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Alternatives and attention in language and reasoning: The case of modals

Abstract In this paper, we employ a novel experimental paradigm using insights from the psychology of reasoning to investigate the question whether certain modals generate and draw attention to alternatives. We present two experimental studies on English modals *might* and *allowed to*, suggesting that whereas epistemic *might* is conventionally associated with alternatives, deontic *allowed to* is not. Based on our findings, we, first, discuss theoretical consequences for existing accounts of modals and, second, argue that using reasoning tasks is a powerful diagnostic tool for exploring the issue of which expressions involve alternatives.

Keywords: reasoning, modals, alternatives

1 Introduction

Recent efforts to seek convergence between natural language semantics and the psychology of reasoning have led to articulated theories of interpretive processes and general-purpose reasoning. In particular, the erotetic theory of reasoning of Koralus & Mascarenhas (2013, 2018) incorporates linguistic insights into a variant of mental models theory of reasoning (Johnson-Laird 1983) to account for a wide range of failures of deductive reasoning. At the core of both the mental models and erotetic theories of reasoning is the idea that attentional mechanisms structure our mental representations of states of affairs, and that a semantics of alternatives is required to model the effects of attention in reasoning.

Disjunction and indefinites are generators of alternatives *par excellence*. They induce a particular kind of illusory inference whose extant accounts all agree must be due to the presence of alternatives, driving attention in ways that render tractable the space of possibilities to be considered, but that introduce the possibility of fallacious reasoning under certain well understood conditions (Mascarenhas & Koralus 2017).

In this paper, we advance our understanding of the connection between the semantics of alternatives in language, attention, and failures of reasoning. We show experimentally that epistemic *might* gives rise to illusory inferences with the same

signature as the ones associated with indefinites, while the same is not true for the deontic modal construction *be allowed to*. We explore the theoretical implications of these findings by outlining how different extant theories of modality can account for them. As a result, we provide arguments from the psychology of reasoning that help winnow the conceptual space of theories of possibility modals in novel ways.[1]

The chief goal of this article is not to argue for a new or specific analysis of modals. Rather, our aim is twofold. First, we argue that certain possibility modals are conventionally associated with the generation of alternatives and that theories which incorporate this property in their analysis fare better when accounting for our data. Second, we discuss the methodological implication arising from our data in view of previous findings that reasoning tasks can serve as a diagnostic for where alternatives in natural language arise. As a result, we argue that methodologies from the psychology of reasoning may serve to answer longstanding questions regarding what linguistic constructions generate alternatives. Epistemic and deontic possibility modals constitute two case studies supporting this position.

With these considerations in mind, we conclude by proposing that a theory of epistemic *might* as a generator of alternatives that drive attention holds the greatest promise to account for the relevant facts from semantics and the psychology of reasoning. We argue that a minor variant of the theory by Ciardelli et al. (2009) offers just that.

1.1 Illusory inferences from disjunction

The erotetic theory of reasoning of Koralus & Mascarenhas (2013) incorporates linguistic insights into a variant of mental models theory of reasoning (Johnson-Laird 1983) to account for a class of attractive fallacies known as illusory inferences from disjunction. Consider the example in (1), heavily simplified from the original examples discovered by Walsh & Johnson-Laird (2004).

(1) **Premise 1**: John speaks English and Mary speaks French, or else Bill speaks German.
Premise 2: John speaks English.
Fallacious conclusion: Mary speaks French.

The reasoning problem in (1), as well as multiple structurally identical problems, have acceptance rates between 80% and 85% (Walsh & Johnson-Laird 2004, Mascarenhas & Koralus 2017, Koralus & Mascarenhas 2018). Yet, the inference in (1) is a fallacy. Suppose Bill speaks German (modeling premise 1) and John speaks

1 We use the term *possibility modal* purely descriptively to refer to modals like English *might* and *be allowed to* as opposed to *must*.

English (premise 2), but Mary doesn't speak French. This is a model of the premises but not the conclusion, and the inference in (1) is thus invalid.

1.2 Mental models and illusory inferences

Building on the mental-models explanation of these fallacies by Walsh & Johnson-Laird (2004), the erotetic theory of reasoning offers an account of this and related illusory inferences that proposes that a question-answer dynamic is at the core of these fallacies. For ease of exposition, let us consider the logical structure behind the example in (1).

(2) **Premise 1**: $(a \wedge b) \vee c$
Premise 2: a
Fallacious conclusion: b

Following Hamblin semantics (Kratzer & Shimoyama 2002, Alonso-Ovalle 2006) and inquisitive semantics (Groenendijk 2008, Mascarenhas 2009), the erotetic theory of reasoning takes the disjunction in the first premise of (2) to *raise an issue*, putting forth two alternatives: $a \wedge b$ and c.[2] The reasoner is now effectively entertaining a question, and she will seek to find the most expedient way of answering it or dispelling it. As it turns out, the second premise offers a hint of an answer: the second premise a is *related* to the first alternative $a \wedge b$ rather than the second c, and the reasoner rushes to pick an answer: the right alternative from the first premise is $a \wedge b$, whence b follows by a mental-models analog of conjunction elimination.[3]

2 Strictly speaking, the Hamblin semantics tradition postulates an existential quantifier over alternatives at the very top of any logical form, ensuring that classical truth conditions are delivered instead of sets of alternatives. It is inquisitive semantics that dispenses with this final step. With that said, we think it's reasonable to slightly adjust standard Hamblin semantics and do without existential closure at the top, maintaining existential closure for embedded clauses.

3 This informal exposition of the erotetic theory of reasoning suffices for the purposes of this article, but it is crucial to note that the theory is given in a fully explicit form by Koralus & Mascarenhas (2013). They give a full regimentation of mental model theory in terms of truth-maker semantics (van Fraassen 1969, Fine 2012) with an inquisitive / Hamblin semantics for disjunction, and they define a small set of dynamic operations on mental models. The central operation is Q(uestion)-Update, which checks whether the workspace of reasoning contains a mental model with more than one alternative (a question) and attempts to answer it with the information in the mental-model interpretation of the premise being integrated. In the original (2013) formulation of the theory, this operation simply checked for overlap: the alternatives in the question that had the most overlap in content with the information in the answer were kept, while all others dropped from attention. More recent experimental data have shown that overlap is too strong a requirement (Sablé-Meyer & Mascarenhas 2019). The novel data in the present article do not require these more sophisticated theories, so we use the original formulation of the theory.

1.3 Alternatives in language beyond disjunction

If the attractiveness of fallacious schemata as in (2) is to be explained in terms of the presence of alternatives in the interpretation of the first (disjunctive) premise, then we expect other linguistic items that have been argued to raise alternatives of a similar kind to produce similar illusory inferences.

In the traditions of both Hamblin semantics (Kratzer & Shimoyama 2002) and inquisitive semantics (Ciardelli 2009), indefinites are akin to *wh*-questions. An expression such as *some pilot* is assigned roughly the same meaning as *which pilot*, namely the set of all pilots. Accordingly, a sentence like *Some pilot writes poems* is analyzed as the set of all propositions of the shape *x writes poems*, for *x* a pilot. In other words, the meaning of the question *Which pilot writes poems?* Under this analysis, the erotetic theory predicts that the example below should give rise to an illusory inference.

(3) **Premise 1**: Some pilot writes poems.
Premise 2: John is a pilot.
Fallacious conclusion: John writes poems.

Assuming a finite domain, we can think of the meaning of the first premise of (3) as a large disjunction of conjunctions. This is given schematically in (4), for P a predicate for *pilot* and W a predicate for *writes poems*. Notice how structurally we have a generalized case of the illusory inference with disjunction in (2).

(4) **Premise 1**: $(Pa \wedge Wa) \vee (Pb \wedge Wb) \vee \cdots \vee (Pz \wedge Wz)$
Premise 2: Pa
Fallacious conclusion: Wa

Mascarenhas & Koralus (2017) found that inferences like (3) were in fact attractive, with acceptance rates around 35%, significantly above the baseline for mistakes established by invalid controls without alternative-generating elements. This demonstrates the existence of the predicted illusory inferences with alternative-generating linguistic operators besides disjunction.

1.4 Reasoning vs. interpretation

Illusory inferences from *disjunction* as in (1) have acceptance rates around 85%, while illusory inferences with *indefinites* as in (3) are accepted around 35% of the time. Why the different acceptance rates, if both disjunctions and indefinites produce alternatives according to our best semantic theories?

