Asserting Epistemic Modals

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Introduction

Consider the following sentence:

(1) Paul might have been at the party last night.

I'll refer to sentences of this form as *MIGHT*-CLAIMS. The classic Kratzerian contextualist account of *might*-claims is that they quantificationally relate the proposition denoted by the sentence without the modal (its PREJACENT) to a contextually-given body of knowledge:

(2) $[\![\text{might}]\!]^i = \lambda p.\lambda w.[\exists w' : w' \in f_i(w)]p(w') = 1$
(adapted from Kratzer 1977)

That is to say: the context of utterance determines a conversational background i,[1] which includes, potentially among other things, a function f_i mapping each world to the set of all worlds compatible with what is known in that world; the *might*-claim is true in a world w iff the set of all worlds compatible with what is known in w contains at least one world in which the prejacent is true.[2] All this talk of 'what is known' immediately raises a question: known by who?[3] That is to say, what body of knowledge is relevant to the interpretation of BARE epistemic modals, i.e. unembedded epistemic modals unaccompanied by overt modifiers (e.g. *given what John believes*)?

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[1] Note that here and throughout I've suppressed orthographic representation of all other aspects of the context parameter, and all other parameters of evaluation, for the sake of legibility.

[2] This is the case on the simplest semantics for modals, which I'll call the 'simple quantificational account' of modals. Kratzer (1981, 1991) went on to develop an account of modals in which they are interpreted relative to both a modal base and an ordering source; the 'ordering semantics' will be discussed in detail below.

[3] Note that this question is not specific to frameworks in which modals are quantifiers over worlds—it's also pertinent to probabilistic accounts of epistemic modality (Swanson 2011, 2015, Lassiter 2011, 2016, 2015, Moss 2015). The question for these theories becomes: probable in whose estimation?

An intuitive answer to the question 'known by who' is: known to the speaker. The crucial intuition is that a speaker's assertion of a *might*-claim is licensed by consideration only of their own knowledge, not by the presumption that they have authoritative information about any other knowledge state. To make this answer more formal: we could answer this question by saying that the conversational background i determined by the context of utterance of a sentence containing a bare epistemic modal always represents the speaker's epistemic state; f_i, in these cases, maps each world to the set of worlds compatible with what the speaker knows in that world. Let's call this view SOLIPSISTIC CONTEXTUALISM, following Yanovich (2014).

Despite the intuitive appeal of solipsitic contextualism, a surprisingly broad consensus has emerged that disagreement data like (3), the relevance of which was first noted by Hawthorne (2004), are fatal to it.

(3) **A**ndrea: Paul might have been at the party last night.
Bertrand: {You're wrong, No way, Thats false}, he was in Barbados.

The reason why data like (3) have been taken to be problematic for solipsistic contextualism is this: if **A**'s claim is a claim about **A**'s own knowledge state, i.e. it is true iff Paul having been at the party last night is an epistemic possibility available to **A** at the time of utterance, then **B**'s response of *that's false* should mean 'your epistemic state does not actually allow for that possibility'. But that's not what is going on in (3). Rather, **B**'s response expresses that the prejacent is not an epistemic possibility *for him* (and goes on to specify why that is so). Intuitively, an interlocutor's disagreement with a *might*-claim is licensed by the relationship of that *might*-claim to their own epistemic state, not the speaker's, an intuition that is seemingly at odds with the intuition that underlies solipsistic contextualism. As Stalnaker (2014) puts it, "The puzzle is that no way of pinning the relevant knowledge state down seems to be able to explain both why we are in a position to make the epistemic "might" claims we seem to be in a position to make, and also why it is often reasonable to disagree with "might" claims made by others" (pp. 137-138).

The responses to this problem (in addition to other problems posed by discourse data, q.v. Yalcin 2011, Swanson 2011, Knobe & Yalcin 2014, MacFarlane 2014, and, for a useful overview, Cariani 2019) that have been proposed in recent literature are wide-ranging.[4] Some proposals hold onto the basic Kraterzian contextualist picture, while pursuing non-solipsistic approaches to the choice of modal base that unmodified and unembedded epistemic modals quantify over—for instance Egan et al. (2005) and Dowell (2011) associate the relevant modal base with a group rather than any particular individual, von Fintel & Gillies (2011) propose that epistemic modals are ambiguous with respect to what modal base they are to be interpreted relative to, giving rise to a 'cloud' of possible denotations, Yanovich (2014) proposes that the denotation of epistemic modals is sensitive to a broader set of contextual factors like the practical goals of the current conversation, and Mandelkern (2019) proposes that the semantics of epistemic modals refers directly to the Common Ground. Other proposals, which we could call, speaking loosely, 'relativist' or 'domain' approaches (Egan et al. 2005, Egan 2007), treat epistemic modals as sensitive to parameters of interpretation that vary independently of the context of utterance, whether those parameters are taken to represent judges (Stephenson 2007), information states (Yalcin 2007, Stalnaker 2014), or contexts of assessment (MacFarlane 2011). Still

[4]For older precedents for some of these ideas, see e.g. Hacking (1967), Teller (1972), DeRose (1991), a.o.

other proposals, which we could call, speaking loosely, 'expressivist' or 'nonfactualist' approaches (Yalcin 2011, Rothschild 2012, Swanson 2015), embrace views in which *might*-claims don't have standard propositional denotations at all, treating *might* as a syncategorematically-defined update operator (Veltman 1996) that indicates that its prejacent is a 'live possibility' (Willer 2013) or expresses lack of belief in the complement of its prejacent (Hawke & Steinert-Threlkeld 2016, 2020).

In this paper, I argue that the conclusion that data like (3) are incompatible with the solipsistic contextualist view was premature. I show that disagreement data like (3) are not problematic for the solipsistic contextualist view *simpliciter*, but rather are problematic for the solipsistic contextualist view *in conjunction with* some standard Stalnakerian assumptions about how to formalize the discourse update carried out by assertions. Therefore, the problem could be solved by changing the semantics of epistemic modals, or by changing the formalization of assertive updates (or both). Prior literature has focused overwhelmingly on the former path; I develop here a solution following the latter path.

I present here a change of camera angle on the standard Stalnakerian formalization of assertive update, which stays close to the core intuitions that motivated the classic view, and behaves identically to Stalnaker (1978) for all sentences whose semantics is not affected by the choice of epistemic conversational background, but which accurately derives the disagreement behavior of bare epistemic modals from their solipsistic contextualist interpretation, on which the epistemic conversational background is interpreted as the speaker's epistemic state. The informal version of this formalization of assertive update, which is developed formally below, is that when a speaker asserts a sentence, they are presenting themselves as though they know it to be true—i.e. they are predicating something of their epistemic state—and they are proposing that the context be modified such that it will be a member of the same predicate of states they've indicated their own epistemic state to be a member of. Crucially, despite its similarity in conception to the classic Stalnakerian view of assertion, this formalization does not deliver the result that any successful assertion add the propositional denotation of the sentence used to carry out that assertion to the Common Ground. On this view of assertion, the explanation for the *prima facie* unexpected (given solipsistic contextualism) discourse behavior of epistemic modals follows from how their epistemicity interacts with the epistemic nature of assertion.

The account of assertion put forward in this paper delivers two primary results about *might*-claims: first, it derives Veltman's (1996) 'consistency test' semantics for *might* (q.v. Yalcin's 2007 'informational consequence'-based proposal for the update potential of *might*-claims) from Kratzer's (1977) simple quantificational semantics for *might*; second, it derives a novel update from Kratzer's (1981, 1991) ordering semantics for *might*: that of adding its prejacent to the ordering source. Both are examples of how the proposed account of assertion allows us to build a bridge from a conventional truth-conditional semantics to 'expressivist' update effects without sacrificing a univocal formalization of assertive update that applies equivalently to all declarative sentences—this allows us to unify the intuitions behind both classes of accounts, while avoiding the *ad hoc* stipulations that threaten to render many expressivist accounts unconstrained and unexplanatory.

Though I take the possibility of a solution to the problem of disagreement over epistemic modals that is situated entirely within the theory of assertion to defang some arguments for the desirability of the proposals cited above, and find the proposal's derivation of the discourse behavior of epistemic modals from their very epistemicity to be explanatorily satisfying, I stop short of arguing that situating a solution here is superior to the semantic approaches cited above on strict grounds of empirical

adequacy. However, I hope to show that the dividing lines between the various views on epistemic modals that have emerged over the last decade or so are oversimple—we can derive 'relativist' effects with a relativistic illocutionary machinery, instead of a relativistic semantic machinery, and likewise we can derive 'expressivist' updates from traditional denotations without stipulating construction-specific discourse effects.

The structure of this paper is as follows: In §1 I briefly articulate the role played by (sometimes implicit) assumptions about the formal mechanism of assertion in the problem (3) poses for the solipsistic contextualist interpretation of bare epistemic modals. In §2, I present my positive proposal, a reframing of the Stalnakerian account of assertion that derives the disagreement behavior of bare epistemic modals from their solipsistic contextualist interpretation while preserving standard Stalnakerian update for assertions of sentences that are not sensitive to the epistemic conversational background. I show how the account deals with *might* on both the simple quantificational semantics of Kratzer (1977), and the ordering semantics of Kratzer (1981, 1991). In §3 I present four extensions of the account, showing how it can deal with embedded epistemic modals, overtly relativized epistemic modals, both the simple quantificational and ordering semantics of *must*, and probabilistic accounts of epistemic modals. §4 concludes.

