# Towards a principled logic of anaphora*

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In the absence of alternatives with comparable empirical coverage, the *dynamic* approach to anaphora has shown an impressive longevity, having been refined and extended in the decades since Heim (1982) and Kamp's (1981) foundational work. Like the dynamic approach to presupposition projection, the dynamic approach to anaphora can be criticized on the grounds of explanatory adequacy — dynamic semantics tailors the entry of each of the logical operators in order to derive the desired accessibility generalizations. Furthermore, dynamic semantics can be criticized on empirical grounds — it fails to account for, e.g., double negation and bathroom sentences. There has long been an intuition that a more explanatory account of anaphora is possible, using the same tools that have been developed for presupposition projection (George 2007, 2008, Schlenker 2008, 2009, a.o.). In this paper, I develop a simple, predictive logic of anaphora — *Dynamic Alternative Semantics* — framed as an extension of Groenendijk & Stokhof's (1991) *Dynamic Predicate Logic*, using a strong Kleene trivalent semantics as the logical substrate. I argue that the resulting theory provides a much more principled treatment of the dynamics of the logical connectives, and furthermore captures data that is problematic for previous theories. I'll also demonstrate that some of the accessibility generalizations in Groenendijk & Stokhof 1991, assumed in much subsequent work, are confounded by pragmatic factors such as ignorance inferences.

## 1 Introduction

Since its inception in the 1980s (Heim 1982, Kamp 1981), Dynamic Semantics (DS) has been an extremely rich research enterprise, with important results in the domains of, e.g., anaphora, presupposition projection, and epistemic modality. Initially, DS was motivated by the observation that singular pronouns can co-vary with singular indefinites in a broader range of environments

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than a classical semantics would lead us to expect. Concretely, the two phenomena motivating DS are *discourse anaphora* and *donkey anaphora*, as illustrated in (1) and (2) respectively:[1]

(1) #I haven't met any$^1$ philosopher; she$_1$ didn't attend the talk.

(2) Everyone who invited a$^1$ philosopher was relieved that she$_1$ came.

The phenomena in (1) and (2) have been taken to motivate a logical system in which *Egli's theorem* and its corrolary hold:

**Observation 1.1** (Egli's theorem). $(\exists^n \phi) \wedge \psi \Leftrightarrow \exists^n (\phi \wedge \psi)$

**Observation 1.2** (Egli's corrolary). $(\exists^n \phi) \rightarrow \psi \Leftrightarrow \forall^n (\phi \rightarrow \psi)$

There are many varieties of DS that fulfill this desideratum, and two separate traditions: *dynamic interpretation* (initiated by Heim's *File Change Semanitcs*) and *dynamic representation* (initiated by Kamp's *Discourse Representation Theory*).[2] In this paper, I'll be focusing on dynamic interpretation theories, and specifically Groenendijk & Stokhof's (1991) Dynamic Predicate Logic (DPL). DPL has been extremely influential in the dynamic literature, and many theories which extend DS to a broader range of empirical phenomena extend DPL (see, e.g., Groenendijk, Stokhof & Veltman 1996 on epistemic modality, and van den Berg 1996 on generalized quantifiers and discourse plurals, etc.). In the next section, we'll consider some of the more prominent issues for DS as a theory of anaphora to singular indefinites.

## 1.1 Double negation and bathrooms

It can be observed that negation renders singular indefinites *inaccessible* as antecedents for subsequent pronouns. We can show this most easily by using an Negative Polarity Item (NPI) to disambiguate scope:[3]

(4) #It's not true that any$^1$ philosopher attended this talk. She$_1$ was unwell.

We can use negative indefinites to make the same point, on the assumption that negative indefinites can be decomposed into sentential negation and existential quantification.

[1] By convention, I'll decorate sentences of English with superscript and subscript indices to indicate the logical binder and bound expression(s) respectively.

[2] The terminology here is borrowed from Yalcin 2013.

[3] NPIs nevertheless license discourse and donkey anaphora, as illustrated by the examples in (3a) and (3b) respectively:

(3) a. Everyone [who read any$^1$ of these books and subsequently critized it$_1$] is a charlatan.

b. Everyone [who read any$^1$ of these books] recommended it$_1$ to their friends.

(5) #No[1] philosopher attended this talk. $\text{She}_1$ was unwell.

In DS, the semantics of negation in tailored to derive this. Without going into the details of, e.g., the DPL interpretation schema, the intuitive idea is that indefinites introduce Discourse Referents (DRS), but negation eliminates any DRS in its scope; in the parlance of DS, we say that negation is *externally static*. We'll refer to this as a "destructive" semantics for negation. An immediate consequence of destructive negation is that, once dead, a DR cannot be resurrected. This means that, in DS, doubly-negated sentences can't introduce DRS.

As has long been recognized (Groenendijk & Stokhof 1991, Krahmer & Muskens 1995), this doesn't seem to be a good prediction — an indefinite in the scope of two negative operators can antecede a subsequent pronoun. This is illustrated by example (6).[4]

(6) It's not true that NO[1] philosopher is attending this talk;
$\text{She}_1$'s sitting in the back!

This suggests that, perhaps we want an underlying logical system in which Double Negation Elimination (DNE) is valid — we can frame the issue for DS in the following way: it strays too far from the classical, thereby rendering certain desirable logical principles no longer valid. It is however worth mentioning the claim (Gotham 2019) that (6) comes with an additional inference that its positive counterpart (7) lacks — namely, that *exactly one* philosopher attended the talk. This suggests that perhaps we want our logic to only validate a limited form of DNE — we'll come back to this point in §4.1.

(7) A[1] philosopher is attending this talk; $\text{She}_1$'s sitting in the back.

The problem of double negation affects the account of other data too. For example, consider Partee's famous *bathroom sentences*. Bathroom sentences demonstrate a parallel between presupposition projection and anaphora in disjunctive sentences; in DS, the fact that the presupposition introduced by *the bathroom* fails to project is taken to indicate that the second disjunct is interpreted in the context of the negation of the first, and the presupposition of the second disjunct is thereby locally satisfied (Beaver 2001). Although less often discussed, the licensing of anaphoric pronouns completely parallels presupposition projection in this respect, as illustrated by the acceptability of (8b).

(8) a. Either there is no bathroom, or the bathroom is upstairs.
   b. Either there is no[1] bathroom, or $\text{it}_1$'s upstairs.

Naturally, we'd like to extend the intuitive explanation for the presuppositional case to the anaphoric case, but due to destructive negation, interpreting the second disjunct in the context

[4] Emphasis is indicated by small caps — although I don't think that this is *essential* for (6) to be a felicitous utterance, my judgement is that this results in a more natural-sounding sentence. Double-negation is clearly a marked option, and seems to be subject to additional, poorly-understood discourse requirements. This is unsurprising, given the availability of the positive counterpart as a competitor. The discourse conditions allowing for doubly negated sentences, and how this affects their prosody, is something that requires further investigation; I will abstract away from these questions in this paper.

of the negation of the first doesn't help explain the availability of anaphora. In other words, we'd like to explain the possibility of anaphora in (8b) in terms of anaphora in (9), but due to the design features of DS, this move is blocked.

(9) Either there is no bathroom, or there isn't $\text{no}^1$ bathroom and $\text{it}_1$'s upstairs.