Mascarenhas (2014) showed that the disjunction inferences are amenable to a pragmatic account. As an alternative to mental-models accounts, we can predict

the observed conclusion assuming an entirely classical reasoning module, acting on pragmatically strengthened meanings of the premises. The computations are far from easy, but on most modern theories of scalar implicature (e.g. Sauerland 2004, Spector 2007) the first premise of the illusory inference from disjunction in (5a) is predicted to be interpreted as in (5b), a fact Spector (2007) had already observed outside the context of reasoning problems.

(5) a. $(a \wedge b) \vee c$
b. $(a \wedge b \wedge \neg c) \vee (c \wedge \neg a \wedge \neg b)$

Assuming (5a) is interpreted as in (5b), and incorporating the second premise a, the conclusion b is no fallacy at all. It follows classically from the conjunction of the two premises. Crucially, no absolving implicature is predicted by any theory of scalar implicature on the market for illusory inferences with *indefinites*. Picat (2019) argues that, rather than being competing accounts, the erotetic theory and the scalar-implicature account are two possible routes leading to the same inference-making behavior in the case of the illusory inference from *disjunction*. For the indefinites case, only the erotetic theory predicts a fallacy, the scalar-implicature route is blocked, explaining its lower endorsement rate.[4]

1.5 Which alternatives?

Alternatives of more than one kind play a role in semantics and pragmatics. It is important to stress that this article concerns only those linguistic operators that encode alternatives in the sense of inquisitive semantics and Hamblin semantics. For example, we make no predictions for inference patterns with premises involving focus alternatives (Rooth 1996) or the alternatives that enter formal pragmatics computations (Sauerland 2004, Chierchia et al. 2012).

It is clear however that this issue deserves more attention. As explained above, the erotetic theory holds that it is the question-raising abilities of Hamblin/inquisitive alternatives that drives the illusory inferences of interest. Focus in particular can raise or at least prime questions, making it a legitimate and interesting question whether we can find some analog of these illusory inferences with focus constructions. We must leave the question unanswered for now.

4 Picat (2019) gives corroborating experimental evidence for this proposal, showing that the disjunction case is affected by cognitive load, reducing pragmatic interpretations and decreasing the rate of fallacious responses, while the indefinite case is unaffected by this manipulation.

1.6 Anatomy of illusory inferences with disjunction-like elements

The most conspicuous property of the illusions of interest is of course drawing the target fallacious conclusion. But these inferences have a more complex signature that has been studied in some detail, and that is important to properly diagnose new fallacies, as we will do in this article. We take four points to be diagnostic of illusory inferences as introduced above.

A "Fallacy". More fallacious conclusions than for invalid controls. The hallmark of these illusory inferences is a fallacious conclusion that picks one of the available possibilities (alternatives) raised by the first premise on the basis of merely partial overlap with the information in the second premise. As is standard in the psychology of deductive reasoning, we consider that a fallacious conclusion is a phenomenon in need of explanation only if it is drawn to a significantly greater extent than fallacious conclusions are drawn in control problems of comparable syntactic complexity. For example, Koralus & Mascarenhas (2018) diagnose illusory inferences of the shape $(a \wedge b) \vee (c \wedge d)$, a by comparing the rate of fallacious conclusions to that prompted by disjunctive syllogisms of the form $(a \wedge b) \vee (c \wedge d)$, $\neg a$.

B "Order". Fewer fallacious conclusions in reverse order. The erotetic theory of reasoning explains these fallacies in terms of a question-answer dynamic. Reasoners consider the disjunctive or indefinite first premise to raise a question, to which the second premise provides a hint at an answer. We thus expect that it should be harder for participants in these experiments to engage in question-based reasoning if the order of the two premises is reversed. Accordingly, illusory inferences with disjunction and with indefinites both show a drop in acceptance of about 10 percentage points when the order of the premises is reversed (Mascarenhas & Koralus 2017, Koralus & Mascarenhas 2018). Crucially, the same experimental studies did not find a drop in acceptance in control inferences with premises of comparable syntactic complexity. Note that we do not expect the effect to disappear entirely, and indeed it does not. It should be easy enough for most participants to realize that simply rereading the premises in a different order offers a more congruent discourse that provides an attractive justification to say "yes, the conclusion follows."

C "Dynamics". No fallacious conclusions without dynamics. A much more radical mitigation of the phenomenon should occur if the dynamic structure of the stimulus is altogether destroyed. This is a novel criterion we propose in the studies in this article: reverse the order of the premises and combine them into a single premise

with a conjunction. We expect next to no fallacious conclusions in such a situation, for the work required to unpack this "flat" structure into a question-answer dynamic should be too onerous for most participants. We found preliminary evidence of the validity of this prediction in a precursor to this article (Mascarenhas & Picat 2019).

D "Rate". Rate of fallacious conclusions depends on available mechanisms. We expect there to be a lower rate of fallacies if the mechanism behind arriving at the conclusion is solely based on erotetic reasoning. As explained above, natural language expressions evoking alternatives allow for reasoning based on question-answer dynamics and make the fallacy more attractive. Given previous results on reasoning with indefinites, the rate of fallacies expected if this is the only mechanism involved is around 35%. For disjunctions, the rate of fallacies is much higher, given that, additionally, pragmatic reasoning and interpretation mechanisms are involved. There is a strengthened meaning of the premise with the disjunction which makes the conclusion not only attractive but valid. We expect a much higher rate of fallacies if both reasoning and interpretation mechanisms are involved.

We take it that any fallacies that do not display these properties are either the product of experimental noise, or derived by entirely different mechanisms. We will regularly return to these four points when spelling out our predictions for the experiments in this article.

2 Illusory inferences with modals

Disjunctions and indefinites raise alternatives in the sense of inquisitive semantics and Hamblin semantics, and accordingly they produce illusory inferences with alternatives as characterized in the preceding section. To test our hypothesis that there is a deep connection between generating alternatives and producing these illusory inferences, we turn our attention to epistemic and deontic possibility modals. We do this for two reasons.

Firstly, most theories analyze epistemic and deontic modals very much on a par. For example, a classical, quantificational account of modality (Kratzer 1977, 1991, 2012) proposes a core semantics for epistemic and deontic modals that is the same up to the choice of modal base and ordering source. Relatedly, many languages of the world (e.g. Indo-European languages) use some of the same lexical items for epistemic and deontic modality. This suggests that epistemic and deontic modals are closely related in their form and their meaning. They would thus make for an appropriate minimal pair through which to study a phenomenon like illusory inferences, provided that there is some reason to expect the two modals to behave differently with respect to these illusory inferences.

Secondly, such an expectation is to some extent warranted. In the tradition of inquisitive semantics, Ciardelli et al. (2009) analyze *might*-sentences as involving *attentive content*, drawing hearer attention to a single proposition. Consider the sentence in (6).

(6) John might be in London.

Ciardelli et al. (2009) argue for a version of inquisitive semantics where a sentence may give rise to alternatives displaying proper inclusion between them. The technical details are of no consequence for the purposes of this article, it will suffice to understand this non-standard version of inquisitive semantics as being in fact closer to Hamblin semantics, which typically does not impose constraints on alternative sets: anything that is a set of expressible propositions can in principle be the meaning of a sentence in Hamblin semantics. Ciardelli et al. (2009) propose that the meaning of (6) should be seen as equivalent to (7a) in their version of inquisitive semantics, where $\top$ is the tautology. Accordingly, they argue that (6) gives rise to the alternative set in (7b).

(7) a. John is in London $\vee \top$
b. {John is in London, $\top$}

Crucially, the sentence as a whole is not a tautology. It is informationally idle in their view, that is, an update with this sentence will exclude no possibilities from any common ground.[5] Nevertheless, it contains two distinct alternatives, it is not identical to the interpretation of the tautology, and therefore not equivalent to it in inquisitive terms. Ciardelli et al. (2009) argue that the semantic contribution of a *might* sentence is not to provide information or raise a *bona fide* question, but simply to *draw attention* to a single possibility. Since one of the alternatives is the trivial alternative, to which it makes little or no sense to draw attention, a *might*-sentence in this semantics offers *one* alternative to focus on. Namely in the case of (6), the proposition that *John is in London*.[6]

Consider now a conjunction embedded under *might* as in (8a), which under the view just presented is to be analyzed as (8b).

5 The claim in the paper more generally is that non-maximal alternatives never contribute to the representation of inquisitive or informational content. But this is a consequence of their definitions of these two notions; there is no in-principle argument that such alternatives somehow shouldn't contribute to inquisitive content, quite the contrary. We will return to this point in the discussion below.

6 Ciardelli (2009), Ciardelli et al. (2009) stress that their analysis is not supposed to capture epistemic uses of *might*. They argue that this may be an advantage of their analysis, as *might* has been argued to differ from other epistemic operators. We will return to this point in the discussion.