1 The Role of Assertion in the Disagreement Problem

Let's revisit the problem of disagreement over *might*-claims and work through the explanation of why it is a problem for solipsistic contextualism a little more carefully.

Consider, first, a disagreement that doesn't involve epistemic modals.

(4) **A**: Paul was at the party last night.
B: {You're wrong, No way, That's false}, he was in Barbados.

One intuitive account of what is going on here is that in this interaction, when **A** asserts p, she is presenting herself as if it is true given her knowledge. When **B** rejects p, he is presenting himself as if p is incompatible with his knowledge. The same intuitive account of what's going on seems to extend to disagreement over *might*-claims as well. Consider (3), repeated here as (5):

(5) **A**: Paul might have been at the party last night.
B: {You're wrong, No way, That's false}, he was in Barbados.

We can characterize this interaction along the same lines as the disagreement over a non-modal claim above: when **A** asserts *might*-p, she is presenting herself as if p is a possibility given her knowledge. When **B** rejects *might*-p, he is presenting himself as if p is not a possibility given his knowledge. This seems like the same thing as is going on with non-modal disagreements. So what is the problem?

Though the informal characterizations of the two disagreements given above seem parallel to each other, only the first follows from the standard Stalnakerian formalization of assertion, as originally outlined in Stalnaker (1978). That formalization begins with an assumption about the representation of a conversational context (q.v. Lewis's 1979 notion of a 'conversational scoreboard'):

(6) Stalnakerian contexts
A conversational context c includes at least:

a. The Common Ground (CG_c), the set of all propositions representing the public mutual knowledge of all interlocutors

b. The Context Set (CS_c), the set of all worlds compatible with all propositions in the Common Ground (i.e. $\forall c$, $CS_c = \bigcap CG_c$)

When we say that the Common Ground represents public mutual knowledge, we mean that it is the set of all propositions such that all interlocutors are publicly committed to behave for the purposes of the conversation as though they know those propositions to be true, and as though they take each other to know those propositions to be true, and so on recursively. In a perfectly cooperative conversation, the public mutual knowledge of the interlocutors will be the same as the private mutual knowledge of interlocutors, but assertions can be used to lie, or to tell made-up stories for each others' entertainment, and so on.

To make an assertion is to propose that the propositional content of the uttered sentence be added to CG; i.e., that all interlocutors agree to behave as though they know the actual world to be a member of the proposition that sentence denotes. Abstracting away from the 'proposal' aspect of this (though see Farkas & Bruce 2010 for a defense of its importance), the standard Stalnakerian account of successful assertive update can be formalized as a function from context to contexts:

(7) Stalnakerian update
An assertion of p is a function from an input context c_i to an output context c_o, such that $CG_{c_o} = CG_{c_i} + p$ (and, therefore, $CS_{c_o} = CS_{c_i} \cap p$)

That is to say: the standard Stalnakerian formalization of assertion treats assertions as proposals that CS be intersected with the asserted proposition, resulting in a representation of the interlocutors' mutual information that does not include any worlds not contained in that proposition.[5]

Finally, because CS is a representation of mutual knowledge, rejection of an assertion of p is licensed iff the context set that would result isn't compatible with (i.e., has an empty intersection with) the rejector's epistemic state.[6]

So, how does this standard formalization deal with the two cases with which this section started? In the former case, that of non-modal disagreement in (4), the speaker has proposed that CS be intersected with the proposition p, the result of which would be that all of the worlds in CS would be p-worlds. The addressee's rejection of that proposed update, then, indicates that she knows the actual world to not be a p-world, and so doesn't want to exclude all non-p worlds from the context set, as that would make it an inaccurate representation of the mutual knowledge of the interlocutors.

[5] Note that this conception of update is a common starting point for theories of dynamic semantics, e.g. Heim's (1982, 1992) Context Change Semantics and Veltman's (1996) Update Semantics.

[6] Note that we are restricting our attention here entirely to cases of outright or 'hard' rejection (*you're wrong, that's false*, and so on), rather than expressions of skepticism prompting further discussion (*are you sure, do you really know, where'd you hear that*, and so on). A speaker may push back against an assertive update without outright rejecting it if the asserted proposition is compatible with but not entailed by their epistemic state.

In the latter case, that of disagreement over a *might*-claim in (5), the speaker has proposed that the context set be intersected with the proposition *might-p*, which, assuming the solipsistic contextualist interpretation of bare epistemic modals, is the set of all worlds in which the speaker's knowledge is compatible with p. So the result of the speaker's proposed update would be the exclusion from the context set of all worlds in which the speaker's epistemic state rules out p. Given this, **B**'s rejection of the proposed update would indicate that she does not think that the actual world is a world in which **A**'s epistemic state is compatible with p. In other words, **B** should be understood to be rejecting **A**'s description of her own epistemic state, and claiming that **A** in fact knows p to be false: it should be roughly equivalent to: 'No, you know that he *wasn't* at the party'. That this interpretation of the disagreement in (5) is unavailable, and that the actual interpretation of that disagreement is that **B** believes the *might*-claim's prejacent to be false, are both wholly unexpected on this view.

What I want to stress about the above explanation is that the solipsistic contextualist truth conditions for *might* don't render the disagreement in (5) unexpected all by themselves. It's only when they're plugged into a theory of what a speaker is *doing* with a sentence's denotation when they assert it that we make any predictions whatsoever about what information that assertion is putting forward—i.e. what update to the conversational context is being proposed—and what it means to disagree with with that assertion—i.e. under what circumstances an interlocutor is justified in rejecting the proposed context update, and how to interpret what the rejection indicates about the rejector's epistemic state.

In the next section, I present a formal proposal for a change of camera angle on Stalnaker's account of assertion that captures the parallelism of the initial intuitive accounts given for the two disagreements above. On this formalization of assertive update, rather than assertion proposing that a propositional denotation be added directly to the Common Ground, when a speaker makes an assertion she presents herself as though she knows the asserted sentence to be true—i.e. she predicates of her epistemic state that the asserted sentence is true at every world in it—and proposes that the context be altered in as conservative a fashion as possible such that the resulting context is guaranteed to be a member of the same predicate of states as her own. Though this captures much the same intuition as Stalnaker's account—that an assertor presents herself as though she knows a sentence to be true, and proposes that that knowledge be made mutual—unlike Stalnaker's formalization, it does not guarantee that all assertions propose that the propositional content of the sentence that has been uttered are intersected directly with the Context Set. Sentences containing bare epistemic modals will have somewhat different update potentials from other sentences because of how their epistemicity interacts with the epistemicity of assertion. The feasibility of this account demonstrates that the empirical threat to solipsistic contextualism that disagreement data poses hinges on the technical details of how the intuition that assertions use propositional content to build mutual knowledge is implemented; as such, that threat appears much weaker than it once did.

2 The Account

In this section I will present a revised formalization of the Stalnakerian account of assertion, and show how it deals with *might*-claims on both the simple quantificational semantics of Kratzer (1977) and the subsequent ordering semantics, familiar from Kratzer (1981, 1991). I show that the proposal accurately predicts the observed disagreement behavior of *might*-claims on both semantics, given

solipsistic contextualism. In §2.1, I present the revised formalization of the Stalnakerian account of assertion, and apply it to the assertion of a sentence whose semantics makes no reference to the speaker's epistemic state, showing that it derives the same results as Stalnaker (1978): intersection of the context set with the proposition denoted by the asserted sentence. Then, in §2.2, I show how the proposal deals with the simple quantificational semantics for epistemic modals. Specifically, I will show that the proposed formalization of assertion derives exactly the update proposed by Veltman (1996) for *might*-claims: the context set will be left unaltered if it has a non-trivial intersection with the prejacent, and anomaly will result otherwise (q.v. Yalcin 2007's 2007 discussion of an update potential for *might*-claims based on 'informational consequence'). Because this implementation derives Veltman's update, it inherits the problems of that proposal (see criticisms in Willer 2013, Rudin 2018)—it is inherently uninformative. The ordering semantics was designed in part to render possibility modals more informative, and so in §2.3, I show how the proposal deals with the ordering semantics for epistemic modals, extending the representation of contexts from the Stalnakerian context set to a tuple of a context set and an ordering source. The proposal derives a to my knowledge wholly novel update potential for assertions of *might-p*: such an assertion is a proposal that p be added to the context's ordering source (cf. Portner's 2004, 2007 proposal that imperatives serve to update prioritizing ordering sources). The result that these two updates can be derived from the simple quantificational semantics for *might* and the ordering semantics for *might*, respectively, without sacrificing the basic intuition behind the standard Stalnakerian account of assertion, is the core contribution of this paper.