## 1.2 Explanatory adequacy

DS more broadly has often been criticized on the grounds of explanatory adequacy, although the discussion tends to revolve more around presupposition projection than anaphora (Soames 1989). This is an especially forceful objection in the domain of presupposition projection, since there are competing, less stipulative theories of presupposition projection which make equivalent, if not superior, empirical predictions to a Heimian dynamic approach (see, e.g., Schlenker 2008, 2009, George 2008, 2007). The point, however, can be made for the dynamic approach to anaphora too, which, arguably has no competitors which cover all the same data.

In DS, the directionality of the flow of referential information is regulated by the semantics of the logical connectives. For example, the semantics of conjunctive sentences in DS in essence stipulates that the second conjunct is interpreted in the context of the first, thus predicting a linear asymmetry in anaphoric licensing. The problem, in a nutshell, is that it's easy to give an alternative semantics for conjunction which interprets the first conjunct in the context of the second, while still maintaining the truth-conditional contribution of conjunction. Therefore, despite purporting to account for the contrast in (10), DS operates at a highly descriptive level.

(10) a. A $\text{philosopher}^1$ is attending this talk and $\text{she}_1$'s sitting in the back.

b. # $\text{She}_1$'s sitting in the back and $\text{a}^1$ philosopher is attending this talk.

What would count as a more explanatory DS? Arguably one on which the dynamic entries for the logical connectives can be derived in a systematic way from their static counterparts — see, e.g., George 2008 for a simple trivalent theory of presupposition projection which has this character. As of yet, there is no especially prominent approach to anaphora which has the same empirical coverage as DS, while being less stipulative in just the way suggested here.[5]

The largely conceptual issue of explanatory adequacy may seem at first blush to be completely independent of the empirical issues with negation and disjunction in DS. As we'll see however, developing a dynamic logic on a firmer footing will, as a consequence, at least partially address these issues.

The paper will proceed as follows: in the next section, I'll develop a new dynamic logic, which I'll call Dynamic Alternative Semantics (DAS), starting out with the basic building blocks of DPL. As we'll see, DAS is somewhat more expressive than DPL, allowing us to distinguish between the positive vs. negative information conveyed by a given sentence. This will make it well-suited to tackling the problem of negation in DS. I'll argue that it's possible to make almost all of the same empirical predictions as standard DS, by simply lifting the strong Kleene connectives into

[5]See Rothschild 2017 and Mandelkern 2020 for two notable exceptions. I discuss these works briefly in §4.2.

a dynamic setting in a systematic way. Furthermore, DAS goes beyond the empirical coverage of standard dynamic theories, and accounts for double negation and bathroom sentences. In the remainder of the paper, I'll demonstrate that some of the apparently problematic predictions of DAS, specifically regarding disjunction are in fact *good* predictions — the data motivating some of the accessibility generalizations assumed by, e.g., Groenendijk & Stokhof were confounded by pragmatic factors. In order to demonstrate this, I'll intensionalize DAS, and ground it in a Stalnakerian pragmatics. Finally, I conclude by comparing DAS to some similar recent proposals.

## 2 Dynamic alternative semantics

### 2.1 Basic building blocks

Much like Groenendijk & Stokhof (1991) we'll proceed by giving a dynamic interpretation for a simple first-order predicate calculus. Following van den Berg's (1996: ch. 2) presentation, we'll assume that the syntax is that of standard predicate logic. We'll simply use the natural numbers $n \in \mathbb{N}$ as variable symbols. The interpretation of sentences is given relative to a first-order model $M \coloneqq \langle D, I, T \rangle$, where $D$ is a non-empty set of individuals, $T$ is the set of truth-values, and $I$ assigns interpretations to predicates as sets of tuples of individuals in a standard way. Since we'll be developing a *trivalent* semantics, $T$ consists of *true,false* ($\top, \bot$) and third truth-value #, which we'll call *maybe*, to reflect it's role in the trivalent substrate. Departing somewhat from DPL, in DAS the interpretation of a sentence relative to an assignment $g$, and a model $M$ is a *set of truth-value/assignment pairs* (we'll omit the model parameter wherever possible). This will afford the system more expressive power than classical DPL, and the significance of this move will become apparent later.[6]

### 2.2 Atomic sentences

We'll assume that assignments may be partial; this means that atomic sentences may return a #-tagged output. We'll formalize this idea using Beaver's (2001) $\delta$-operator, which converts *false* to *maybe*.[7]

| $\delta$ | |
|---|---|
| 1 | 1 |
| 0 | # |
| # | # |

Table (1): Beaver's (2001) $\delta$-operator

**Definition 2.1** (Atomic sentences)**.** We provide provisions here for dealing with a monadic predicate and a single term, with separate clauses for variables and individual constants. These are

[6]This presentation is inspired by Charlow's (2014, 2019) monadic dynamic semantics.

[7]Note for concreteness that we assume a weak Kleene semantics for meta-language conjunction ($\wedge$), i.e., if any conjuncts in the meta-language are *maybe*, then the entire conjunctive statement is *maybe*.

generalized to sequences of terms in the obvious way.

$$[\![P\ n]\!]^g := \{ (\delta\ (n \in \mathsf{dom}\ g) \wedge g_n \in I(P), g) \}$$

$$[\![P\ c]\!]^g := \{ (I(c) \in I(P), g) \}$$

It will frequently be illustrative to consider the interpretation of a sentence relative to a privileged assignment: the *initial assignment* $g_\top$, which is the unique assignment whose domain is the empty set. An atomic sentence with free variables interpreted relative to the initial assignment will always return the maybe-tagged input assignment, as illustrated in (11a). As long as every variable is in the domain of the input assignment, the sentence will return the true- or false-tagged input assignment depending on the model; this is illustrated in (11b).

(11) a. $[\![P\ 1]\!]^{g_\top} = \{ (\#, g_\top) \}$

b. $[\![P\ 1]\!]^{[1 \mapsto a]} = \{ (a \in I(P), [1 \mapsto a]) \}$

## 2.3 Random assignment

We follow van den Berg (1996: ch. 2) in introducing DRS via a privileged tautology — *random assignment* ($\varepsilon^n$). In DAS, random assignment indexed $n$, given an input g, returns a set of true-tagged modified assignments $g^{[n \mapsto x]}$,[8] for each individual $x$ in the domain.

**Definition 2.2** (Random assignment)**.**

$$[\![\varepsilon^n]\!]^g = \{ (\top, g^{[n \mapsto x]}) \mid x \in D \}$$

Assuming a simple domain of individuals $D := \{ a, b, c \}$, the effect of random assignment is illustrated in (12).

(12) $[\![\varepsilon^1]\!]^{g_\top} = \{ (\top, [1 \mapsto a]), (\top, [1 \mapsto b]), (\top, [1 \mapsto c]) \}$

## 2.4 Conjunction, negation, and positive closure

In order to define the logical connectives in DAS, we'll take the strong Kleene trivalent connectives as a starting point. We'll begin by considering the strong Kleene truth tables for negation and conjunction, and come back to disjunction and material implication later. We take this to be a reasonable starting point, since the strong Kleene truth-tables can be derived by taking the third truth-value to represent *uncertainty*, i.e. *either true or false*.[9] Strong Kleene negation simply projects uncertainty; strong Kleene conjunction is true iff the first and second conjunct are true,

[8] $g^{[1 \mapsto x]}$ is the unique assignment exactly like $g$, except which maps 1 to $x$.

