

When pragmatics matters more for truth-value judgments: An investigation of quantifier scope ambiguity

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Abstract Investigations of linguistic meaning rely crucially on truth-value judgments, which assess whether a sentence can truthfully describe a given scenario. In investigations of language acquisition, truth-value judgments are used to assess both the target knowledge adults have and the developing knowledge children have at different ages. On the basis of truth-value judgments, researchers have concluded that differences between how children resolve ambiguous utterances and how adults do so persist until at least age five. Current explanations compatible with the experimental data attribute these differences to both grammatical processing and pragmatic factors. Here, we use computational cognitive modeling to formally articulate the ambiguity-resolution process that underlies child and adult judgments in a truth-value judgment task; crucially, the model can separate out the individual contributions of specific grammatical processing and pragmatic factors to the resulting judgment behavior. We find that pragmatic factors play a larger role than grammatical processing factors in explaining children's non-adult-like ambiguity resolution behavior, and the computational modeling framework allows us to understand exactly why that is. Interestingly, the model predicts qualitative similarity between child and adult ambiguity resolution. Given this prediction, we then extend our model to show how the same processes are active in adult ambiguity resolution. This result supports continuity in the development of ambiguity resolution, where children do not qualitatively change how they resolve ambiguity in order to become adult-like. We discuss the implications of our results for acquisition more generally, including both theories of development and methods for assessing that development.

Keywords: truth-value judgments; ambiguity; development; pragmatics; Rational Speech Act model

1 Introduction

How should we characterize the meaning of sentences, and how do we (as speakers) learn that meaning? These questions call into focus the intersection of two traditions of inquiry: the semantics of natural language and language development. One of the key empirical methodologies for questions at this intersection is the truth-value judgment task (Crain & McKee 1985; Crain & Thornton 1998). Here, we use a complementary methodology to investigate how to interpret truth-value judgment behavior in specific cases where the truth-value judgment task has been used. More specifically, we model the cognitive processes, both linguistic and extra-linguistic, that deliver truth-value judgment task behavior in precise experimental contexts. This computational cognitive modeling allows us to separate out the contributions from these different cognitive processes, in contrast with behavioral contexts where these processes interact.

1.1 Truth-value judgments for assessing meaning

Knowing the meaning of some sentence S means knowing the conditions required for S to be true—the *truth conditions* of S . A sentence’s truth conditions might not exhaust the meaning of that sentence; they eschew connotative and social elements of meaning. Still, semanticists agree that truth conditions are a key component of sentence meaning: if you know what a sentence means, then you can identify the sorts of situations it describes. Therefore, one way of diagnosing sentence meaning is to map out the situations a sentence can describe (i.e., those situations in which the sentence is true) and those it cannot. In other words, one way of diagnosing sentence meaning is to consult one’s truth-value judgments for a range of situations. Those situations where the sentence is judged as a true description are then compatible with the sentence’s meaning. Semanticists are constantly engaged in investigations of this sort: imagine a situation and evaluate whether a sentence of interest is true in that situation. However, individuals without this sophisticated linguistic training—naive adults and children—often need to be helped with (i.e., tricked into) this reasoning. Enter the truth-value judgment task.

Rather than asking someone to imagine situations and the sentences that describe them, truth-value judgment tasks provide this information explicitly. In particular, to successfully engage children in the necessary reasoning, child truth-value judgment tasks often involve fairly elaborate setups that try to imitate natural conversational contexts. The hope is that more natural con-

versational contexts will mitigate any unusual pragmatics that would interfere with children’s reasoning (Crain & McKee 1985; Crain & Thornton 1998).

In a typical child truth-value judgment task implementation, a story is acted out using figures and props (e.g., a story about horses jumping over things like logs and fences). At the end of the story, an observer (often a puppet so the child won’t be intimidated) describes the outcome of the story with a statement (e.g., *None of the horses jumped over the fence*). This statement is the test sentence, and the child is meant to evaluate that sentence against the story scenario. The child then is asked to decide whether what the observer (puppet) said was okay (i.e., “yes” or “no”)—that is, whether the child would endorse the puppet’s statement as a reasonable thing to say, given the story scenario. A puppet is used, rather than an adult experimenter, because children are less hesitant to disagree with a puppet who they think said something wrong than with an adult who they think said something wrong (Crain & McKee 1985; Crain & Thornton 1998).

The tacit linking hypothesis assumes that when children endorse the observer’s description, they judge the sentence as true in the story scenario; when they choose not to endorse the description, they judge it as false. Typically, a child’s response (i.e., endorsing with “yes” or not endorsing with “no”) is followed up with an explicit question about why the child answered the way she did—this questioning also helps to ensure that the child is saying “yes” or “no” because the child thinks the observer’s description is appropriate or not, respectively.

We reiterate that all these accommodations in the truth-value judgment task aim to mitigate any unusual pragmatics that children might bring to the experimental scenario, given that this task is still a rather unnatural conversational situation. In particular, the truth-value judgment task does not ask children to simply interpret an utterance (as they would do in a natural conversation), by inferring the state of the world that the speaker in describing. Instead, in the truth-value judgment task, the state of the world is already known by both the child participant and the observer who produces the utterance; so, the child’s task is not to infer the state of the world, but rather to decide whether the observer’s utterance aptly describes that state of the world. A simple way for children to make this judgment is to decide if they themselves would produce it, given the observed state—an odd kind of production task (Degen & Goodman 2014). The reasoning involved is fairly sophisticated, so child implementations of the truth-value judgment task are constantly being improved to facilitate children’s ability to successfully perform this reasoning and demonstrate their underlying linguistic knowledge (Thornton 2017).

A truth-value judgment task can of course also be used for adult participants. The special design of child truth-value judgment tasks is meant to facilitate reasoning about the truth-value of specific statements. So, adults can benefit from the same truth-value judgment design features (though adults may lose patience with the more child-like aspects, such as listening to a puppet).

1.2 A concrete truth-value judgment task example: Quantifier scope ambiguity

At this point, it will be useful to consider a concrete example and the motivating case study for our investigation of truth-value judgments: universally-quantified sentences with negation, such as *Every horse didn't jump over the fence*. Such sentences are interesting from a theoretical perspective because they typically allow ambiguity (at least in English), with different interpretations conditioned by the scope of the logical operators introduced by *every* and negation. Such a sentence allows two interpretations (shown in (1)). Under the SURFACE interpretation, the logical scope of the operators corresponds to their scope at surface structure (*every* over *n't*: $\forall > \neg$); under the INVERSE interpretation, the logical scope inverts the surface scope (*n't* over *every*: $\neg > \forall$). Each scope option therefore leads to a different interpretation: surface scope corresponds to a “none” interpretation while inverse scope corresponds to a “not all” interpretation.

- (1) Every horse didn't jump over the fence.
- a. SURFACE SCOPE ($\forall > \neg$):
None of the horses jumped over the fence.
 - b. INVERSE SCOPE ($\neg > \forall$):
Not all of the horses jumped over the fence.

Truth-value judgment data have demonstrated differences between adults and children when it comes to their judgments of sentences like (1) in certain story scenarios. To appreciate these differences, consider the story scenario in Figure 1: there are two horses, and one jumped over the fence while the other did not. So, the surface interpretation of (1) is false: it is false that none of the horses jumped over (because horse 1 did in fact jump over). However, the inverse interpretation is true: it is true that not all of the horses jumped over (horse 2 didn't).

In a not-all scenario of this sort, adults readily endorse statement (1) (90-100% acceptance) while five-year-old children typically do not (10-20% acceptance; Musolino 1998; Lidz & Musolino 2002; Musolino & Lidz 2006;

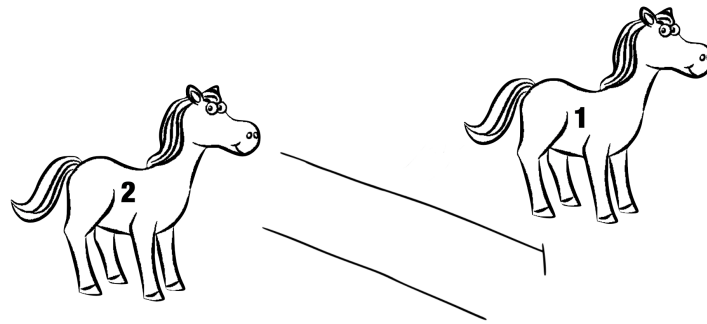


Figure 1: Example not-all scenario in which horse 1 jumps over the fence but horse 2 does not. In this scenario, (1b) is true (not all of the horses jumped) but (1a) is false (it is not the case that no horses jumped).

Musolino 2006; Viau et al. 2010; Tieu 2015). Following the implicit linking hypothesis mentioned above for the truth-value judgment task, these child judgments have been interpreted to mean that children struggle to access the inverse interpretation of sentences like (1) the way that adults can. The interesting question is why (and perhaps even whether) children struggle to access that interpretation, and there are several possibilities that have been discussed in the literature cited above. Perhaps children are unable generate the inverse interpretation at all because their semantic knowledge is still developing (a grammatical factor). Perhaps children can generate the inverse interpretation, but not access it in the truth-value judgment task because of their developing processing abilities (a grammatical factor). Perhaps children can in fact generate and access the inverse interpretation, but choose not to endorse the test sentence for other—typically pragmatic—reasons (e.g., children don't believe the sentence is a reasonable thing to say, given the story scenario); this resistance to endorsement would be due to one or more pragmatic factors.