2.1 The Neo-Stalnakerian Formalization of Assertion

On the Stalnakerian account, assertions use the propositional content of a sentence to update a representation of public mutual knowledge. Adding a proposition to the representation of public mutual knowledge entails consideration of the individual public knowledge of the interlocutors: an assertor represents herself as though she knows the asserted proposition to be true, and asks interlocutors to do the same, by way of proposing that it be added to the interlocutors mutual knowledge.[7]. The standard formalization of the Stalnakerian account of assertion, from Stalnaker (1978) forward (presented above in §1), focuses on the way assertions update public mutual knowledge, and lets the assertor's publicization of her individual knowledge go unformalized, though it's an obvious consequence of what is formalized. We can think of this as a choice of camera angle: it places the updating of mutual knowledge at the center of the frame, with the assertor's publicization of her individual knowledge out of focus in the periphery. What I propose here is a change of camera angle on the same scene: a formalization of the Stalnakerian account of assertion that places the assertor's publicization of her individual knowledge at the center of the frame. On this formal account, which I'll call the Neo-Stalnakerian Formalization of assertion (NSF), propositional content enters the picture with respect to what the assertor predicates of her epistemic state; update to the representation of mutual knowledge is then derived from that predication.

To get off the ground with this formalization, I'll first specify the representation I'll assume for epistemic states. Throughout this subsection, I'll outline the minimal representation of epistemic states necessary to give an implementation of the simple quantificational semantics for epistemic modals;

[7] For a clear explanation of the relation between individual public knowledge and mutual public knowledge in the Stalnakerian worldview, see Farkas & Bruce (2010).

in 2.3, I change the representation of an epistemic state to be more congruent with the ordering semantics for epistemic modals, which is implemented therein.

I assume for now that an epistemic state i is simple a set of worlds, representing the set of all worlds compatible with what the state-holder knows:

(8) Epistemic states (simple version):
$D_i = \wp W$

That is to say, the domain of epistemic states D_i is simply the set of all subsets of the set of all possible worlds W.

I assume, following Stalnaker (1978), a formalization of conversational contexts in terms of the public mutual knowledge of the interlocutors.[8] That is to say, the context is itself an epistemic state, that representing what is publicly mutually known by the interlocutors.

(9) NSF conversational contexts
The domain of contexts $D_c = D_i$

That is to say: c is eligible to be a conversational context iff c is an epistemic state. I will adopt standard Stalnakerian terminology in sometimes referring to a context as the context set. Because in the NSF, an assertion does not directly add a proposition to the contextual representation of public mutual knowledge, a contextual representation like the Common Ground will not be useful; rather, assertive update will be modeled directly in terms of how it affects the context set.

When a speaker asserts a sentence s, they present themself as though they know s to be true.[9] That is to say, they present themself as though s is true at every world in their epistemic state's modal base. We can generate the set of all such states by using a meta-intensionalization function MI that maps sentences to the set of all epistemic states characterized by knowledge of the truth of that sentence:

(10) Meta-Intensionalization (simple version):
$MI = \lambda s.\{\, i : [\forall w : w \in i][\![s]\!]^i(w) = 1 \,\}$

Note that the conversational background variable i covaries with the epistemic states we're building a set of here because of the solipsistic contextualist assumption that the epistemic conversational background is determined by the context of utterance to be the speaker's epistemic state. As this function returns the set of states the speaker could be in if they know s to be true in the context

[8] Note that I focus here only on the context qua representation of mutual knowledge, not on the role of the context as a parameter of interpretation; the only aspect of contextual parameterization I'm interested in here is that it delivers an epistemic conversational background i. For more finely elaborated models of conversational context, see e.g. Roberts (1996), Ginzburg (1996), Farkas & Bruce (2010). I assume this extremely simple notion of context for presentational simplicity, as the elaborations provided by the models above are irrelevant to the phenomena I discuss here.

[9] A note on terminology: it's typical to say that a speaker asserts a proposition, rather than that a speaker asserts a sentence. It would perhaps be more kosher to say that a speaker 'makes an assertion by way of uttering a sentence s' or some comparable phrasing. Any readers who are bothered by describing assertion as something that is done with a sentence, rather than with the semantic content of a sentence, can feel free to mentally swap out my simpler phrasing in the main text for the more complicated but more pedantically accurate phrasing discussed here.

of utterance, if the speaker holds epistemic state i_1, then we are in a context that delivers i_1 as the epistemic conversational background for the interpretation of s; if they hold epistemic state i_2, then we are in a context that delivers i_2 as the epistemic conversational background for the interpretation of s, and so on.

When a speaker asserts s, then, they present themself as though their epistemic state is a member of the predicate of states derived by meta-intensionalizing s:

(11) NSF ASSERTIVE LICENSING:
An agent a is licensed to assert s iff $i_a \in MI(s)$
Where i_a is a's epistemic state

In the NSF, context update is derived from the predicate of states $MI(s)$ rather than through direct interaction between the propositional denotation of s and the context. Informally, the speaker is proposing that the context be modified such that it become a member of the same set of states that licensed her assertion; i.e., that it be updated to reflect the same knowledge that licensed her assertion.

(12) NSF ASSERTIVE UPDATE (informal version):
When a speaker asserts s in c, she is proposing that c be modified in the most conservative possible way such that the resulting context becomes a member of $MI(s)$

This notion of update requires some unpacking. I assume, following Stalnaker (1978) and Veltman (1996) among many others, that information growth in a well-behaved conversation is monotonic—if a conversational context requires non-monotonic modification, that is a signal that something has gone wrong, and the conversation has been defective in some way. Because a context c is just a set of worlds, there is only one way to monotonically increase the information represented by c: to remove worlds from it.[10] I will rely on the following notion of refinement in enforcing monotonicity of update:

(13) REFINEMENT: (simple version)
A state i' is a refinement of a state i ($i' \leq_r i$) iff $i' \subseteq i$

Monotonicity of update can be enforced with the following stipulation:

(14) MONOTONICITY CONDITION ON CONTEXT UPDATE:
For any context update mapping an input context c_i to an output context c_o, $c_o \leq_r c_i$

This defines the space of possible modifications that can be made to the context—so what exactly does it mean to modify the context 'in the most conservative possible way'?

(15) UPDATE (formal version):
When a speaker asserts s in c, she is proposing that an s-commensurate and s-conservative function be applied to c

[10] Note, again, that in §2.3 the representation of epistemic states will be altered by inclusion of an ordering source that is not defined in terms of the modal base, expanding the possible ways of increasing the information represented by the context to either removing worlds from the context set or adding propositions to the ordering source.

There are three concepts that need to be defined to make sense of this update condition.

(16) Commensurativity:
For any function f, sentence s, f is s-commensurate iff $[\forall c : c \text{ is } s\text{-compatible}] f(c) \in MI(s)$

Informally: the context update associated with an assertion made using a sentence s must result in the context becoming a member of the set of epistemic states that licensed the assertion.

(17) Conservativity:
For any function f, sentence s, f is s-conservative iff $[\forall s' : [\forall c : c \text{ is } s\text{-compatible}] f(c) \in MI(s')][\![s]\!]^c \subseteq [\![s']\!]^c$

Informally: the context update associated with an assertion made using s must not guarantee that the resulting context entail anything that isn't already entailed by s.

(18) Compatibility (simple version):
For any context c, sentence s, c is s-compatible iff $[\exists c' : c' \leq_r c] c' \in MI(s) \wedge c' \neq \varnothing$

Finally: when we are thinking about s-commensurativity and s-conservativity, we should restrict our attention to contexts that have non-empty refinements that are members of $MI(s)$—i.e., contexts that could be updated to reflect the speaker's epistemic state without resulting in anomaly. I take compatibility to also be a condition on the cooperativity of an assertion: an assertion of s is only cooperative relative to a s-compatible context, as if the context is not s-compatible, it will be impossible for the update to be successful.

(19) Anomalous update (simple version):
For any context c such that c is not s-compatible:
$c[s] = \varnothing$

Read $c[s]$ as 'c updated by an assertion with s'. This states that incompatible updates are anomalous, resulting in the absurd context $\varnothing$.

The overall view of update we get from the definition in (15) is that an assertion of s is assigned an update potential that guarantees that the resulting context be a member of $MI(s)$ without guaranteeing that the context entail anything not entailed by s.

Before we move on to a concrete example, it is important that we define the licensing condition for rejection of an assertion. I define the licensing condition for rejection in terms of $MI(s)$, the set of all possible epistemic states the speaker could have if they indeed know the sentence they've asserted to be true:

(20) Rejection:
A speaker is licensed in outright rejecting an assertion made using s iff her epistemic state is not s-compatible.

In other words, when a speaker A rejects with an assertion of s by a speaker B, A is presenting herself as though there is no way in which she could monotonically update her epistemic state to make it share the property that B has predicated of her own epistemic state by virtue of her assertion: that of being a member of $MI(s)$. This follows from the role the context is taken to play in a Stalnakerian framework: the representation of public mutual knowledge. If the proposed update would result in a context that is incompatible with an interlocutor's epistemic state, then there is no way for her to monotonically update her epistemic state to bring it into congruence with the representation of public mutual knowledge that would result, and she is licensed in rejecting that update.