[9] See George 2014 for an overview; to quote Rothschild 2017: p. 1: "[...] when the dust has settled, this remains the simplest viable treatment of presupposition projection on the market.".

and false if either conjunct is false; where these truth/falsity conditions are silent, the result is maybe.

| $\neg^s$ | |
|---|---|
| 1 | 0 |
| 0 | 1 |
| # | # |

| $\wedge^s$ | 1 | 0 | # |
|---|---|---|---|
| 1 | 1 | 0 | # |
| 0 | 0 | 0 | 0 |
| # | # | 0 | # |

Table (2): Negation and conjunction in strong Kleene

Negation in DAS is simply strong Kleene negation lifted into a dynamic setting — we write $\neg^s$ in the meta-language for *strong Kleene negation*. In sharp departure from DPL, and similar dynamic theories, this means that negation is *externally dynamic*, as we'll see later.[10]

**Definition 2.3** (Negation)**.**

$$[\![\neg\,\phi]\!]^g = \{\,(\neg^s\, t, h) \mid (t,h) \in [\![\phi]\!]^g\,\}$$

In order to get to our final semantics for conjunction, we'll first defined *lifted strong Kleene conjunction* ($\triangle$) as an auxiliary operator.[11]

**Definition 2.4** (Lifted strong Kleene conjunction)**.**

$$[\![\phi \triangle \psi]\!]^g = \{\,(t \wedge^s\ u, i) \mid \exists h[(t,h) \in [\![\phi]\!]^g \wedge (u,i) \in [\![\psi]\!]^h]\,\}$$

DAS will swiftly become difficult to reason about, so at this stage it will be useful to define two extremely helpful auxiliairy notions: the *positive* and *negative* extension of a sentence. As one might expect, the positive extension of $\phi$ relative to $g$ is simply all of the assignments in the interpretation of $\phi$ relative to $g$ tagged true, and likewise but tagged false for the negative extension.

[10]We refer to this entry as "lifted", since DAS implicitly uses the `State.Set` monad; negation can be derived by *mapping* strong Kleene negation into `State.Set`. See, e.g., Charlow (2019) for discussion of how to lift "ordinary" truth-functional operators into a dynamic setting via `State.Set`. For concreteness, the operation we use to map negation into DAS is as follows, where $f$ is a function from truth-values to truth-values, and $m$ is a dynamic proposition (a function from assignments to sets of truth-value assignment pairs):

(13) $\textbf{map}\ f\ m \coloneqq \lambda g\,.\,\{\,(f\ t, h) \mid (t,h) \in m\ g\,\}$

To simplify the presentation, we define *map* as an operation applying directly to extensions, where the assignment parameter of the interpretation function is equivalently factored out as an argument of the extension. Applying map to strong Kleene negation gives back lifted strong Kleene negation.

[11]As before, we refer to this semantics as "lifted", since it can be derived by mapping strong Kleene conjunction into `State.Set` in a systematic way. For concreteness, the operation we use to lift truth-functional connectives into DAS is as follows, where $f$ is a curried function from pairs of truth-values to truth-values, and $m, n$ are dynamic propositions:

(14) $\textbf{lift}_2\ f\ m\ n \coloneqq \lambda g\,.\,\{\,(f\ t\ u, i) \mid \exists h[(t,h) \in m\ g \wedge (u,i) \in n\ h]\,\}$

**Definition 2.5** (Positive and negative extension)**.**

$$\llbracket \phi \rrbracket^g_+ = \{\, h \mid (\top, h) \in \llbracket \phi \rrbracket^g \,\}$$
$$\llbracket \phi \rrbracket^g_- = \{\, h \mid (\bot, h) \in \llbracket \phi \rrbracket^g \,\}$$

For completeness, we can also define the *maybe extension*:

$$\llbracket \phi \rrbracket^g_u = \{\, h \mid (\#, h) \in \llbracket \phi \rrbracket^g \,\}$$

It's helpful to think of DAS as consisting of two DPL-like logics, computing the positive and negative information conveyed by a sentence in tandem. Using the notion of positive and negative extension, we can already establish some useful equivalences involving negation:

**Observation 2.1.** *Since all that negation does is flip the classical truth values, the positive extension of a negated sentence is the negative extension of the contained sentence, and the negative extension of a negated sentence is the positive extension of the contained sentence. The maybe extension of a negated sentence is the same as that of the contained sentence, since strong Kleene negation projects uncertainty.*

$$\llbracket \neg\, \phi \rrbracket^g_+ = \llbracket \phi \rrbracket^g_-$$
$$\llbracket \neg\, \phi \rrbracket^g_- = \llbracket \phi \rrbracket^g_+$$
$$\llbracket \neg\, \phi \rrbracket^g_u = \llbracket \phi \rrbracket^g_u$$

**Observation 2.2** (Double negation)**.** *Due to observation 2.1, it's obvious that a double negated sentence will be equivalent to its positive counterpart.*[12]

$$\llbracket \neg\,\neg\, \phi \rrbracket^g_+ = \llbracket \neg\, \phi \rrbracket^g_- = \llbracket \phi \rrbracket^g_+$$
$$\llbracket \neg\,\neg\, \phi \rrbracket^g_- = \llbracket \neg\, \phi \rrbracket^g_+ = \llbracket \phi \rrbracket^g_-$$

Now that we have the notions of positive and negative extension, we can also reason about the positive and negative extension of complex sentences with lifted strong Kleene conjunction. In order to do this, we consider the different ways in which strong Kleene conjunction may return true, i.e., only if both conjuncts are true. We therefore compute the relational composition of the positive extensions of the conjuncts (i.e., DPL conjunction).[13]. This is indicated in (15a). In order to compute the *negative extension*, we consider the different ways in which strong Kleene conjunction may return false, i.e., if either conjunct is false. We therefore compute the relational composition of the positive/negative/maybe-extension of the first conjunct and the negative extension of the second, and the relational composition of the negative extension of the first conjunct, and the positive/negative/maybe-extension of the second, and gather up the results, as indicated in (15b).

[12]We call two sentences $\phi$ and $\psi$ "equivalent" in this paper iff $\llbracket \phi \rrbracket^g_+ = \llbracket \psi \rrbracket^g_+$ and $\llbracket \phi \rrbracket^g_- = \llbracket \psi \rrbracket^g_-$.

[13]N.b., we use $*$ in the meta-language as a wildcard ranging over truth-values.

(15) a. $[\![\phi \vartriangle \psi]\!]^g_+ = \{\, i \mid \exists h[h \in [\![\phi]\!]^g_+ \wedge i \in [\![\psi]\!]^h_+]\,\}$

b. $[\![\phi \vartriangle \psi]\!]^g_- = \{\, i \mid \exists h[h \in [\![\phi]\!]^g_- \wedge (i, *) \in [\![\psi]\!]^h]\,\}$
$\cup \{\, i \mid \exists h[(h, *) \in [\![\phi]\!]^g \wedge i \in [\![\psi]\!]^h]\,\}$

For a number of reasons, the operators we have defined so far will not result in a reasonable dynamic logic, and we need to define an auxiliary operator to arrive at the final semantics for conjunction (and the other logical connectives).[14] We'll call this operator *positive closure*, and treat it syntactically as a one-place sentential operator †.[15] The output of positive closure is guaranteed to either deliver a set of true-tagged assignments, or singletons of either false- or #-tagged assignments. This ensures that DRs are introduced only by the positive extension of the contained sentence.