(8) a. $might(a \wedge b)$
b. $(a \wedge b) \vee \top$

The logical structure in (8b) is a special case of the first premise of standard illusory inferences from disjunction reviewed above: $(a \wedge b) \vee c$. If this analysis is on the right track, the we expect the schema in (9) to give rise to illusory inferences.

(9) **Premise 1**: $might(a \wedge b)$
Premise 2: a
Fallacious conclusion: b

Crucially, to our knowledge no theory of *deontic* possibility modals exists that argues that they are alternative generators in the sense introduced above. In sum then, epistemic and deontic possibility modals are an excellent testing ground for our proposal in this article. First, they are closely related in their form and meaning; second there is a theory on the market arguing that epistemics are alternative generators, but no such claim has been made for deontics; third, the question of whether either epistemic or deontic possibility modals are alternative generators is still very much an open question.

We investigated epistemic *might* and deontic *be allowed to* in two experimental studies. Our goal was to apply methodologies from the psychology of reasoning to answer the question whether either of these operators generate alternatives, using convergence with the theoretical landscape as a means to confirm the validity of our strategy.

2.1 Experiment 1 — *might*

2.1.1 Methods

Design and materials We used reasoning tasks to test the hypothesis that *might* introduces alternatives and thus invites illusory inferences. Our target reasoning problems had the structure in (9), instantiated in (10).

(10) Miranda might play the piano and be afraid of spiders.
Miranda plays the piano.
Fallacious conclusion: Miranda is afraid of spiders.

Following the signature of illusory inferences expounded in section 1.6, we tested (A) inferences with the pattern in (10), (B) inferences as in (10) but with the order of the two premises reversed, and (C) "flat" versions of (10) that combine the truth-conditional information in the two premises into a single sentence. In (11) we give the schemata for reversed and flat versions.

(11) a. REVERSED TARGET
a
$might(a \wedge b)$
Fallacious conclusion: b

b. FLAT
$a \wedge might(b)$
Fallacious conclusion: b

Notice that the flat version is plausibly truth-conditionally equivalent to the canonical and reversed versions. In the context of an assertion of a in the first conjunction, a second conjunct $might(a \wedge b)$ is equivalent to $might(b)$.[7]

Additionally, we considered the possibility that the first premise alone might suffice to prompt the conclusion, a plausible hypothesis since the first premise certainly raises the probability of the embedded conjunction. If fallacious conclusions in the canonical case in (10) were in fact entirely attributable to the first premise alone, then these fallacious conclusions would not constitute a case of the illusory inferences of interest.

Summing up, we tested the following four conditions in a between-subjects design, due to the high degree of similarity between the stimuli in the conditions.

(12) a. CANONICAL $might(a \wedge b), a \vdash b$
b. P1 $might(a \wedge b) \vdash b$
c. FLAT $a \wedge might(b) \vdash b$
d. REVERSED $a, might(a \wedge b) \vdash b$

We further tested valid and invalid controls based on *modus ponens*, as schematized in (13). Invalid controls play a central role in the experimental psychology of deductive reasoning. They consist of invalid inference patterns that do not rely on the functional item of interest, in this case modals. Invalid controls thus establish a baseline of noisy error that is crucial to diagnosing fallacious inference-making behavior. Only target items whose fallacious conclusions are accepted significantly more than the baseline established by invalid controls constitute explananda.

(13) a. VALID MP if a then b, $a \vdash b$
b. INVALID MP if a then b, $c \vdash b$

Participants and Procedure We recruited 210 subjects on Amazon Mechanical Turk, 66% were female and their mean age was 36 (ranging from 18 to 74, $\sigma = 11.5$). Participants were assigned to one of the four target conditions. They each solved

7 The reader skeptical of this argument will find a more persuasive version of the flat condition in Experiment 2, where we used the straightforwardly equivalent schema $a \wedge might(a \wedge b)$ instead.

14 reasoning problems, 8 targets and the rest controls (see Appendix A in 5.1 for materials). In each case, participants were given one or two premises (according to the exact items) and a proposed conclusion, and were asked whether the conclusion followed from the premises. We gave participants two examples in the instructions to the experiment, displaying one valid and one invalid inference, with materials that did not occur elsewhere in the experiment.

Predictions If *might*-sentences give rise to illusory inference we expect to observe the pattern described in Section 1.6 above. We repeat these points below and how they translate into specific predictions for Experiment 1.

A "Fallacy" More fallacious conclusions than for invalid controls in canonical order

- canonical and reversed targets should be accepted significantly more than the baseline for mistakes established by invalid controls, diagnosing a fallacy

B "Order" Fewer fallacious conclusions in reversed order

- canonical targets should be somewhat more attractive than reversed targets, instantiating the order effects observed for other illusory inferences;

C "Dynamics" No fallacious conclusions without dynamics

- the acceptance of canonical and reversed targets should depend on the presence of the second premise, so that P1 targets should be lower than canonical and reversed targets;
- flat targets should be much lower than canonical and reversed targets, and indeed indistinguishable from the baseline for mistakes established by invalid controls

D "Rate" Higher rate of fallacious conclusions in the presence of additional sources

- we expect the rate of fallacies to be similar to that of indefinites, as no additional pragmatic mechanisms are involved

The last prediction requires explication. Recall that with disjunctive illusory inferences there were two possible routes to the observed inference-making behavior, a reasoning route and an interpretation route. The reasoning analysis says that

alternatives generated by the first premise pose a question to which reasoners find a hint at an answer in the second premise. The interpretation route is tacit about the first premise's alternative-generation potential, and assumes no question-answer dynamics, instead deriving the "fallacy" as a sound conclusion from pragmatically strengthened interpretations of the premises. With indefinites however, only the reasoning route is available, pragmatics does not derive an absolving interpretation for either premise or the conclusion. Illusory inferences with epistemics are like those with indefinites in this respect, they lack an absolving interpretation. Indeed, in order to validate the putative conclusion of interest, we would need for the first premise of the inference to have *at least* the implicature in (14a), or perhaps more naturally that in (14b).

(14) a. $\Diamond(a \wedge b) \wedge \neg\Diamond(a \wedge \neg b)$, equivalently $\Diamond(a \wedge b) \wedge \Box(a \rightarrow b)$
b. $\Diamond(a \wedge b) \wedge \Box(a \leftrightarrow b)$

We know of no theory of pragmatics that would predict such an implicature for the first premise of our inferences with *might*. Accordingly, we expect the rate of fallacious conclusions to pattern with indefinites (about 35%) rather than with disjunctions (about 85%).

2.1.2 Results

We excluded from the analysis participants who reported having taken at least one graduate-level course in natural language semantics or pragmatics, and participants who reported using notes or diagrams frequently during the experiment. We further excluded participants based on their performance in control problems when their response patterns clearly revealed that they paid little or no attention to the task. Specifically, we excluded participants that gave only "yes"-responses and those that got less than 50% of controls overall correct (i.e. who answered correctly to at most one "yes"- and one "no"-control). In total, we excluded 41 participants (19.5%). We analyzed the data from the remaining 169 participants using a generalized linear mixed-effects model with the `glmer` function in `R` (Bates et al. 2015). We fitted participants' answers to no-controls and target inferences into a binomial linear mixed-effects model predicting the answer from STRUCTURE with 5 levels (canonical, reversed, P1, flat, no-control). The maximal converging model included random intercepts for participants and items.[8] To calculate contrasts between conditions, we used pairwise comparisons with the `emmeans` package in R. We used Holm

8 More precisely, the model we ran was INFERENCE-TYPE + (1 | SUBJECT) + (1 | ID). The more complex model including random slopes and intercepts for subjects and items created a singular fit, revealing correlations between random effects.

Comparison	Est.	Std.Err	p-value	Corrected p-value
Canonical vs. no-control	2.2	0.4	$p < 0.0001$	$p < 0.0001$
Reverse vs. no-control	1.2	0.4	$p < 0.01$	$p < 0.01$
flat vs. no-control	-0.5	0.412	$p = 0.1923$	
Canonical vs. P1	1.3	0.5	$p < 0.01$	$p < 0.05$
Canonical vs. flat	2.8	0.5	$p < 0.0001$	$p < 0.0001$
Reverse vs. flat	1.8	0.5	$p < 0.001$	$p < 0.01$
Canonical vs. reverse	1.0	0.4	$p < 0.05$	$p < 0.05$

Table 1 Details of the statistical analysis; all comparisons were analyzed using the `glmer` function in `R`, with and random intercepts for participants and items

correction for multiple comparisons. The details of the statistical tests we ran are in Table 1. Figure 1 summarizes the results.

