionist techniques to analyse question disjunction, and we have seen why this is problematic for our purposes.

Inquisitive semantics   Inquisitive semantics (Ciardelli, Groenendijk, et al. 2013) is a framework in which questions and assertions are analysed as having the same type. Both are sets of sets of worlds, construed as the epistemic states that resolve the question or make the assertion true. Concretely, a proposition p is the set of sets of worlds such that p is uniformly true in the classical sense, as in (41a). Here "$s \vdash p$" is to be read as "s supports p" and means that all worlds in s are p-worlds. I will write P for the inquisitive denotation to distinguish it from the classical one, and I will use Q, Q', etc. for abstract variables denoting inquisitive propositions, even though that is not standard practice. A polar question over proposition P is assumed to have the structure $?P$, where $?$ is an operator defined in (41b), that adds to its argument all the sets that have no intersection with anything in it. The result is given in (41c). Notice that P is yes/no-asymmetric, but $?P$ is not.

(41)   a. $P = \{s \mid s \vdash p\}$
        b. $? = \lambda Q. Q \cup \{s \mid \forall s' \in Q. s \cap s' = \varnothing\}$
        c. $?P = \{s \mid s \vdash p\} \cup \{s \mid s \vdash \neg p\}$

The structure of the analysis is actually extremely similar to our version of H/K semantics. The basic denotation of a question/assertion is yes/no-asymmetric and derives directly from the corresponding proposition. Then, an additional operator adds the negative case. (42) gives an helpful translation table from H/K semantics to inquisitive semantics; notice that the main innovation at this point is the loss of the distinction between propositions and proto-questions.

(42) p $\qquad\qquad\quad \longrightarrow P$
        $?p = \{p\}$ $\qquad\quad \longrightarrow P$
        $\mathrm{C}[?p] = \{p, \neg p\}$ $\quad \longrightarrow ?P$

Taking the maximal elements (under set inclusion) in an inquisitive proposition, called the *alternatives*, in fact lets us perform the opposite translation to H/K denotations. The operator ALT defined in (43) does just that.

(43)   a. $\mathrm{ALT}(Q) := \{s \mid s \in Q \wedge \neg \exists s' \in Q. s \subsetneq s'\}$
        b. $\mathrm{ALT}(P) = \{p\}$
        c. $\mathrm{ALT}(?P) = \{p, \neg p\}$

Unlike H/K semantics, inquisitive semantics allows for parallel definitions of conjunction and disjunction as set intersection and set union respectively, which as we already discussed is desirable from a theoretical perspective. Relative to the H/K definitions, the inquisitive connectives do the same thing to alternatives in simple cases, and where they differ, the inquisitive behaviour has been argued to be preferable (Ciardelli, Roelofsen,

et al. 2017). The definitions are given in (44).

(44)   a. $Q \wedge Q' = Q \cap Q'$
         b. $Q \vee Q' = Q \cup Q'$

Unfortunately, even though I made such uniform definitions part of our desiderata, in this case the uniformity does not bring us any closer to understanding presupposition projection. In fact, the discussion of H/K semantics above carries above almost entirely to inquisitive semantics, translating the structures as per (42) and using ALT to recover the predicted answers. For conjunctive questions, $?P \wedge ?Q$ is the quadripartition, and we need $?(P \wedge ?Q)$ to get the desired tripartition and transparency to obtain. The treatment of disjunctive questions is again not uniform with that of conjunctive questions: $?P \vee ?Q$ has to be ruled out on semantic or pragmatic grounds (Hoeks and Roelofsen 2019) and we most naturally analyse alternative questions as $P \vee Q$ and open disjunctive questions as $?(P \vee Q)$ (Roelofsen and Farkas 2015). Either way, we do not straightforwardly get transparency.

In fact, by switching to inquisitive semantics we might even have lost our explanation of presupposition projection in assertions. When we derive why the Transparency Theory predicts a presupposition to the effect that $\neg p$ is true to be satisfied in the second disjunct of $p \vee q$, the following property is crucial: $p \vee \top = p \vee \neg p$, where $\top$ is a tautology (cf. Schlenker 2008). This property does not hold in inquisitive semantics (here Ω is the set of all worlds and $\mathcal{P}$ maps sets to their powerset).

(45)   a. $P \vee \top = \{s \mid s \vdash p\} \cup \mathcal{P}(\Omega) = \mathcal{P}(\Omega)$
         b. $P \vee \neg P = \{s \mid s \vdash p\} \cup \{s \mid s \vdash \neg p\} = ?P \neq \mathcal{P}(\Omega)$

A similar problem will arise when deriving local contexts in the manner of Schlenker (2009) or when deriving trivalent connectives in the manner of George (2014). The natural way around this issue is to look at the logical properties of $!(P \vee Q)$ rather than $P \vee Q$; in inquisitive semantics, $!(P \vee Q)$ is essentially a classical disjunction. However, since the presence of the operator ! is assumed to be specific to assertions, predicating our derivation of projection facts upon it makes it so that our analysis does not extend to questions.

Categorial theories   The final approach to question semantics that I will mention is the categorial approach. In categorial theories, questions are assigned a complex functional type, that depends on the kind of question we are looking at. Thus a polar question might have a type isomorphic to a proposition, or a more complex type (e.g. $((st)st)st$ for Krifka (2001)), while a *who* question would be a one-place property of individuals (type $e \rightarrow s \rightarrow t$ or $s \rightarrow e \rightarrow t$). The appeal is, among other things, to allow for finer-grained distinctions than other theories. In particular, categorial theories always avoid yes/no-symmetry.

The main challenge for categorial theories is that questions behave in a way that suggests they all have the same type: they can be relatively freely coordinated, and a number of attitude verbs can embed any kind of questions. In order to solve the former problem, proponents of categorial approaches usually resort to the reductionist view in its higher-order type variant (see in particular Krifka 2001 and Xiang 2020), with the problems that we have seen. Alternatively, they fall back to one of the above theories to account for coordination and embedding. Categorial theories thus do not help us towards understanding our phenomenon.

3   A proposal: trivalent homogeneous questions

3.1   Introduction and background

In this section, I propose a novel view of what kind of object questions denote, which is rich enough to let us derive presupposition projection patterns while avoiding the issues we have encountered in our earlier attempts.

While this choice is not a necessity, it will make the motivation for the proposal and the exposition clearer if we adopt a trivalent view of propositions and presuppositions — in Section 4.2 we will see how the system can function with a bivalent propositions as well. The trivalent approach consists in reifying presuppositions by assuming that there are three truth values: 0, to which propositions map worlds where they are false, 1, to which propositions map worlds where they are true, and #, to which propositions map worlds where their presuppositions are not satisfied. Presupposition projection can then be derived from a trivalent semantics for logical operators (Peters 1979; Beaver and Krahmer 2001; George 2014).

It is customary in the formal semantics literature to entertain two perspectives on propositions at the same time: propositions are functions from worlds to truth values but they are also sets of worlds. In a bivalent setting, these two domains are isomorphic and the mapping between them is trivial. In a trivalent setting, the domain of functions from worlds to (trivalent) truth values is richer than the domain of sets of worlds, so in what follows I will distinguish between the functional type $s \to t$ of trivalent propositions and the set type $\{s\}$ of sets of worlds, as that will make certain things clearer. The operator $\mathbb{1}$ maps a trivalent proposition to its truth set:

(46) $\mathbb{1}(p) := \{w \mid p(w) = 1\}$

Finally, the trivalent perspective lets us define an operator that maps a trivalent proposition to "its" presupposition. This operator will be denoted as π; it is defined in (47a). Note that what π returns is a set of worlds (there is no sense in which the presupposition of a proposition could itself have a presupposition). The identity in (47b) will be useful to have in mind.[28]

(47)   a. $\pi(p) := \{w \mid p(w) \in \{0, 1\}\}$
         b. $\forall p.\, \pi(p) = \mathbb{1}(p) \cup \mathbb{1}(\neg p)$

3.2   Trivalent homogeneous polar questions

The system I present here is a trivalent theory of polar questions. We consider a polar question $?p$ to be a predicate over sets of worlds (construed as epistemic states). This predicate is potentially undefined: we take $?p$ to be defined at a state s if and only if s settles p, i.e. s supports p or $\neg p$. Thus:

(48) $?p = \lambda s.\begin{cases} 1 & \text{if } s \vdash p, \\ 0 & \text{if } s \vdash \neg p, \\ \# & \text{in all other cases.} \end{cases}$

Here p is a trivalent proposition. As before, $s \vdash p$ is read as "s supports p" and means

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[28]Here $\neg$ denotes trivalent negation: it maps 0 to 1, 1 to 0, and # to #.

that p maps all worlds in s to 1. Thus $s \vdash \neg p$ will hold when s maps all worlds in s to 0. If s does not support the presuppositions of p, that is if there are #-worlds in s, neither $s \vdash p$ nor $s \vdash \neg p$ can hold. There are therefore two reasons why we can have $?p(s) = \#$: (i) if s includes both 1-worlds and 0-worlds, i.e. s does not settle p; (ii) if s includes #-worlds, i.e. s does not satisfy the presuppositions of p. These two reasons are of course compatible with each other.

It is helpful to compare our denotation to what is assumed in inquisitive semantics. Inquisitive semantics also assumes questions are predicates over sets of worlds, but it does not distinguish positive and negative answers. Thus, the standard inquisitive treatment of polar questions, when converted from set notation to functional notation, is essentially what is given in (49):

$$(49) \qquad ?_{\mathrm{inq}}p = \lambda s. \begin{cases} 1 & \text{if } s \vdash p \text{ or } s \vdash \neg p, \\ 0 & \text{otherwise.} \end{cases}$$

The system I propose here can therefore be seen as an extension of the inquisitive account of polar questions. Unlike the basic inquisitive account, it assigns to them a yes/no-asymmetric denotation: $?p$ and $?(\neg p)$ are different objects.[29] Also unlike the basic inquisitive account, it is integrated within a trivalent logic, making it easy to develop a theory of presupposition projection based on it, as we will see. One may wonder how the usage of the third value (#) in question denotations relates to its usual usage: is the system based on the idea that questions presuppose something? Tentatively, the fact that undecided states are mapped to # can be related to the pragmatic effect of questions, which require the addressees to address them before the conversation can move on. Thus questions arguably do presuppose that certain states will not be undecided, specifically they presuppose that *future Common Grounds* will have addressed them (see also Footnote 37). At this point, however, I do not have a definitive answer as to how to interpret the distinction between positive answers — I certainly do not intend here to develop a system where questions can be "true" or "false". While I believe that this distinction could be linked to certain pragmatic phenomena, this will not be discussed here. Instead, we can consider for now that the distinction is relevant to nothing else than question coordination. The most conservative interpretation would be that it is merely a formal device.[30]

The inquisitive denotation can be defined in terms of ours; we call it the "domain" of a question. Intuitively, as in inquisitive semantics, the domain is the set of states where the question is resolved. Also as in inquisitive semantics (cf. (43)), we can define the "alternatives" of a question, the least specific states where the question is resolved,

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[29]The basic idea of adding yes/no-asymmetry to inquisitive semantics has already been explored by Roelofsen and Farkas (2015) but their system is conceptually quite different. Roelofsen and Farkas (2015), propose a two-dimensional theory where $?p$ has its usual inquisitive denotation on the ordinary dimension (essentially $\{p, \neg p\}$). On the second dimension, called the "highlighting" dimension, $?p$ essentially denotes p.