(16) $[\![\dagger\, \phi]\!]^g = \{\, (\top, h) \mid h \in [\![\phi]\!]^g_+ \,\}$
$\cup \{\, (\bot, g) \mid [\![\phi]\!]^g_+ = \varnothing \wedge [\![\phi]\!]^g_- \neq \varnothing \,\}$
$\cup \{\, (\#, g) \mid [\![\phi]\!]^g_+ = [\![\phi]\!]^g_- = \varnothing \wedge [\![\phi]\!]^g_u \neq \varnothing \,\}$

This is even more perspicuous if we consider the positive and negative extension of a sentence subject to positive closure. Note that positive closure has no effect on the positive extension of the sentence. The negative extension, on the other hand, is only ever the input or the empty set.

(17) a. $[\![\dagger\, \phi]\!]^g_+ = [\![\phi]\!]^g_+$

b. $[\![\dagger\, \phi]\!]^g_- = \{\, g \mid [\![\phi]\!]^g_+ = \varnothing \wedge [\![\phi]\!]^g_- \neq \varnothing \,\}$

We now give our final entry for conjunction, in terms of strong Kleene conjunction and positive closure. The other logical connectives will be defined in exactly the same way.

**Definition 2.6** (Conjunction)**.**

$$\phi \wedge \psi \Leftrightarrow \dagger\, (\phi \vartriangle \psi)$$

It will be useful to consider the positive and negative extension of a conjunctive sentence; the positive extension is just the positive extension of lifted strong Kleene conjunction, but the negative extension is the input, just in case the positive extension of lifted strong Kleene conjunction is empty, but the negative extension is non-empty.

(18) a. $[\![\phi \wedge \psi]\!]^g_+ = [\![\phi \vartriangle \psi]\!]^g_+$

b. $[\![\phi \wedge \psi]\!]^g_- = \{\, g \mid [\![\phi \vartriangle \psi]\!]^g_+ = \varnothing \wedge [\![\phi \vartriangle \psi]\!]^g_- \neq \varnothing \,\}$

[14]This is essentially because lifted strong Kleene conjunction fails to validate *Egli's theorem* for negative extensions.

[15]We could instead have defined the semantics of conjunction in DAS directly, but seperating out these two components leads to a more perspicuous presentation of the logic.

## 2.5 Egli's theorem

It should be clear at this point that DAS will deal easily with standard cases of discourse anaphora, and validate Egli's theorem with respect to positive extensions — this is because positive extensions of conjunctive sentences completely mimic DPL. The validity of Egli's theorem in terms of negative information may not be as obvious, but it turns out that this goes through too, due to the semantics of conjunction. We don't give a full proof here, but only a brief demonstration in terms of the sentences in (19).

(19) a. $(\varepsilon^1 \wedge P\,1) \wedge Q\,1$

b. $\varepsilon^1 \wedge (P\,1 \wedge Q\,1)$

(20) a. $[\![(\varepsilon^1 \wedge P\,1) \wedge Q\,1]\!]_-^g$

b. $= \{\, g \mid [\![(\varepsilon^1 \wedge P\,1) \wedge Q\,1]\!]_+^g = \varnothing \wedge [\![(\varepsilon^1 \wedge P\,1) \wedge Q\,1]\!]_-^g \neq \varnothing \,\}$

c. $= \left\{ g \;\middle|\; (I(P) \cap I(Q)) = \varnothing \wedge \exists i \begin{bmatrix} \exists h[h \in [\![\varepsilon^1 \wedge P\,1]\!]_+^g \wedge i \in [\![Q\,1]\!]_-^g] \\ \vee\, \exists h[h \in [\![\varepsilon^1 \wedge P\,1]\!]_-^g \wedge (*, i) \in [\![Q\,1]\!]^h] \\ \vee\, \exists h[(*, h) \in [\![\varepsilon^1 \wedge P\,1]\!]^g \wedge i \in [\![Q\,1]\!]_-^h] \end{bmatrix} \right\}$

d. $= \left\{ g \;\middle|\; (I(P) \cap I(Q)) = \varnothing \wedge \begin{array}{l} \exists x[x \in I(P) \wedge x \notin I(Q)] \\ \vee\, I(P) = \varnothing \\ \vee\, \exists x[x \in I(P) \wedge x \notin I(Q)] \end{array} \right\}$

e. $= \{\, g \mid (I(P) \cap I(Q)) = \varnothing \,\}$

(21) a. $[\![\varepsilon^1 \wedge (P\,1 \wedge Q\,1)]\!]_-^g$

b. $= \{\, g \mid [\![\varepsilon^1 \wedge (P\,1 \wedge Q\,1)]\!]_+^g = \varnothing \wedge [\![\varepsilon^1 \wedge (P\,1 \wedge Q\,1)]\!]_-^g \neq \varnothing \,\}$

c. $= \{\, g \mid (I(P) \cap I(Q)) = \varnothing \,\}$

## 2.6 Accessibility and negation

We're now in a position to understand how DAS captures some of Groenendijk & Stokhof's (1991) observations regarding accessibility, despite maintaining an externally dynamic negation. DAS straightforwardly predicts that an indefinite (which we translate via random assignment and conjunction) in the scope of negation fails to introduce a DR.

(22) a. It's not true that anyone[1] is here.

b. $\neg\, (\varepsilon^1 \wedge H\,1)$

First, we compute the positive extension of the contained sentence. Since the positive extension of a conjunctive sentence is essentially just computed via relational contribution, as in DPL, we elide the details.[16]

(24) $[\![\varepsilon^1 \wedge H\ 1]\!]^g_+ = \{\, g^{[1\mapsto x]} \mid x \in I(H)\,\}$

We can now compute the negative extension of the contained sentence in terms of the positive extension, and the negative extension of lifted strong Kleene conjunction. Note that since random assignment is a tautology, so its negative/maybe extensions are always empty.

**Observation 2.3** (The positive/negative extension of random assignment)**.**

$$[\![\varepsilon^n]\!]^g_+ = \{\, g^{[1\mapsto x]} \mid x \in D\,\}$$
$$[\![\varepsilon^n]\!]^g_- = \varnothing$$

Based on observation 2.3, we therefore only need to concern ourselves with the case in which the first conjunct is true and the second is false.

(25) a. $[\![\varepsilon^1 \wedge H\ 1]\!]^g_-$
b. $= \{\, g \mid [\![\varepsilon^1 \wedge H\ 1]\!]^g_+ = \varnothing \wedge [\![\varepsilon^1 \vartriangle H\ 1]\!]^g_- \neq \varnothing\,\}$
c. $= \{\, g \mid I(H) = \varnothing \wedge \exists x[x \notin H\ 1]\,\}$
d. $= \{\, g \mid I(H) = \varnothing\,\}$

The positive extension of the negated sentence can now be computed directly in terms of the negative extension of the contained sentence. The result of course is the input assignment, just so long as its true that nobody is here. We therefore successfully capture that an indefinite in the scope of negation fails to pass along anaphoric information, without building this directly into the semantics of negation itself, as in DPL!