Our findings are in line with the view that the fallacious conclusions we observe are illusory inferences from alternatives. Regarding point A "Fallacy": Canonical and reversed targets were significantly more accepted than mistakes were made on no-controls. This is diagnostic of a reasoning fallacy. The difference between canonical and reverse targets was significant (point B "Order"). Moreover, canonical and reversed targets were significantly more accepted than flat targets, which were no different from no-controls, suggesting that dynamics did play a role (point C "Dynamics"). Furthermore, canonical targets were significantly more accepted than P1 targets. Thus, the fallacy cannot be explained by the first premise alone. And last, the rate of fallacies is comparable to those yielded by indefinites (point D "Rate"), which suggests the fallacy is based on erotetic reasoning alone.

2.2 Experiment 2 — *be allowed to*

Experiment 1 showed that structures containing the epistemic modal *might* give rise to illusory inferences from alternatives, in line with our reading of Ciardelli et al.'s (2009) inquisitive-semantics account of *might*. To our knowledge, no analogous claim has been made for deontic possibility modals. Yet, rich parallelisms between epistemic and deontic modality have motivated unified analyses of epistemics and deontics, raising the question whether deontic modals are also generators of alternatives in the relevant sense.

The goal of our Experiment 2 is to directly compare both modal types and explore the possibility of the deontic modal *allowed to* giving rise to illusory inferences from alternatives in the same way as *might*. This is a truly exploratory study, where we

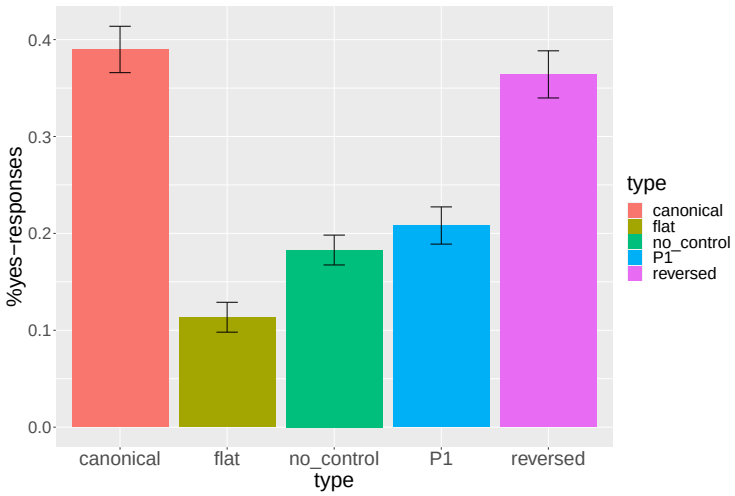

Figure 1 Mean yes-responses by condition; error bars indicate the standard error of the mean

look at illusory inferences from alternatives to diagnose the presence or absence of alternatives in deontic possibility modals.

2.2.1 Methods

Design and Materials As in Experiment 1, we used an inference-making task to test a series of predictions related to the alternative-generating power of epistemic and deontic possibility modals.

The target reasoning problems involved one of two modals, epistemic *might* or deontic *allowed to*. They instantiated one of the three structures in (15). We treated STRUCTURE as a between-subjects factor due to the sentences in the three conditions being very similar. We fully crossed this factor with the factor MODALITY, which we treated as a within-subjects factor. As a result, a given participant saw only one of the structure conditions in (15), but saw both modals with different items.

(15)
a. CANONICAL might/is allowed to $(a \wedge b)$, $a \vdash b$
b. FLAT $a \wedge$ might/is allowed to $(a \wedge b) \vdash b$
c. REVERSED a, might/is allowed to $(a \wedge b) \vdash b$

We minimally changed the *flat* structure condition compared to Experiment 1. This was because the more extreme version of the flat condition *a and* MODAL*(b)* used in Experiment 1 prompts additional inferences in the deontic case that are absent from the epistemic case. Specifically, *a and allowed to b* may give rise to an

implicature that *not allowed to a* given that *a* is a salient excludable alternative. This readings is not available for *might* as *not might a* contradicts *a*, and therefore cannot be excluded innocently (Fox 2007).

The analytical role of the flat structural variant however remains the same. The dynamics of erotetic reasoning ought to be markedly disturbed by this new flat version of the premises. It is harder to construe the two premises as a question-answer pair when conjoined directly.[9]

We give an example of each structure with epistemic and deontic modality in (16) and (17), respectively.

(16) a. John might have stolen from the rich and given to the poor; John stole from the rich ⊢ John gave to the poor. CANONICAL
b. John stole from the rich, and he might have stolen from the rich and given to the poor ⊢ John gave to the poor. FLAT
c. John stole from the rich; John might have stolen from the rich and given to the poor ⊢ John gave to the poor. REVERSED

(17) a. John was allowed to steal from the rich and give to the poor; John stole from the rich ⊢ John gave to the poor. CANONICAL
b. John stole from the rich, and he was allowed to steal from the rich and give to the poor ⊢ John gave to the poor. FLAT
c. John stole from the rich; John was allowed to steal from the rich and give to the poor ⊢ John gave to the poor. REVERSED

We also varied which of *a* vs. *b* from the first premise *might/is allowed to*$(a \wedge b)$ was introduced in the second premise, controlling for any potential order effects from the embedded conjunction. This last factor ORDER was tested within subjects, that is participants saw half of the items with *a* as the second premise, and the other half with *b* as the second premise.

In sum, we manipulated 3 factors: MODALITY with 2 levels (deontic or epistemic), STRUCTURE of the reasoning problem with 3 levels (canonical, flat, or reversed) and the relative order of second premise/conclusion with 2 factors (the second premise being the first or second conjunct of the argument of *might* in the first premise). Thus, we had $2 * 2 * 3 = 12$ conditions. We created 6 items per within-subjects condition (4 in total, DEONTIC–LEFT, DEONTIC–RIGHT, EPISTEMIC–LEFT, EPISTEMIC–RIGHT). These 24 critical items were distributed across 4 experimental lists (specifying the sentence stimuli that a participants sees and in which order)

9 A further change in the materials compared to Experiment 1 was the change of tense to past form. This was done to avoid the ambiguity of present tense in English, and a possible interference with type of modality. We did not expect this change to affect epistemic *might* and the observations for Experiment 1.

in a Latin square design. Across participants (and experimental lists) each critical item is seen equally often in all its (within-subject) conditions. This was done for each group factor (STRUCTURE) separately, resulting in 12 sub-experiments that participants were randomly assigned to.

Besides target items, we used both control items and baseline items. Control items served the purpose of eliminating unfocused participants. They used valid and invalid conclusions based on *modus ponens* and *disjunctive syllogism*. Baseline items used conjunction elimination and were used to establish a baseline for error rates. In (18) we give the structure of control and baseline items.

(18)

a.	VALID MP	if a then b; $a \vdash b$
b.	INVALID MP	if a then b; $c \vdash a$
c.	VALID DS	a or b; $\neg a \vdash b$
d.	INVALID DS 1	a or b; $c \vdash d$
e.	INVALID DS 2	a or b; $\neg b \vdash c$
f.	VALID BASELINE	$a \wedge b \vdash a$
g.	INVALID BASELINE 1	$a \vdash a \wedge b$
h.	INVALID BASELINE 2	$a \wedge b \vdash a \wedge c$

In addition to 24 target items per list, there were the same 12 baseline and the same 12 control item in each list.

Participants and Procedure We recruited 183 participants via Prolific, 56% were female and their mean age was 33 (ranging from 20 to 60, $\sigma = 13.2$). Participants were assigned to one of 12 experimental lists. They each solved 48 reasoning problems; 24 targets, 12 controls and 12 baselines (see Appendix B in 5.2). The procedure used was the same as for Experiment 1. Participants were presented with premises and a proposed conclusion. They had to evaluate whether the conclusion followed from the premises. Before the core of the experiment began, we explained and exemplified the concepts of valid and invalid conclusion.

The exclusion criteria we used were the same as for Experiment 1. We excluded people who reported having often taken notes, having taken at least one graduate-level course in semantics/pragmatics, or answered less than 50% of control questions correctly. With these exclusion criteria we removed 26 participants (14.2%) and analyzed data from 157 participants.

Predictions The goal of Experiment 2 was to test whether we observe the same type of illusory inferences with deontic as with epistemic modals. On theoretical grounds, we had some reason to expect there to be no illusory inferences of the relevant kind with deontics. While one theory argued that *might* generates alternatives,

the inquisitive account by Ciardelli et al. (2009), no theory to our knowledge had made a similar claim about deontics.