Interestingly, strategic changes to the truth-value judgment task setup lead to more adult-like behavior, such that children more readily endorse sentences like (1) in a not-all scenario as in Figure 1 (Viau et al. 2010). However, despite the carefully-designed manipulations of the experimenters, it often remains unclear which factors are responsible for children's differing behavior: grammatical factors, pragmatic factors, or both. This is where computational cognitive modeling can help us.

1.3 Computational cognitive modeling of the truth-value judgment task

Computational cognitive models implement cognitive theories concretely. In particular, a computational cognitive model articulates a hypothesis of how different components of underlying knowledge interact to produce observable behavior (e.g., Pearl & Sprouse 2013; Goodman & Frank 2016; Pearl & Mis 2016; Pearl 2017; Pearl et al. 2017; Pearl in press; Pearl & Sprouse 2018; Scontras et al. *electronic*). Here, we use computational cognitive modeling to implement cognitive theories of ambiguity resolution in context, specifically how a participant (child or adult) would resolve a quantifier scope ambiguity like (1) in story contexts like Figure 1. By articulating how different cognitive components interact—both grammatical and pragmatic components—a computational cognitive model can predict not only which factors contribute to the observed truth-value judgment endorsement behavior, but also how much each factor contributes. Doing so allows us to transparently untangle the separate contributions of each factor.

So, by applying a modeling approach to the question of truth-value judgments for scopally-ambiguous utterances, we can identify which cognitive factors lead to adult-like judgments and how they do so. We can then see if these same factors can lead to child-like judgments, and, if so, how they do so. When we have potential explanations for both adult behavior and child behavior, we can then articulate a developmental theory: what needs to change for children to become adult-like. More generally, when we understand how the underlying cognitive factors can yield observable endorsement behavior in the truth-value judgment task, we better understand the truth-value judgment task itself, and how to interpret truth-value judgment results.

An additional benefit of computational cognitive models that predict observable behavior (such as the endorsement behavior in a truth-value judgment task) is that model predictions can then be tested by further behavioral work. If we find that the modeling predictions match what humans do (e.g., truth-value judgment endorsement rates for other contexts and/or test sentences), we have strong support for the model-implemented theory of how the underlying components interact.

1.4 The rest of this paper

This paper is structured as follows. We begin with an overview of the empirical facts concerning children’s ambiguity-resolution behavior in truth-value judgment tasks, together with the relevant task manipulations that make children

more adult-like. We then present our computational cognitive model of utterance endorsement in the truth-value judgment task, which is conceived within the Bayesian Rational Speech Act modeling framework (Goodman & Frank 2016; Scontras et al. *electronic*). With this modeling approach, we demonstrate how both child and adult truth-value judgment endorsement behavior can be captured using a single model with different parameter settings. In other words, we show how the same cognitive factors can interact in the same way to produce either child-like or adult-like behavior. The differences are quantitative in nature, such that different values for the same factor yield diverging behavior. So, our model predicts that adults and children are in fact performing the same pragmatic calculus when evaluating ambiguous utterances in the truth-value judgment task.

Given this finding, our model predicts that adults should be affected by the same sorts of task manipulations that have been found to modulate children's behavior. We explore this prediction, considering truth-value judgment data for a case of ambiguity that leaves adults looking like children. We demonstrate how our model can be extended to capture this adult truth-value judgment task behavior, underscoring how the same underlying cognitive variables interacting in the model-specified way can account for a variety of truth-value judgment task data. More generally, our findings suggest that child and adult truth-value judgment behaviors are driven by the same underlying cognitive factors, but with different values for those factors. So, to become adults, children do not need to qualitatively change how they resolve scope ambiguity; this finding supports the continuity hypothesis in the development of scope-ambiguity resolution from childhood into adulthood. We conclude by synthesizing our findings and discussing their implications for our understanding of language development and methods that can be fruitfully used to assess that development.

2 Children on the truth-value judgment task

Children's behavior with scopally-ambiguous utterances in the truth-value judgment task has been shown to be sensitive to manipulations of the experimental context. In the basic task, children are presented with a background story about the agents—for example, horses engaging in some activities. After this background story, children watch as the agents attempt to complete an action, such as jump over a fence. The critical not-all result state meant to prompt the inverse scope interpretation is illustrated in Figure 1, where horse 1 jumps over the fence and horse 2 does not. In this scenario, the surface interpretation of the sentence in (1), repeated in (2), is false (again, because horse 1 did jump;

therefore, none jumped is false), and the inverse scope interpretation is true (again, because horse 2 didn't jump; therefore not all jumped is true).

- (2) Every horse didn't jump over the fence.
- a. SURFACE SCOPE ($\forall > \neg$):
None of the horses jumped over the fence.
 - b. INVERSE SCOPE ($\neg > \forall$):
Not all of the horses jumped over the fence.

A puppet then produces an utterance, such as the sentence in (2), and the child is asked to state if the puppet is right. That is, the child is asked whether she would endorse the puppet's utterance as a true description of the scenario. Typically, children refuse to endorse the puppet's utterance in inverse-verifying scenarios like Figure 1, saying that the puppet is wrong; in contrast, adults readily endorse the utterance in this context. This behavior has been interpreted as children failing to access the inverse scope interpretation that would make the utterance true. That is, if children could access the inverse scope interpretation, they would recognize that *not all of the horses jumped over the fence* is true in this scenario, and therefore they should endorse the scopally-ambiguous utterance in (2). But given that children typically do not endorse the utterance in this scenario, children's behavior is interpreted as evidence that they must not access the inverse scope interpretation.

Previous accounts of children's scope-interpretation behavior have recognized that both processing and pragmatic factors may contribute to non-adult-like behavior. Musolino (1998; 2006) observed that the surface scope interpretation in (2a) may be easier to process because the scope relationship at logical form (i.e., $\forall > \neg$) aligns with the linear order of these elements in the utterance (i.e., *Every* precedes *n't*). In contrast, for the inverse scope interpretation in (2b), this parallelism does not hold, with the scope relationship (i.e., \neg scopes over \forall) opposite the linear order of the elements in the utterance. Musolino hypothesized that this lack of parallelism would make the inverse scope interpretation more difficult to access. In line with this prediction, Conroy et al. (2008) found that when adults are time-restricted, they favor the surface scope interpretation. We thus see a potential role for processing factors in children's inability to access the inverse scope. Perhaps children, with their still-developing processing abilities, are unable to allocate sufficient processing resources to reliably access the inverse scope interpretation.

In addition to this processing factor, Gualmini et al. (2008) noted that discourse properties, such as what children consider to be the question under discussion (QUD), may impact their scope-interpretation behavior. Formal

theories of pragmatics suggest that all discourse transpires with respect to some QUD, whether implicit or explicit; utterances in the discourse need to (at least partially) answer the QUD to be pragmatically felicitous (Roberts 2012). Gualmini and colleagues (Hulsey et al. 2004; Gualmini et al. 2008) suggest that children are very sensitive to this requirement. In particular, children may be able to access the inverse scope interpretation but nonetheless choose the surface scope interpretation because it better answers the perceived QUD in the contrived experimental setups. So, children’s observed behavior would derive from a still-developing ability to manage the contextual information available and correctly infer the intended QUD.

Interestingly, various alterations to the truth-value judgment task setup have yielded more adult-like behavior in children—namely, greater rates of endorsing the puppet’s ambiguous utterance in not-all scenarios. For example, Musolino & Lidz (2006) hypothesized that negation in an utterance might require certain felicity conditions to be met. In particular, negated utterances require a preceding affirmative context with which to contrast (Wason 1965). Musolino & Lidz augmented the basic truth-value judgment task to include an additional contrast condition in which the puppet precedes its negative scopally-ambiguous utterance with a contrasting affirmative clause. This additional clause describes a previous successful story action (an “early success”), as in *Every horse jumped over the log, but every horse didn’t jump over the fence*. This early-success contrast manipulation increased children’s willingness to accept the scopally-ambiguous utterance in the not-all scenario: children in the baseline condition endorsed the puppet’s statement just 15% of the time, while children in the early-success condition endorsed the puppet’s statement 60% of the time (N=20). Viau et al. (2010) replicated this increase in utterance endorsement using only an early-success story context (N=24). That is, the higher utterance endorsement rate was maintained by an early-success story context alone; children did not need an explicit contrast clause in the test utterance (instead hearing only a scopally-ambiguous utterance like *Every horse didn’t jump over the fence*, just as in the original experiments by Musolino & Lidz).