[30]Under the proposal presented here, answers to a complex question will all be assigned a certain category (positive or negative). For instance, $p \wedge q$ will be a positive answer to $?p \wedge ?q$, while $p \wedge \neg q$ will be a negative one. As an anonymous reviewer points out, there is no clear intuition on how to make this distinction beyond simple polar questions that we could evaluate the proposal against. My claim is that this particular way of dividing up "positive" and "negative" answers will lead to an adequate account of question coordination and presupposition projection, which is enough to justify it. Unless we ascertain that another phenomenon involves a more fine-grained distinction or a different one, there is no reason to be concerned about the premise of categorizing answers beyond what what is intuitively obvious.

which correspond intuitively to the possible answers. For a polar question $?p$, $\text{ALT}(?p)$ will be $\{\mathbb{1}(p), \mathbb{1}(\neg p)\}$, i.e. the denotation that is generally assumed in Hamblin-Karttunen semantics.[31]

(50) $\text{DOM}(Q) := \{s \,|\, Q(s) \in \{0, 1\}\}$

(51) $\text{ALT}(Q) := \{s \,|\, s \in \text{DOM}(Q) \wedge \neg \exists s' \in \text{DOM}(Q). \, s \subsetneq s'\}$

(52)   a. $\text{DOM}(?p) = \{s \,|\, s \vdash p\} \cup \{s \,|\, s \vdash \neg p\}$
         b. $\text{ALT}(?p) = \{\mathbb{1}(p), \mathbb{1}(\neg p)\}$

Finally, we define an operator that maps a question to its *informational commitment*, the union of all states where the question is defined. Intuitively, this is (a superset of) what the questioner takes for granted (the intuition is that the questioner is assuming that the question can be resolved; we will formalize it in Section 3.4).

(53) $\text{INFO}(Q) := \bigcup \text{DOM}(Q)$

In the case of a polar question $?p$, INFO will return the union of p's truth worlds and p's falsity worlds, leaving out the #-worlds. This is essentially the presupposition of p:

(54) $\text{INFO}(?p) = \pi(p)$

This identity will let us derive the fact that a polar questions has the same presuppositions as the corresponding assertion.

To give a concrete illustration of how all the definitions I just introduced work, consider (55a). This sentence presupposes that John once smoked. We therefore assign to it, under a trivalent view of presuppositions, the denotation in (55b). The corresponding polar question in (56a) is assigned the denotation in (56b). In (57) we see that the domain of the question is the union of the set of 1-states and the set of 0-states; these are all the subsets of the set of 1-worlds ($\mathbb{1}(p)$) and the set of 0-worlds ($\mathbb{1}(\neg p)$) respectively. The domain therefore has two maximal elements, the set of 1-worlds and the set of 0-worlds, and these constitute the alternatives as seen in (58). Finally, the informational commitment, seen in (59), is the union of the domain, which is equivalent in our case to the union of the alternatives: the set of worlds where John used to smoke and either stopped or still does, i.e. the set of worlds where John used to smoke.

(55)   a. p: John stopped smoking.
         b. $p = \lambda w. \begin{cases} 1 & \text{if John used to smoke and stopped in } w, \\ 0 & \text{if John used to smoke and still does in } w, \\ \# & \text{if John never smoked in } w. \end{cases}$

(56)   a. $?p$: Did John stop smoking?
         b. $?p = \lambda s. \begin{cases} 1 & \text{if John used to smoke and stopped throughout } s, \\ 0 & \text{if John used to smoke and still does throughout } s, \\ \# & \text{in all other cases.} \end{cases}$

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[31]In the possible extension to constituent questions based on $\exists_G$ that we are going to discuss in Section 4.4, ALT will specifically return (roughly) the Hamblin-Karttunen answers to the question, as opposed to propositions corresponding to the partition cells in the sense of Groenendijk and Stokhof (1984). For polar questions, there is no difference between Hamblin answers and partition cells.

(57) $\textsc{dom}(?p) = \{s \mid ?p(s) = 1\} \cup \{s \mid ?p(s) = 0\}$
$= \mathcal{P}(\{w \mid p(w) = 1\}) \cup \mathcal{P}(\{w \mid p(w) = 0\})$
$= \mathcal{P}(\mathbb{1}(p)) \cup \mathcal{P}(\mathbb{1}(\neg p))$
$= \mathcal{P}(\{w \mid \text{John used to smoke and stopped in } w\})$
$\quad \cup \mathcal{P}(\{w \mid \text{John used to smoke and still does in } w\})$

(58) $\textsc{alt}(?p) = \{\mathbb{1}(p), \mathbb{1}(\neg p)\}$
$= \left\{ \begin{array}{l} \{w \mid \text{John used to smoke and stopped in } w\}, \\ \{w \mid \text{John used to smoke and still does in } w\} \end{array} \right\}$

(59) $\textsc{info}(?p) = \bigcup \textsc{dom}(?p)$
$= \left(\bigcup \mathcal{P}(\mathbb{1}(p))\right) \cup \left(\bigcup \mathcal{P}(\mathbb{1}(\neg p))\right)$
$= \mathbb{1}(p) \cup \mathbb{1}(\neg p)$
$= \pi(p)$
$= \{w \mid \text{John used to smoke in } w\}$

3.3   Defining the connectives

The extension of the system to conjunctions and disjunctions of polar questions is straightforward. The type we assign to questions is conjoinable, and we are going to assume that *and* and *or* apply to it directly. In a trivalent theory, there are several sensible candidates for conjunction and disjunction. Peters (1979) argues that presupposition projection patterns suggest we should adopt left/right-asymmetric connectives. These connectives are known as *Peters connectives* or *Middle Kleene* connectives (in contrast to Strong and Weak Kleene, cf. Beaver and Krahmer 2001); their definitions are given in the tables below (the left-hand argument is in rows).[32]

(60)

$\wedge$	0	1	#
0	0	0	0
1	0	1	#
#	#	#	#

$\vee$	0	1	#
0	0	1	#
1	1	1	1
#	#	#	#

Our assumption will be that question coordination obeys Middle Kleene rules. From this we immediately get a denotation for conjunctive and disjunctive questions, and we can compute their domain and therefore their resolution conditions (under the assumption that a question is resolved when a proposition in its domain is known to be true).

Here we see that the distinction between positive and negative answers, which has played no role so far, will crucially affect the denotation of coordinated questions.

3.4   Deriving presupposition projection

To give an account of presupposition projection in our system, we need a version of Stalnaker's bridge for questions, or in other words, we need to specify what in a question's denotation makes it felicitous to ask or not in a given context.

As mentioned above, we are going to identify the presuppositions of a question with the output of INFO. The motivation for this view goes as follows: in general, questions do

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[32]George (2014) motivates the MK connectives by showing how they can be derived from classical bivalent logic together with some general assumptions about definedness conditions.

not modify the set of worlds in the context set, but they restrict what continuations are allowed, in the sense that unless the participants contribute an assertion that addresses the question, the conversation will be at an impasse.[33] It seems natural then to consider that a speaker should not ask a question if it is not established (in the Common Ground) that the participants are capable in principle of contributing such an assertion. It should be necessary, then, that when a question is asked, the Common Ground entail that there exist a proposition that is true at the actual world and addresses the question. Going back to our system, the speaker should only utter Q if it is established that there exist states in $\text{DOM}(Q)$ that contain the actual world, or equivalently that the Common Ground supports $\text{INFO}(Q)$.

We have already seen that $\text{INFO}(?p) = \pi(p)$. Thus, we predict a polar question to presuppose the same thing as the corresponding assertion. This is a well-known fact, and is sometimes used as a test or even a definition for presuppositions.[34]

3.5 Conjunctive questions

We can now see what our system predicts conjunctive questions to mean.

Denotation and presuppositions The denotation of a conjunction of questions $?p \wedge ?q$ is as follows:

(61) $?p \wedge ?q = \lambda s. \begin{cases} 1 & \text{if } s \vdash p \text{ and } s \vdash q, \\ 0 & \text{if } s \vdash \neg p \text{ or if } s \vdash p \text{ and } s \vdash \neg q, \\ \# & \text{in all other cases.} \end{cases}$

From this we can compute the domain and the alternatives. The domain being the downwards closure of the alternatives, it is sufficient to give the alternatives:

(62) $\text{ALT}(?p \wedge ?q) = \{\mathbb{1}(\neg p), \mathbb{1}(p \wedge \neg q), \mathbb{1}(p \wedge q)\}$

The alternatives are mutually exclusive. If p has no presupposition, they form a three-cell partition of the logical space. The three cells are essentially those I called the tripartition when discussing H/K semantics, and we have seen that they correspond intuitively to the possible answers to our presuppositional conjunctive question (4).

We can compute the predicted presupposition of a conjunctive question, as follows:

(63) $\begin{aligned} \text{INFO}(?p \wedge ?q) &= \bigcup \text{DOM}(?p \wedge ?q) \\ &= \bigcup \text{ALT}(?p \wedge ?q) \\ &= \mathbb{1}(\neg p) \cup (\mathbb{1}(p) \cap \mathbb{1}(q)) \cup (\mathbb{1}(p) \cap \mathbb{1}(\neg q)) \\ &= \mathbb{1}(\neg p) \cup (\mathbb{1}(p) \cap (\mathbb{1}(q) \cup \mathbb{1}(\neg q))) \\ &= \mathbb{1}(\neg p) \cup (\mathbb{1}(p) \cap \pi(q)) \\ &= (\mathbb{1}(\neg p) \cup \mathbb{1}(p)) \cap (\mathbb{1}(\neg p) \cup \pi(q)) \\ &= \pi(p) \cap (\mathbb{1}(\neg p) \cup \pi(q)) \end{aligned}$

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[33]This is of course an idealization where we ignore, among other things, the possibility of avowing complete or partial ignorance ("I don't know", "Probably", etc.), the possibility of rejecting the question ("who cares?"), the possibility of providing an implicit answer ("A: Did John come? B: He was sick."), the existence of rhetorical questions, and so on.