2.1.1 A simple example

The above comprises the entire system put forward in this paper as it applies to frameworks whose representation of epistemic states is a simple set of worlds. In §2.3, I'll extend the system to include ordering sources, which will affect anything above labeled 'simple version'. I'll now walk through a concrete example to make the operation of the system clear.

Consider a simple sentence like *John is dead*. I assume the following propositional denotation for this sentence, abstracting away from irrelevant complexities:

(21) $[\![\text{John is dead}]\!]^i = \lambda w.dead'(j)(w)$

When a speaker asserts this sentence, she presents herself as though her epistemic state is a member of $MI(\text{John is dead})$ and proposes that the context be modified so as to become a member of that set as well.

(22) $MI(\text{John is dead})$
$= \lambda s.\{\, i : [\forall w : w \in b_i][\![s]\!]^i(w) = 1 \,\}(\text{John is dead})$
$= \{\, i : [\forall w : w \in b_i][\![\text{John is dead}]\!]^i(w) = 1 \,\}$
$= \{\, i : [\forall w : w \in b_i]dead'(j)(w) = 1 \,\}$

The meta-intensionalized denotation in (22) is a set of epistemic states. In this case, the set of epistemic states the speaker could possibly hold if she knows the sentence *John is dead* to be true is the set of all subsets of the proposition denoted by that sentence in the context of utterance. To see this, imagine that the set of all possible worlds W contains only three worlds, w_1, w_2, and w_3. The proposition $\lambda w.dead'(j)(w)$ maps w_1 and w_2 to 1, and w_3 to 0. Relative to this set of worlds, (22) is the following set:

(23) $\left\{ \begin{array}{c} \{\text{w}_1, \text{w}_2\} \\ \{\text{w}_1\} \quad \{\text{w}_2\} \\ \varnothing \end{array} \right\}$

Call the proposition denoted by *John is dead* in the context of utterance p. In this case, $MI(s)$ returns the set of all epistemic states that contain only p-worlds—the downward closure of p.[11] It's easy to see that a comparable result will obtain for all sentences whose denotation does not vary with the

[11] The downward closure of a set is the set containing all of its subsets.

epistemic conversational background variable i: for any sentence s such that $[\![s]\!]^i = [\![s]\!]^{i'} = p$ for all i,i', for any i, $[\![s]\!]^i$ is true at every w in i iff i is a subset of p. However, we will see in §2.2 that this will not be the case for sentences whose denotation varies with the epistemic conversational background variable i.[12]

2.1.2 Update potential

Given that in this case, $MI(s)$ is the set of all epistemic states that are subsets of the proposition p denoted by s in the context of utterance, it is trivial to derive the update condition associated with the assertion: it is Stalnakerian update, i.e. intersection of the context set with p.

(24) Update potential for non-epistemic sentences (simple version):
For any c, and for any s such that for all i, i', $[\![s]\!]^i = [\![s]\!]^{i'}$,
$c[\![s]\!] = c \cap [\![s]\!]^c$

This states that for sentences whose denotation is unaffected by the epistemic conversational background, the update potential of an assertion of that sentence is the intersection of the context set with the proposition denoted by that sentence in the context of utterance. This is not an ad hoc stipulation; rather, it is a special case of the general formalization of update potential given in (15).

To see this, recall the update condition from (15). It specifies that the update potential of an assertion of s is an operation that guarantees that any s-compatible context to which it is applied will become a member of $MI(s)$, but doesn't guarantee that the resulting contexts will entail anything not entailed by s. Recall as well that we've assumed that updates must take the form of monotonic information increase, i.e. the only maneuver available to us is to remove worlds from the context set. Given that we're dealing with a sentence s that denotes the same proposition p regardless of the epistemic conversational background, whose meta-intensionalized denotation is simply the set of all epistemic states that are a subset of p, this means that we need an operation that will render the context set a subset of p (s-commensurativity), without removing any worlds that aren't necessary to achieve that (s-conservativity)—an operation that will remove all and only those worlds from the context that prevent it from being a subset of p. Intersection with p is precisely such an operation. Because this reasoning follows from the characteristic downward-closed structure of (23),

[12] A brief comment on the fact that the proposed formalization of assertion can be productively discussed in terms of the generation of a set of sets of worlds.

The use of sets of sets of worlds in the modeling of context update is a familiar feature of a variety of proposals—see for instance Beaver (2001), Willer (2013). However, I have not proposed that the context itself be lifted to a set of sets of worlds, as Beaver and Willer do, nor have I proposed that sentences themselves denote sets of sets of worlds, as in Inquisitive Semantics (Ciardelli et al. 2013, 2019). Rather, in my proposal these sets of sets of worlds enter the picture only as part of the formalization of the licensing and update potential of assertions, in terms of cashing out what it means to say that the speaker is presenting herself as though she knows the asserted sentence to be true, and in terms of how that property of states is used to propose that the context be updated. The model of contexts and how they are updated that I am presenting here is still quite close to the familiar Stalnakerian one: there is a context set that is simply a set of worlds, and it is updated by having worlds removed from it.

There is an especially strong parallel between the meta-intensionalized denotations used here, which often result in sets that are downward closed, and the sentential denotations of Inquisitive Semantics, which treats all sentential denotations as downward closed sets of sets of worlds. However, closure under subset is not an inherent property of the sets of sets of worlds generated by the meta-intensionalization function, and in §2.2 we'll see that that operation generates sets with quite different properties when applied to *might*-claims.

it follows that the NSF derives classical Stalnakerian intersective updates as the update potential of all sentences s that denote the same proposition p regardless of the epistemic conversational background, and whose meta-intensionalized denotations therefore comprise the set of all epistemic states whose modal base component is a subset of p—i.e., all sentences whose denotation makes no reference to the contextually-determined epistemic conversational background. In §2.2, we'll see how a sentence whose denotation *is* sensitive to the epistemic conversational background can have a meta-intensionalized denotation that does not have the property of being closed under subset, and therefore does not license classical Stalnakerian intersective update.

2.1.3 Rejection

Finally, consider what a speaker does when they reject an assertion of *John is dead*: they represent themself as though their epistemic state is not s-compatible. Recalling the definition of s-compatibility from (18), this means that in rejecting an assertion of s, the rejector represents themself as though there is no refinement of their epistemic state (other than the absurd state, the empty set) that is a member of $MI(s)$. As in the case at hand $MI(s)$ is simply the set of all epistemic states that are a subset of the proposition p denoted by s, the rejector therefore represents herself as though there are no p-worlds in her epistemic state—i.e., as though she knows p to be false.

In the next section we'll see that if the sentence includes a bare epistemic modal (given the solipsistic contextualist semantics), its meta-intensionalized denotation will not be the set of all epistemic states that are a subset of its propositional denotation in the context of utterance, and so update and rejection will both proceed somewhat differently.

2.2 Implementing the Simple Quantificational Semantics

In this section, I show how the NSF interacts with the simple quantificational semantics for *might* (Kratzer 1977). I'll show that the NSF derives the 'consistency test' semantics for *might* (proposed by Veltman 1996 in the context of Update Semantics; see also Yalcin's 2007 discussion of an 'informational consequence' account of the update potential of *might*-claims) as the update potential of *might*-claims given the simple quantificational semantics. This will suffice to show that the NSF can account for the disagreement behavior of *might*-claims given the solipsistic contextualist semantics for *might*. However, the consistency test semantics has been criticized for predicting all assertions of *might*-claims to be informationally trivial (see e.g. Willer 2013, Rudin 2018). In §2.3, I show how the NSF deals with the ordering semantics of Kratzer (1981, 1991), which assigns a stronger semantics to *might*, deriving a novel, informative update for *might*-claims.

The simple quantificational semantics for *might* which we will be concerned with in this section is given in (2), repeated here as 25:

(25) $[\![\text{might}]\!]^i = \lambda p.\lambda w.[\exists w' : w' \in f_i(w)]p(w') = 1$
(adapted from Kratzer 1977)

On this semantics, a *might*-claim is true in a world w relative to a contextually-determined epistemic conversational background i iff there is at least one world w' epistemically accessible from w via the epistemic accessibility function f_i derived from i such that the prejacent proposition is true in w'.

I call this the simple quantificational semantics because it treats *might*-claims as simply existentially quantifying over a set of epistemically accessible worlds.[13]

In order to fully understand what's going on in (25), we'll need to make an assumption about how an epistemic state i determines a corresponding epistemic accessibility function f_i. I'll assume, following Gillies (2004), von Fintel & Gillies (2010), that epistemic states are CLOSED:

(26) For any $i \in D_i$,
$[\forall w : w \in i] f_i(w) = i$

That is to say, any epistemic state i determines an epistemic accessibility function f_i that maps every world in i to i; informally, the set of worlds epistemically accessible with respect to an epistemic state does not vary between worlds in that epistemic state.

Given the solipsistic contextualist semantics on which the conversational background for bare epistemic modals is always determined by the context of utterance to be the epistemic state of the speaker, (25) gives us a semantics where any *might*-claim will denote the set of all worlds in which its prejacent is compatible with the epistemic state of the speaker, as determined by the context of utterance.