(26) $[\![\neg\,(\varepsilon^1 \wedge H\ 1)]\!]^g_+ = [\![\varepsilon^1 \wedge H\ 1]\!]^g_- = \{\, g \mid I(H) = \varnothing\,\}$

Now, briefly, consider the sentence below:

[16] One interesting thing to note is that, even though DAS is built on a strong Kleene substrate, which doesn't encode linear asymmetries between arguments, nevertheless the system derives a linear bias in the licensing of anaphora. In other words, 1 will be free in the following sentence:

(23) $P\ 1 \wedge \varepsilon^1$

This is because, in order to compute the positive extension of a conjunctive sentence, we do relational composition of the positive extensions of the conjuncts, just as in DPL, and relational composition is non-commutative. The linear asymmetry therefore comes from the *dynamics* of passing referential information. It is therefore unnecessary to adopt a proposal such as George's (2007, 2008, 2014), where strong Kleene is incrementalized — at least, not for the purposes of deriving linear asymmetries with anaphora.

(27) a. It's not true that nobody is here.

b. $\neg(\neg(\varepsilon^1 \wedge H\,1))$

Based on observation 2.2, we know that the positive extension of a doubly negated sentence is the positive extension of the positive counterpart, i.e., DNE is valid. This predicts that a doubly negated sentence can introduce a DR, just like its positive counterpart.

(28) $[\![\neg(\neg(\varepsilon^1 \wedge H\,1))]\!]^g_+ = [\![\varepsilon^1 \wedge H\,1]\!]^g_+ = \{\, g^{[1\mapsto x]} \mid x \in I(H)\,\}$

## 2.7 Bathroom sentences

Now that we've gone through some relatively straightforward sentences involving conjunction and negation, we'll turn out attention towards disjunctive sentences, which, as we'll see, will give rise to some complexities. In DAS, the logical connectives are derived in a systematic manner — we first define the lifted strong Kleene connective, and then define the connective in DAS in terms of its lifted counterpart, and positive closure. We'll do this now for disjunction. First, consider the truth table — strong Kleene disjunction is true if either of the disjuncts are true, and false only if both disjuncts are false; uncertainty projects in the obvious way.

| $\vee^s$ | 1 | 0 | # |
|---|---|---|---|
| 1 | 1 | 1 | 1 |
| 0 | 1 | 0 | # |
| # | 1 | # | # |

Table (3): Disjunction in strong Kleene

Now, as before, we define lifted strong Kleene disjunction ($\underline{\vee}$).

**Definition 2.7** (Lifted strong Kleene disjunction)**.**

$$[\![\phi \,\underline{\vee}\, \psi]\!]^g = \{\, (t \vee^s u, i) \mid \exists h[(t,h) \in [\![\phi]\!]^g \wedge (u,i) \in [\![\psi]\!]^h]\,\}$$

Now, let's consider the positive and negative extension of sentences with lifted strong Kleene disjunction separately.

(29) a. $[\![\phi \,\underline{\vee}\, \psi]\!]^g_+ = \{\, i \mid \exists h[h \in [\![\phi]\!]^g_- \wedge i \in [\![\psi]\!]^h_+]\,\}$
$\cup \{\, i \mid \exists h[h \in [\![\phi]\!]^g_+ \wedge (i,*) \in [\![\psi]\!]^h]\,\}$
$\cup \{\, i \mid \exists h[(*,h) \in [\![\phi]\!]^g \wedge i \in [\![\psi]\!]^h_+]\,\}$

b. $[\![\phi \vee \psi]\!]^g_- = \{\, i \mid \exists h[h \in [\![\phi]\!]^g_- \wedge i \in [\![\psi]\!]^h_-]\,\}$

Finally, as before, disjunctive sentences are defined in terms of lifted strong Kleene and positive closure:

**Definition 2.8** (Disjunction)**.**

$$\phi \vee \psi \Leftrightarrow \dagger\,(\phi \veebar \psi)$$

This gives us the following positive and negative extensions:

(30) a. $[\![\phi \vee \psi]\!]^g_+ = [\![\phi \veebar \psi]\!]^g_+$

b. $[\![\phi \vee \psi]\!]^g_- = \{\, g \mid [\![\phi \veebar \psi]\!]^g_+ = \varnothing \wedge [\![\phi \veebar \psi]\!]^g_- \neq \varnothing \,\}$

Since one of the verification conditions for lifted strong Kleene involves passing the negative extension of the first disjunct into the positive extension of the second, we can now immediately account for Partee's bathroom disjunctions.

(31) a. Either there is no bathroom, or it's upstairs.

b. $(\neg\,(\varepsilon^1 \wedge B\,1)) \vee U\,1$

It will be helpful to start by giving the positive/negative extensions of each of the disjuncts to begin with.

(32) a. $[\![\neg\,(\varepsilon^1 \wedge B\,1)]\!]^g_+ = \{\, g \mid I(B) = \varnothing \,\}$

b. $[\![\neg\,(\varepsilon^1 \wedge B\,1)]\!]^g_- = \{\, g^{[1 \mapsto x]} \mid x \in I(B) \,\}$

c. $[\![U\,1]\!]^g_+ = \{\, g \mid 1 \in \mathsf{dom}\ g \wedge g_1 \in I(U) \,\}$

d. $[\![U\,1]\!]^g_- = \{\, g \mid 1 \in \mathsf{dom}\ g \wedge g_1 \notin I(U) \,\}$

Now to compute the positive extension of the disjunctive sentence, we take the union of the positive extension of the first disjunct, and the result of passing the negative extension of the first disjunct into the second (we only consider this case, since passing the positive/maybe extension of the first disjunct into the second will always result in an empty positive extension, since anaphora won't be licensed):

(33) $[\![\neg\,(\varepsilon^1 \wedge B\,1) \vee U\,1]\!]^g_+ = \{\, g \mid I(B) = \varnothing \,\} \cup \{\, g^{[1 \mapsto x]} \mid x \in I(B) \wedge x \in I(U) \,\}$

We thereby successfully account for anaphoric licensing in bathroom sentences! The sentence is predicted to be true iff there is no bathroom, or there is a bathroom and it's upstairs.

An apparent problem with this semantics is that we predict a disjunctive sentence to be externally dynamic, which contradicts the standard assumption in DS. In §3, we return to this question, and argue that disjunction *is* in fact externally dynamic by dint of its semantics. A similar issue will arise with implication, which we turn to next.

## 2.8 Donkey anaphora

We haven't yet said anything about donkey anaphora, as in (34). This is one of the central empirical motivations for classical DS, and DPL-like systems predict strong, universal truth-conditions for sentences like (34).

(34) If anyone$^1$ is here, then they$_1$ are unhappy.

In order to consider the predictions made in DAS, let's first consider the semantics for strong Kleene material implication $\rightarrow^s$; strong Kleene material implication is true just so long as the either the antecedent is false, or the consequent is true, and false only if the antecedent is true and the consequent is false. Uncertainty projects in the obvious way.

| $\rightarrow^s$ | 1 | 0 | # |
|---|---|---|---|
| 1 | 1 | 0 | # |
| 0 | 1 | 1 | 1 |
| # | 1 | # | # |

Table (4): Material implication in strong Kleene

We can derive the meaning for the conditional operator in DAS by the same procedure as before; namely, we lift strong Kleene implication into a dynamic setting, and apply the positive closure operator. Skipping over the details, we end up with the following positive and negative extensions for implicational sentences in DAS.