[34]I am ignoring here the various pragmatic presuppositions that are associated to asking a questions: that the facts have not been established before in the discourse, that the answer is accessible, etc.

The end result is an intersection of two terms. The first term is $\pi(p)$, the presupposition of p: for $?p \wedge ?q$ to be felicitous, all worlds in the context therefore have to satisfy the presuppositions of p. The second term means that worlds in the context set should either make p false or satisfy the presuppositions of q. Essentially, this is equivalent to saying that they should satisfy the material conditional $p \rightarrow \pi(q)$. These two conditions on the context are exactly the ones that have been argued since Karttunen (1973; 1974) to describe presupposition projection in conjunctive assertions, and that I have argued in Section 1 also describe presupposition projection in conjunctive questions. This is not surprising, because once we have derived the tripartition, this projection pattern follows in a fairly theory-agnostic way.

The main difference with H/K semantics and inquisitive semantics here is that the way we derive the tripartition is significantly different. In these theories, a conjunction of polar questions $?p \wedge ?q$ most naturally corresponds to what I called the quadripartition, and the tripartition can only be derived by assuming a left/right-asymmetric structure ($?(p \wedge ?q)$ in inquisitive semantics), if it can be derived at all. We get the tripartition from a left/right-symmetric structure because our conjunction is left/right-asymmetric in its semantics.

Contrast with earlier accounts   The reason that earlier accounts derive the quadripartition is of course not just that it naturally falls out from the formalism, but also that it is thought to be an empirically adequate characterisation of what conjunctive questions mean in general, absent presuppositional effects. It is of course desirable that we maintain this empirical coverage.[35]

There are several ways in which we can derive the quadripartition in our system. First, we can assume that *and* may also denote a Weak Kleene trivalent conjunction. Second, we can assume there is an operator T, defined as in (64), that makes all the answers positive. Then, $\mathrm{T}(?p) \wedge ?q$ (or $\mathrm{T}(?p) \wedge \mathrm{T}(?q)$) would have the alternatives of the quadripartition. Finally, we can assume that whenever we think we observe the quadripartition, what we are actually seeing is two separate questions and the coordination takes place at the level of attitude or speech acts. The result of high-level coordination is indeed virtually indistinguishable in most cases from the quadripartition: if I know the answer to both conjuncts, I know the answer to the quadripartition. This is the basis of what we have called reductionist theories (e.g. Hirsch 2017; Xiang 2020).

(64) $\mathrm{T}(Q) := \lambda s. \begin{cases} 1 & \text{if } Q(s) = 1 \text{ or } Q(s) = 0 \\ \# & \text{if } Q(s) = \# \end{cases}$

Whatever solution we adopt, we end up with a system where conjunctive questions can have the resolution conditions of either the tripartition, or the quadripartition. We assume that examples of presuppositional filtering involve the tripartition; recall that

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[35]H/K semantics and inquisitive semantics generate a variety of other readings, as we have seen. In particular, there is a polar reading ($\{p \wedge q, \neg(p \wedge q)\}$), and a reverse tripartition ($\{\neg q, p \wedge q, \neg p \wedge q\}$). Without further assumptions, our system generates none of these either. In principle, the polar reading can be generated if we assume that *and* can take scope below ?, or if we have an operation that collapses the 0-alternatives together. The reverse tripartition can be generated if we assume that *and* can also denote a right-to-left rather than left-to-right Middle Kleene connective, or equivalently that it can combine either with the first or the second conjunct first. The reverse tripartition is unattested, and, pending further investigation, my judgement is that the polar reading is not available either (cf. Footnote 23), so these extra assumptions are not necessary.

because it is yes/no-symmetric, the quadripartition cannot be the basis of an analysis of this phenomenon. When there is no presuppositional trigger that forces the tripartition, in principle, conjunctive questions should be ambiguous. This raises the following question: why do we generally observe quadripartitive readings, and are there counterexamples?

I would like to claim that, in an appropriate context, conjunctive questions can have the answerhood conditions of the tripartition, even when no presupposition is present to force the reading. An example is given in (65). (65a) has the form $?p \wedge ?q$, and a fully knowledgeable speaker may answer it felicitously with $\neg p$, even though that should be under-informative relative to the quadripartition. This tends to show that the tripartition does exist. The tripartition also predicts that $\neg q$ as an answer should be judged uncooperative. What might in fact be the case, even though the judgment is not clear, is that $\neg q$ as an answer implicates p. While the present account does not offer an immediate explanation for this fact, it would be impossible to derive it from the quadripartition due to its yes/no-symmetry; the tripartition does not have this problem.[36] Additionally, the quadripartition would predict that if p is replaced by p' which is equivalent to $\neg p$, as in (65b), then we should see no difference. What we observe is that p' becomes an uncooperative answer.

(65)   Context: *Mary has been working in an underground garage. John has been outside so he knows the weather, and he was the last person to use the grill so he knows about the charcoal.*

    a.   Mary: Is the weather nice outside, and is there any charcoal left?
       (i)   John: It's raining outside. ($\neg p$)
           $\not\rightsquigarrow$ There is no charcoal left.
           $\not\rightsquigarrow$ There is some charcoal left.
       (ii) ?John: There is no charcoal left. ($\neg q$)
           $\overset{?}{\rightsquigarrow}$ The weather is nice. (p)
    b. # Mary: Is it raining outside, and is there any charcoal left?
       John: It's raining outside. ($p' \equiv \neg p$)

A related example is given in (66a). Here the context guarantees that the quadripartition cannot possibly be addressed by the participants in the conversation, while the tripartition can. A conjunctive question is perfectly felicitous, suggesting that the issue it raises is the tripartition. An advocate of the quadripartition will of course argue that we have not independently established that it is infelicitous to ask a conjunctive question that cannot be fully addressed, but if conjunctive questions always denote the quadripartition and not being able to tell between some of the cells is fine, there is no reason why reversing the order of the conjuncts as in (66b), or replacing the left-hand nucleus with its negation as in (66c) leads to infelicity.

(66)   Context: *Mary has applied for a grant. The decision is due to come by physical mail; none of the participants has any other way of finding out about it. Someone asks:*

    a.   Has the mail arrived, and did Mary get her grant?
    b. #Did Mary get her grant, and has the mail arrived?

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[36]One could in particular use the fact that $p \wedge \neg q$ is a good answer and entails $\neg q$. Asserting $\neg q$ always raises the contextual probability of $p \wedge \neg q$ relative to the other two cells, from which an inference that $p \wedge \neg q$ holds might follow as a relevance implicature.

c. #Has the mail failed to arrive, and did Mary get her grant?

Thus, the tripartition can be observed in non-presuppositional cases. What both our examples have in common is that the reason the speaker is not interested the quadripartitive issue is clear from the context: in (65), it is more information than they need to establish whether they can carry out their plan or not, and in (66a), it is more information than the listener can possibly be knowledgeable about.[37] This suggests that the reason that most examples found in the literature lend themselves to quadripartitive readings is purely pragmatic: in the absence of an explicit context, the quadripartitive reading is more natural because it requires less specific assumptions on the speaker's motivations. Indeed, if p and q are arbitrary, unrelated propositions, it is hard to see why a speaker would be interested in the tripartition formed from $?p \wedge ?q$, while being interested in the quadripartition is just being interested in both $?p$, and $?q$, possibly for different reasons.

It should be noted as a counterpoint that most speakers seem to judge matrix conjunctive questions somewhat degraded out-of-the-blue. This is puzzling if conjunctive questions can easily denote the quadripartition, which I have just argued should be easy to accept without context. It would be consistent, however, with the view that conjunctive questions always denote the tripartition, and that the quadripartition is an artefact of high-scope coordination, which might be unavailable or marked in matrix questions.

Another counterpoint is that apparent quadripartitive readings of conjunctive questions do not necessarily constitute strong evidence that these questions denote the quadripartition. First, it is in general felicitous for answerers to provide more information than the questioner was explicitly asking for. (67) is a felicitous dialogue and is not taken to provide evidence that B's utterance is a semantic answer to A's question. Thus the fact that a conjunctive question $?p \wedge ?q$ can generally be answered with $\neg p \wedge \neg q$ is not necessarily incompatible with a tripartitive analysis. In contrast, being less precise than was asked is not felicitous in general, which is why (65) is puzzling from a quadripartitive perspective.

(67)   A: Is the weather nice outside?
        B: Yes, but there is no charcoal left.

3.6   Disjunctive questions

Disjunctive questions work very similarly.

Denotation and preuppositions   We derive for them the following denotation and alternatives:

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[37]One might in fact argue that both (65) and (66a) do involve presupposition filtering, but with pragmatic presuppositions. In (65), the presupposition that the second question is relevant to some greater issue is not satisfied in the context: it only matters whether there is charcoal if the weather is nice, as the goal is to take out the grill. In (66a), similarly, the presupposition that knowledge of the answer to the second question is accessible to the participants is not satisified, but what is true is that if the mail has arrived, participants have a chance knowing whether Mary got her grant. This is consistent with the interpretation of the trivalent system offered in Section 3.2: a polar question $?q$ on its own presupposes that future Common Grounds will entail q or $\neg q$, but when it is in the second conjunct of $?p \wedge ?q$, this presupposition can be filtered, and it is fine if the question is uninteresting or unknowable when p is false.

(68) $\quad ?p \vee ?q = \lambda s. \begin{cases} 1 & \text{if } s \vdash p \text{ or if } s \vdash \neg p \text{ and } s \vdash q, \\ 0 & \text{if } s \vdash \neg p \text{ and } s \vdash \neg q, \\ \# & \text{in all other cases.} \end{cases}$

(69) $\quad \text{ALT}(?p \vee ?q) = \{\mathbb{1}(p), \mathbb{1}(\neg p \wedge q), \mathbb{1}(\neg p \wedge \neg q)\}$

The predicted presupposition is derived as follows:

(70) $\quad \begin{aligned}\text{INFO}(?p \vee ?q) &= \bigcup \text{DOM}(?p \vee ?q) \\ &= \bigcup \text{ALT}(?p \vee ?q) \\ &= \mathbb{1}(p) \cup (\mathbb{1}(\neg p) \cap \mathbb{1}(q)) \cup (\mathbb{1}(\neg p) \cap \mathbb{1}(\neg q)) \\ &= \mathbb{1}(p) \cup (\mathbb{1}(\neg p) \cap (\mathbb{1}(q) \cup \mathbb{1}(\neg q))) \\ &= \mathbb{1}(p) \cup (\mathbb{1}(\neg p) \cap \pi(q)) \\ &= (\mathbb{1}(p) \cup \mathbb{1}(\neg p)) \cap (\mathbb{1}(p) \cup \pi(q)) \\ &= \pi(p) \cap (\mathbb{1}(p) \cup \pi(q)) \end{aligned}$

The only difference with conjunction is that we have p instead of $\neg p$ in the second term. Thus, we again predict the presuppositions of p to project, but the material conditional that should hold throughout the context set is now $\neg p \rightarrow \pi(q)$ rather than $p \rightarrow \pi(q)$. Once again, these contextual requirements are exactly what Karttunen (1973; 1974) argues should be derived for disjunctive assertions, and what I argued in Section 1 to be the observed pattern in disjunctive questions.