Still, we considered the question to be properly an open question, given the scarcity of theoretical and experimental work on modals as generators of alternatives, and given the well-known parallelisms between epistemic and deontic modality.

Consequently, one very live hypothesis is that they should behave completely alike, that is, that we observe the same pattern for both modal types in Experiment 2 as we did for epistemic modals in Experiment 1. Specifically, in line with point A "Fallacy" in Section 1.6, we might find there to be more yes-responses to canonical target items than to no-baseline items. We might expect there to be fewer fallacies with reverse order structures than canonical order (point B "Order" in Section 1.6); no fallacies with flat structures (point C "Dynamics" in Section 1.6); and a rate of fallacies at about 30% (point D "Rate" in Section 1.6).

2.2.2 Results

We analyzed the results with generalized linear mixed effects models, using the `lme4` package and `glmer` function in `R`. We report the most complex converging model. We give details of the random-effects structure in the tables reporting the statistical results. Fixed factors were MODAL, ORDER, STRUCTURE, depending on the analysis, see more details below. We calculated contrasts with pairwise comparisons using the `emmeans` package in `R`. We used Holm correction for multiple comparisons, and report both corrected and uncorrected p-values.

First, we looked at whether deontic modals give rise to fallacious inferences of the form *modal(a and b); a* $\vdash$ *b* (point A). We compared the rate of fallacies of canonical structures with no-baselines for both modals separately.[10] Our analysis shows that both modals give rise to these inferences in the canonical form. That is, the rate of "yes"-responses differs significantly from "no"-baselines for both types of modality, see Figure 2 and Table 2.

To test whether the two modals are affected differently by STRUCTURE, we first looked at the interaction between STRUCTURE and MODAL in our analysis.[11] We see that for both levels *deontic/epistemic:reversed/canonical* and *deontic/epistemic:flat/canonical* this interaction is significant, as shown in Table 3 and Figure 5.

10 Specifically, we used a subset of the data containing no-baseline items, canonical structures for epistemics, and canonical structures for deontics and ran the maximally converging model: RESP $\sim$ INFERENCE-TYPE + (1+INFERENCE-TYPE | SUBJECT) + (1 | ID).

11 Specifically, we ran the maximally converging model RESP $\sim$ MODAL*STRUCTURE + (1 | SUBJECT) + (1 | ID) with canonical/deontic as the reference level.

Comparison	Est.	Std.Err	p-value	Corrected p-value
Deontic canon. vs. no-base.	6.9	1.7	$p < 0.0001$	$p < 0.001$
Epist. canon. vs. no-base.	6.2	1.7	$p < 0.0001$	$p < 0.01$

Table 2 Responses to "no"-baselines, canonical deontics and epistemics were fitted into a model with inference type as fixed effect, and random intercepts for participants and items, as well as a random slope for inference type grouped by participant

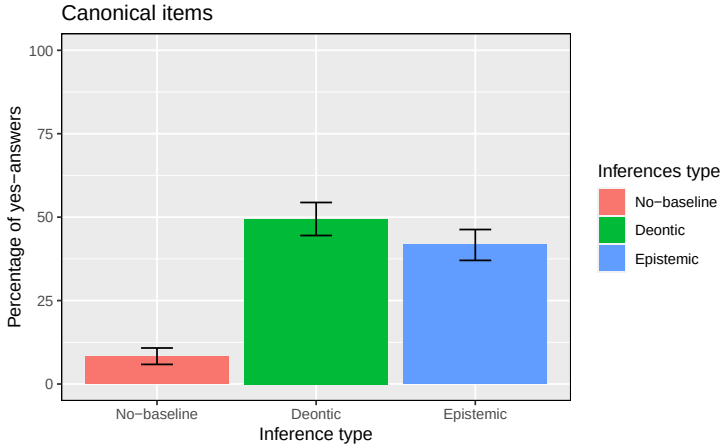

Figure 2 Mean yes-responses by inference type; error bars indicate the standard error of the mean

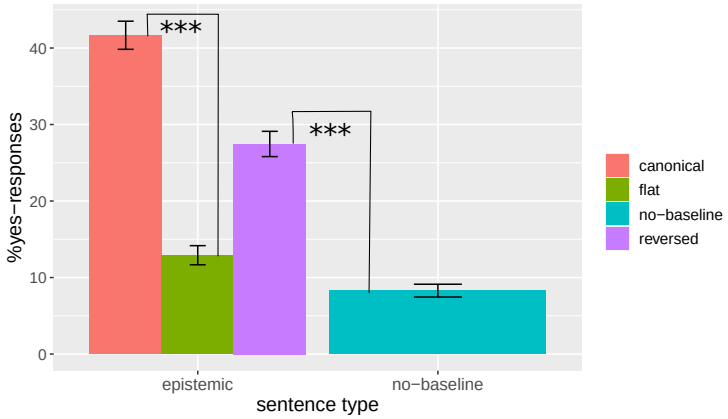

Figure 3 Mean yes-responses by STRUCTURE versus no-baselines for epistemics; error bars indicate the standard error of the mean. Black lines show significant contrasts.

The interactions show that structure affects epistemic and deontic modals differently. Zooming in on contrasts,[12] we observe a significant difference between canonical versus flat structures for both modals, with the effect being more pronounced for epistemics. Canonical and reversed structures, however, do not differ significantly from each other for deontics. The contrast is only marginally significant for epistemics (see table 4). This result for epistemics is consistent with previous experimental results Mascarenhas & Koralus (2017) showing that the effect of order is very subtle, and only visible with a very high number of participants. The presence of a significant interaction together with the difference in estimates, however, are suggestive of epistemics being affected by reverse versus canonical order, whereas deontics are not.

Zooming in further, we looked at the contrast between "no"-baseline items and flat structure items, and "no"-baseline items versus reversed target items, for both modals separately.[13] We see that, for epistemics, flat structures do not differ significantly from "no"-controls (see Table 5). However, for deontics, they do (see Table 6). Furthermore, there is a significant difference between reversed targets and "no"-controls for both modals. The contrasts we find for each modal are displayed and highlighted in Figure 3 and 4.

12 We calculated contrasts using the `emmeans` package and `contrasts` function in R. We used Holm's correction for multiple comparisons.

13 We created a subset for each model. Then, we used the same model as before: model is RESP ∼ INFERENCE-TYPE + (1+INFERENCE-TYPE | SUBJECT) + (1 | ID)).

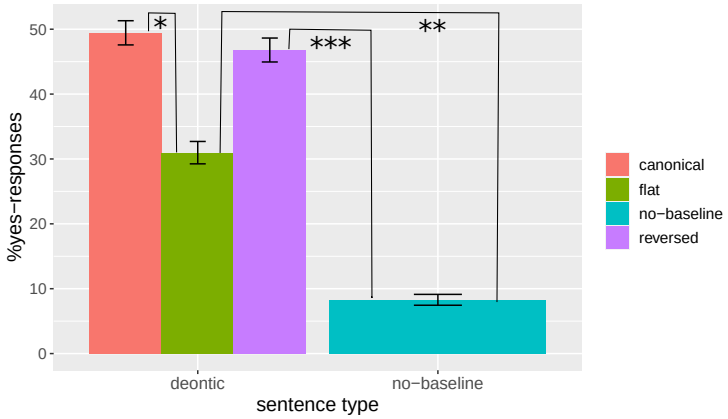

Figure 4 Mean yes-responses by STRUCTURE versus no-baselines for deontics; error bars indicate the standard error of the mean. Black lines indicate significant contrasts.

Coefficient	**Estimate**	**Std.Err**	***p*-value**	**Corrected *p*-value**
Intercept	-0.3	0.4	$p = 0.6$	
Epistemic	-0.6	0.2	$p < 0.001$	
Flat	-1.8	0.6	$p < 0.01$	
Reversed	-0.1	0.6	$p = 0.9$	
Epistemic:Flat	-1.7	0.3	$p < 0.0001$	$p < 0.0001$
Epistemic:Reversed	-1.1	0.2	$p < 0.0001$	$p < 0.0001$

Table 3 Responses to deontics and epistemics targets were fitted into a model with MODALITY, STRUCTURE and the interaction between the two as fixed effects, and random intercepts for participants and items.