However, if we meta-intensionalize a *might*-claim on the solipsistic contextualist, we do not derive the set of all subsets of that proposition, as we for a sentence whose denotation is insensitive to the epistemic conversational background in §2.1. Rather, we get the following:

(27) a. $[\![\text{John might be dead}]\!]^i = \lambda w.[\exists w' : w' \in f_i(w)] dead'(j)(w') = 1$
b. $MI(\text{John might be dead})$
$= \{i : [\forall w : w \in i][\![\text{John might be dead}]\!]^i(w) = 1$
$= \{i : [\forall w : w \in i][\exists w' : w' \in f_i(w)] dead'(j)(w') = 1$

The same holds for any *might*-claim whose prejacent's denotation does not vary with the epistemic conversational background; I'll say that such a sentence is of the form *might-p*:

(28) a. $[\![\text{might-}p]\!]^i = \lambda w.[\exists w' : w' \in f_i(w)] p(w') = 1$
b. $MI(\text{might-}p)$
$= \{i : [\forall w : w \in i][\![\text{might-}p]\!]^i(w) = 1$
$= \{i : [\forall w : w \in i][\exists w' : w' \in f_i(w)] p(w') = 1$

Recall that meta-intensionalization in a way of representing the set of epistemic states the speaker could hold if they indeed know the sentence they've asserted to be true. By virtue of solipsistic contextualism, whatever epistemic state the speaker is in is the value of the epistemic conversational background parameter. So given any arbitrary state i, if i is a state in which the speaker knows the *might*-claim she's asserted to be true, that means that the *might*-claim is true given i as the epistemic conversational background. By virtue of the simple quantificational semantics, the set of all such states is simple the set of all states containing at least one p-world:

[13] Note that the orthographic presentation of the account here is quite similar to Yalcin's (2007) 'domain semantics' for epistemic modals. Despite the notational similarity, the account I give here is indeed Kratzer's: I intend the parameter i to be determined by the context parameter, with other aspects of contextual parameterization suppressed for readability; Yalcin is quite clear in rejecting a contextualist interpretation of how his domain parameter is set.

(29) a. For any i such that $i \cap p \neq \varnothing$:
$[\forall w : w \in i][\exists w' : w' \in f_i(w)]p(w') = 1$

Proof:

$i \cap p \neq \varnothing$	PREMISE
$w \in i \rightarrow f_i(w) = i$	(26)
$[\forall w : w \in i] f_i(w) \cap p \neq \varnothing$	$\therefore$

b. For any i such that $i \cap p = \varnothing$:
$[\forall w : w \in i] \neg [\exists w' : w' \in f_i(w)]p(w') = 1$

Proof:

$i \cap p = \varnothing$	PREMISE
$w \in i \rightarrow f_i(w) = i$	(26)
$[\forall w : w \in i] f_i(w) \cap p = \varnothing$	$\therefore$

c. For any s of the form *might-p*, $MI(s) = \{\, i : i \cap p \neq \varnothing \,\}$

Proof:

$i \in MI(\text{might-}p) \leftrightarrow [\forall w : w \in i][\exists w' : w' \in f_i(w)]p(w') = 1$	(28)
$[\forall w : w \in i][\exists w' : w' \in f_i(w)]p(w') = 1 \leftrightarrow i \cap p \neq \varnothing$	(29a), (29b)
$MI(\text{might-}p) = \{\, i : i \cap p \neq \varnothing \,\}$	$\therefore$

Assuming the simple quantificational semantics for *might*, the meta-intensionalized denotation for *might-p* is the set of all epistemic states that contain at least one p-world. Imagine again a set of worlds W containing w_1, w_2, and w_3, where the proposition denoted by *John is dead* maps w_1 and w_2 to 1, and w_3 to 0. Relative to this set of worlds, (27b) looks like this:

(30) $$\left\{ \begin{array}{ccc} & \{w_1, w_2, w_3\} & \\ \{w_1, w_2\} & \{w_1, w_3\} \quad \{w_2, w_3\} & \\ & \{w_1\}\,\{w_2\} & \end{array} \right\}$$

Note that this set has a different structure from the previous meta-intensionalized denotation that we saw in (23). We saw in the previous section that the meta-intensionalized denotation of a sentence whose denotation is not affected by the choice of epistemic conversational background will simply be the set of all subsets of that propositional denotation. What we've seen here is that when a sentence has a denotation that varies depending on the choice of epistemic conversational background, meta-intensionalizing it does not necessarily result in the set of all subsets of that proposition denoted by that sentence in the context of utterance, not by any special stipulation but entirely from the interaction between the sentence's interpretational dependence on the epistemic conversational background, and the way that MI manipulates that epistemic conversational background. In other words, this difference is due to the interaction between the speaker-oriented epistemicity of bare epistemic modals and the speaker oriented epistemicity of the NSF model of assertion.

2.2.1 Rejection

The formulation of the licensing of rejection given in (20) specifies that in rejecting an assertion of s, the rejector presents herself as though her epistemic state is not s-compatible. The meta-intensionalized denotation of *might-p*, given in (28b), is the set of all epistemic states including at least one p-world. Given the definition of compatibility in (18), a speaker's epistemic state is not *might-p*-compatible relative to the denotation in (30) iff it has no p-worlds in it. This captures precisely the intuitive meaning of disagreement over *might*-claims: when a speaker rejects an assertion of *might-p*, she presents herself as though p is incompatible with her epistemic state. The NSF predicts that rejection is licensed by the rejector's lack of epistemically accessible p-worlds, and that in rejecting *might-p* the rejector indicates that she knows p to be false, even though the semantics of *might* is sensitive only to the assertor's epistemic state.

2.2.2 Update potential

In the section §2.1, I showed that the NSF derives intersective update as the update potential for all sentences whose denotation is not affected by the epistemic conversational background, and whose meta-intensionalized denotation therefore comprises the set of all subsets of the proposition it denotes in the context of utterance. This is because intersection is the most conservative operation that guarantees that one set will become a subset of another. That logic does not go through for a meta-intensionalized denotation that is not closed under subset, like (30).

So what will the update potential for a sentence of the form *might-p* be, if not intersection? To see the answer to this, consider first which contexts are compatible (in the sense of (18)) with the denotation in (30). All contexts either have a p-world in them, or have no p-worlds in them. Contexts with no p-worlds in them are not compatible with this denotation: no refinement of them is a member of (30). Contexts with p-worlds in them are already members of (30), as it is the set of all sets of worlds containing at least one p-world. In other words, it is a logical necessity that any context is either incompatible with (30), or is already a member of it. For contexts of the latter class, no further modification is necessary to make it a member of (30), so the most conservative update is to leave it alone. For contexts in the former class, no monotonic modification can make it a member of (30), so the update fails.[14].

(31) Update potential for *might-p* (simple version):
For any c,
$$c[\text{might-}p] = \begin{cases} c \text{ if } c \cap p \neq \varnothing \\ \varnothing \text{ otherwise} \end{cases}$$

Again, this is not an ad hoc stipulation, but a special case of the general formalization of update potential given in (15), as applied to sentences whose meta-intensionalized denotation has the characteristic structure of (30). Recall the condition on anomalous update in (19) for an explanation of the 'otherwise' case.

[14]Peter Klecha (p.c.) notes that this result follows only given the assumption that all updates are monotonic. If information growth were allowed to be non-monotonic, i.e. if updates were allowed to add worlds to the context, the update potential of *might-p* relative to a context with no p-worlds in it could be to add p-worlds to the context. A fuller discussion of the possibilities made available by non-monotonic update systems, and the problems posed by them, is outside the scope of this paper.

To summarize: the update potential of a sentence of the form *might*-p, given the simple quantificational semantics for *might*, is exactly that proposed by Veltman (1996): it leaves the context untouched if it has a p-world in it, and it is anomalous otherwise. It's worth noting that this result, which Veltman (1996) achieved by analyzing *might* is a syncategorematically-defined update operator, is derived from the interaction between the fully general proposal for the update potential of assertions and the simple quantificational semantics of *might* given by Kratzer (1977)—the special behavior of *might*-claims comes without needing to treat them as denoting anything other than a standard proposition.

One immediate problem for the Veltmanian consistency test semantics for *might* is that it renders *might*-claims intrinsically uninformative (Willer 2013). The ordering semantics for modality (Kratzer 1981, 1991) was designed in part to address informativity issues related to modal claims, strengthening possibility modals and weakening necessity modals. In the next section, I show how the NSFcan be extended to implement the ordering semantics. I do this by extending the notion of contexts and epistemic states from a set of worlds to a tuple of a set of worlds and an ordering source; I show that this extension has no effect on the update potential of sentences whose denotation doesn't vary with the choice of ordering source, which still update contexts by intersecting the context set with the proposition they denote in the context of utterance, and then I show how it derives a novel, informative update effect for *might*-claims, without sacrificing the feature that speakers are licensed to disagree with *might*-claims when they have no p-worlds in their epistemic modal base.