Contrast with earlier accounts: the open reading   Per (69), we predict that the issue raised by a disjunctive question involves three mutually-incompatible possibilities, corresponding essentially to the answer set $\{p, \neg p \wedge q, \neg p \wedge \neg q\}$. This is intuitively a good match for the open reading of disjunctive questions, which also has three good answers. However, our partition cells do not exactly match what Roelofsen and Farkas (2015) derive in their inquisitive account (and what we derived earlier in our H/K reconstruction of it). Roelofsen and Farkas (2015) assume for open disjunctive questions the structure $?(p \vee q)$, from which we get the equivalent of $\{p, q, \neg p \wedge \neg q\}$. Thus our alternatives exhibit a left/right asymmetry that is not found in their account. Concretely, they predict that the question is resolved when the participants know the positive answer to either disjunct, even though they might be ignorant about the other one, while in our system the participants need to know for sure what the truth value of p is.

It is not clear which prediction is more correct: an example like (71) shows some left/right asymmetry in the resolution conditions, as we predict: if the answerer says q, we draw the inference that $\neg p$, which would be consistent with the idea that the partition cell being identified here really is $\neg p \wedge q$. With a symmetric approach, it is formally impossible to account for the contrast. The characterization that the question has not been resolved if one is ignorant about John might nevertheless be too strict.

(71) $\quad$ A: Did John↗ arrive or did Mary?↗

$\qquad$ a. $\quad$ B: John arrived. $\rightsquigarrow$ Mary might or might not have arrived.

$\qquad$ b. $\quad$ B: Mary arrived. $\overset{?}{\rightsquigarrow}$ (B thinks) John didn't arrive.

We can add that, in some other cases, no asymmetry can clearly be detected:[38]

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[38]I thank an anonymous reviewer for this example.

(72) A: Can I call you tomorrow at 3pm↗ or on Friday at 1pm?↗

    a. B: You can call me tomorrow. ⤳ B does not want A to call them on Friday.

    b. B: You can call me on Friday. ⤳ B does not want A to call them tomorrow.

This could suggest that, in at least some cases, $\{p, q, \neg p \wedge \neg q\}$ really is the answer set that we want. We can generate it within our system by assuming that disjunction can denote a Strong Kleene, rather than Middle Kleene connective, or that it always does. If that is the case, we expect to observe presupposition filtering not just from the right-hand disjunct through the left-hand one, but also the other way round. Recall that the data is unclear on this point: we do observe a left-right contrast, but it is much less clear than for conjunction, and in the case of assertions, Rothschild (2011) among others has argued that the data does not support an asymmetric view of filtering. In a way, our discussion whether the resolution conditions of open disjunctive questions are left/right-asymmetric or not reflects the earlier debate whether presupposition filtering in disjunctive assertions is left/right-asymmetric or not. We will leave both questions open, but venture that they should receive a common answer.

Contrast with earlier accounts: the closed reading Disjunctive questions also have a closed reading (or alternative question reading). In an alternative question like (73), the two propositions p and q inside each disjunct are presupposed to be exhaustive ($p \vee q$ holds) and exclusive ($p \wedge q$ does not hold). In H/K semantics, alternative questions are generally assigned the denotation $\{p, q\}$; the equivalent move in inquisitive semantics is to assign to them the structure $p \vee q$. Alternative questions can be resolved by asserting either of the two disjuncts, p or q.

(73) Did John↗ arrive or did Mary?↘

To derive a two-element answer set from the denotation of $?p \vee ?q$, it seems natural to assume a mechanism through which a presupposition that a question can be answered positively can be generated, for instance through an operator POS:

(74) $\text{POS}(Q) := \lambda s : Q(s) = 1. \, Q(s)$

(75) $\text{ALT}(\text{POS}(?p \vee ?q)) = \{p, \neg p \wedge q\}.$

If that is our account of alternative questions, we correctly predict that they have two possible answers, and that p and q are good answers in a context where exhaustiveness ($p \vee q$) and exclusivity ($\neg(p \wedge q)$) hold (since in such a context, q and $\neg p \wedge q$ are equivalent). We also derive the exhaustiveness presupposition. We do not derive the exclusivity presupposition in any way, however, so we have to assume that it arises from a separate mechanism, perhaps as an effect of focus structure in the question nucleus: cf. Section 4.5 for discussion. Under the Strong Kleene variant of the theory, the alternatives are $\{p, q\}$, as in the traditional account, but again the exclusivity presupposition is not derived.

Finally, recall that in inquisitive semantics and H/K semantics, a structure like $?p \vee ?q$ will result in the answer set $\{p, \neg p, q, \neg q\}$, which has to be ruled out one way or another, as there is no reading of disjunctive questions that looks like it. The system here does not generate this problematic answer set.

3.7 Interim conclusion

At this point, I have given an account of conjunctive questions and disjunctive questions such that the presupposition projection patterns that we observed in Section 1 can be easily derived. I have argued that the predicted resolution conditions for conjunctive and disjunctive questions are attested, and that those predicted by earlier accounts can also be generated with simple extra assumptions.

The analysis offers a fully parallel account of conjunctions and disjunctions of polar questions. They are both assumed to have the structure that the surface syntax suggests, that is to say, $?p \square ?q$, where $\square$ is the (generalized) truth-conditional binary connective that you would expect (in its Middle Kleene variant).

Furthermore, assuming that propositions receive a classical trivalent denotation (type $s \to t$ where t is the type of trivalent truth values), we can give a single entry for "and" and for "or" that applies to both questions and propositions. We just have to make them polymorphic truth-conditional connectives, in a way similar to what Partee and Rooth (1983) propose (with the added twist that we understand truth values to be trivalent):

(76) a. $[\![\text{and}]\!] = \lambda Q_{\alpha \to t}. \lambda P_{\alpha \to t}. \lambda x_\alpha. P(x) \wedge Q(x)$
      b. $[\![\text{or}]\!] = \lambda Q_{\alpha \to t}. \lambda P_{\alpha \to t}. \lambda x_\alpha. P(x) \vee Q(x)$
      (type $\forall \alpha. (\alpha \to t) \to (\alpha \to t) \to \alpha \to t$ in both cases)

Thus all our desiderata as listed in Section 2.3 are met.

Of course, the theory presented here is probably not the only way to achieve these desidarata, and slight reformulations as well as radically different (possibly dynamic) theories could probably be offered. I do believe that they would involve some form the following idea: there is some notion of a positive and a negative answer to a question, and the connectives should interact with that in a similar manner to the way they interact with propositions. At an even more abstract level, the way we analyse a question should be more parallel to the way we analyse a similar proposition than it currently is. As I discussed in Section 2, and demonstrated for the case of answer set semantics, the lack of such properties in earlier accounts makes it very difficult to deal with the data presented here.

4 Further predictions and challenges for the trivalent inquisitive theory of questions

This final section explores in more detail the theory we have just developed: we first revisit some data points that we have set aside so far, and show how we are able to deal with them, and then briefly show we could develop the theory into a general theory of questions.

4.1 The effect of *or not*

Recall that, even though an alternative question using *or not* such as (77) has the same resolution conditions as the corresponding polar question, when such questions occur in a coordinated structure, they do not license presupposition filtering (cf. (9b)/(18b)).

(77) Did John come or not?

Within our framework, *or not* questions are naturally analysed as instantiating the structure $?p \vee ?(\neg p)$, which we will abbreviate as $\tilde{?}p$. Thus, they receive the denotation in (78). This makes *or not* questions completely similar to polar questions in terms of their domain, their alternatives, and their informational commitment, as seen in (79).

(78) $\quad \tilde{?}p = \lambda s. \begin{cases} 1 & \text{if } s \vdash p \text{ or } s \vdash \neg p, \\ \# & \text{otherwise.} \end{cases}$

(79)
     a. $\text{DOM}(\tilde{?}p) = \{s \mid s \vdash p \vee s \vdash \neg p\}$
     b. $\text{ALT}(\tilde{?}p) = \{\mathbb{1}(p), \mathbb{1}(\neg p)\}$
     c. $\text{INFO}(\tilde{?}p) = \pi(p)$

The main way in which *or not* questions are predicted to differ from polar questions is that they behave differently with respect to coordination. Consider for instance $\tilde{?}p \wedge ?q$:

(80) $\quad \tilde{?}p \wedge ?q = \lambda s. \begin{cases} 1 & \text{if } s \vdash p \wedge q \text{ or } s \vdash \neg p \wedge q, \\ 0 & \text{if } s \vdash p \wedge \neg q \text{ or } s \vdash \neg p \wedge \neg q, \\ \# & \text{in all other cases.} \end{cases}$

(81)
     a. $\text{ALT}(\tilde{?}p \wedge ?q) = \{p \wedge q, \neg p \wedge q, p \wedge \neg q, \neg p \wedge \neg q\}$
     b. $\text{INFO}(\tilde{?}p \wedge ?q) = \pi(p) \cap \pi(q)$

Unlike $?p \wedge ?q$, $\tilde{?}p \wedge ?q$ evokes a four-cell partition. Furthermore, the presuppositions of the second conjunct are predicted to project no matter what. Thus, we predict (82) (repeated from (9b)) to be infelicitous.

(82)   #Is Syldavia a monarchy or not, and is the Syldavian monarch a progressive?

4.2 Local contexts, triviality, and bivalent propositions

In Section 1, we stated a generalization of the observed data in terms of local contexts. The point was to also capture certain redundancy effects, seen in (83a) and (83b) (repeated from (26a) and (26b)), and in (84a) and in (84b) (repeated from (27a) and (27b)).

(83)
     a. #Is Ann in Paris and is she in France?
     b. #Is Ann in Paris and is she in London?

(84)
     a. #Is Ann away from Paris or is she in France?
     b. #Is Ann away from Paris or is she in London?

The infelicity of all these examples is predicted by a generalization in terms of local contexts (the local context of q in $?p \wedge ?q$ entails p, the local context of q in $?p \vee ?q$ entails $\neg p$) together with a triviality constraint: no constituent should be trivially true or false in its local context.