(35) a. $$[\![\phi \rightarrow \psi]\!]^g_+ = \{\, i \mid \exists h[h \in [\![\phi]\!]^g_+ \wedge i \in [\![\psi]\!]^h_+]\,\} \cup \{\, i \mid \exists h[h \in [\![\phi]\!]^g_- \wedge (i, *) \in [\![\psi]\!]^h]\,\} \cup \{\, i \mid \exists h[(*, h) \in [\![\phi]\!]^g \wedge i \in [\![\psi]\!]^h_+]\,\}$$

b. $$[\![\phi \rightarrow \psi]\!]^g_- = \{\, g \mid [\![\phi \rightarrow \psi]\!]^g_+ = \varnothing \wedge \exists i, h[h \in [\![\phi]\!]^g_+ \wedge i \in [\![\phi]\!]^g_-]\,\}$$

If we apply this semantics to a donkey sentence we predict weak, existential truth-conditions. This is easiest to see if we compute the positive extension of a donkey sentence.

(36) a. If anyone$^1$ is here, then they$_1$ are unhappy.

b. $(\varepsilon^1 \wedge H\ 1) \rightarrow U\ 1$

Consider first the positive/negative extensions of the antecedent and consequent:

(37) a. $[\![\varepsilon^1 \wedge H\ 1]\!]^g_+ = \{\, g^{[1 \mapsto x]} \mid x \in I(H)\,\}$

b. $[\![\varepsilon^1 \wedge H\ 1]\!]^g_- = \{\, g \mid I(H) = \varnothing\,\}$

(38) a. $[\![U\ 1]\!]^g_+ = \{\, g \mid 1 \in \mathsf{dom}\ g \wedge g_1 \in I(H) \,\}$

b. $[\![U\ 1]\!]^g_- = \{\, g \mid 1 \in \mathsf{dom}\ g \wedge g_1 \notin I(H) \,\}$

We can now compute the positive extension of the conditional sentence as in (39). Note that the positive extension will be non-empty just in case there is at least one person who is both here and unhappy. The existence of someone who is here and happy fails to falsify the sentence.

(39) $$[\![(\varepsilon^1 \wedge H\ 1) \rightarrow U\ 1]\!]^g_+ = \{\, g^{[1 \mapsto x]} \mid x \in I(H) \wedge I(U) \,\} \cup \{\, g \mid I(H) = \varnothing \,\}$$

This doesn't match our intuitions regarding the truth-conditions of the sentence under consideration, which imposes a stronger, universal requirement. The falsity conditions we predict for the donkey sentence in (36a) are however strong, which seems to match our intuitions. We predict that (36a) is false, and therefore has a non-empty negative extension, iff nobody is here and unhappy, but someone is here and happy. As is well-known, in fact both weak and strong readings are attested (Chierchia 1995, Kanazawa 1994); our semantics derives the weak reading, and we need to do something extra to derive the strong reading. Our theory therefore diverges sharply for DPL-like theories, which derive the strong reading as basic.[17] How to capture weak vs. strong readings is a thorny issue, and there at least exist proposals which assume that weak readings should be generated in the semantics (see, e.g., Champollion, Bumford & Henderson 2019 for recent work on this topic within a partial dynamic setting), and therefore we leave a more thorough exploration of donkey anaphora in DAS to future work.

## 3 Ignorance, disjunction, and accessibility

There seems to still be a problem with this entry for disjunction. We can make it concrete using the sentence under consideration. Imagine that, in the model, there is exactly one bathroom $b$, so $I(B) = \{\, b \,\}$. In such a model, we predict anaphora to be licensed in (40). This is because, if the extension of *bathroom* is non-empty, the disjunctive sentence will output modified assignments which map 1 to a bathroom upstairs.

(40) $(\neg\,(\varepsilon^1 \wedge B\ 1) \vee U\ 1) \wedge D\ 1$

On the basis of similar observations, Groenendijk & Stokhof (1991) give a semantics for disjunctions in DPL that is *externally static*. This is motivated by data such as the following:

[17] In fact, if we extend DAS to universal quantification in the obvious way, Egli's corrolary won't be validated. Howver, it's possible to demonstrate that a weaker equivalence holds:

$$(\varepsilon^n \wedge \phi) \rightarrow \psi \Leftrightarrow (\varepsilon^n \wedge \phi \wedge \psi) \vee \neg\,(\varepsilon^n \wedge \phi)$$

(41) Either a[1] layperson was in the audience, or we had no press. # They$_1$ enjoyed it.

There are however two problems with this move. As Groenendijk & Stokhof observe, the DPL entry for disjunction fails to capture *Stone disjunctions*, as illustrated in (42). This data would seem to clearly indicate that disjunction is externally dynamic, and that something else is responsible for the impossibility of anaphora in (41).[18]

(42) Either a[1] philosopher is in the audience or a[1] linguist is.
(Either way) I hope she$_1$ enjoys it.

Furthermore, as observed by Rothschild (2017), this move fails to account for anaphora in discourses such as the following.

(43) a. Either a[1] layperson was in the audience, or we had no press.
b. We had some press, so I hope they$_1$ enjoyed it!

The intuition I'd like to pursue, following Rothschild's suggestion is that (41) licenses anaphora only in a context which entails the truth of the first disjunct. Typically, disjunctive sentences are infelicitous when uttered in a context that entails the truth of one of the disjuncts, which serves to explain the impossibility of anaphora in (41).

(44) Context: *It's common ground that a layperson was in fact in the audience.*
#Either a[1] layperson was in the audience or we had no press.

In order to formalize the account, we'll need to intensionalize DAS — fortunately, this is almost complete mechanical; we simply add a world parameter to the interpretation function, and relativize $I$ to the world of evaluation. In an intensional setting, sentences will return world/truth-value/assignment *tuples*, rather that world assignment pairs. This is illustrated below for a simple atomic sentence. Everything else remains as before, except we'll assume that the positive/negative extension in an intensional setting is a set of world-assignment pairs (rather than just assignments).

(45) a. $[\![P\ 1]\!]^{w,g} = \{\, (\delta\,(n \in \mathsf{dom}\ g) \land g_n \in I_w(P), w, g)\,\}$
b. $[\![P\ 1]\!]^{w,g}_{+} = \{\, (w,g) \mid \delta\,(n \in \mathsf{dom}\ g) \land g_n \in I_w(P)\,\}$
c. $[\![P\ 1]\!]^{w,g}_{-} = \{\, (w,g) \mid \delta\,(n \in \mathsf{dom}\ g) \land g_n \notin I_w(P)\,\}$

We'll also outline a simple Stalnakerian pragmatics in the next section, alongside a rule of assertion.

[18] Groenendijk & Stokhof instead suggest that natural language disjunction can either be translated as externally static disjunction, or what they call *program disjunction*. This is obviously an undesirable move.

## 3.1 Pragmatics

In order to account for the illusion of external staticity of disjunction, it will be useful to be concrete about the pragmatics. We'll assume a relatively standard Heimian notion of an information state, consisting of a set of world-assignment pairs, as in Definition 3.1. Such information states can track relative certainty regarding both worldly and referential information. Since assignments are partial, it's natural to treat the initial information state as the product of logical space, and the initial assignment — this represents a scenario in which nothing is known, and nothing has been said.