Notably, the early-success affirmative-context manipulation potentially changes several aspects of the experimental context. First, observing early successes can shift participants’ expectations about successful outcomes more generally in the experimental world. This shift then potentially increases the salience of a QUD targeting this success, such as *did all the horses succeed?* (all?). Recognizing this QUD’s potential significance, Gualmini (2004) attempted to manipulate the experimental context so it favored the all? QUD. With all? as the salient QUD, children’s endorsement of a scopally-ambiguous

utterance that perfectly answered **all?** in the critical not-all scenario increased to 90%. Even for a scopally-ambiguous utterance that does not fully answer the **all?** QUD, children’s endorsement rate was at 50% with the **all?** QUD—markedly higher than the 15% baseline from the original study by Musolino & Lidz (2006). This finding highlights that privileging the **all?** QUD increases children’s utterance endorsement in these scenarios.

In addition to altering expectations about likely states of the world and QUDs, a third potential impact of the early-success affirmative-context manipulation involves scope access. By altering the experimental world expectations and/or expectations about the QUD to increase access to the inverse scope, the inverse scope interpretation may remain more accessible for later use. Viau et al. (2010) term this prolonged increase in accessibility “structural priming”. Children who are better able to access the inverse scope are then more likely to endorse the scopally-ambiguous utterance in subsequent not-all scenarios. Viau et al. investigated structural priming explicitly by attempting to directly alter the accessibility of the inverse scope interpretation. In one modified truth-value judgment task, the authors attempted to prime the access of the inverse scope interpretation; in another modified task, they attempted to directly prime the inverse scope’s logical structure (e.g., $\neg > \forall$).

The first structural priming manipulation was implemented via the now-familiar early-success affirmative-context manipulation. For the first three trials, the prior experimental context indicated successful outcomes; the effect was that children endorsed the scopally-ambiguous utterance 50% of the time. Crucially, the subsequent three trials removed the supportive affirmative-context manipulation, yet children continued to not only endorse the scopally-ambiguous utterance, but to endorse it more than they had before (80% endorsement). Viau et al. (2010) attribute this result to a priming effect of the inverse interpretation from the first three trials: having accessed the inverse structure in the early trials, children are more likely to access that same structure in later trials. However, the increase in utterance endorsement could be due to the privileging of multiple factors that are products of the affirmative-context manipulation: (i) expectations about successful outcomes in the experimental world, (ii) the salience of the **all?** QUD, or (iii) the ease of access to the inverse scope interpretation.

The second structural priming manipulation removed the affirmative-context story in the first three trials. In its place, children were asked whether they would endorse a scopally-unambiguous utterance (e.g., *not every horse jumped over the fence*), whose interpretation had logical operators in the same configuration as the inverse scope interpretation of the scopally-ambiguous utterance (i.e., $\neg > \forall$). Children endorsed this utterance 80% of the time. In the

subsequent three trials, children were asked if they would endorse the scopally-ambiguous utterance in the same experimental scenario—and their endorsement rate remained at 80%. [Viau et al. \(2010\)](#) interpret this effect as priming of the relevant logical form: the inverse scope was easier to access in the scopally-ambiguous utterance because it was recently accessed in the unambiguous utterances. The authors argue that this priming effect proceeded in the absence of manipulations to the pragmatic context; yet, even here there may still be pragmatic factors at work. The unambiguous utterance accomplishes three things: (i) it provides an instance of the $\neg > \forall$ configuration, (ii) it provides information about successful outcomes, and (iii) it suggests the all? QUD, answering it with *no*. Thus, in this attempt to prime the inverse logical form, the authors may have also altered expectations about the pragmatic context of the experiment as it relates to successful outcomes and relevant QUDs.

These experimental studies highlight at least three core factors (two pragmatic, one grammatical processing) that underlie children’s utterance endorsement behavior in the truth-value judgment task: (i) pragmatic: expectations about the experimental world (e.g., how likely successful outcomes are), (ii) pragmatic: expectations about the QUD (e.g., if it is relevant to establish whether all outcomes were successful), and (iii) grammatical processing: the accessibility of the inverse scope (i.e., the ease by which the logical form is accessed). These experimental studies have also supported different theoretical proposals for the source of children’s differences. The proposals split on whether they attribute the differences solely to an inability to manage contextual information (i.e., pragmatic factors; [Gualmini 2008](#)) or whether grammatical processing deficits also significantly contribute (i.e., difficulty accessing inverse scope; [Viau et al. 2010](#)). Importantly, it is not obvious from any of the existing experimental manipulations how to separate the independent contributions of these components. To capture and independently manipulate the contributions of each of the pragmatic and grammatical processing factors, we formalize their role in the interpretation of scopally-ambiguous utterances, using tools from computational cognitive modeling.

3 A computational cognitive model for every-not utterances

We model ambiguity resolution within the Bayesian Rational Speech Act (RSA) modeling framework ([Goodman & Frank 2016](#)), which views language understanding as a social reasoning process. The RSA framework finds broad empirical support from its ability to successfully model a range of pragmatic

language phenomena, from scalar implicature (Goodman & Stuhlmüller 2013) and vague gradable adjectives (Lassiter & Goodman 2013) to generic utterances (Tessler & Goodman 2019) and hyperbole (Kao et al. 2014b). Within the framework, language understanding is modeled by a *pragmatic listener* L_1 who interprets an utterance by reasoning about a cooperative *speaker* S_1 who is trying to inform a *literal listener* L_0 about the world. We note that the Bayesian inference mechanism on which this modeling framework relies is plausible for young children to use; a body of developmental evidence suggests that even very young children are capable of this kind of inference (3 years: Xu & Tenenbaum 2007; 9 months: Gerken 2006; Dewar & Xu 2010; Gerken 2010; 6 months: Denison et al. 2011; among many others).

Our model is a “lifted-variable” extension, in which the ambiguous utterance’s literal semantics is parameterized by interpretation-fixing variables (e.g., whether the scope is surface or inverse). Hearing an ambiguous utterance, a pragmatic listener reasons jointly about the true state of the world (e.g., how many horses jumped over the fence), the scope interpretation that the speaker had in mind (i.e., surface vs. inverse), as well as the likely QUD that the utterance addresses (e.g., *how-many?* vs. *all?*).

To connect our model’s predictions with the available truth-value judgment data, we follow recent suggestions in the literature for how to treat truth-value judgments. In particular, truth-value judgments are not viewed as pure language comprehension behavior, but rather as a form of language production (e.g., Degen & Goodman 2014; Jasbi et al. 2019). Recall from our discussion of the task above that it does not present as a typical comprehension task because both the participant and the speaker are already aware of the true world state. So, the participant is not trying to simply comprehend the utterance, which would involve the participant trying to infer the world state, given the utterance. Instead, participants in the truth-value judgment task are shown a scenario and asked if a specific utterance can accurately describe that scenario. In this way, the truth-value judgment task seems to be asking if a speaker should describe the given scenario with the test sentence.

A simple way for participants to make this decision is to decide if they would produce that utterance, given that scenario. If so, participants should endorse the utterance; if not, participants should not endorse the utterance. So, if participants judge the utterance as a reasonable description because they judge that they themselves could produce that utterance in the scenario, the participants endorse the utterance in the truth-value judgment task. As noted before, this setup means that participants have to effectively reason about their own potential production. Given this understanding of the task, we model participants’ truth-value judgment behavior as the (relative) endorsement of a

pragmatic speaker S_2 for an utterance about an observed situation; S_2 makes this decision by reasoning about the probability that L_1 (who is reasoning about S_1 's reasoning about L_0) would arrive at the correct world state after hearing the utterance. Given that language understanding and language production are modeled as cases of recursive social reasoning between speakers and listeners, there is no production behavior without reasoning about comprehension (i.e., reasoning about how a listener would interpret the utterance), and there is no comprehension behavior without reasoning about production (i.e., reasoning about how a speaker would have chosen the utterance); in this way, the model blurs the boundaries of production vs. comprehension.

3.1 Model specification

We take world states $w \in W$ to correspond to the number of successful outcomes, for example, the horses that successfully jumped over the fence ($W = \{0, 1, 2\}$); the world success baserate b_{suc} determines the probability that any individual will succeed.¹ We assume a simple truth-functional semantics where an utterance u denotes a mapping from world states to truth values ($Bool = \{\mathbf{true}, \mathbf{false}\}$). We parameterize this truth function so that it depends on the scope interpretation $i \in I = \{\mathbf{inverse}, \mathbf{surface}\}$, $[[u]]^i: W \rightarrow Bool$.

We consider two alternative utterances $u \in U$: the **null** utterance (i.e., saying nothing at all, and so choosing not to endorse the utterance) and the scopally-ambiguous utterance **amb** (e.g., “*Every horse didn’t jump over the fence*”); $U = \{\mathbf{null}, \mathbf{amb}\}$. We include no additional alternative utterances because participants are given none when asked to provide truth-value judgments: they can either choose to endorse the ambiguous utterance (i.e., choose to produce it as a description of the scenario) or they can choose to not endorse the utterance. In the latter case of not endorsing the target utterance, we model this choice as the participant deciding that it would be better to communicate no information with their utterance, rather than the (misleading) information conveyed by the target utterance. To communicate no information, the model provides a null tautology, which tells the listener nothing new and leads instead to the listener relying on prior knowledge.

¹ In an earlier formulation of the model, we manipulated the world state prior by assigning probabilities directly to the possible states, rather than using a success baserate to assign those probabilities; the model produced the same behavior we report below for the current model (Savinelli et al. 2017).