Here, we have been adopting a trivalent theory of presuppositions, and we have not mentioned local contexts so far. It is nevertheless straightforward to derive local contexts that are exactly the ones we need, by applying Schlenker's (2009) procedure to our question denotations.[39] The facts above will then be correctly predicted.

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[39]Schlenker (2009) applies his procedure to bivalent propositions, but the procedure is mostly agnostic with respect to the type of what it is looking at and it is straightforward to adapt it to functions from states to trivalent values. The one thing we need to specify is how contextual restriction works with states:

If we derive local contexts in this way, since they can also be used to predict presupposition projection, as shown by Schlenker (2009), then we no longer need trivalent connectives at the propositional level. Thus the Middle Kleene connectives would exclusively serve for questions, and the present proposal would come down to the fact that question coordination obeys some form of Middle Kleene logic, without tying it as closely to presupposition projection. Since the empirical predictions are the same in the cases that we have been looking at so far, the choice between these two approaches has to be made on other grounds.

It should be noted that at least as far as these particular facts are concerned, we do not absolutely need a theory of local contexts. The various infelicitous examples listed above run afoul of plausible pragmatic constraints on question denotation:

- (83a) would be banned by a global redundancy constraint: we predict it to be equivalent to its first conjunct on its own.

- (83b) is predicted to only have 0-alternatives, as its 1-alternative is a contradiction. It is plausible that this is pragmatically deviant: the yes/no-asymmetries in polar questions identified in the literature (Büring and Gunlogson 2000; Roelofsen and Farkas 2015, a.o.) suggest the the positive answer has to have some special significance in the discourse context.[40]

- Within existing H/K approaches,(84a) would be banned by a unique answer constraint: the two disjuncts overlap in natural contexts. In our system, the disjunction is equivalent to "Is Ann away from Paris or not?", and could be banned by a global redundancy constraint as in the case of (83a).

- A unique answer constraint would also ban (84b) in H/K theory. In our system, the disjunction is in fact equivalent to its first disjunct "Is Ann away from Paris?", so a global redundancy constraint would ban it.

4.3 Embedding and the relation between questions and propositions

Predicates like *know* can embed both questions, as in (85a), and propositions, as in (85b). It is of course desirable to analyse these two examples with a single entry for *know*, while accounting for the major difference in meaning between them.

(85) a. John knows whether Mary came.
      b. John knows that Mary came.

Uegaki (2019) compares several approaches, including:

- *Q-to-P approach*: assuming that *know* embeds propositions, and that there is an operator that converts questions into propositions.

- *P-to-Q approach*: assuming that *know* embeds questions, and that there is an operator that converts propositions into questions.

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we take it that if the context set is C, the set of contextually permissible states is the set of subsets of C.

[40]I think the distinction between 1-alternatives and 0-alternatives would be helpful in a theory of question bias, but this is not the place to develop this idea.

- *Uniformitarian approach*: giving the same kind of denotation to propositions and questions.

He argues that the Q-to-P approach cannot account for certain phenomena that are easy to analyze under the P-to-Q and uniformitarian approaches.[41]

At any rate, our theory can in principle be made compatible with all three strategies. Here I will briefly describe an implementation of P-to-Q.[42] The P-to-Q approach can be implemented in our system in a fairly conservative way, mostly following the same lines as what would be done in answer set semantics. We can assume, as we have been doing so far, that propositions have type $s \to t$ where t is the type of trivalent truth values. The following conversion operator converts a proposition to a question (type $\{s\} \to t$, where $\{s\}$ is the type of (non-trivalent) sets of worlds):

(86) $\quad$ THAT$(p) := \lambda s. \begin{cases} 1 & \text{if } s \vdash p, \\ \# & \text{otherwise.} \end{cases}$

As a remark, THAT$(p) = $ POS$(?p)$. Then, the following entry for *know* will deliver the appropriate truth conditions and presuppositions for (85a) and (85b).

(87) $\quad$ $\llbracket \text{know} \rrbracket = \lambda w.\, \lambda Q.\, \lambda x : w \in \text{INFO}(Q).\ \text{Dox}^w(x) \vdash \text{ANS}_S(Q)(w)$

where ANS$_S$ is an answerhood operator that computes the answer to a question:

(88) $\quad$ $\text{ANS}'_S(Q)(w) = \left(\bigwedge \{ p \mid p \in \text{ALT}(Q), p(w) = 1 \} \right)$
$$\land \left(\bigwedge \{ \neg p \mid p \in \text{ALT}(Q), p(w) = 0 \} \right)$$

Applied to a polar question $?p$, ANS$_S$ returns p if p is true and $\neg p$ if p is false. The definition does not have to be as complicated as it is to get this result, but it will also extend to constituent questions as we will see below. In the propositional case, we correctly capture that "know that p" presupposes that p is true, as INFO$($THAT$(p)) = \mathbb{1}(p)$.

4.4 Extending the proposal to constituent questions

I will only sketch an extension of the system to constituent questions here, in order to show what it might look like. I will make a specific assumption on how the denotation of constituent questions should be, though alternative approaches can certainly be pursued. There are of course a number of issues that might prove challenging to tackle, and I will mention a few.

Trivalent existential quantification $\quad$ To extend our system to constituent questions like (89), we need to give them a denotation that is a trivalent predicate upon sets of worlds, as for polar questions.

(89) $\quad$ Who came?

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[41]For discussion of the differences between P-to-Q and uniformitarian approaches, see Roelofsen 2019 and references therein.

[42]Q-to-P can be implemented through answerhood operators; cf. Section (97). The uniformitarian approach can be implemented by assigning to propositions the same type as questions: since the domain of questions is richer it is straightforward to embed the domain of propositions in it.

The assumption that we make here, then, is that constituent questions are existentially quantified-into polar questions. This is consistent with the fact that *wh*-words are cross-linguistically similar to indefinites, and it is going to deliver the alternatives that we want. We can implement the idea as follows:

(90) $[\![\text{who}]\!] = \lambda P. \lambda s. \exists x \in \mathbf{human}.\, ?(P(x))(s)$

(91) $[\![(89)]\!] = \lambda s. \exists x \in \mathbf{human}.\, ?(\mathbf{came}(x))(s)$

For the sake of perspicuity, we will ignore the restriction to humans and adopt the following notation:

(92) $?x.\, P(x) \equiv \lambda s. \exists x.\, ?P(x)(s)$

What the denotation we derive actually comes down to depends on what kind of trivalent existential quantifier we adopt. I propose to adopt the lazy trivalent existential quantifier defined by Beaver and Krahmer (2001) and George (2014), which is described in (93).[43]

(93) $\exists x \in D.\, P(x) = \begin{cases} 0 & \text{if for all } y \in D,\, P(y) = 0, \\ 1 & \text{if there is } y \in D \text{ such that } P(y) = 1, \\ \# & \text{in all other cases.} \end{cases}$

We then derive the following domain, alternatives and informational commitment:

(94) $\text{DOM}(?x.\, P(x)) = \{s \mid [\forall x.\, s \vdash \neg P(x)] \vee [\exists x.\, s \vdash P(x)]\}$

(95) $\text{ALT}(?x.\, P(x)) = \{\bigcap_x \mathbb{1}(\neg P(x))\} \cup \{\mathbb{1}(P(x)) \mid x\}$

(96) $\text{INFO}(?x.\, P(x)) = [\exists x.\, P(x)] \vee [\forall x.\neg P(x)]$

The alternatives are the *mention-some* answers to '(89), plus the negative answer. The presupposition we predict for a constituent question involving a presuppositional predicated, such as (97), is a roughly-existential presupposition similar to what George (2014) predicts for existential quantification: at least one person has a dog and is walking it, or everyone has a dog and nobody is walking their dog.

(97)   Who's walking their dog?

Answerhood operators for constituent questions   Much of the discussion in the literature about the semantics of constituent questions is applicable to our system as long as we are able to redefine the various notions of answers that have been established in the literature. Since the alternatives that we derive are very similar to what is commonly assumed in Hamblin-Karttunen semantics, this is relatively straightforward to accomplish.

As we have already seen, our alternatives already correspond to the *mention-some* answers to the question. A second kind of answers is found in the strongly-exhaustive answers, which provide full information about thetruth of the alternatives. The strongly-exhaustive answer to "who came", for instance, is the proposition asserting that the people who came came, and that the people who did not come did not come. We have already seen

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[43]An natural alternative would be a universally-projecting existential quantifier, such that $\exists x.\, \phi_x$ is defined only if ϕ_x is defined for all x. This would make the alternatives to the questions congruent with its strongly-exhaustive answers, which is unwelcome because there is no obvious way to derive weakly-exhaustive and mention-some answers from strongly-exhaustive answers.

above, in (88), the definition of an operator computing the strongly-exhaustive answer, and an entry for *know* based on it. Finally, some question-embedding predicates like *surprise* are thought to be sensitive to weakly-exhaustive answers. The weakly-exhaustive answer to "who came" is the proposition that the people who came came, with no information about the fact that nobody else did. Thus, (98) means that John was surprised that the people who came came, as opposed to being surprised that the people who did not come did not come.

(98)   It surprised John who came.

It is also straightforward to define an operator that computes the weakly exhaustive answer through a conjunction, as seen in (99). Here we are exploiting the difference between alternatives that are mapped to 0 and to 1; this enables us to go around the fact that our alternatives include an extra element ("nobody came") relative to what is generally assumed in H/K semantics, where it is often assumed that the negative answer is not a semantic answer (e.g. Dayal 1996).

(99) $\text{ANS}_W(Q)(w) := \bigwedge \{p \,|\, p \in \text{ALT}(Q) \wedge Q(p) = 1 \wedge p(w) = 1\}$

To conclude, the system proposed here can be extended to constituent questions in a way that is fairly close to H/K semantics and lets us replicate its basic ingredients, with the main difference (which we can get around if we wish) lying in our assumption that the negative answer to a constituent question is an actual semantic alternative. The parallelism with H/K semantics makes it so that much of the discussion found in the literature is applicable to the trivalent system.

4.5   Some potential challenges

A problem with plurals and Dayal's presupposition   A common alternative to the definition of weakly-exhaustive answers found above involves maximization rather than conjunction, following Dayal (1996). If the domain includes Ann and Bill, and *who* is taken to also range over plural individuals, under standard assumptions a constituent question like (89) should denote the set in (100) in H/K semantics.