**Definition 3.1** (Information state)**.** An *information state* $c$ is a set of world-assignment pairs. Where:

- $c_\top$, the initial information state, is defined as: $/W \times \{ g_\top \}$.
- $c_\varnothing$, the absurd information state is the empty set $\varnothing$

Now we define an *update* operation to model the effect on a context (which we model as an information state) of asserting a sentence; given a sentence $\phi$, update maps information states to information states. Since DAS is distributive, much like DPL, update does some work — namely, it computes the positive extension of the sentence at every point in the information state, and gathers up the results. As usual, update is assumed to be subject to Stalnaker's *bridge principle*, generalized to information states in the obvious way — for update to be defined, the sentence must be either true or false at every point in the input context.

**Definition 3.2** (Update)**.**

$$c[\phi] := \begin{cases} \bigcup\limits_{(w,g)\in c} [\![\phi]\!]_+^{w,g} & \forall (w,g) \in c[[\![\phi]\!]_+^{w,g} \neq \varnothing \vee [\![\phi]\!]_-^{w,g} \neq \varnothing] \\ \varnothing & \text{otherwise} \end{cases}$$

We get Heim's (1991) *familiarity presupposition* for free, from the definedness conditions on atomic sentences, in combination with the universal requirement of bridge, i.e., update of an information state $c$ with a sentence with a free variable $n$ will only be defined if $n$ is defined for *every* assignment in the information state. We say that a variable $n$ is *familiar* in a context $c$, iff $n$ is in the domain of every assignment, s.t., $(*, g) \in c$.[19] It's easy to see that an utterance of a sentence with an indefinite will result in an information state that satisfies the presupposition induced by matching free variable.

## 3.2 Deriving apparent external staticity from ignorance

We're now in a position to account for some of the behaviour observed for disjunctive sentences in §2.7. The first thing to observe is that disjunctive sentences place a requirement on the context

[19]We remain neutral here as to whether to build Heim's novelty condition directly into the semantics of random assignment (see, e.g., van den Berg's 1996 *guarded random assignment*), or to derive it as an implicated presupposition.

— an utterance of a sentence of the form "$P$ or $Q$" is only felicitous if both $P$ and $Q$ are open possibilities, i.e., the context should not entail the truth/falsity of $P$ or $Q$ (see, e.g., Meyer 2013 for recent discussion).

(46) Context: *it's common ground that someone was in the audience.*
\# Either someone was in the audience or the event was a disaster.

We can use this fact to account for the apparent external staticity of disjunction. Consider the following space of logical possibilities:

- $w_{ad}$: $a$ was in the audience, and the event was a disaster.
- $w_{a\neg d}$: $a$ was in the audience, and the event wasn't a disaster.
- $w_{\varnothing d}$: nobody was in the audience, and the event was a disaster.
- $w_{\varnothing\neg d}$: nobody was in the audience, and the event wasn't a disaster.

And consider the sentence under consideration, and a simplified Logical Form:

(47) a. Either someone$^1$ was in the audience, or the event was a disaster.
b. $(\varepsilon^1 \wedge A\ 1) \vee D\ e$

Let's first consider the positive extension of the disjunctive sentence, which we compute by considering the different verification conditions of strong Kleene disjunction, lifted into a dynamic setting, as usual. This is just all the assignments in the positive extension of the first disjunct, together with the result of passing the positive/negative/maybe extension of the first disjunct into the second and gathering up the (positive) results.

(48)
$$[\![(\varepsilon^1 \wedge A\ 1) \vee D\ e]\!]^g_+ = \{\,(w, g^{[1\mapsto x]}) \mid x \in I_w(A)\,\} \cup \{\,(w, g) \mid I_w(A) = \varnothing \wedge I_w(e) \in I_w(D)\,\}$$

We can now consider the result of updating the initial information state with the disjunctive sentence. Note that the bridge principle is trivially satisifed, since the sentence doesn't contain any free variables. We simply dispense with any points not in the positive extension of the sentence, resulting in the following updated context.

(49)
$$\left\{\begin{matrix}(w_{ad}, g_\top),\\ (w_{a\neg d}, g_\top),\\ (w_{\varnothing d}, g_\top),\\ (w_{\varnothing\neg d}, g_\top),\end{matrix}\right\}[(\varepsilon^1 \wedge A\ 1) \vee D\ e] = \left\{\begin{matrix}(w_{ad}, [1 \mapsto a]),\\ (w_{a\neg d}, [1 \mapsto a],\\ (w_{\varnothing d}, g_\top),\end{matrix}\right\}$$

Note, crucially, that the resulting information state is one in which 1 is *not familiar*! This means that the presupposition of a subsequent sentence with a matching free variable won't be satisfied. This derives the (apparent) external staticity, in cases where the independently motivated requirement that the disjuncts are open possibilities is satisfied.[20]

[20] The explanation also goes through for cases in which the indefinite is in the second disjunct.

(50) *Context: total ignorance*
Either someone[1] was in the audience, or the event was a disaster. # $\text{She}_1$ enjoyed it.

This account correctly captures Rothschild's observation: an intermediate assertion can eliminate the world-assignment pair $(w_{\varnothing}, g_{\top})$, thus rendering 1 familiar.

(51) *Context: total ignorance*

a. Either someone[1] was in the audience, or the event was a disaster.

b. (Actually) the event wasn't a disaster.

c. So, I hope $\text{she}_1$ enjoyed it.

What if we entertain an information state identical to the initial state, only with this point removed? The result is an information state which entails that *either a was in the audience, or the event wasn't a disaster*.

$$(52) \quad c' := \left\{ \begin{array}{l} (w_{ad}, g_{\top}), \\ (w_{a\neg d}, g_{\top}), \\ (w_{\varnothing\neg d}, g_{\top}), \end{array} \right\}$$

Updating this context with the disjunctive sentence results in just those worlds in which $a$ was in the audience, paired with assignments mapping 1 to $a$. In other words, an update of $c'[(\varepsilon^1 \wedge A\ 1) \vee D\ e]$ is contextually equivalent to an update by just the first disjunct $c'[\varepsilon^1 \wedge A\ 1]$. We assume that the utterance of the disjunctive sentence is odd in such a context, as illustrated below:

(53) Context: *if nobody is in the audience, then the event wasn't a disaster*
#Either nobody was is the audience, or the event was a disaster.

### 3.3 Stone disjunctions

Stone disjunctions are not particularly problematic for DAS, and will certainly not motivate an alternative notion of program disjunction, as in DPL. The ignorance requirement on disjunctive assertions allows for subsequent anaphora in such cases. To illustrate, consider the following:[21]

(54) a. Either a[1] linguist is here, or a[1] philosopher is.
$(\varepsilon^1 \wedge L\ 1 \wedge H\ 1) \vee (\varepsilon^1 \wedge P\ 1 \wedge H\ 1)$

Now, we compute the positive extension of the disjunctive sentence using the logic of lifted strong Kleene.[22] Note that the output set *only* contains assignments at which 1 is defined.

[21] Note that, since conjunction is associative in DAS (Egli's theorem), we can omit parentheses in sentences involving multiple conjuncts.

[22] We assume here that the random assignment in the second disjunct is uncertain in the verification condition where we also consider the positive extension of the first (i.e., the novelty condition).

(55) $[\![(\varepsilon^1 \wedge L\,1 \wedge H\,1) \vee (\varepsilon^1 \wedge P\,1 \wedge H\,1)]\!]^g_+ = \{\, g^{[1\mapsto x]} \mid x \in I(L) \wedge x \in I(H) \,\} \cup \{\, g^{[1\mapsto x]} \mid x \in I(P) \wedge x \in I(H) \,\}$

To illustrate concretely, consider the following logical space, where subscripts indicate, exhaustively, who is here ($l$, a linguist, and $p$, a philosopher): $W := \{\, w_{lp}, w_l, w_p, w_\varnothing \,\}$. Updating the initial information state with the stone disjunction results in an information state where familiarity presupposition induced by a matching free variable is satisfied, due to introduction of a DR that is either a linguist or a philosopher.