The utterance semantics appears in (3)², where the interpretation parameterization only impacts the truth value for utterance **amb** (since only **amb** has multiple interpretations available). If **inverse** is active, **amb** receives a “not-all” reading and is true so long as not all (two) outcomes were successful (i.e., $w \neq 2$). If **surface** is active, **amb** receives a “none” reading, which is true only in a world with zero successes (i.e., $w = 0$).

- (3) *Utterance semantics* $\llbracket u \rrbracket^i$:
- a. $\llbracket \text{null} \rrbracket^i = \lambda w. \text{true}$
 - b. $\llbracket \text{amb} \rrbracket^i = \text{if } i = \text{inverse}, \text{ then } \llbracket \text{inverse} \rrbracket, \text{ else } \llbracket \text{surface} \rrbracket$
- where: $\llbracket \text{inverse} \rrbracket = \lambda w. w \neq 2$
 $\llbracket \text{surface} \rrbracket = \lambda w. w = 0$

The literal listener L_0 hears some utterance u (e.g., *Every horse didn't jump over the fence*) with intended interpretation i (e.g., **inverse**) and returns a uniform distribution over those world states w that are compatible with the literal semantics of u (e.g., $w \in \{0, 1\}$, so the normalized $p(w=0)=p(w=1)=0.5$).³ The function $\delta_{\llbracket u \rrbracket^i(w)}$ maps a Boolean truth value to a probability, 1 or 0 (e.g., **true** maps to 1). So, for instance, for the **inverse** interpretation where the interpretation is true for $w=0$ or 1 and false for $w=2$, $\delta_{\llbracket u \rrbracket^i(0)}=\delta_{\llbracket u \rrbracket^i(1)}=1$, while $\delta_{\llbracket u \rrbracket^i(2)}=0$.

$$P_{L_0}(w|u, i) \propto \delta_{\llbracket u \rrbracket^i(w)}$$

We consider three QUDs $q \in Q$: (i) “How many horses succeeded?” (**how-many?**), (ii) “Did all of the horses succeed?” (**all?**), and (iii) “Did none of the horses succeed?” (**none?**). The QUDs serve as projections from the inferred world state to the relevant dimension of meaning, $q : W \rightarrow X$ (Kao, Wu, Bergen & Goodman 2014b; Kao, Bergen & Goodman 2014a). In practice, the QUDs establish partitions on the possible world states, as shown in (4): **how-many?** is an identity function on world states, **all?** returns **true** only if both outcomes were successful, and **none?** returns **true** only if none of the outcomes were successful.

- (4) *QUD semantics* $\llbracket q \rrbracket$:

² We use notation that maximizes transparency to the implementation in the publicly-available code base at [URL TO BE INSERTED].

³ We are presenting a version of RSA where L_0 does not take into account the state prior $P(s)$ in calculating the posterior over states, which is a departure from the original formulation. For more on this choice, including empirical justification, see Qing & Franke (2015); Scontras et al. (electronic).

- a. $\llbracket \text{how-many?} \rrbracket = \lambda w. w$
- b. $\llbracket \text{all?} \rrbracket = \lambda w. w = 2$
- c. $\llbracket \text{none?} \rrbracket = \lambda w. w = 0$

To capture the notion that communication proceeds relative to a specific QUD q , L_0 must infer not only the true world state w , but also the value of the QUD applied to that world state, $\llbracket q \rrbracket(w) = x \in X$. When q is *how-many?*, X ranges over W ; otherwise, X ranges over $Bool$. In other words, when q is *how-many?*, L_0 infers whether x is 0, 1, or 2; when q is *all?*, L_0 infers whether x is *true* (i.e., $w \in \{2\}$) or *false* (i.e., $w \in \{0, 1\}$); when q is *none?*, L_0 infers whether x is *true* (i.e., $w \in \{0\}$) or *false* (i.e., $w \in \{1, 2\}$).⁴

$$P_{L_0}(x|u, i, q) \propto \sum_w \delta_{x=\llbracket q \rrbracket(w)} \cdot P_{L_0}(w|u, i)$$

The speaker S_1 chooses an utterance u in proportion to its utility. Utterance utility concerns the chance of successfully communicating q 's answer (i.e., the answer to the QUD) to L_0 . Thus, S_1 chooses utterances by maximizing the probability that L_0 arrives at the intended x from u . This selection is implemented via a softmax function (*exp*) and free temperature parameter α , which controls how “rational” or “greedy” the speaker will be in utterance selection; as α increases, S_1 is more likely to choose utterances with higher utility. One way to think about α is as a contrast parameter that controls how the modeled speaker views relative probabilities in a probability distribution. When $\alpha=1$, the modeled speaker views the true relative probabilities (e.g., 0.6 vs. 0.4 utility); when $\alpha<1$, the contrast is decreased, and so the differences between relative probabilities are smoothed away (e.g., 0.55 vs. 0.45 utility); when $\alpha>1$, the contrast is increased, and so the differences between relative probabilities are sharpened (e.g., 0.7 vs. 0.3 utility). In this way, $\alpha>1$ leads S_1 to choose utterances with higher utility more often—the relative probability of a higher utility utterance is increased (e.g., from 0.6 to 0.7).⁵

⁴ We note that by partitioning the possible world states, QUDs allow the modeled listener to shift the probabilities determined by the literal semantics. In fact, QUD manipulations were originally proposed within the RSA framework to handle non-literal language, where, by necessity, the probability determined by the literal semantics must shift; see Kao et al. (2014a; b) for additional discussion.

⁵ See Zaslavsky et al. (2020) for a recent exploration of the role of α in RSA. RSA models also factor in the cost of the utterance, such that S_1 's utility seeks to minimize utterance cost. We assume that our utterances are equally costly—neither response in the truth-value judgment task imposes a greater cost, as the participant is saying either “yes” or “no”—so the cost term cancels out.

$$P_{S_1}(u|w, i, q) \propto \exp(\alpha \cdot \log(L_0(x|u, i, q)))$$

Utterance interpretation happens at the level of the pragmatic listener L_1 , who interprets an utterance u to jointly infer the world state w , the interpretation i , and the QUD q . We therefore model ambiguity resolution as pragmatic inference over an under-specified utterance semantics (i.e., the interpretation variable i ; Scontras & Goodman 2017). To perform this inference, L_1 inverts the S_1 model by Bayes’ rule, and so the joint probability of w , i , and q is proportional to the likelihood of S_1 producing utterance u given world state w , interpretation i , and QUD q , as well as the priors on w , i , and q .

$$P_{L_1}(w, i, q|u) \propto P_{S_1}(u|w, i, q) \cdot P(w) \cdot P(i) \cdot P(q)$$

To model the utterance endorsement implicit in truth-value judgment behavior, we need one more level of inference. The pragmatic speaker S_2 observes the true world state w and selects u by inverting the L_1 model, thus maximizing the probability that a pragmatic listener would arrive at w from u by summing over possible interpretations i and QUDs q that accompany world w . In other words, S_2 chooses u to communicate w by simulating how L_1 would resolve i and q for each of the possible utterances.

$$P_{S_2}(u|w) \propto \exp(\log \sum_{i,q} P_{L_1}(w, i, q|u))$$

3.2 Model predictions

To generate model predictions, we must fix various model parameters. The S_1 speaker rationality parameter $\alpha > 0$ is set to 1 (i.e., no scaling of S_1 ’s utility), although we find the same qualitative patterns with higher values of α . The priors $P(w)$ and $P(q)$ correspond to expectations for the discourse context (i.e., likely world states or QUDs). In the default case, we set these priors so that the individual success baserate b_{suc} is set to 0.5 (i.e., horses have a 50% chance of success) and the relevant QUDs have equal probability (i.e., $P(\text{how-many?}) = P(\text{all?}) = P(\text{none?}) = \frac{1}{3}$). The interpretation prior $P(i)$ corresponds to how easy it is to access the inverse scope interpretation. In the default case, $P(\text{inverse}) = P(\text{surface}) = 0.5$. Because model priors correspond to the beliefs that conversational participants bring to bear on the communication scenario instantiated by a truth-value judgement task, these default settings model a case where a conversational participant has maximal uncertainty about the relevant parameter values.

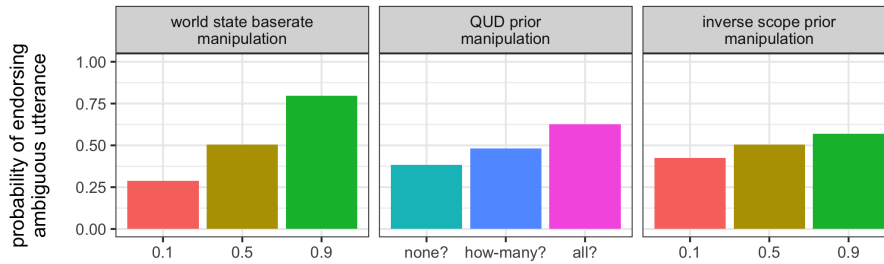


Figure 2: Model predictions for ambiguous utterance endorsement (e.g., *Every horse didn't jump over the fence*) in a not-all scenario (e.g., 1-of-2 horses jump over the fence). Lower endorsement probability corresponds to less adult-like (i.e., more child-like) behavior. For the QUD factor, the favored parameter value receives most of the prior probability weight ($P(\textit{favored}) = 0.9$). For the processing variable (scope), the prior corresponds to how strongly the inverse scope is favored.