(100) $\{\text{Ann came}, \text{Bill came}, \text{Ann and Bill came}\}$

Because the set is closed under conjunction, the weakly-exhaustive answer is always in the set, and it can be recovered without a conjunction operation. Instead, it can be defined as the maximal element (under entailment) within the true answers:

(101) $\text{ANS}_W(Q_{HK})(w) := \iota p.\, p \in Q \wedge p(w) \wedge [\forall p' \in Q.\, p'(w) \rightarrow p \vDash p']$

The expression above is only defined if there is a single p verifying the scope of ιp. It is frequently assumed that questions in fact presuppose that the actual world makes the expression above defined: this is *Dayal's presupposition*, also known as the *strongest answer* presupposition. Under this assumption, among other niceties, the difference in presuppositions between "which student came?" (assumed to be as (102)) and "which students came?" (assumed to be as (100)) follows naturally.

(102) $\{\text{Ann came}, \text{Bill came}\}$

Unfortunately, this line of thought is not directly transferable to our system. Indeed,

because we define alternatives as maximal elements under set inclusion, we are committed to the fact that alternatives are logically independent. Thus, we cannot have an alternative set that looks like (100). The set of positive alternatives for (89) can only look like (102), whether the *wh*-word ranges over singular individuals only or over both singular and plural individuals.

This has at least two consequences. First, we need to define weakly-exhaustive answers in the conjunctive manner above. Second, the uniqueness presupposition of *which student came* cannot be derived from a more general presupposition of questions. Instead, it has to be analyzed as a presupposition specific to *which* questions.[44]

Deriving the exclusivity inference in alternative questions A related issue is that, as we have seen above, we do not derive the exclusivity presupposition of alternative questions. Recall that an alternative question or closed disjunctive question carries the presupposition that exactly one of the disjuncts is true. The presupposition can be further decomposed into exhaustiveness (at least one disjunct is true: $p \vee q$) and exclusivity (both disjuncts are not true together: $\neg(p \wedge q)$). With the denotation that is traditionally assumed in H/K semantics, $\{p, q\}$, both exhaustiveness and exclusivity can be derived from a strongest answer constraint in the spirit of Dayal (1996). In our system, assuming that a closed disjunctive question denotes $\text{POS}(?p \vee q?)$ (as predicted by a Middle Kleene view of disjunction), the alternatives are $\{p, \neg p \wedge q\}$. A strongest or unique answer constraint, applied to our alternatives, derives exhaustiveness but not exclusivity. Worse than that, it is in fact impossible to construct a constraint that derives exclusivity from $\{p, \neg p \wedge q\}$, because the desired presupposition ($\neg(p \wedge q)$) is not expressible as a function of p and $\neg p \wedge q$. The Strong Kleene view of disjunction is more promising: the alternatives for $\text{POS}(?p \vee ?q)$ are $\{p, q\}$, and a unique answer constraint will derive the exclusivity presupposition. Adding a unique answer presupposition will not affect our analysis of conjunctive cases (where the alternatives form a partition anyway). We will need to assume that it is not operative in open disjunctive questions, which do not seem to presuppose anything, but this problem is already present in earlier accounts. If we want instead to maintain the Middle Kleene view, which as we have seen helps explain certain left/right asymmetries, one solution might be to adopt a similar approach as in the case of *which* questions, consisting in proposing that the individual alternatives entail the exclusivity inference. However, unlike in the case of *which*, there is no independent motivation for the move.

5   Conclusion

In this long paper, we have identified a striking parallelism between the presupposition projection patterns observed when a trigger is in the second of two coordinated polar questions, and when a trigger is in the second of two coordinated assertions. We have seen that existing views on the semantics of coordinated questions struggle to explain the projection pattern in itself, and that they would have an even harder time explaining the parallelism, because they do not analyse coordinated questions in a way that adequately

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[44]Champollion et al. (2017) defend a localist account of *which*'s presupposition within the framework of inquisitive semantics, where the same problem occurs. Decoupling the presuppositions of *which* questions from answerhood operators is also argued for by Uegaki (2020) and Hirsch and Schwarz (2019) within H/K semantics. An alternative solution to the issue consists in making singular and plural *which* questions different again through extensions of the inquisitive system; see for instance the dynamic inquisitive proposal of Dotlačil and Roelofsen (2020).

relates them to coordinated assertions. The system proposed here, a trivalent extension of inquisitive semantics, solves the initial problem of deriving the projection facts by making polar questions more closely related to assertions. In doing so, we also make polar questions yes/no-asymmetric, and we give a uniform account of question conjunction and question disjunction, two things that have proven problematic for some theories of questions. While we do not have the space to do it here, a natural next step will be to identify which other phenomena the resulting system might help us with, possibly including question bias for instance. Additionally, while I think I have shown here a defect in existing theories of questions taken as-is, there certainly exist other ways of extending various theories, following the same basic conceptions that I followed when extending inquisitive semantics, that would deliver the same welcome results, and we will have to see whether some of them might not improve on the present system in various respects.

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A Formal companion to section 2.4

This appendix contains a formal development that demonstrates the problems encountered when combining answer set semantics with either the Transparency Theory or trivalent theories, as cursorily explained in Section 2.4. We will derive the following facts: the observed presupposition filtering pattern is expected in conjunctive questions if they are analysed as the tripartition, but not if they are analysed as the quadripartition. Meanwhile, the observed pattern in disjunctive questions is not derived in any natural way.

Note that throughout the appendix we work with question denotations, rather than with declarative sentences containing embedded questions, and therefore the results might only directly apply to matrix questions. In principle, at least under the Transparency Theory, we could predict different filtering patterns under certain embeddings (in particular we might expect more filtering predictions). While I do not think this would in fact be an issue, there is no attempt to prove it here.

When the proof of a result is ommitted, it is because it is immediate.

A.1 Equivalence relations on questions

The Transparency theory requires a notion of equivalence: we define several natural ones below. As in the main text, we identify questions with Hamblin denotations (sets of propositions). We will use Q, Q', etc. to name our abstract variables representing questions. In order to show that the results extend to Karttunen denotations (functions from worlds to sets of propositions), we will also refer to Karttunen denotations as $\hat{Q}$, $\hat{Q}'$, etc. The following relations let us map between the two kinds:

Definition 1 (Relation between Hamblin and Karttunen denotations). For a question Q, we define:

$$\hat{Q} := \lambda w. \{p \in Q \mid p(w)\}$$

As long as Q does not contain a logical contradiction, the following relation lets us reverse the mapping:

$$Q = \bigcup_w \hat{Q}(w)$$

Note that it is not true in general that all Karttunen questions can be derived from a Hamblin question (i.e. that they can be written as $\hat{Q}$ for some Q). However, all the questions that we consider here are representable as Hamblin questions.

In the main text, we left the notion of equivalence we were using somewhat implicit. Here we define several reasonable notions of equivalence. These notions are all relativised to a context set C (they are notions of contextual equivalence). The purpose of this wealth of definitions is to show that the issues outlined in the main text do not depend on specific implementational choices within the general framework of H/K semantics. In particular, whether we adopt H-equivalence, which the most natural approach from the perspective of a Hamblin-style theory, or K-equivalence, which is the most natural approach from the perspective of a Karttunen-style theory, does not matter, showing that our discussion applies to Hamblin- and Karttunen-style accounts equally.

For the notions based on a presuppositional answerhood operator, I implement a variety of ways of dealing with the unique answer presupposition. The first approach treats

it essentially as an entailment, a second one only considers *worlds* where it is true, and a third one only considers *contexts* where it is true.[45]

Non-contextual versions of the non-presuppositional notions can be obtained by taking C to be the set of all possible worlds.

Definition 2 (Pointwise contextual restriction)**.**

(i) For a proposition p and a context set C, $p_{|C}$ is the function from C to truth values that is identical to p at all points (the *contextual restriction* of p to C).

(ii) For a question Q and a context set C, the contextual restriction of Q to C is $Q_{|C}$, where:
$$Q_{|C} := \{p_{|C} \mid p \in Q\}$$

Definition 3 (Hamblin equivalence)**.** Q and Q' are *H-equivalent* relative to context set C iff $Q_{|C} = Q'_{|C}$.

Definition 4 (Karttunen equivalence)**.** Q and Q' are *K-equivalent* relative to context set C iff:
$$\forall w \in C. \, [\hat{Q}(w)]_{|C} = [\hat{Q}'(w)]_{|C}$$

Definition 5 (Unique-answer answerhood operator)**.** We define:
$$\text{ANS}_U := \lambda Q. \, \lambda w. \, \iota p. \, p \in Q \wedge p(w)$$

Definition 6 (Unique answer equivalence: entailed presupposition)**.** Q and Q' are *U-equivalent* relative to a context set C iff:

(i) the unique answer presuppositions of Q and Q' are equivalent in C,

(ii) calling C' the subset of C where the presuppositions are met, we have:
$$\forall w, w' \in C'. \, \text{ANS}_U(Q)(w)(w') = \text{ANS}_U(Q')(w)(w')$$

Definition 7 (Unique answer Strawson-equivalence — world-level version)**.** Q and Q' are *W-equivalent* relative to a context set C iff for any world w in C such that the unique answer presuppositions of both Q and Q' are true at w, we have:
$$\forall w' \in C. \, \text{ANS}_U(Q)(w)(w') = \text{ANS}_U(Q')(w)(w')$$

Definition 8 (Unique answer Strawson-equivalence — context-level version)**.** Q and Q' are *C-equivalent* relative to a context set C iff for any subset C' of C where the unique answer presuppositions of both Q and Q' are true, we have
$$\forall w, w' \in C'. \, \text{ANS}_U(Q)(w)(w') = \text{ANS}_U(Q')(w)(w')$$

Result 1 (H-equivalence and K-equivalence are almost the same thing)**.**

(i) If C is a context set and Q and Q' are two questions, then if Q and Q' are H-equivalent in C, they are K-equivalent in C.

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[45]Both the second and third approach are reminiscent of the definition of Strawson-entailment proposed by von Fintel (1999).

(ii) If C is a context set and Q and Q' are two questions that do not contain any proposition that is false throughout C, then if Q and Q' are K-equivalent in C, they are H-equivalent in C.

Result 2 (Order of the answer-based equivalences by strength)**.** If C is a context set and Q and Q' are two questions, then if Q and Q' are U-equivalent in C, they are W-equivalent in C, and if they are W-equivalent in C, they are C-equivalent in C.

Result 3 (K-equivalence is stronger than unique answer equivalences)**.** If C is a context set and Q and Q' are two questions, then if Q and Q' are K-equivalent in C they are U-equivalent and therefore also W-equivalent and C-equivalent in C.

Result 4 (Collapse of the equivalences)**.** If C is a context set and Q and Q' are two questions whose unique answer presuppositions are satisfied in C, then for any pair of letters α, β within H, K, U, W, and C, Q and Q' are α-equivalent in C iff they are β-equivalent in C.