(56) $$\left\{ \begin{matrix} (w_{lp}, g_\top), \\ (w_l, g_\top), \\ (w_p, g_\top), \\ (w_\varnothing, g_\top), \end{matrix} \right\} [(\varepsilon^1 \wedge L\,1 \wedge H\,1) \vee (\varepsilon^1 \wedge P\,1 \wedge H\,1)] = \left\{ \begin{matrix} (w_{lp}, [1 \mapsto l]), (w_{lp}, [1 \mapsto p]) \\ (w_l, [1 \mapsto l]), \\ (w_p, [1 \mapsto p]) \end{matrix} \right\}$$

This is a marked improvement over, e.g., DPL, where Stone disjunctions are captured by positing an ambiguity in natural language disjunction.

## 3.4 Internal staticity and logical independence

Groenendijk & Stokhof (1991) observe that disjunction appears to be internally static; an indefinite in an initial disjunct can't license anaphora in a subsequent disjunct.

(57) #Either someone[1] is in the audience, or they're sitting down.

Groenendijk & Stokhof build this behaviour directly into the semantics of disjunctive sentences, but Simons (1996) suggests that the reason that (57) is bad is because the pronoun *they* is interpreted as a covert definite description (the "e-type" strategy). On this assumption, the pronoun stands in for the description *the linguist in the audience*, and therefore the second disjunct Strawson entails the first, violating the requirement that two disjuncts are logically independent. We'll essentially adopt this explanation, only without resorting to an e-type account of pronouns — instead, we'll maintain our assumption that pronouns are simply variables. Consider the translation of (57):

(58) $(\varepsilon^1 \wedge A\,1) \vee (S\,1)$

The only condition under which the second disjunct could be true, is if the first disjunct is also true; if the first disjunct is false, no DR is introduced and the second disjunct is maybe. This means that every context in which the second disjunct is true, will be one in which the first is also true. In order to cash out logical independence in a dynamic setting, we assume that disjunctions are subject to the following constraint:

(59) $\ulcorner \phi \vee \psi \urcorner$ is odd relative to $g$ if $[\![\neg\, \phi \wedge \psi]\!]^g_+ = \varnothing \vee [\![\phi \wedge \neg\, \psi]\!]^g_+ = \varnothing$

(57) is independently ruled out by logical independence; $[\![\neg\, (\varepsilon^1 \wedge A\,1) \wedge S\,1]\!]^g_+ = \varnothing$.

# 4 Problems and prospects

## 4.1 Uniqueness and universal inferences

An apparent problem with the current system is that it fails to capture Krahmer & Muskens's (1995) intuition that bathroom sentences have strong, universal truth-conditions, as mentioned earlier in the paper. What is responsible is that the logic we have developed here derives weak, existential truth-conditions for donkey anaphora, and this carries over to bathroom sentences.

One thing to observe is that weak readings of bathroom sentences are in fact attested, so the fact that our theory can at least generate this reading should not count against it. Presumably, whatever mechanism is responsible for deriving strong readings for donkey anaphora could derive strong readings for bathroom sentences too. The weak reading is illustrated in the following example:

(60) Everyone who [either has $\text{no}^1$ credit card or paid with $\text{it}_1$] has left the restaurant.

Clearly, anyone with at least one credit card and paid with it has left — whether or not they have other credit cards which they did/didn't pay with is irrelevant to the truth of the sentence.

As for Gotham's claim that double negation and disjunctive sentences are associated with a uniqueness inference, this is directly counter-exemplified in (60) for bathroom sentences; for double-negation, i'm skeptical that uniqueness is the right characterization of the facts, Simon Charlow (p.c.) notes that an indefinite under double negation also licenses *maximal* plural anaphora:

(61) Logan doesn't have $\text{no}^1$ credit card. $\text{They}_1$'re on the table.

The conditions governing putative uniqueness inferences are poorly understood, and the judgements are not completely stable. We leave a further investigation of these facts to future work.

## 4.2 Related work

There are a number of proposals which directly inspired the current work, such as Krahmer & Muskens's (1995) *double negation Discourse Representation Theory (DRT)* and Gotham's (2019) work on the status of double negation and disjunction in DPL. Although these proposals clearly relate to the current work — especially Krahmer & Muskens's bivalent semantics — these are not direct competitors, since they rely on stipulated dynamic connectives, as in orthodox DS.

Probably the most directly relevant is Rothschild 2017, which aims to give a unified account of presupposition projection and anaphora in terms of a trivalent semantics for the logical operators. Rothschild departs much further from standard dynamic semantics than we do here, and makes one crucial assumption that we can do without — in order to capture, e.g., bathroom sentences, Rothschild assumes the free insertion of classically transparent conjuncts. The nature of this insertion process is somewhat mysterious. Furthermore, in order to capture linear asymmetries, Rothschild notes that he would have to adopt an incrementalized version of the strong Kleene connectives (see George 2007, 2008, 2014). In DAS, simple strong Kleene alongside the logic of referential information passing derives linear asymmetries straightforwardly.

Similarly, Mandelkern (2020) develops an extremely interesting system he dubs *pseudo-dynamics*, which seems to make largely the same predictions as DAS. Unlike DAS however, pseudo-dynamics is static, and rests on an eliminative notion of update. Although I don't discuss the proposal in depth here, i'll simply note that there are some conceptual issues for *pseudo-dynamics* that DAS skirts — for example, in pseudo-dynamics indefinites carry a disjunctive presupposition, which unlike other presuppositions, is (somewhat mysteriously) assumed to be automatically accommodated. In DAS, on the other hand, the same result is achieved via positive closure, which simply ensures that DRS are only introduced in the positive extension of a given sentence. Nothing special need be said about the logic of presupposition.

### 4.3 Conclusion

In this paper, we've developed an alternative dynamic logic for anaphora: DAS, which improves upon competitors in a number of ways. DAS essentially layers the mechanics of referential information passing on top of a trivalent substrate, based on the logic of Strong Kleene; the logic is *predictive*, in the sense that a strong Kleene semantics can be derived for any logical operator via the logic of uncertainty (see Krahmer 1998 and George 2014 for discussion). I showed that, as well addressing a prominent conceptual objection to DS, DAS very much improves the empirical coverage of orthodox dynamic theories, specifically in the domain of double negation and bathroom sentences. The predictive nature of DAS came at an apparent cost — certain accessibility generalizations observed by Groenendijk & Stokhof (1991) failed to fall out. In the latter half of the paper, I showed that these generalizations were largely illusory, and rather arose from a failure to take seriously the pragmatic component.

I take DAS to be, not the final word, but a *starting point* for a new, predictive approach to the dynamics of anaphora, using the logic of strong Kleene as a foundation. There are many obvious extensions to be explored, such as generalized quantifiers, strong readings of donkey anaphora, and other phenomena within the purview of DS more broadly construed, such as quantificational subordination and discourse plurals. I'm optimistic that taking a *predictive* approach as a starting point will help illuminate the role of semantics vs. pragmatics in the explanation of linguistic phenomena such as anaphora.

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