MODAL	**Comparison**	**Est.**	**Std.Err**	***p*-value**	**Corrected *p*-value**
Deontics	Canon. vs. rev.	0.1	0.6	$p = 0.9$	$p = 1$
	Canon. vs. flat	1.8	0.6	$p < 0.01$	$p < 0.01$
Epistemics	Canon. vs. rev.	1.1	0.6	$p < 0.05$	$p = 0.13$
	Canon. vs. flat	3.4	0.6	$p < 0.0001$	$p < 0.0001$

Table 4 Contrasts of interest of the model reported in table 3

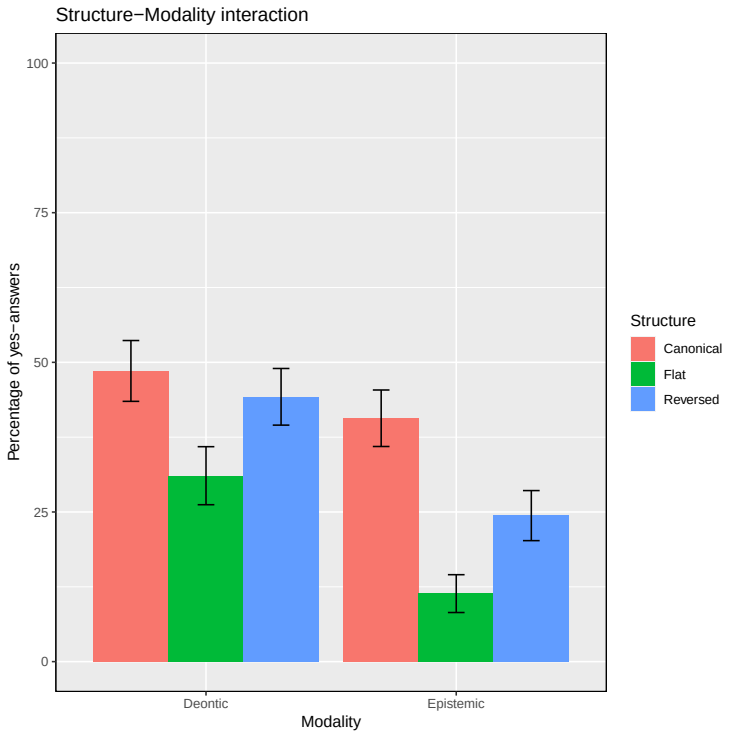

Figure 5 Mean yes-responses by MODAL and STRUCTURE; error bars indicate the standard error of the mean. Lines indicate the significant interactions.

Comparison	**Est.**	**Std.Err**	***p*-value**	**Corrected *p*-value**
Flat vs. no-baseline	-1.1	1.9	$p = 0.6$	$p = 1$
Reversed vs. no-baseline	4.9	1.3	$p < 0.001$	$p < 0.01$

Table 5 Responses to no-baseline, reversed and flat epistemics were fitted into a model with inference type as fixed effect, and random intercepts for participants and items and random slopes for inference type grouped by participant

Comparison	Est.	Std.Err	p-value	Corrected p-value
Flat vs. no-baseline	4.1	1.3	$p < 0.01$	$p < 0.01$
Reversed vs. no-baseline	5.9	1.3	$p < 0.0001$	$p < 0.0001$

Table 6 Responses to no-baseline, reversed and flat deontics were fitted into a model with inference type as fixed effect, and random intercepts for participants and items and random slopes for inference type grouped by participant

Comparison	STRUCTURE	Est.	Std.Err	p-value	Corrected p
Epist. vs. deont.	Canonical	0.6	0.2	$p < 0.001$	$p < 0.01$
	Flat	2.2	0.2	$p < 0.0001$	$p < 0.0001$
	Reversed	1.6	0.2	$p < 0.0001$	$p < 0.0001$

Table 7 Contrasts between epistemic (reference level) and deontic modality for each structure.

We also investigated the contrast between modals for each structure, based on the model reported in Table 3. We summarize this contrast in Table 7. We observe that there are significantly more "yes"-responses for deontics than for epistemics for each structure.

We also checked the effect of ORDER. We had no specific prediction regarding this effect, but wanted to control for it. Consequently, we expected that any order effect should affect both modals in the same way. However, we find a significant and surprising interaction between ORDER and MODALITY, see Table 8.[14] With deontics, fallacies are more attractive when premise 2 is the left conjunct of premise 1, i.e. when premise two is *a* if premise one is *was allowed to a and b*. For epistemics, fallacies are more attractive when the second premise was the right conjunct of premise one, i.e. *b* if premise one was *might have a and b*. This further underlines the difference between modals. Since we had no predictions for this comparison, and this analysis was post-hoc, we will not interpret this finding further.

2.3 Summary of experimental results

Summing up the results of Experiment 1, we see that structures containing the epistemic modal *might* give rise to fallacies carrying the signature of illusory inferences

14 We used the following model RESP $\sim$ MODAL*ORDER + (1 + MODAL*ORDER | SUBJECT) + (1 + ORDER | ID), which was the most complex converging one. The reference level was deontic/left conjunct.

Coefficient	Estimate	Std.Err	p-value
Intercept	-1.4	0.4	$p < 0.001$
Epistemic	-1.3	0.4	$p < 0.001$
First conjunct	0.7	0.3	$p < 0.05$
Epistemic:second conjunct	-0.9	0.4	$p < 0.05$

Table 8 Responses to deontics and epistemics targets were fitted into a model with MODALITY, ORDER and the interaction between the two as fixed effects, and all possible random effects

with alternatives. Both reversed and canonical targets show a higher rate of fallacies than "no"-baselines (point A "Fallacy"). These cannot be explained as resulting from the first premise alone given the difference we observe between canonical targets and P1 targets. Furthermore, illusory inferences seem to require a dynamic operation as evidenced by the absence of fallacies for flat targets (point C "Dynamics"), and fewer fallacies for reversed compared to canonical targets (point B "Order").

The findings of Experiment 2 show that deontics also give rise to fallacious conclusions. However, we also see that the signatures of the fallacies we observe for epistemics and deontic modals differ. We replicated our findings for epistemics from Experiment 1. The fallacies we observe have the characteristics of illusory inferences with alternatives in that they are sensitive to the exact reasoning structure they are embedded in. There are fewer fallacies for reversed and flat targets than there are for canonical targets (points B and C about "Order" and "Dynamics"). Furthermore, reversed and canonical targets differ from "no"-baselines (point A "Fallacy"), whereas flat targets do not (point C "Dynamics").

We observe a significant rate of fallacies with deontics as well, as evidenced by a significantly higher rate of "no"-responses to canonical targets than "no"-baselines. However, the rate is significantly higher than for epistemics and they are much less sensitive to structure. Regarding point D "Rate", we see that deontics neither pattern with epistemics, nor indefinites, nor disjunction regarding the rate of fallacies, suggesting that entirely different mechanisms are at play. Regarding point B "Order", we observe no order effects with deontics. There is no difference between reversed and canonical targets, which contrasts with the findings for epistemics, as shown by a significant interaction between these structures and type of modality. Regarding point C "Dynamics", we moreover find that for deontics flat structures attract the fallacy as well, as they differ significantly from the "no"-baseline. So do reverse structures. Dynamics thus seem to play less of a role. Still, we do observe that flat structures behave differently from canonical and reverse targets. Given the overall

picture we see, lacking the full signature of illusory inferences from alternatives, the root of this effect is plausibly not alternative generation and erotetic reasoning.

3 General Discussion

An analysis of *might* as generating alternatives in combination with the propositional erotetic account of reasoning of Koralus & Mascarenhas (2013) can explain the pattern we observe for epistemic modals in Experiments 1 and 2. What does that mean, specifically?

As discussed in the introduction to this article, Ciardelli et al. (2009) offer an attentive semantics for *might* where

$$might(\varphi) \equiv \varphi \vee \top \ .$$

The main proposal is that *might*-sentences contribute neither informative nor inquisitive content, but merely draw attention to a single proposition. According to their definition of *inquisitive*, *informative* and *attentive* content, non-maximal alternatives like φ above never contribute to inquisitive or informative content and only manifest attentive content. However, there is no principled reason to exclude the possibility of having non-maximal alternatives represent inquisitive content. Indeed our data suggest that *might* sentences not only draw attention to an alternative, but also raise an issue of whether this alternative holds, as evidenced by their potential to prompt illusory inferences. Assuming *might* only draws attention to an alternative does not explain why we observe a difference between reversed and canonical target sentences. Assuming that the second premise is addressing an issue raised by the first one does explain this difference. We suggest that a minimally updated version of Ciardelli et al. (2009)'s proposal that allows for *might* sentence to raise an issue whether φ holds makes the right predictions.

Ciardelli et al. (2009) further distinguish between an attentive and epistemic versions of *might*, and propose that their analysis only applies to the attentive version. They argue that *might* behaves differently from other epistemic operators, discussing in particular observations by Zimmermann (2001) that (19a) and (19b) are equivalent in meaning whereas (20a) and (20b) are not.