To better understand children's utterance endorsement behavior with scopally-ambiguous utterances, we can independently manipulate the values of the priors on W , Q , and I —modeling the possibility that different participants would enter the communication scenario with different beliefs—and observe their impact on utterance endorsement in not-all scenarios. That is, we can systematically manipulate the relevant priors to test how pragmatic factors (W and Q) and processing factors (I) contribute to non-adult-like utterance endorsement behavior in the truth-value judgment task.

To investigate the effect of manipulating the world state prior (Figure 2, *left panel*), we systematically alter the success baserate b_{suc} ; in the horse context, b_{suc} controls beliefs about how likely horses are to succeed at jumping. Holding the QUD and scope priors at their default values, we see a marked increase in endorsement of the ambiguous utterance in the not-all scenario as beliefs about horse success increase. Utterance endorsement is at its lowest (0.29) when prior knowledge suggests that horses are particularly unlikely to succeed at jumping (i.e., that b_{suc} is 0.1); utterance endorsement is at its highest (0.80) when we believe horses are very likely to succeed (i.e., that b_{suc} is 0.9).

Just as with the world state prior, we can systematically manipulate the QUD prior (Figure 2, *center panel*). Favored QUDs receive a prior probability of 0.9; other QUDs receive a prior probability of $0.1/2 = 0.05$. Holding the other priors at their default values, we see an increase in utterance endorsement from the *none?* (0.38) to *how-many?* (0.48) to *all?* (0.63) QUD. So, utterance endorsement is at its lowest when we believe the QUD is about whether none

of the horses jumped; utterance endorsement is at its highest when we believe the QUD is about whether all of the horses jumped.

Finally, for the binary scope prior (Figure 2, *right panel*), we systematically manipulate the prior probability of **inverse** scope from 0.1 to 0.9. Holding the other priors at their default values, we see a monotonic increase in utterance endorsement as the probability of **inverse** increases. The model predicts an endorsement probability of 0.57 when the prior probability of **inverse** is at its highest (0.9)—at its lowest (0.1), endorsement drops to 0.42. So, the more accessible the inverse interpretation, the more utterance endorsement increases—though notably, the change is less than the endorsement rate changes that occur by altering the pragmatic factors.

To summarize, the world state and QUD priors have a more dramatic impact on utterance endorsement than the scope prior. There are two main reasons for these predictions. First, for the world state prior, when expectations favor success, the ambiguous utterance is maximally informative regardless of the scope interpretation it receives: **amb** communicates to a listener that prior expectations do not hold (i.e., *None/Not all of the horses succeeded* goes against the expectation that all (two) horses would succeed, which is what high b_{suc} entails). So, **amb** is particularly useful for communicating about the *a priori* unlikely not-all world states that appear in the experimental scenarios. Second, for the QUD manipulation, when **all?** is favored, either interpretation of **amb** fully resolves the QUD: whenever **amb** is true (i.e., whether none or not all of the horses succeeded), it is not the case that all of the horses succeeded. A pragmatic speaker recognizes the utility of **amb** as an answer to **all?** in a not-all world state, irrespective of the intended scope interpretation. More generally, both pragmatic factors highlight that either scope interpretation will suffice if the right pragmatic context is present (a high b_{suc} or favoring the **all?** QUD). Thus, the model predicts that the grammatical processing factor (i.e., the inverse scope prior) should matter very little if both pragmatic factors are set so that either scope interpretation is informative. We demonstrate that this prediction is indeed true in Figure 3.

In particular, Figure 3 shows the interaction of all three factors for utterance endorsement when $b_{suc} = 0.9$ and **all?** are favored. We see the combined effects of the world state and QUD priors; together, they lead to near-total endorsement of the ambiguous utterance. We also see more clearly the relatively small contribution of the scope prior, where changing the prior probability of **inverse** from 0.1 to 0.9 leads to just a 0.002 change in endorsement probability. Thus, we see how the priors on the pragmatic factors overwhelm the processing factor of scope access. When the optimal QUD and world state are favored, even when **inverse** is highly inaccessible (i.e., $P(\text{inverse}) = 0.1$), we still

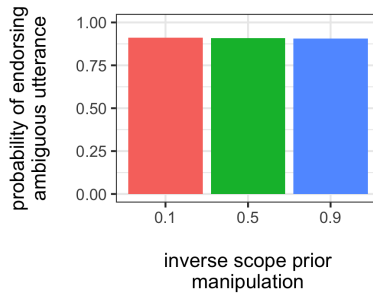


Figure 3: Model predictions for ambiguous utterance endorsement when total-success world states are favored ($b_{suc} = 0.9$) and the optimal QUD is favored ($P(\text{all?}) = 0.9$).

predict high utterance endorsement (0.91). That is, even if the inverse scope is very inaccessible, the model predicts high rates of endorsement for the truth-value judgment task when a supportive pragmatic context is present.

3.3 Discussion

Our results suggest that when it comes to understanding non-adult-like behavior in the truth-value judgment task, there is a stronger role for the pragmatics of context management (as realized in priors on world state and QUD) than for grammatical processing (as realized in the prior on scope interpretations), although there may be a role for both. So, the observed failure of children to endorse scopally-ambiguous utterances in not-all scenarios likely stems more from children’s beliefs about the world of the experiment (e.g., whether horses are *a priori* likely to succeed) and about the topic of conversation (e.g., whether the conversational goal is to determine if all the horses succeeded) than an inability to grammatically derive or access the inverse scope interpretation. Indeed, our model predicts the highest rates of utterance endorsement when resolving the scope ambiguity is irrelevant for communicating successfully about the not-all world. In other words, the model predicts high endorsement whenever the pragmatic context is supportive—either because expectations favor total success or the QUD asks if **all?** of the horses succeeded—irrespective of how difficult it is to access the inverse scope. This prediction arises because both scope interpretations serve to inform a listener, either that the *a priori* likely $w = 2$ does not hold, or that the answer to the **all?** QUD is *no*. So, the non-adult truth-value judgment task behavior we see in children is predicted to stem from an inability to manage the pragmatic context as effectively as

adults do; to become more adult-like in these scenarios, children must learn to adapt to less supportive pragmatic contexts.

Still, we might wonder if children’s inability to access or reason about alternatives plays a non-trivial role in their truth-value judgment behavior here. Indeed, the growing literature on this topic highlights children’s inability to access or reason with alternatives as a primary source of the divergences between child and adult pragmatic language behavior (e.g., Barner et al. 2011; Bale & Barner 2013). In the case of the judgment data that we model, perhaps there are utterance alternatives better-suited to describing the not-all scenarios, and adults—but not children—incorporate these alternatives into their truth-value reasoning. In the judgment data, any preference for these alternative utterances gets absorbed into the probability associated with choosing not to endorse the *every-not* test utterance. According to this reasoning, a participant would identify a better way of describing the not-all scenario and therefore choose to not endorse the utterance provided. This hypothesis therefore predicts that participants who are better able to reason about alternative utterances (i.e., adults) should endorse the scopally-ambiguous test utterance less often in the not-all scenario. Similarly, participants who are less able to reason about alternative utterances (i.e., children) should endorse the test utterance more often. Yet, the behavioral patterns do not support this prediction: adults endorse the test utterance more than children do, not less. Therefore, we believe that children’s inability to reason about a broader set of utterance alternatives is unlikely to explain the behavioral patterns we have captured with our model.

If adults and children are using the same mechanism to resolve scope ambiguity in context (as implemented in our RSA model), we should find that similar contextual pressures affect endorsement behavior in both groups. In particular, less supportive pragmatic contexts in these scenarios—due to the priors on world states or QUDs—should lead to lower endorsement rates also in adults. Preliminary behavioral results suggest that adults are sensitive to QUD manipulations for *every-not* utterances precisely as our model predicts: endorsement rates are at their highest when **all?** is privileged, intermediate for **how-many?**, and lowest for **none?** (Song et al. 2020; 2021). We build on this finding in the following section by exploring a case of ambiguity where adults start behaving like children.

4 Two-not

Over the course of three truth-value judgment tasks, Musolino & Lidz (2003) demonstrated that adults are sensitive to some of the same experimentally-manipulated factors as children when it comes to endorsing scopally-ambiguous utterances. Rather than looking at *every-not* sentences, Musolino & Lidz investigated sentences with negation and cardinal numerals like *two*, as in (5). As with *every-not*, these *two-not* sentences admit two interpretations, corresponding to the relative scope of the logical operators introduced by the numeral and negation.

- (5) Two horses didn't jump over the fence.
- a. SURFACE SCOPE ($\exists > \neg$):
There are two horses that didn't jump over the fence.
 - b. INVERSE SCOPE ($\neg > \exists$):
It's not the case that there are two horses that jumped over the fence.