A.2   The Transparency Theory

Here we derive the announced results under Schlenker's (2008) Transparency Theory. Schlenker (2008) proposes that in a context C, a presuppositional clause is acceptable if and only if the proposition being presupposed is *transparent* in the clause's position. Transparency is defined as follows (some details are simplified or left implicit; the reader is referred to Schlenker 2008):

Definition 9 (Transparency (Schlenker 2008))**.** Let $\alpha\beta\gamma$ be a sentence where β is an embedded clause, and δ be another clause denoting a proposition d. d is *transparent* in the position of β if and only if for any completion γ' that makes $\alpha\beta\gamma'$ well-formed, and any clause β', then $\alpha\beta'\gamma'$ and $\alpha(\delta$ and $\beta')\gamma$ are contextually equivalent in C.

The definition of Transparency depends on a notion of contextual equivalence; it is straightforward to apply it to questions as long as we have defined equivalence over them. Thus we can in principle define H-transparency, K-transparency and so on in terms of what is above.

The relations between our notions of equivalence immediately translate into a relation between our notions of transparency:

Result 5 (Relation between the transparencies)**.**

(i) H-transparency implies K-transparency.

(ii) K-transparency implies U-transparency.

(iii) U-transparency implies W-transparency.

(iv) W-transparency implies C-transparency.

We can now derive the results that we are interested in. From this first result, it follows that under the quadripartitive account, no presupposition filtering should ever be observed when the trigger is in the second conjunct of a conjunctive question.

Result 6 (The quadripartition). Let C be a context set, and p a proposition such that there exist a proposition q_0 that is not related by contextual entailment to p or $\neg p$.[46] A proposition d is H-transparent (as well as K/U/W/C-transparent) in the position of q in $\textsc{c}[?p] \wedge \textsc{c}[?q]$ if and only if C supports d.

Proof. The direction "if C supports d, then d is transparent" is immediate.

For the other direction, it suffices to show that if d is C-transparent, then C supports d, as C-transparency is the weakest form of transparency. Note that the definedness condition of $\textsc{ans}_U$ is always met if the answers form a logical partition as they do here, and therefore we do not need to consider subcontexts at all.

Assume then that d is C-transparent. We will write $Q(q) = \textsc{c}[?p] \wedge \textsc{c}[?q]$; the fact that d is C-transparent means that $\textsc{ans}_U(Q(q))(w)$ is equivalent in C to $\textsc{ans}_U(Q(d \wedge q))(w)$ for any q and any $w \in C$ (taking β' to be q, γ' to be the empty string).

It follows from the existence of q_0 that p is not trivially true or false in C. Let then w, w' be two worlds such that $p(w) = 1$ and $p(w') = 0$. We have $\textsc{ans}_U(Q(\top))(w) = p \wedge \top = p$ and $\textsc{ans}_U(Q(\top))(w') = \neg p \wedge \top = \neg p$.

- If $d(w) = d(w') = 0$, then $\textsc{ans}_U(Q(d))(w) = p \wedge \neg d$, from which it follows that $p \wedge \neg d$ is equivalent to p, i.e. that p entails $\neg d$. We also have $\textsc{ans}_U(Q(d))(w') = \neg p \wedge \neg d$, from which it follows similarly that $\neg p$ entails $\neg d$. The only way these two entailments can hold is if d is a trivial falsehood. But then, we have $\textsc{ans}_U(Q(d \wedge q_0))(w) = p \wedge \neg(d \wedge q_0)$, which is equivalent to p, so p is equivalent to $\textsc{ans}_U(Q(q_0))(w)$ which is either $p \wedge q_0$ or $p \wedge \neg q_0$. This contradicts the assumption that p and $q_0/\neg q_0$ are not related by entailment.

- If $d(w) = 1$ and $d(w') = 0$, then $\textsc{ans}_U(Q(d))(w) = p \wedge d$ and $\textsc{ans}_U(Q(d))(w') = \neg p \wedge \neg d$. It follows that p entails d and that $\neg p$ entails $\neg d$. Hence p is equivalent to d. Then, we have $\textsc{ans}_U(Q(d \wedge q_0))(w') = \neg p \wedge \neg(d \wedge q_0)$ which is equivalent to $\neg p$, so $\neg p$ is equivalent to $\textsc{ans}_U(Q(q_0))(w')$, which is either $\neg p \wedge q_0$ or $\neg p \wedge \neg q_0$. It follows that $\neg q_0$ or q_0 entails $\neg p$, against assumption.

- If $d(w) = 0$ and $d(w') = 1$, the same reasoning as in the previous case applies, replacing any occcurrence of p with $\neg p$ and vice-versa, and replacing w' with w.

The only remaining possibility is that $d(w) = d(w') = 1$. Since w can be any p-world and w' can be any $\neg p$-world, d has to be a trivial truth in C. $\square$

The next result shows that if conjunctive questions denote the tripartition, then we predict the observed pattern of presupposition filtering in the second conjunct.

Result 7 (The tripartition). Let C be a context set, and p a proposition such that there exist a proposition q_0 that is not related by contextual entailment to p or $\neg p$. A proposition d is H-transparent (as well as K/U/W/C-transparent) in the position of q in $\textsc{c}[?p \wedge \textsc{c}[?q]]$ if and only if C supports the material conditional $p \to d$.

Proof. We define: $Q(q) := \textsc{c}[?p \wedge \textsc{c}[?q]] = \{\neg p, p \wedge \neg q, p \wedge q\}$.

Assume that C supports $p \to d$, and let q be an arbitrary proposition. $p \wedge d \wedge q$ is equivalent in C to $p \wedge q$, and this relation can equivalently be written as $[p \wedge d \wedge q]_{|C} = [p \wedge q]_{|C}$. Moreover, we have $p \wedge \neg(d \wedge q) = (p \wedge \neg d) \vee (p \wedge \neg q)$, which is equivalent

to $p \wedge \neg q$ (as $p \wedge \neg d$ is a contextual contradiction). It follows that $Q(q)$ and $Q(d \wedge q)$ are H-equivalent in C. Then, for any completion γ, $Q(q)\gamma$ and $Q(d \wedge q)\gamma$ are also H-equivalent in C.[47] Therefore, d is H-transparent in the position of q in $Q(q)$, and it is also K/U/W/C-transparent.

In the other direction, assume that d is C-transparent in the position of q in $Q(q)$ in C. As before, it follows from the existence of q_0 that p is not trivially true or false in C. Let us then take $w \in C$ such that $p(w) = 1$. If $d(w) = 0$, since $\mathrm{ANS}_U(Q(\top))(w) = p$ and $\mathrm{ANS}_U(Q(d))(w) = p \wedge \neg d$, p and $p \wedge \neg d$ are contextually equivalent, which is equivalent to saying that C supports $p \to \neg d$. Assume without loss of generality (as we could replace q_0 by $\neg q_0$) that $q_0(w) = 1$. Then, we have $\mathrm{ANS}_U(Q(d \wedge q_0))(w) = p \wedge (\neg d \vee q_0)$. Since p entails $\neg d$, it also entails $\neg d \vee q_0$, and therefore $\mathrm{ANS}_D(Q(d \wedge q_0))(w)$ is e The direction "if C supports d, then d is transparent" is immediate. quivalent to p. By C-transparency, p is thus equivalent to $\mathrm{ANS}_U(Q(q_0))(w)$, i.e. p is equivalent to $p \wedge q_0$, or equivalently p contextually entails q_0, against assumptions. Therefore this case is impossible, and $d(w) = 1$. Since this holds for any w such that $p(w) = 1$, C supports $p \to d$. $\qquad\square$

Moving on to disjunction, the following result shows that no filtering is predicted to be possible with H/K/U-transparency. The next result is an extension to W-transparency (that an additional condition is needed is essentially a bug in the definition). A final result shows that C-transparency derives a degenerate pattern where everything is transparent.

Result 8 (No filtering in alternative questions (H/K/U))**.** Let C be a context set. A proposition d is H/K/U-transparent in the position of q in $?p\vee?q$ in C if and only if C supports d.

Proof. The direction "if C supports d, then d is transparent" is immediate.

We define $Q(q) := ?p\vee?q = \{p, q\}$. Assume that d is U-transparent in C. The unique answer presupposition of $Q(\top)$ is equivalent to $\neg p$, from which it follows that the unique answer presupposition of $Q(d)$ is equivalent to $\neg p$, i.e. that $p \veebar d$ is equivalent to $\neg p$ ($\veebar$ represents an exclusive disjunction). This can be verified to be equivalent to the fact that d is trivially true in C. $\qquad\square$

Result 9 (No filtering in alternative questions (W))**.** Let C be a context set, and let d be a proposition that does not contextually entail p.[48] d is W-transparent in the position of q in $?p\vee?q$ if and only if C supports d.

Proof. Once again, define $Q(q) := ?p\vee?q = \{p, q\}$.

The direction "if C supports d, then d is transparent" is immediate.

Assume d is W-transparent. Let w be a world such that $p(w) = 0$ and $d(w) = 1$ (such a world exists by assumption). $\mathrm{ANS}_U(Q(\top))(w) = \top$ and $\mathrm{ANS}_U(Q(d)) = d$, so d is contextually equivalent to $\top$. $\qquad\square$

Result 10 (C-equivalence derives a degenerate pattern in alternative questions)**.** Let C be a context set, and let d be a proposition. d is C-transparent in the position of q in $?p\vee?q$.

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[47]We would need to define an explicit fragment to properly prove this fact, but I think it is reasonable to take it for granted.

[48]If d contextually entails p, then the unique answer presupposition of $\{p, d \wedge q\}$ will be contradictory, whatever q is. d is actually W-transparent in this case, but not in an interesting way.

Proof. Once again, define $Q(q) := ?p \vee ?q = \{p, q\}$.

Take q an arbitrary proposition, and call C' the set of worlds in C such that $p \vee (d \wedge q)$ and $\neg(p \wedge q)$ are true. C' is the biggest subset of C such that the unique answer presupposition of both $Q(q)$ and $Q(d \wedge q)$ is met. What we need to prove is that $\text{ANS}_U(Q(q))$ and $\text{ANS}_U(Q(d \wedge q))$ define the same two-place predicate over C', or equivalently that for any $w \in C'$, both operators return equivalent propositions when applied to w, with equivalence being taken in C'.[49] Take then w in C'.

- If $p(w) = 1$, then $\text{ANS}_U(Q(q))(w) = \text{ANS}_U(Q(d \wedge q))(w) = p$.

- If $p(w) = 0$, from the definition of C' we have $d(w) = q(w) = 1$. It follows that $\text{ANS}_U(Q(q))(w) = q$ and $\text{ANS}_U(Q(d \wedge q))(w) = d \wedge q$. Due to the way C' is defined, both q and $d \wedge q$ are equivalent to $\neg p$ in C' and therefore to one another, as desired.