(19) a. John might be in Paris or in London.
b. John might be in Paris and he might be in London

(20) a. It is consistent with my beliefs that John is in London or that he is in Paris.
b. It is consistent with my beliefs that John is in London and it is consistent with my beliefs that he is in Paris.

Ciardelli et al. (2009) propose a new notion of meaning where sentences make proposals instead of denoting truth-conditions. They argue that the basic observation about *might*, that it talks about a possibility — which under a standard view is captured by its being an existential modal operator — can be accounted for in pragmatic terms (do not say anything that you do not consider to be possible).

We propose that our findings and Ciardelli et al.'s (2009) observations can be reconciled with a traditional view of *might* as an epistemic operator, if one assumes that epistemic *might* introduces alternatives. The differences between (19) and (20) are accounted for by not assuming that *all* constructions expressing epistemic states will raise alternatives. Rather, we take it to be a lexical property of certain operators: *might* raises alternatives, *it is consistent with my beliefs* does not.[15] Moreover, the observations about the poor or inexistent truth-conditional contribution of attentive *might* sentences can be framed in general terms for epistemic *might*, based on work on epistemic modals that sees them as expressing content, rather than asserting it (Yalcin 2008, Anand & Hacquard 2012).

It is also possible that more conservative approaches to the semantics of epistemic modality can be adapted to make similar predictions. Under a standard view from philosophical logic, possibility modals are existential quantifiers over possible worlds. They thus a share a common feature with indefinites, as illustrated by the gloss in (21).

(21) John might be in London.
'Some epistemically accessible world is a "John is in London" world.'

Now for the novel assumption: if we take the indefinite that occurs in the classical interpretation of modals to be of an inquisitive nature, then modals raise a question, as in (22a). The actual world is a candidate for such a world assuming reflexivity, as is standard in epistemic semantics. By asserting that one of the conjuncts is true in the actual world as the second premise (22b) does, all the preconditions for an illusory inference according to the erotetic theory are met, and we expect the fallacious conclusion in (22c).[16]

15 This idea connects to observations by Anand & Hacquard (2012), Hacquard & Wellwood (2012) on embedding epistemic operators. In a nutshell, it is extremely deviant to embed *might* under bouletic attitudes (?? John wishes that it might rain), while this is not true for all weak epistemic constructions (John wishes that it were consistent with his beliefs that it should rain). These peculiarities of *might* lend plausibility to the idea that *might* can raise alternatives but not all truth-conditionally close epistemic constructions will.

16 More formally, a sentence like 'might (*a* and *b*)' could be construed as a (possibly infinitary) disjunction the same way as indefinites in the nominal domain can be, see schematized in (ia). Thus, the modal claim is similar in structure to the type of disjunctions illusory inferences have originally have been observed for.

(22) a. John might be in London and visit his sister.
'Which epistemically accessible world w is such that John is in London in w and visiting his sister in w?'
b. John is in London.
'John is in London in the actual world.'
c. John is visiting his sister.
'John is visiting his sister in the actual world.'

Indeed maintaining an analysis of *might* as existential quantifier over possible worlds also captures the fact that the rate of illusory inferences we observe is similar to the ones previously found for indefinites in the nominal domain (but lower than for disjunction). Following Picat & Mascarenhas (2019), we expect these inferences to have only one source, erotetic reasoning. That is, there should be no scalar implicature of either premise that derives the "fallacy" as a result of valid reasoning. Indeed, in order to validate the observed conclusion, we would need for the first premise of the inference to have *at least* the implicature in (23a), or perhaps more naturally that in (23b).

(23) a. $\Diamond(a \wedge b) \wedge \neg\Diamond(a \wedge \neg b)$, equivalently $\Diamond(a \wedge b) \wedge \Box(a \rightarrow b)$
b. $\Diamond(a \wedge b) \wedge \Box(a \leftrightarrow b)$

We know of no theory of pragmatics that would predict such an implicature for the first premise of our inferences with *might*, confirming our expectations.

To our knowledge there is no theory of deontic modality holding that deontics are inquisitive or raise Hamblin-style alternatives per se. This is in line with our findings that the fallacies we observe with deontics do not have the full signature of illusory inferences with alternatives. At first glance, this is surprising under a Kratzerian view where both deontic and epistemic possibility modals are existential quantifiers over possible worlds. Following a quantificational analysis, the same

(i) a. $\big(a(w_1) \wedge b(w_1)\big) \vee \big(a(w_2) \wedge b(w_2)\big) \vee \big(a(w_@) \wedge b(w_@)\big) \vee \cdots$
b. $a(w_@)$
c. **Fallacious conclusion**: $b(w_@)$

Despite the purely formal parallel, there are important differences to note between existential quantification in the nominal and modal domain, however. Whereas a statement like "John is a pilot" clearly identifies an individual, John, which can then be taken to refer to and designate an individual in the domain, this is less transparent for factual claims referring to the actual world. Worlds are neither uniquely identifiable designators nor overtly expressed. Whereas we offer a sketch of how our data could be captured within a more standard approach to modality, cashing out the details is much harder. As the main goal of the current paper is not to argue for a specific approach to modality, we have to leave this for future research.

parallel to existential quantification in the nominal domain is expected. As a result, the reasoning pattern in (24) is predicted to occur.

(24) **Premise 1**: *allowed to* $(a \wedge b)$
Premise 2: a
Fallacious conclusion: b

The explanation for the fallacious conclusion in (24) follows the one for epistemic modals: the meaning of the first premise can be paraphrased as the indefinite statement 'Some deontically accessible world is an a- and b- world'. Given the inquisitive nature of indefinites, the issue raised by this statement is which world is the described world. By asserting 'a' next attention is drawn to the actual world as being this world. As a result, 'b' follows.

However, an important contrast is observed in the literature on possibility modals which is that reflexivity need (and maybe even should) not be assumed for deontic modals. If this assumption is not made the reasoning as described above will not succeed. More precisely, the set of disjunctive alternatives raised by P1 in (24) does not contain the actual world under this assumption. Pointing out that 'a' holds in the actual world will thus not draw any attention to one of the alternatives. As a result, the fallacious conclusion is not attractive.

There is a further relevant observation in the literature, which is that the inference in (25a) does not necessarily go through for the sentence in (25) due to the *package deal* effect deontic modals are associated with (Merin 1992, van Rooy 2000).

(25) John is allowed to steal from the rich and give to the poor.
a. $\not\rightarrow$ John is allowed to steal from the rich.

The effect can be described as a sentence 'allowed to a and b' only giving permission to do both or neither 'a and b', which van Rooy (2000) calls *bi-conditional permission*. Deriving the package deal effect using a pragmatic strengthening mechanism seems unattractive for two reasons: first, as noted above, the same mechanism is not available for epistemic modals. It is unclear why this should be so, and would break the parallels observed between the two types of modality. Second, the *package deal* reading is argued to be the empirically rarer case (Merin 1992, van Rooy 2000). If a strengthening mechanism is generally available, the question is why it would not occur more reliably. However, van Rooy (2000) argues that there are cases where the *both-or-neither* reading is a salient one, most notably for cases like (25), where pragmatically it is more plausible that one (stealing) is not allowed without the other (giving). He proposes a performative analysis of permission where permission

sentences like *may A* update the permissibility set *P* by contracting $\neg A$.[17] According to van Rooy (2000), the different readings — with and without *package deals* — are the result of the performative update happening after or before type shifting, thus deriving two parses $P_{update}(a)$ *and* $P_{update}(b)$ and $P_{update}(a \text{ and } b)$.[18] The minimal element the permissibility set is updated with in the latter case is *a and b*. As a result, the only worlds where *a* is allowed are also worlds where *b* is allowed, and a *package deal* effect emerges. In the former case, the permission update operates on *a* and *b* separately, thus the permissibility set also contains *a and* $\neg$ *b* and *b and* $\neg$ *a* worlds. There is no *package deal* effect in this case.

That the same effect is not observed for epistemics is unsurprising, as they are not seen as performative in the same way as deontics, and, as a result, the two operations for updating are not available.

Our findings can be made sense of in view of van Rooy's (2000) analysis of *package deals*. The relevant reading is, in principle, available for all structures we embedded conjunctive permission in, repeated in (26) below, and will make the conclusion 'John gave to the poor' necessarily more attractive.

(26) a. John stole from the rich and he was allowed to steal from the rich and give to the poor.
b. John stole from the rich. He was allowed to steal from the rich and give to the poor.
c. John was allowed to steal from the rich and give to the poor. He stole from the rich.