One scenario that distinguishes between these interpretations is shown in Figure 4, where there are four horses total and two (horses 1 and 2) jumped over the fence while another two (horses 3 and 4) did not. Here, the surface interpretation is true: there are in fact two horses, horses 3 and 4, that did not jump over the fence. In contrast, the inverse interpretation is false: there are two horses that jumped over the fence (horses 1 and 2).

In the first task of Musolino & Lidz (2003), adults heard *two-not* sentences in a context where both interpretations were true. For example, the scenario might have one out of three horses jumping over a fence; the surface interpretation is true because there are two horses who did not jump; the inverse interpretation is also true because it is not the case that there are two horses who did jump. After deciding whether to endorse the utterance, participants then justified their response so that their scope interpretation could be inferred. For example, if their explanation referred to the two horses that did not jump, then it was assumed that participants accessed the surface interpretation (there are two horses that didn't jump). However, if the explanation referred to only one horse jumping, then it was assumed that participants accessed the inverse interpretation (only one horse jumped, so it's not the case that two did). Musolino & Lidz found that all participants endorsed the utterance, and the explanations provided indicated a strong surface scope bias (75% surface, 7.5% inverse, 17.5% unclear from explanation). The authors interpreted this finding

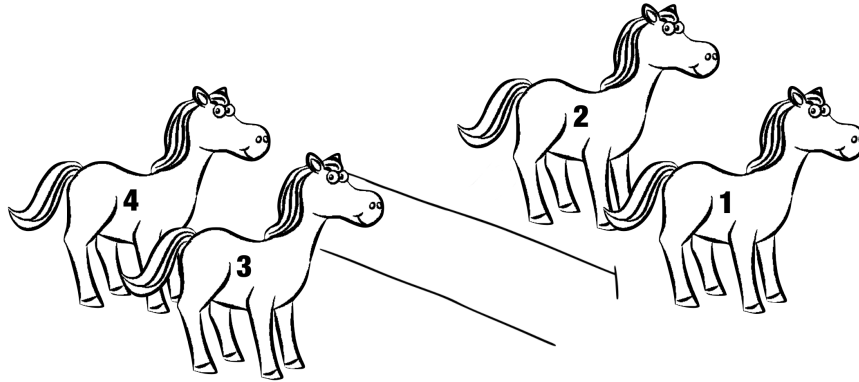


Figure 4: Example 2-of-4 scenario in which horses 1 and 2 jump over the fence but horses 3 and 4 do not. In this scenario, only the surface interpretation of the *two-not* sentence is true.

as evidence that adults prefer the surface interpretation of *two-not* utterances when both interpretations are true in context.

In the second task, adults heard a *two-not* sentence in two different contexts. The first context included two actors (e.g., horses), with one actor successfully completing the action (as in Figure 1; e.g., horse 1 jumped while horse 2 didn't). In this 1-OF-2 context, the surface interpretation is false (only one horse didn't jump, so it is false that two horses didn't jump), but the inverse interpretation is true (only one horse did jump, so it is indeed not the case that two horses jumped). Adults exhibited low endorsement (27.5%) for these 1-OF-2 contexts.

In the second context, there were four actors. For example, four horses attempted to jump over a fence; two jumped and two did not, as in Figure 4. In this 2-OF-4 context, the surface interpretation of the scopally-ambiguous *two-not* utterance is true: there are two horses that did not jump (horses 3 and 4 in Figure 4). However, the inverse interpretation is false because there are two horses that did jump (horses 1 and 2). In these contexts, adults had an endorsement rate of 100%.

Musolino & Lidz interpreted this asymmetry in endorsement rates between the two types of contexts, 1-OF-2 vs. 2-OF-4, as a strong surface scope preference in adults. According to this explanation, non-endorsement occurs in the 1-OF-2 context because only the inverse scope is true; in contrast, endorsement occurs in the 2-OF-4 context because the surface scope is true. That is, both patterns arise because adults favor the surface interpretation. While we find this account compelling, we note that there are other differences between the

two contexts that might lead to the observed asymmetry. For example, it could be that the seemingly benign change from two to four total actors affects the pragmatic context. Another variable is the potential ambiguity present in the numeral semantics, which only becomes relevant in the 2-OF-4 context—we return to this ambiguity in the following subsection. In either case, exploring the effects of these factors in a formal model of truth-value judgment behavior like the one we implemented above can clarify the process underlying utterance disambiguation. Before presenting such a model, we review one additional experiment that investigates the impact of different experimental context manipulations on adult judgments. In particular, [Musolino & Lidz](#) set out to determine whether adults are affected by the same factors as children when it comes to increasing utterance endorsement for scopally-ambiguous utterances.

In their third task, the [Musolino & Lidz](#) tested adults in 1-OF-2 contexts using an early-success manipulation familiar from the child truth-value judgment experiments reviewed above. With an early-success manipulation, adults saw a positive contrasting clause describing successful outcomes before the utterance of interest, as in (6).

- (6) **Two horses jumped over the rock, but**
two horses didn't jump over the fence.

Adults responded just as the children did to the early-success contexts, shifting to strong endorsement (92.5%; cf. 27.5% endorsement without the explicit contrast). However, as [Musolino & Lidz](#) note, it is not obvious *why* the adult endorsement rate increases when the early-success contrast is present.

Here is where our model of utterance endorsement might be able to help: just as we did with *every-not* utterances, we can model utterance endorsement for *two-not* utterances in an attempt to formally explicate the contribution of context to the observed endorsement behavior. In the process, we can also test the hypothesis of continuity in the development of scope ambiguity resolution: if the same model architecture can capture both child and adult behavior, we have strong support for the hypothesis that children and adults are employing the same disambiguation mechanism.

4.1 Model specification

Our *two-not* model is a direct extension of the *every-not* model presented above. As before, we take world states $w \in W$ to correspond to the number of successful outcomes; the world success baserate b_{suc} determines the probability that an individual will succeed. We continue to assume a simple truth-functional semantics where an utterance u denotes a mapping from world states to truth

values. As before, we parameterize this truth function so that it depends on the scope interpretation $i \in I = \{\text{inverse}, \text{surface}\}$, $\llbracket u \rrbracket^i: W \rightarrow \text{Bool}$. We consider two alternative utterances $u \in U$: the **null** utterance (i.e., saying nothing at all, which we take as equivalent to choosing *not* to endorse the utterance) and the scopally-ambiguous *two-not* utterance **amb** (e.g., *Two horses didn't jump over the fence*).

To fix the utterance semantics, we must consider potential ambiguity introduced by the numeral in cases where the number of relevant individuals n exceeds the numeral's value. For example, consider the positive utterance *Two horses jumped over the fence*. If we assign an exact ($=$) semantics to the utterance, it will be true only when two horses succeeded. If we assign an at-least (\geq) semantics, the sentence will be true when two or more horses succeeded. In worlds with only two horses, the exact vs. at-least distinction makes no difference: the sentence will be true in the world where both horses succeed, and false in all other worlds. However, in a world with four horses, the numeral semantics will define different truth-functional mappings. With the exact semantics, the sentence is true in any world where two horses—but not more—succeed. With the at-least semantics, the sentence is true in a larger set of worlds, where two or more horses succeed.

To evaluate the potential contribution of utterance semantics to the 1-OF-2 vs. 2-OF-4 asymmetry, we consider two different sets of utterance alternatives, one with **amb₌** and another with **amb_≥**. So, $U_{=} = \{\text{null}, \text{amb}_{=}\}$ and $U_{\geq} = \{\text{null}, \text{amb}_{\geq}\}$. The utterance semantics in (7) shows that scope parameterization i only impacts the truth conditions for **amb** utterances.

- (7) *Utterance semantics* $\llbracket u \rrbracket^i$:
- a. $\llbracket \text{null} \rrbracket^i = \text{true}$
 - b. $\llbracket \text{amb}_{=/\geq} \rrbracket^i = \begin{cases} \text{if } i = \text{inverse, then } \llbracket \text{inverse}_{=/\geq} \rrbracket, \\ \text{else } \llbracket \text{surface}_{=/\geq} \rrbracket \end{cases}$

where:

$$\begin{aligned} \llbracket \text{inverse}_{=} \rrbracket &= \lambda w. w \neq 2 \\ \llbracket \text{surface}_{=} \rrbracket &= \lambda w. \text{if } \max(W) = 2, \text{ then } w = 0, \text{ else } w = 2 \\ \llbracket \text{inverse}_{\geq} \rrbracket &= \lambda w. w < 2 \\ \llbracket \text{surface}_{\geq} \rrbracket &= \lambda w. w < 3 \end{aligned}$$

In our horse-jumping scenario, the **inverse₌** interpretation returns **true** just in case the number of horses that jumped is not equal to two (so $w \neq 2$, which means the number could in fact be 3 or 4, or 0 or 1). Similarly, **surface₌** returns **true** just in case the number of horses that did not jump is equal to two; in a world with two horses, this requirement means that zero horses

jumped ($w=0$), and in a world with four horses, this requirement means that exactly two horses did jump ($w=2$). For the at-least interpretations, **inverse**_≥ returns **true** just in case the number of horses that jumped is less than two. That is, if it is not the case that at least two horses jumped, then zero horses or only one horse jumped (and so $w \in \{0,1\}$, which is equivalent to $w < 2$). The at-least **surface**_≥ returns **true** just in case the number of horses that jumped is less than three. That is, if at least two horses did not jump, then two, three, or four did not jump, which means two, one, or zero did jump (so $w \in \{0,1,2\}$, which is equivalent to $w < 3$).