Thus, d is C-transparent. $\qquad\square$

These results are essentially preserved if we look at $\{p, q, \neg(p \vee q)\}$ rather than $\{p, q\}$; the proofs are ommitted to save space.

A.3   Trivalent theories: compositional approach

We now turn to trivalent theories. The simplest way of adapting trivalent theories to our problem is to consider that conjunction and disjunction of propositions are trivalent, and that question conjunction and disjunction just do what they usually do in H/K semantics, except that questions are sets of *trivalent* propositions. Below we define such a system, called the Simple Trivalent Answer set Theory (STAT). It is straightforward to verify that under STAT, essentially the same results as with the Transparency Theory obtain: the conjunctive case can be dealt with through the tripartition, while the disjunctive case is puzzling.[50]

Concretely, assume the following:

Definition 11 (STAT)**.**

(i) Questions are sets of trivalent propositions.

(ii) Question conjunction is pointwise Middle Kleene conjunction.

(iii) Question disjunction is set union.

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[49]The fact that we look at equivalence in C' rather than in C here is the crucial difference between C-equivalence and W-equivalence.

[50]An entirely parallel result obtains in the case of the Simple Dynamic Answer set Theory (SDAT):

Definition 10 (SDAT)**.**

(i) Questions are sets of dynamic propositions denoting Context Change Potentials (CCPs), as defined in Heim 1983.

(ii) Question conjunction is pointwise CCP composition.

(iii) Question disjunction is set union.

(iv) Questions are felicitous only if all their answers are defined at the context.

Li (2019) proposes a system of this kind.

(iv) Questions are felicitous only if all their answers return 0 or 1 at all worlds in the context.

Result 11 (Predictions of STAT). Under the assumptions given in Definition 11:

(i) If a conjunctive question (schema: $?p \wedge ?q$) denotes the quadripartition $\{p \wedge q, p \wedge \neg q, \neg p \wedge q, \neg p \wedge \neg q\}$, it should presuppose $\pi(p)$ and $\pi(q)$.

(ii) If a conjunctive question (schema: $?p \wedge ?q$) denotes the tripartition $\{p \wedge q, p \wedge \neg q, \neg p\}$, it should presuppose $\pi(p)$ and $p \to \pi(q)$.

(iii) Whether a disjunctive question (schema: $?p \vee ?q$) denotes $\{p, q\}$ or $\{p, q, \neg p \wedge \neg q\}$, it should presuppose $\pi(p)$ and $\pi(q)$.

In order to deal with disjunction, one might think that case (iv) in Definition 11 should be relaxed to an existential presupposition: *at least one* answer should be defined at a given world in the context. We can in fact unify the constraint with the independent assumption, generally made in H/K theories, that questions presuppose that one of their answers is true:

Definition 12 (Existential variant of STAT).

(iv') Questions presuppose that at least one answer is defined and true and each world in the context.

The observed case of filtering is immediately predicted, but we derive no order effects.

A.4 Trivalent deployment of the connectives

Another way of extending trivalent theories to questions is to apply the methodology of "Peters-Kleene deployment", as described by George (2014), to derive trivalent meanings for the question connectives from their bivalent meaning. Below we provide an implementation of George's idea, and derive the same results as with the Transparency theory: presupposition filtering is predicted for the tripartition but not for the quadripartition as far as conjunction is concerned, and not at all as far as disjunction is concerned (only the case of $\{p, q\}$ is treated as using $\{p, q, \neg p \wedge \neg q\}$ instead does not make a difference).

The idea behind Peters-Kleene deployment is as follows: assume you want to derive how potential undefinedness in p will project in the environment $F(p)$. What you know is the specification of F in the bivalent world (i.e. you know what $F(p)$ is for a total proposition p). Then, define the *repair set* of a trivalent proposition p: it is the set of all propositions that agree with p wherever p is defined.

Definition 13 (Repair set of a trivalent proposition). If p is a trivalent proposition, the *repair set* of p is written as p^R and given by:

$$p^R := \{p' \in \{0,1\}^\Omega \mid \forall w.\, p(w) \in \{0,1\} \to p'(w) = p(w)\}$$

(Here Ω is the set of possible worlds, and therefore $\{0,1\}^\Omega$ is the set of total/bivalent propositions.)

From the repair set of p, we derive the deployment of F, F^D. Unlike $F(p)$, $F^D(p)$ is potentially defined for some trivalent inputs p. Those inputs are those where no matter how p is repaired into a bivalent proposition p', $F(p')$ is the same.

Definition 14 (Deployment of a functor)**.** If F is a function whose input is a bivalent proposition, the *deployment* of F is written as F^D and given by:

(i) If there is an output X such that for all $p' \in p^R$, $F(p') = X$, then $F^D(p) = X$.

(ii) Otherwise, $F^D(p) = \#$.

In our case, F will be the function from q to the denotation of the question schematized as $?p \wedge ?q$ or $?p \vee ?q$. The desired result is that even if q is potentially undefined, $F(q)$ might be defined in some cases.

Applying this methodology is most naturally done within a Karttunen-style account. Thus, the output of K will be a Karttunen question (type $s(st)t$). The definition of deployment above presupposes a notion of equality on the outputs of F; the natural choice in our case is the usual definition of set equality.[51]

We can now prove the results promised above:

Result 12 (Deployment of conjunction (quadripartition))**.** For total propositions p and q, define $F_p(q)$ to represent the Karttunen denotation of a quadripartitive conjunctive question:

$$F_p(q) := \lambda w. \lambda r_{st}. r \in \{p \wedge q, p \wedge \neg q, \neg p \wedge q, \neg p \wedge \neg q\} \wedge r(w)$$

Let p be a total proposition that is true in at least two worlds and false in at least two worlds.[52] $F_p^D(q)$ is defined if and only if q is total.

Proof. It is clear that if q is total, $F_p^D(q)$ is defined (it is just $F_p(q)$).

Assume that q is not total, i.e. that there is w such that $q(w) = \#. =$ Take $q' \in q^R$ such that $q'(w) = 1$, and $q'' \in q^R$ that is exactly the same as q', except that $q''(w) = 0$.

- If $p(w) = 1$, then $F_p(q')(w) = \{p \wedge q'\}$ and $F_p(g'')(w) = \{p \wedge \neg q''\}$. Let w' be another world where p is true: we have $[p \wedge q'](w') = q'(w')$ and $[p \wedge q''](w') = q''(w')$. By construction, $q'(w') \neq q''(w')$, so $F_p(q')(w) \neq F_p(g'')(w)$.

- If $p(w) = 0$, then $F_p(q')(w) = \{\neg p \wedge q'\}$ and $F_p(q'')(w) = \{\neg p \wedge \neg q''\}$. Let w' be another world where p is false: we have $[\neg p \wedge q'](w') = q'(w')$ and $[\neg p \wedge q''](w') = q''(w')$. By construction, $q'(w') \neq q''(w')$, so $F_p(q')(w) \neq F_p(g'')(w)$.

Either way, $F_p^D(q)$ is not defined. By contraposition, if $F_p^D(q)$ is defined, q is total. $\square$

Result 13 (Deployment of conjunction (tripartition))**.** For total propositions p and q, define $F_p(q)$ to represent the Karttunen denotation of a tripartitive conjunctive question:

$$F_p(q) := \lambda w. \lambda r_{st}. r \in \{p \wedge q, p \wedge \neg q, \neg p\} \wedge r(w)$$

Let p be a total proposition that is true in at least two worlds. $F_p^D(q)$ is defined if and only if all $\#$-worlds for q are 0-worlds for p.[53]

Proof. Assume that q is only undefined at worlds where p is false. Take q', q'' in q^R, and let w be a world.

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[51]Note that we could relativize everything to a context set, as is done in the definition of K-equivalence, while preserving all the results.

[52]As before, this condition should be met in any realistic example.

[53]This can also be stated as "p entails $\pi(q)$", where π is defined as in (47).

- If $p(w) = 0$, then $[p \wedge q'](w) = [p \wedge q''](w) = 0$, and $[p \wedge \neg q'](w) = [p \wedge \neg q''](w) = 0$

- If $p(w) = 1$, then q is defined at w and $q'(w) = q''(w) = q(w)$. If $q(w) = 1$, we have $[p \wedge q'](w) = [p \wedge q''](w) = 1$ and $[p \wedge \neg q'](w) = [p \wedge \neg q''](w) = 0$. If $q(w) = 0$, we have $[p \wedge q'](w) = [p \wedge q''](w) = 0$ and $[p \wedge \neg q'](w) = [p \wedge \neg q''](w) = 1$.

Then, $p \wedge q' = p \wedge q''$ and $p \wedge \neg q' = p \wedge \neg q''$, from which the fact that $F_p(q') = F_p(q'')$ immediately follows. Therefore $F_p^D(q)$ is defined.

In the other direction, the same reasoning as in the previous result shows that if there is w such that $q(w) = \#$ and $p(w) = 1$, then $F_p^D(q)$ is not defined. By contraposition, if $F_p^D(q)$ is defined, there is no such w. $\qquad\square$

Result 14 (Deployment of disjunction)**.** For total propositions p and q, define $F_p(q)$ to represent the Karttunen denotation of a disjunctive question:

$$F_p(q) := \lambda w. \, \lambda r_{st}. \, r \in \{p, q\} \wedge r(w)$$

Let p be a total proposition that is true in at least two worlds. $F_p^D(q)$ is defined if and only if q is total.

Proof. It is clear that if q is total, $F_p^D(q)$ is defined.

Assume there is w such that $q(w) = \#$. Take $q' \in q^R$ such that $q(w) = 1$, and $q'' \in q^R$ that is the same as q', except that $q''(w) = 0$.

- If $p(w) = 0$, we have $F_p(q')(w) = \{q'\}$ and $F_p(q'')(w) = \varnothing$.

- If $p(w) = 1$, we have $F_p(q')(w) = \{p, q'\}$ and $F_p(q'')(w) = \{p\}$. If $q' \neq p$, $F_p(q')(w) \neq F_p(q')(w)$. If $q' = p$, take w' such that $p(w') = 1$. We have $F_p^D(q')(w') = \{p, q'\} = \{p\}$ and $F_p(q'')(w') = \{p, q''\}$ (recall that q'' agrees with q', and therefore with p, at w'). Since $q'' \neq q'$ by construction, $F_p(q')(w') \neq F_p(q'')(w')$.

Either way, $F_p^D(q)$ is not defined. By contraposition, if $F_p^D(q)$ is defined, q is total. $\qquad\square$