Before moving on, note that the account put forward in this section assigns an update potential to assertions of bare epistemic modal claims on which there is no proposition that they propose to add to the stockpile of public mutual knowledge. However, it is nonetheless a solipsistic contextualist account of the semantics of bare epistemic modals: relative to any context, a *might*-claim denotes the set of worlds in which the prejacent is compatible with the speaker's knowledge. It's just not the case that the NSF is a theory on which any assertion serves to add the propositional content of the uttered sentence directly to the stockpile of public mutual knowledge. That such a theory of assertion is a possibility worth taking seriously is noted in several recent papers. For instance, Ninan (2012) has this to say:

> Even [once] we've settled all of the relevant details of our compositional semantic theory, we still face a choice about what to take the information communicated by a sentence at a context to be. §5, p.412

And Rabern (2012) has this to say:

> …we utter words with certain meanings (and certain syntax) in order to say the things we say. This platitude, however, does not call for the *identification* of the two notions—all it calls for is that the assertoric content of a sentence in a context should be systematically *determined* by its compositional value. §2, p.88

The NSF can be seen as one way of formally cashing out what is called for by Rabern's platitude: a system in which assertive updates are systematically determined by the propositional content that is the compositional semantic value of the uttered sentence, but not equivalent to simply adding that proposition to the stockpile of mutual knowledge.

Note as well that the divergence in update effect between sentences whose denotations are and are not affected by the choice of epistemic conversational background is not traceable to any construction-

specific stipulations; both kinds of update potential fall out as special cases of the same general update potential in (15).

2.3 The Ordering Semantics

In this section, I extend the NSF to include ordering sources in its representation of contexts and epistemic states. The ordering semantics for modality (Kratzer 1981, 1991) is predicated on just such an extension—it takes the interpretation of epistemic modals to be sensitive to not just a set of worlds, but a set of worlds plus a set of propositions from which an ordering over them is induced. I extend the concept of an epistemic state to incorporate both an epistemic modal base and an ordering source:

(32) Epistemic states (ordering version):
$D_i = \{ \langle b_i, o_i \rangle : b_i \in D_{st} \wedge o_i \in \wp D_{st} \}$

An epistemic state is a pair of an epistemic modal base b_i (a set of worlds) and an epistemic ordering source o_i (a set of propositions). We maintain the assumption that $D_c = D_i$, i.e. that a conversational context is an epistemic state with a particular interpretation (representing the public mutual knowledge of the interlocutors), meaning that now a conversational context is also a pair of a modal base and an ordering source.

An ordering source is used to induce an ordering over worlds in the modal base. Simplifying things via the limit assumption (q.v. Portner 2009 p.66), an ordering source returns the subset of the modal base containing only worlds which are maximal with respect to that ordering:

(33) $\textsc{Best}_{b_i,o_i} = \{ w : w \in b_i \wedge \neg\exists w' \in b_i \text{ s.t. } w' >_{o_i} w \}$
Where $\forall w, w'$, $w \geq_{o_i} w'$ iff $\forall p \in o_i$, $w' \in p \rightarrow w \in p$ ($w >_{o_w} w'$ iff $w \geq_{o_i} w'$ and $w' \not\geq_{o_i} w$)

A world w is a Bestworld with respect to a modal base and an ordering source if no other world in that modal base is a member of a proper superset of the ordering propositions that w is a member of. We can think of an epistemic ordering source as a set of propositions that are most normal or most likely, giving the set of salient 'live' possibilities relative to that epistemic state (q.v. Willer 2013 for a discussion of the role of 'live' possibilities in the interpretation of epistemic modals). So the Bestworlds with respect to some epistemic state i are the epistemically possible worlds compatible with the largest number of 'live' possibilities in i.

I assume that, in addition to an accessibility function f_i, an epistemic state determines an ordering source function g_i, a function from worlds to ordering sources giving the epistemic ordering source in each world. I assume that ordering sources are closed in the same way as accessibility functions:

(34) $[\forall w : w \in b_i] g_i(w) = o_i$

This gives us everything we need to understand the ordering semantics. The ordering semantics for *might* takes it to express what Kratzer calls human possibility, presented here:

(35) Human Possibility:
$[\![\textit{might}]\!]^i = \lambda p.\lambda w.[\exists w' : w' \in \textsc{best}_{f_i(w),g_i(w)}]p(w') = 1$

On the ordering semantics, the contextually-given epistemic conversational background determines a function from worlds to a modal base and an ordering source; *might* can then existentially quantify over a special subset of that epistemic modal base: the set of all worlds in that modal base that are most compatible with the 'live' possibilities given by the ordering source. This allows for more informative *might*-claims—rather than entailing that the prejacent is merely not ruled out by the contextually-given modal base, they entail that the prejacent is represented among the most normal/likely options highlighted by the relevant epistemic state.[15] The solipsistic contextualist interpretation of the ordering semantics for epistemic modals works the same as on the simple quantificational semantics: the epistemic conversational background i relative to which bare epistemic modals are interpreted is always fixed by the context of utterance as the speaker's epistemic state.

The addition of ordering sources to the conception of a context means that there are now two possible ways to monotonically increase the information expressed by a context: to remove worlds from the context set, or to alter the ordering source.[16] I revise the notion of refinement below, to capture this:

(36) Refinement (ordering version):
A context c' is a refinement of a context c ($c' \leq_r c$) iff $b_{c'} \subseteq b_c$

Because contexts and epistemic states are now tuples, of which a modal base is only one component, we will need to make small adjustments to three previous definitions:

(37) Compatibility (ordering version):
For any context c, sentence s, c is s-compatible if $[\exists c' : c' \leq_r c]c' \in MI(s) \wedge b_{c'} \neq \varnothing$

(38) Anomalous update (ordering version):
For any context c such that c is not s-compatible:
$c[s] = \langle \varnothing, o_i \rangle$

[15] Note that probabilistic semantics for modals also allow for stronger possibility modals. Probabilistic accounts generally treat epistemic modals as entailing that their prejacents are associated with epistemic probability above some threshold. This analysis has been applied to a variety of phenomena: for example to statements of comparative probability by Lassiter (2015), to strengthening inferences associated with statements of epistemic possibility by Rudin (2016), and to free choice inferences by Santorio & Romoli (2017). (Cf. Holliday & Icard 2013, who argue that it is more difficult to pull apart the empirical predictions made by probabilistic and Kratzerian accounts than is often claimed.) I show how the NSF could be extended to implement a probabilistic semantics for epistemic modals below.

[16] Note that the definition in (36) allows an ordering source to be altered in any way as a context is refined. Readers might wonder why a monotonicity condition on context update shouldn't restrict ways in which the ordering source can be altered, say, by requiring that propositions be added to it, not removed from it. But observe that live possibilities can both appear and disappear in the course of monotonic information increase. For instance, in a context in which Paul is an indecisive partygoer, it could be a live possibility that Paul was at the party, and a live possibility that he wasn't. Later, we can gain the information that he indeed was present, rendering the latter possibility no longer a live possibility, and indeed no longer a possibility at all. That live possibilities can become dead possibilities only through the monotonic increase of information shows that a monotonicity condition on context update should not preclude propositions being removed from the ordering source.

(39) Meta-Intensionalization (ordering version):
$MI = \lambda s.\{\, i : [\forall w : w \in b_i][\![s]\!]^i(w) = 1 \,\}$

In these definitions, reference to a context and an epistemic state, respectively, have been changed to explicitly reference the modal base component of that context and epistemic state. I will refer to any context whose modal base component is empty as 'the absurd context' All other definitions from §2.1 persist unchanged.

2.3.1 Non-epistemic propositions

In the ordering version of the NSF, sentences whose denotation does not vary with the choice of epistemic conversational background will behave the same as before: they will intersect the context set with the proposition they denote in the context of utterance (and leave the ordering source unchanged). To see this, let's revisit the example of *John is dead*. Meta-intensionalization will proceed just as in (22):

(40) MI(John is dead)
$= \{\, i : [\forall w \in b_i][\![\text{John is dead}]\!]^i(w) = 1 \,\}$
$= \{\, i : [\forall w \in b_i] dead(j)(w) = 1 \,\}$

Because we are now treating epistemic states i as tuples $\langle b_i, o_i \rangle$, rather than as simple sets of worlds, this meta-intensionalized denotation is a set of such tuples: all such tuples in which the proposition denoted by *John is dead* is true at every world in the modal base. Because the sentence's denotation is always the same proposition p regardless of the epistemic conversational background, each i in (40) has a modal base component that is a subset of p, for the reasons discussed in §2.1. Note that the ordering source has no effect on the truth of this proposition, and so the speaker is not placing any restrictions on what the ordering source component of their epistemic state is like by virtue of their assertion—the set of all epistemic states they could hold contains all states whose modal base is a subset of p, irrespective of what the ordering source is:

(41) (40) $= \{i : b_i \subseteq \{w : dead'(j)(w) = 1\} \,\}$

The update potential associated with an assertion of *John is dead*, then, will be the same Stalnakerian update potential as before we added ordering sources into the mix: intersection of the context set with p.