We consider five potential QUDs $q \in Q$, three from the *every-not* model: (i) “How many horses succeeded?” (**how-many?**), (ii) “Did all of the horses succeed?” (**all?**), and (iii) “Did none of the horses succeed?” (**none?**). We also consider two additional QUDs specific to the *two-not* utterance: (iv) “Did exactly two horses succeed?” (**two=?**), and (v) “Did at least two horses succeed?” (**two**_≥**?**). We add the **two?** QUDs under the assumption that by explicitly mentioning a numeral, that cardinality may be directly relevant to the topic of conversation. The QUDs behave as in (8).

- (8) *QUD semantics* $\llbracket q \rrbracket$:
- a. $\llbracket \text{how-many?} \rrbracket = \lambda w. w$
 - b. $\llbracket \text{all?} \rrbracket = \lambda w. w = \max(W)$
 - c. $\llbracket \text{none?} \rrbracket = \lambda w. w = 0$
 - d. $\llbracket \text{two=?} \rrbracket = \lambda w. w = 2$
 - e. $\llbracket \text{two}_{\geq}\text{?} \rrbracket = \lambda w. w \geq 2$

4.2 Model predictions

To generate model predictions for adult sensitivity to the pragmatic contrast manipulation and the 1-OF-2 vs. 2-OF-4 asymmetry, we fix various model parameters. For 1-OF-2 data, we set the number of individuals to 2 (i.e., $\max(W) = 2$); for 2-OF-4 data, we set the number of individuals to 4 ($\max(W) = 4$). The S_1 speaker rationality parameter $\alpha > 0$ is set to 1. As before, the priors $P(w)$ and $P(q)$ correspond to expectations for the discourse context. In the default case, we set the individual success baserate b_{suc} to 0.5 and we set $P(q)$ so that the relevant QUDs have equal prior probability. The interpretation prior $P(i)$ corresponds to how easy it is to access the **inverse** scope interpretation. In the default case, $P(\text{inverse}) = P(\text{surface}) = 0.5$. As with the *every-not* model, we can independently manipulate the values of the priors on W , Q , and I , and observe their impact on utterance endorsement in order to better understand utterance endorsement behavior with scopally-ambiguous utterances.

Recall the empirical phenomena we are trying to capture: (i) the dramatic increase in endorsement rates in the 1-OF-2 context when an explicit contrast is present, and (ii) the stark asymmetry in utterance endorsement rates between 1-OF-2 and 2-OF-4 contexts. We report results for each phenomenon in turn.

4.2.1 The explicit contrast effect for 1-of-2

We can attempt to capture the increase in ambiguous utterance endorsement rates by systematically manipulating the pragmatic and processing factors, as implemented in the relevant priors. In a 1-OF-2 context, the *two-not* model predictions are identical to the predictions of the *every-not* model in Figure 2 above—the models align because the ambiguous *two-not* and *every-not* utterances, for both scope interpretations, wind up true of exactly the same world states when $W = \{0, 1, 2\}$. That is, the **surface** interpretation for the *every-not* utterance holds that all (i.e., two) horses failed to jump over the fence (i.e., $w = 0$); the **surface** interpretation for the *two-not* utterance is the same: two (i.e., all) horses failed to jump ($w = 0$). The situation is similar for the **inverse** interpretation: *every-not* is true when not all of the horses jumped over the fence (i.e., $w = 0, 1$), and *two-not* is true when the number of horses that jumped is not two ($w = 0, 1$)

By replicating the results of our manipulations for the *every-not* model, each prior manipulation for the *two-not* model qualitatively captures the response pattern from Musolino & Lidz (2003). In particular, as before, the pragmatic factors controlling world and QUD beliefs have a more pronounced effect than the grammatical processing factor controlling scope access; the model’s world prior baserate manipulation comes closest to capturing the experimentally-observed effect of explicit contrast manipulation (i.e., 27.5% base endorsement vs. 92.5% endorsement with the explicit contrast).

Just as before, we can also amplify the effect of the world baserate manipulation by allowing it to interact with the other factors. Specifically linking this manipulation to the experimental context, the early-success explicit contrast manipulation possibly affects two aspects of the disambiguation calculus. First, it could increase expectations for success (i.e., a high b_{suc} if all (two) horses recently succeeded at jumping over something); second, it could shift the topic of conversation to whether total success was achieved again (i.e., a high prior on the **all?** QUD). To model the gangup of factors, Figure 5 plots the interaction of the world and QUD priors, together with the effect of scope.

The right side of Figure 5 replicates Figure 3: we see that access to the inverse scope has very little impact on the endorsement rate. This contrasts with the left side of Figure 5, where a low baserate ($b_{suc}=0.1$) and a uniform prior on

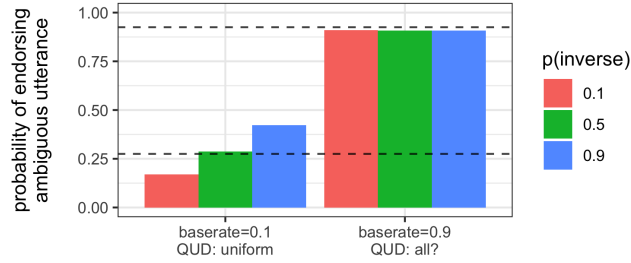


Figure 5: Model predictions for ambiguous *two-not* utterance endorsement in a 1-OF-2 context when multiple factors interact. Dashed lines represent experimentally-observed endorsement behavior in the absence (lower) vs. presence (upper) of an explicit contrast.

QUDs (so *all?* isn't favored) lead access to the inverse scope to have a noticeable effect, with endorsement increasing with the prior probability of the *inverse* interpretation. Given these predictions, it seems that the empirically-observed low-endorsement baseline (27.5%) most likely results from low expectations for success ($b_{suc} = 0.1$) and QUD uncertainty (QUD: *uniform*), together with a moderate-to-low probability of accessing the *inverse* scope ($P(inv) = 0.1$ or 0.5). From this baseline, we can implement the effect of the explicit contrast manipulation by increasing success expectations ($b_{suc} = 0.9$) and shifting the topic of conversation to whether total success occurred (QUD: *all?*). This manipulation results in a dramatic increase in utterance endorsement, irrespective of scope.

To summarize, if the explicit contrast clause impacts a listener's beliefs about the horses' chance of success (increasing b_{suc}) or the QUD (favoring *all?*), then the model predicts the endorsement rate should increase. Notably, both of these manipulations make the *two-not* scopally-ambiguous utterance more informative for a listener. In the case of the the world state manipulation, *two-not*—under either scope interpretation—informs the listener that her prior beliefs about total horse success do not hold. Similarly with the QUD manipulation favoring *all?*, both scope interpretations answer this question in the negative (i.e., it is not the case that all (two) horses succeeded).

4.2.2 The 1-of-2 vs. 2-of-4 asymmetry

Our model predicts that these factors should be active in utterance disambiguation more generally; therefore, the same model implementation used to capture 1-OF-2 endorsement behavior should be able to capture 2-OF-4 endorsement

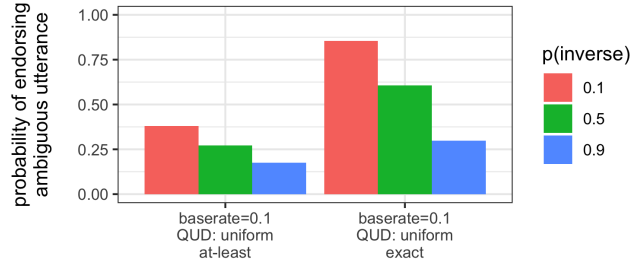


Figure 6: Model predictions for ambiguous *two-not* endorsement in a 2-OF-4 context with a low baserate of success is low (0.1) and QUD uncertainty (uniform QUD prior). On the left, we see predictions for an at-least semantics; on the right, we see predictions for an exact semantics.

behavior. More specifically, we would expect the very same factors and values to additionally capture the high endorsement rate in the 2-OF-4 context *without* the explicit contrast. If so, we would have computational modeling evidence that the factors identified for capturing the experimentally-observed effect of the explicit contrast are indeed active in utterance disambiguation more generally.

Recall the baseline 1-OF-2 parameter values most likely to lead to low endorsement: low expectations for success ($b_{suc} = 0.1$) and QUD uncertainty (QUD: `uniform`). To model the 2-OF-4 context, we change the number of actors n to 4 and additionally manipulate whether the exact ($=$) or at-least (\geq) utterance semantics applies, as predictions diverge when there are more than two actors in the context (recall the discussion in Section 4.1). This decision impacts both the utterance semantics and the relevant set of QUDs (e.g., if the at-least semantics gets used, then the `two≥?` QUD is included in the set of potential QUDs).