The fact that the effect was less pronounced in the flat condition may be due to the ambiguity regarding temporal relation between 'a' and the permission of 'a and b'. In the flat condition, there seems to be a reading where permission follows the event of John stealing from the rich. The finding may also be rooted in the different interactions of update operators and sentence structure. That is, the performative analysis of permission in the flat condition may be less available given that the first conjunct is an assertion. This may be less problematic when both are construed as separate sentences. We must leave exploring this relationship between temporal/sentence structure and permission for further research. However, we suggest that *package deals* may be the source of the fallacious conclusions we observe for deontics.

17 By contrast, Merin (1992) gives up on the idea of permission update as contraction using *package deals* as one argument.

18 This is highly simplified, as the formal details are not crucial for the current discussion, but see van Rooy (2000) for more details.

4 Conclusions

Our results show that epistemic and deontic possibility modals are sensitive to alternatives in different ways. Insights from reasoning tasks suggest that it is part of the semantics of epistemic modals that they draw attention to certain propositions by raising questions in the same way as indefinites do. This is in line with both an existential quantifier view a la Kratzer, as well as an inquisitive semantics analysis following Ciardelli (2009). However, overall the account by Ciardelli (2009) seems to fare better in making this exclusively about *might*. Treating *might* as a special operator, having built attention to alternatives into its semantics, and treating *allowed to* on a par with other free choice permission items fully captures the picture we observe. Importantly, we have shown that reasoning tasks can be instrumental in answering questions regarding the involvement of alternatives in natural language expressions, and inform theories of modality.

One speculative but promising conjecture emerges from this work. It is clear that linguistic expertise is essential in the study of the psychology of human reasoning. Virtually the totality of work in this field uses linguistically presented reasoning and decision problems to diagnose failures of reasoning, so a thorough understanding of interpretive processes is indispensable. But the methods of the psychology of reasoning can be illuminating to the field of natural language semantics as well. If our theories and our interpretation of these experimental results are on the right track, then illusory inferences of the sort discussed here can serve as novel diagnostic tools for the presence of alternative-generating elements in semantics.

5 Appendices

5.1 Appendix A

Targets

(27) P1: Peter might be part of a band and study archeology
P2: Peter is part of a band
C: Peter studies archeology

(28) Linda might drink coffee every morning and play videogames at night
Linda drinks coffee every morning
Linda plays videogames at night

(29) John might learn Chinese and learn Finnish
John is learning Chinese
John is learning Finnish

(30) Owen might eat candy and walk to school every day
Owen eats candy
Own walks to school every day

(31) Chris might bike on the weekends and read late at night
Chris bikes on the weekends
Chris reads late at night

(32) Paul might get a tattoo and move to Michigan
Paul is getting a tattoo
Paul is moving to Michigan

(33) Karen might be in basketball team and invite friends over to her house
Karen is in the basketball team
Karen invites friends over to her house

(34) Tracy might hang out with Susan and take a Swedish class
Tracy hangs out with Susan
Tracy is taking a Swedish class

Controls

(35) If yesterday was Wednesday, Carol went to the theater
Yesterday was Wednesday
Carol went to the theater

(36) If George dyed his hair, Mary was delighted
George dyed his hair
Mary was delighted

(37) Daniel ate an apple or a pear
Daniel did not eat an apple
Daniel ate a pear

(38) Kit or Rose learned Latin at school
Rose did not learn Latin at school
Kit learned Latin at school

(39) If Claire is at home, Ann is at the movies
Olivia might have a new car
Ann is at the movies

(40) If Brian was brave, he asked Carry out
The weather might be cold
Brian asked Carry out

(41) Selina or Robin came early this morning
Robin did not come early
Edward came early this morning

(42) Clint or Wanda ate the whole cake
Clint loves chamber music
Kevin ate the whole cake

5.2 Appendix B

Targets

(43) John might have stolen from the rich and given to the poor
John stole from the rich
John gave to the poor

(44) Daniel was allowed to skip a day of work and go to a march in Washington
Daniel skipped a day of work
Daniel went to a march in Washington

(45) George might have skipped afternoon classes and attended team practice
George skipped afternoon classes
George attended team practice

(46) Audrey was allowed to go to the mall and buy a book
Audrey went to the mall
Audrey bought a book

(47) Linda might have left the base and called her parents
Linda left the base
Linda called her parents

(48) Riley was allowed to take Mary's car and go grocery shopping
Riley took Mary's car
Riley went grocery shopping

(49) Alexander might have had his friends over and played video games
Alexander had his friends over
Alexander played video games

(50) Bob was allowed to go to a bar and talk to a friend
Bob went to a bar
Bob talked to a friend

(51) Sam might have skipped school and visited his grandmother in the hospital
Sam skipped school
Sam visited his grandmother in the hospital

(52) June was allowed to visit her boyfriend and do her homework
June visited her boyfriend
June did her homework

(53) Paul might have taken his parents' car and picked up dinner
Paul took his parents' car
Paul picked up dinner

(54) Jeremy was allowed to leave school and get a job
Jeremy left school
Jeremy got a job

(55) John might have stayed at school till late and used the library
John stayed at school till late
John used the library

(56) Thomas was allowed to take a painkiller and call a friend
Thomas took a painkiller
Thomas called a friend

(57) Bill might have gone fishing and cooked dinner
Bill went fishing
Bill cooked dinner

(58) James was allowed to join the army and study medicine
James joined the army
James studied medicine

(59) Peter might have called Mary and talked about church
Peter called Mary
Peter talked about church

(60) Heather was allowed to buy a lottery ticket and sell her car
Heather bought a lottery ticket
Heather sold her car

(61) Laura might have gone to a concert and drunk beer
Laura went to a concert
Laura drank beer

(62) Charlotte was allowed to take the bus and go to the dentist
Charlotte took the bus
Charlotte went to the dentist

(63) Brittany might have gone to France in summer and sold her guitar
Brittany went to France in the summer
Brittany sold her guitar

(64) Owen was allowed to go to a baseball game and apply for a job
Owen went to a baseball game
Owen applied for a job

(65) Jean might have run a marathon and bought tickets for the super bowl
Jean ran a marathon
Jean bought tickets for the super bowl

(66) Nathan was allowed to open a bank account and get a SIM card
Nathan opened a bank account
Nathan got a SIM card

Controls

(67) If yesterday was Wednesday, Carol went to the theater
Yesterday was Wednesday
Carol went to the theater

(68) If Arthur's favorite team won the game, he partied all night long
Arthur's favorite team won the game
Arthur partied all night long

(69) If George dyed his hair, Mary was delighted
George dyed his hair
Mary was delighted

(70) Daniel ate an apple or a pear
Daniel did not eat an apple
Daniel ate a pear

(71) Kit or Rose learned Latin at school
Rose did not learn Latin at school
Kit learned Latin at school

(72) Sally or Norman came to the party
Sally did not come to the party
Norman came to the party

(73) If Brian was brave, he asked Lydia out
Brian got a cat
Brian asked Lydia out

(74) If Sam was hungry, he ate three cheeseburgers
Sam bought a new computer
Sam ate three cheeseburgers

(75) If Bruce went to Tokyo, he took a lot of pictures
Bruce spent the weekend with his cousin
Bruce took a lot of pictures

(76) Selina or Robin came early this morning
Robin did not come early
Selina got engaged

(77) Lois knows how to juggle with 4 balls or how to do a backflip
Lois does not know how to juggle with 4 balls
Lois knows how to breathe fire

(78) Clint or Wanda ate the whole cake
Clint did not eat the whole cake
Wanda loves chamber music

Baslines

(79) Luke moved to New York and bought a new phone
Luke moved to New York

(80) Joan visited her mother and bought new shoes
Joan bought new shoes

(81) Diego went to a basketball game and stopped at the gas station
Diego went to a basketball game

(82) Roberta picked out a wedding present and paid her bills
Roberta paid her bills

(83) Kim made coffee and greeted a colleague
Kim made coffee

(84) Janine played chess with her brother and went grocery shopping
Janine went grocery shopping

(85) Hugo watched a play with a friend
Hugo watched a play with a friend and went to a dinner party

(86) Sina handed out flyers for a restaurant
Sina handed out flyers for a restaurant and picked up a book from the library

(87) Carlos picked up his daughter from school
Carlos picked up his daughter from school and mowed the lawn

(88) Estelle signed a lease and called her landlord
Estelle signed a lease and went for a run

(89) Vaughn booked a vacation and changed the oil in his car
Vaughn went on a date and changed the oil in his car

(90) Fiona went to Starbucks and registered for classes
Fiona met her neighbour and went to Starbucks

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