(42) Update potential for non-epistemic sentences (ordering version):
For any c, and for any s such that for all i, i', $[\![s]\!]^i = [\![s]\!]^{i'}$,
$c[s] = \langle b_c \cap [\![s]\!]^c, o_c \rangle$

For any such sentence, any context that has no p-worlds in its context set will not be s-compatible, as there is no way to either subtract worlds from the context set or alter the ordering source that will result in the context set becoming a (non-empty) subset of p. Assuming that we're dealing with an s-compatible context, with at least one p-world in its context set, altering the context's ordering source could not possibly change whether it is a member of the set given in (41), as altering the

ordering source would have no effect on whether the context set is a subset of p. That leaves us with the strategy of removing worlds from the context set. Again, the most conservative way to remove worlds from the context set to guarantee that it will be a subset of p is to intersect it with p.

Rejection of assertions of such sentences also works the same as before: the rejector is presenting herself as though her epistemic state is not s-compatible; as discussed above, an epistemic state is not compatible with (41) iff its modal base has no p-worlds in it. Therefore, the disagreer is presenting herself as though she knows p to be false.

2.3.2 *Might*-claims

We've seen that assertions of sentences whose denotation is not affected by the epistemic conversational background work the same in the ordering version of the NSF as they did in the version without ordering sources. What about assertions of *might*-claims?

First, the meta-intensionalized denotation of a *might*-claim, given the ordering semantics in (35):

(43) a. $[\![\text{John might be dead}]\!]^i = \lambda w.[\exists w' : w' \in \textsc{best}_{f_i(w),g_i(w)}]dead'(j)(w') = 1$

b. $MI(\text{John might be dead})$
$= \{\, i : [\forall w : w \in b_i][\![\text{John might be dead}]\!]^i(w) = 1 \,\}$
$= \{\, i : [\forall w : w \in b_i][\exists w' : w' \in \textsc{best}_{f_i(w),g_i(w)}]dead'(j)(w) = 1 \,\}$

The same hold for any *might*-claim whose prejacent's denotation does not vary with the epistemic conversational background; for any such sentence of the form *might*-p:

(44) a. $[\![\text{might-}p]\!]^i = \lambda w.[\exists w' : w' \in \textsc{best}_{f_i(w),g_i(w)}]p(w') = 1$

b. $MI(\text{might-}p)$
$= \{\, i : [\forall w : w \in b_i][\![\text{might-}p]\!]^i(w) = 1 \,\}$
$= \{\, i : [\forall w : w \in b_i][\exists w' : w' \in \textsc{best}_{f_i(w),g_i(w)}]p(w) = 1 \,\}$

Given the assumption that epistemic states are closed (26, 34), for any $w \in b_i$, $f_i(w) = b_i$, and $g_i(w) = o_i$. Therefore, the set in (44b) is equivalent to the following:

(45) $\{\, i : [\forall w : w \in b_i][\exists w' : w' \in \textsc{best}_{b_i,o_i}]dead'(j)(w) = 1 \,\}$

That is to say, meta-intensionalizing *might*-p, given the ordering semantics, returns the set of all epistemic states whose ordering source picks out at least one p-world from its modal base as a Bestworld.

2.3.3 Update potential

How can we determine the update potential of the denotation in (45)? Let's first divide the set of all possible contexts into those whose context bassete contains at least one p-world, and those whose context set contains no p-worlds. All contexts of the latter kind are incompatible with (45)—there can be no p-worlds in $\textsc{best}_{b_i,o_i}$ if there are no p-worlds in b_i, and no refinement of such a context can

reintroduce p-worlds into b_i, by (36). Because all such contexts aren't *might*-p-compatible, we can safely restrict our attention to the set of all contexts whose context set contains at least one p-world. Contexts in this set may or may not already be members of (45), depending on whether their ordering source picks out a p-world as maximal. The question facing us is: what operation will guarantee that all such contexts will become a member of (45) (s-commensurativity) without guaranteeing that the resulting context will entail anything stronger than (45) (s-conservativity)?

Adding p to the context's ordering source has exactly those properties. It is commensurate with (45) by definition (16), meaning that adding p to the ordering source of any context with a p-world in its context set will result in that context being a member of (45). To see this, consider the definition of maximality relative to an ordering source given in (33), repeated here:

(46) $\textsc{Best}_{b_i,o_i} = \{\, w : w \in b_i \wedge \neg\exists w' \in b_i \text{ s.t. } w' >_{o_i} w \,\}$
Where $\forall w, w'$, $w \geq_{o_i} w'$ iff $\forall p \in o_i$, $w' \in p \rightarrow w \in p$ ($w >_{o_w} w'$ iff $w \geq_{o_i} w'$ and $w' \not\geq_{o_i} w$)

A world w in c's context set is maximal with respect to that context's ordering source (i.e., is a member of $\textsc{best}_{b_i,o_i}$) iff there are no worlds in the context set that are members of a proper superset of the propositions in the ordering source that w is a member of. This has the logical consequence that for any proposition p in o_i, as long as there is at least one p-world in b_i, there will be a p-world in $\textsc{best}_c$. The only way a p-world w could fail to be maximal with respect to an ordering source that includes p is if there is some other world that is a member of a proper superset of the ordering propositions that w is a member of, guaranteeing that such a world is also a p-world—i.e., if p is in the ordering source, the only thing that can be ranked strictly higher than a p-world is another p-world.

This demonstrates that adding p to the ordering source of a context whose modal base contains at least one p-world guarantees that that context will become a member of (45), the set of all contexts c such that $\textsc{best}_{b_i,o_i}$ contains a p-world.

It should also be clear that adding p to the ordering source is conservative with respect to (45) given definition (17), satisfying the other half of the update condition. Adding p to the ordering source does not guarantee that any non-p world will become maximal with respect to the ordering for any arbitrary *might*-p-compatible context, and so the update does not guarantee that contexts updated with it will entail anything other than what is already entailed by *might*-p.

(47) Update Potential for *might*-p (ordering version):
For any c
$$c[\text{might-}p] = \begin{cases} \langle b_c, o_c + p \rangle \text{ if } b_c \cap p \neq \varnothing \\ \langle \varnothing, o_c \rangle \text{ otherwise} \end{cases}$$

This update is informative: relative to context whose ordering source did not already contain p, the resulting context is a proper refinement of the context prior to the update. On this account, an assertion of a *might*-claim is a proposal that the context be made to pick out the prejacent as a live possibility (q.v. the discussion of 'live possibility' in Willer 2013).

Note that Portner (2004, 2007) develops an account of imperatives on which they serve to update a priority ordering source. Portner (2007) observes that there does not appear to be a parallel in the epistemic domain:

> ……we have a canonical mechanism for building up the [epistemic] modal base (declaratives) and a canonical mechanism for building up the ordering source for deontic modals (imperatives); it would provide a nice symmetry if we could find a grammatical mechanism for helping to determine the ordering source for epistemic modals as well. p.353–354

Portner goes on to speculate that evidentials might supply such a grammatical mechanism. On the ordering implementation of the NSF, *might*-claims serve a function directly parallel to Portner's proposed update for imperatives: they serve to add a proposition to an epistemic ordering source. On the proposal developed here, only *some* assertions of declarative sentences serve to restrict an epistemic modal base; because of the interaction between the speaker-oriented epistemicity of *might* and the speaker-oriented epistemicity of the NSF model of assertion, *might*-claims serve a different function.

2.3.4 Rejection

Rejection of a *might*-claim given the ordering semantics is predicted to work exactly as it did for the simple quantificational semantics. As we saw in the discussion above, the only contexts that are incompatible with the meta-intensionalized denotation of *might*-p are those whose modal base has no p-worlds in it. As rejection is licensed by the rejector's epistemic state being incompatible with that meta-intensionalized denotation, a rejector is presenting themselves as though they have no p-worlds epistemically accessible to them, just as in the implementation of the simple quantificational semantics for *might*.

3 Extensions

The core results of this paper have already been encountered: that changing the camera angle on the classical Stalnakerian account of assertion is enough to dissolve the problem that disagreement over *might*-claims poses for the solipsistic contextualist account of epistemic modals. I've shown that the NSF (i.) derives Veltmanian update from the solipsistic contextualist interpretation of the simple quantificational semantics for *might*, and (ii.) derives the novel update of adding a proposition to the ordering source from the solipsistic contextualist interpretation of the ordering semantics for *might*. in this section, I dot i's and cross t's, showing four extensions of the NSF that alert readers might be wondering about.

In the above, I've focused only on *might*-claims, as they have been the empirical focus of literature on the disagreement problem; in §3.1, I discuss *might*'s dual, *must*, showing what the NSF predicts the update potential of *must*-claims to be on both the simple quantificational and the ordering semantics.

In the above, I've focused only on bare epistemic modals; in §3.2 and §3.3, I give brief sketches of how the system here can deal with embedded epistemic modals, and with overt relativizers (e.g. *given what John believes*).

Finally, the above has provided implementations only of accounts that treat modals as quantifiers over worlds. In §3.4, I give a brief sketch of how to implement a probabilistic semantics for epistemic

modals in the NSF, treating epistemic states as finitely additive probability measures and update as Jeffrey conditioning.

3.1 *Must*-claims

The literature on the disagreement problem has focused predominantly on *might*; hence my focus on *might* in §2. However, any account of epistemic modals must be able to account not just for *might*, but also for its dual, *must*. In this section I show the update potentials the NSF derives for *must* on both the simple quantificational semantics (§3.1.1) and on the ordering semantics (§3.1.2).