As shown in Figure 6, we do indeed predict high endorsement with the same parameter value baseline, but only with exact utterance semantics and a fairly low probability of accessing the inverse scope ($P(inv) = 0.1$). This prediction is shown on the right side of Figure 6, where a high endorsement rate is predicted with the pragmatic factors identified above ($b_{suc} = 0.1$, QUD prior is `uniform`), as long as the numeral *two* has an exact semantics and access to `inverse` scope is low ($P(inv) < 0.5$). In contrast, when *two* has an at-least semantics (left side of Figure 4), the model predicts low endorsement with these pragmatic factors.

4.3 Discussion

Our model of *two-not* utterances—a straightforward extension of the *every-not* model—captures the effect of the early-success explicit-contrast manipulation observed in adults. Notably, we saw that the *every-not* model captures the same effect in children. This parallelism—sensitivity to the pragmatic context in both children and adults across different contexts—suggests that the same disambiguation mechanism is active in both children and adults. Adults seem better able to charitably interpret less-supportive pragmatic contexts (i.e., the original *every-not* scenarios; cf. the Principle of Charity from Gualmini et al. 2008); yet, there remain scenarios (i.e., the 1-OF-2 *two-not* contexts) where even adult abilities are exceeded. We interpret the common underlying mechanism as support for developmental continuity in scope ambiguity resolution; no qualitative shift is required for five-year-old children to become adult-like in how they resolve scope ambiguity in context.

The model also captures Musolino & Lidz’s results from the 2-OF-4 context: with the very same parameter values that yield low endorsement rates for 1-OF-2 contexts, the model predicts the high endorsement observed for 2-OF-4 contexts. The only change is increasing the number of relevant individuals from two to four. This exploration of the 1-OF-2 vs. 2-OF-4 contexts allows us to refine our understanding of the potential sources of child and adult behavior. Our findings from the *every-not* model suggested that pragmatic factors alone are capable of capturing the non-adult-like behavior in children, and the extension in the current model captures the explicit-contrast effect in adults; however, it appears that the processing factor of scope access (in particular, disfavoring the inverse scope) is needed to account for Musolino & Lidz’s adult 2-OF-4 results. This finding supports the conclusion of Musolino & Lidz, namely that adults have a strong preference for surface interpretations of *two-not* utterances. Combined with the appropriate pragmatic context, that preference has the potential to drive the endorsement asymmetry between the 1-OF-2 and 2-OF-4 contexts. Whether this surface-interpretation preference for *two-not* utterances is also something children share remains an open empirical question; experimental results for *every-not* do not answer this question definitively (Viau et al. 2010).

Interestingly, the current model requires one more ingredient to account for the 1-OF-2 vs. 2-OF-4 difference in adult behavior: an exact semantics for utterances with numerals (in contrast to an at-least semantics; for discussion, see Geurts 2006; Breheny 2008; Spector 2013; Kennedy 2015). While the underlying utterance semantics is not something easy to manipulate in an experiment, it is exactly the kind of variable we can systematically explore in a computational cognitive model. By doing so here, we are able to show

the necessity of an exact semantics in generating observable adult behavior. This result provides empirical support, coming from computational cognitive modeling, for theories about the semantics and pragmatics of numerals. In particular, we account for the observed adult behavior by assuming that adults interpret *two-not* utterances as meaning exactly two and not at least two.

5 General discussion

Truth-value judgments serve a critical role in diagnostics of linguistic meaning, yet the cognitive processes involved in generating these judgments—particularly the precise impact of context on pragmatic reasoning—have rarely been formally examined. Here, we have formally investigated the cognitive underpinnings of the truth-value judgment methodology. We used as our case study the phenomenon of scope ambiguity, where children’s behavior often deviates noticeably from that of adults; yet, both child and adult behavior can be profoundly affected by changes to task setups. Using the methodology of computational cognitive modeling, we advanced precise hypotheses about how linguistic knowledge, world knowledge, and general social reasoning interact to deliver observed behavior in the truth-value judgment task. To the extent that our model captures the data we set out to predict, we have found support for the hypothesis our model encodes, which specifies how pragmatic and processing factors interact to generate observed truth-value judgment behavior. While we believe it is possible (and perhaps even likely) that other models with different assumptions may also be able to capture the judgment behavior, our aim here has been to test the viability of our hypothesis—an existence proof—rather than performing model comparisons. An exciting area for future work is to specify alternative hypotheses via computational cognitive models, and see if those models too can capture the behavior patterns that our hypothesis here does.

Our model of utterance endorsement in the truth-value judgment task predicts the lowest rates of utterance endorsement for ambiguous utterances in not-all scenarios (as in Figure 1) when neither interpretation—surface or inverse—is useful for successful communication. The interpretation could be less useful because the interpretation is false; for example, the surface interpretation *every-not* is false in the not-all scenario. The interpretation could also be less useful because beliefs about the pragmatic context render the interpretation uninformative; for example, if we expect horses should be unsuccessful at jumping or we are uncertain about the conversational topic in a not-all scenario, the inverse interpretation of *every-not* is frustratingly

imprecise. For instance, if we thought horses were going to be unsuccessful, then we expected zero horses to be successful; if all we know from the utterance is that not all of the horses were successful, we have learned very little about whether it was really the case that zero horses were successful. Similarly, if we thought the topic of conversation was about whether none of the horses were successful, hearing that not all the horses were successful is only somewhat helpful—we again do not know for sure whether none of the horses were successful.

Given these observations, the utterance non-endorsement behavior that has been previously used to demonstrate children’s difficulty with inverse scope calculation in fact requires no disambiguation at all if the goal is informative communication (as mentioned above, both interpretations can be uninformative in certain pragmatic contexts). Instead, children (and adults) simply need the ability to manage the pragmatic context so they can recognize the potential informativity of these ambiguous utterances. Notably, considerations of pragmatic context have long played a role in the design and interpretation of the truth-value judgment task (e.g., [Crain et al. 1996](#)). Here we have taken the extra step of formally articulating specific pragmatic factors and the role they play in children’s apparent difficulty with ambiguous utterances in the truth-value judgment task. In this way, we can specify how changing the experimental context impacts the pragmatic factors that underlie children’s truth-value judgment endorsement behavior.

We saw that two aspects of the pragmatic context have an outsized effect on utterance endorsement, and for similar reasons. When the QUD is such that the ambiguous utterance provides a full answer under either scope interpretation, we recognize the ambiguous utterance as a useful thing to say, and so participants are more likely to endorse it as a communicative act. For example, in a not-all horse-jumping scenario, if we care about whether all of the horses jumped, either interpretation is useful—both the surface and the inverse interpretations tell us that the answer is “no”. When prior beliefs about the world context and what counts as a likely state of affairs are contradicted by the ambiguous utterance—again, under either scope interpretation—the utterance is potentially very informative, which makes it more useful and thus more likely to be endorsed. For example, in a not-all scenario, if we think horses nearly always succeed in jumping, we would expect the world where all the horses are successful to be most likely; here, either interpretation is useful because both the surface and the inverse interpretations tell us that the world where all the horses are successful is in fact not the one we are in.

So, in order to endorse the ambiguous utterance, a truth-value judgment experimental participant must be able to manage the pragmatic context in

such a way that allows them to recognize the potential utility of the ambiguous utterance; in our modeling framework, managing the pragmatic context amounts to using priors for QUDs and world states that yield a true and informative statement, even if they may not be supported already by the immediate discourse context. With *every-not* utterances, adults are able to manage the pragmatic context in this way, while children require additional support. Given the differing amounts of life (and language) experience between children and adults, it seems plausible that the two groups would arrive at different priors for the pragmatic factors in our model. However, with *two-not* utterances, even adults require additional support to manage the pragmatic context in certain scenarios.

Our findings underscore the complexity of information involved in interpreting scopally-ambiguous utterances, including the literal semantics of the utterances involved, processing factors that affect interpretation accessibility, pragmatic factors that affect the potential informativity of the utterance, and the recursive social reasoning between speakers and listeners. Over the course of two applications—explaining children’s non-adult-like behavior with *every-not* utterances and adults’ child-like behavior with *two-not* utterances—we find evidence for the impact of both pragmatic and processing factors on truth-value judgment behavior; in particular, we see how a specific confluence of values for these factors yields the observed utterance endorsement behavior in multiple contexts. The fact that the same pragmatic factors can have such a pronounced effect on both child and adult behavior highlights the developmental continuity in pragmatic reasoning from childhood to adulthood. Moreover, the fact that the processing factor of scope access is crucial for explaining adult behavior in certain contexts motivates experimental work with children to see if their behavior is likewise affected by this processing factor in similar contexts.

More broadly, we have demonstrated how computational cognitive modeling can help us refine our theories about different aspects of language, including theories of language understanding, language development, and language representation. Importantly, we have shown how analytic results allow for a better understanding of behavior in the truth-value judgment task, thereby allowing for a better understanding of the task itself and thus a cleaner mapping between our cognitive theories of ambiguity resolution and the data that test them. The moral is as follows: before we can effectively interpret truth-value judgment behavior with respect to our theories of processing, development, and representation, we must understand the pragmatics involved; the current work offers a path toward that understanding.

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