3.1.1 The simple quantificational semantics

On the simple quantificational semantics, *must*, being the dual of *might*, quantifies universally over the same set of worlds over which *might* quantifies existentially:

(48) $[\![\text{must}]\!]^i = \lambda p.\lambda w.[\forall w' : w' \in f_i(w)]p(w') = 1$
(adapted from Kratzer 1977)

This being the simple quantificational semantics, I'll derive the update potential for this version of the semantics of a *must*-claim using the simple version of the NSF from §2.1, as ordering sources will serve no purpose here.

For any sentence of the form *must-p* (i.e., a *must*-claim whose prejacent denotes p regardless of the epistemic conversational background):

(49) $MI(\text{must-}p)$
$= \{\, i : [\forall w : w \in i][\![\text{must-}p]\!]^i(w) = 1 \,\}$
$= \{\, i : [\forall w : w \in i][\forall w' : w' \in f_i(w)]p(w') = 1 \,\}$

Given the assumption that epistemic states are closed (26), this is simple the set of all subsets of p:

(50) (49) $= \{\, i : i \subseteq p \,\}$

Proof:
$[\forall w : w \in i][\forall w' : w' \in f_i(w)]p(w') = 1 \leftrightarrow [\forall w : w \in i][\forall w' : w' \in i]p(w') = 1$ (26)
$[\forall w : w \in i][\forall w' : w' \in i]p(w') = 1 \leftrightarrow [\forall w : w \in i]p(w) = 1$ vacuous quantification
$\{\, i : [\forall w : w \in i][\forall w' : w' \in f_i(w)]p(w') = 1 \,\} = \{\, i : i \subseteq p \,\}$ $\therefore$

That is to say: the meta-intensionalized denotation of *must-p*, given the simple quantificational semantics, is *identical* to the meta-intensionalized denotation of its prejacent. Therefore, *must-p* will have the same update potential as a non-epistemic sentence denoting p, for the reasons discussed in §2.1.

(51) Update Potential for *must-p* (simple version):
For any c,
$c[\text{must-}p] = c \cap p$

This is *prima facie* a very undesirable result: surely, asserting that p *must* be true accomplishes something different *in some way or another* than simply asserting that p *is* true. For context, however, note that it is currently up for debate whether *must-*p indeed entails p, or whether it has a weaker meaning. In defense of the former view, see von Fintel & Gillies (2010, 2021); in defense of the latter view, see Lassiter (2014, 2016) and Goodhue (2017), building on part on arguments made by Karttunen (1972) and Kratzer (1991). For those convinced by the latter view, that the strong semantics for *must* in (48) derives the undesirable result in (51) would just provide further support for a weaker semantics for *must*. Those convinced of the former view, however, have proposed that the intuitive weakness of *must*-claims is due not to their logical strength, but to their carrying an evidential meaning supplementing the classic semantics in (48).

To be concrete, von Fintel & Gillies (2010) propose that *must* presupposes that its prejacent is not entailed by the 'kernel' representing what is known by direct observation (or whatever may count as direct evidence in the context). If this is the case, then the *prima facie* undesirability of treating the update potential of *must-*p as the same as the update potential of its prejacent would be defused: *must-*p would be differentiated from its prejacent by way of its evidential presupposition. If one were so inclined, one could develop a model of the NSF in which the notion of an epistemic state were extended to include a kernel K, and derive an update potential for *must-*p on which it may manipulate K_c. I leave such an extension as an exercise to the reader.

In the following section, I show what the ordering version of the NSF derives for the ordering semantics of *must*, which delivers a weaker meaning for *must*-claims.

3.1.2 The ordering semantics

On the ordering semantics, *must*, as the dual of *might*, quantifies universally over the same set of Bestworlds that *might* quantifies over existentially:

(52) Human Necesity:
$[\![\text{must}]\!]^i = \lambda p.\lambda w.[\forall w' : w' \in \textsc{best}_{f_i(w),g_i(w)}]p(w') = 1$

That is to say: *must-*p is true, on the ordering semantics, iff every world that is most compatible with the live possibilities given the epistemic conversational background is a p-world.

Meta-intensionalizing a sentence of the form *must-*p gives us the following:

(53) $MI(\text{must-}p)$
$= \{\, i : [\forall w : w \in b_i][\![\text{must-}p]\!]^i(w) = 1 \,\}$
$= \{\, i : [\forall w : w \in b_i][\forall w' : w' \in \textsc{best}_{f_i(w),g_i(w)}]p(w') = 1 \,\}$

Given the assumption that epistemic states are closed (34), (53) is equivalent to the following:

(54) $\{\, i : [\forall w : w \in b_i][\forall w' : w' \in \textsc{best}_{b_i,o_i}]p(w') = 1 \,\}$

Because $\textsc{best}_{b_i,o_i}$ is always a subset of b_i (33), we can simply this like so:

(55) (53) $= \{\, i : [\forall w : w \in \textsc{best}_{b_i,o_i}]p(w) = 1 \,\}$

That is to say: the meta-intensionalized denotation of *must-p*, given the ordering semantics, is the set of all epistemic states all of whose BESTworlds are p-worlds.

The general formalization of NSF update in (15) specifies that the update potential of any assertion in c of a sentence s is that an s-commensurate and s-conservative function be applied to c. In the case of the meta-intensionalized denotation in (53), we need a function that will render any *must-p*-compatible context a member of that set (*must-p*-commensurativity) without guaranteeing that every context thus modified will entail anything not already entailed by *must-p* (*must-p*-conservativity). The function with these properties is that function which adds p to a context's ordering source, and removes from that ordering source all propositions disjoint with p.

As a preliminary, note that any context whose context set contains at least one p-world is *must-p*-compatible. By the definition of refinement in (36) and the definition of BEST in (33), any such context has a refinement in which all its BESTworlds are p-worlds: that refinement such that the ordering source is the singleton set $\{p\}$. So we restrict our attention to such contexts in the following.

Consider the first portion of this update function: the addition of p to the ordering source. Any update function that does not add p to the ordering source of the context being updated cannot be *must-p*-commensurate. To see this, consider the following scenario:

(56) $c_1 = \langle b_{c_1}, o_{c_1} \rangle$
$p = \{w_1, w_2\}$
$b_{c_1} = \{w_1, w_2, w_3\}$
$o_{c_1} = \varnothing$
$\text{BEST}_{b_{c_1}, o_{c_1}} = \{w_1, w_2, w_3\}$

In this scenario, c_1 is *must-p*-compatible, by virtue of having p-worlds in its context set. Removing w_3 from the context set would result in a context that entails *must-p*, but it would also entail p, which is not entailed by *must-p* (on the ordering semantics), thereby violating *must-p*-conservativity. The only other path available is to add a proposition to the ordering source, and that proposition must be p, as the addition of any other proposition would likewise violate *must-p*-conservativity. Therefore, any *must-p*-commensurate update operation must involve adding p to the ordering source.

But that alone is not enough. Consider the following scenario:

(57) $c_2 = \langle b_{c_2}, o_{c_2} \rangle$
$p = \{w_1, w_2\}$
$q = \{w_3\}$
$b_{c_2} = \{w_1, w_2, w_3\}$
$o_{c_2} = \{q\}$
$\text{BEST}_{b_{c_2}, o_{c_2}} = \{w_3\}$

In this scenario, c_2 is *must-p*-compatible, by virtue of having p-worlds in its context set. But simply adding p to the ordering source will not deliver a new context relative to which all BESTworlds are p-worlds. By virtue of the presence of the p-disjoint proposition q in the ordering source, the non-p-world w_3 will remain in the set of BESTworlds, as there is no world in a proper superset of the ordering propositions it is a member of. The disjoint proposition q must therefore be removed from

the ordering source in order to guarantee that all Bestworlds are p-worlds. This is the case for all and only p-disjoint propositions in the ordering source for any *must*-p-compatible context. Ordering propositions overlapping at least partially with p are unproblematic; by virtue of their overlap with p, only worlds in their intersection with p are possible Bestworlds, as their non-p-worlds will be in a proper subset of the ordering propositions that their p-worlds are members of.

This reasoning delivers the following update potential for *must*-p, given the ordering semantics:

(58) Update Potential for *must*-p (ordering version):
For any c:
$$c[\text{must-}p] = \begin{cases} \langle b_c, (o_c + p) \cap \{i : i \cap p \neq \varnothing\}\rangle \text{ if } b_c \cap p \neq \varnothing \\ \langle \varnothing, o_c\rangle \text{ otherwise} \end{cases}$$

That is to say: the update potential the NSF derives for *must*-p, given the ordering semantics, is that it results in a context such that every live possibility is compatible with p. Put differently: it proposes that p be taken to be a live possibility (if it wasn't already), and that all possibilities disjoint with p no longer be taken to be live possibilities (if they were before).

3.2 Embedded epistemic modals

The interpretation of epistemic modals is quite constrained in both matrix and embedded contexts. Speakers seem to assert sentences with matrix epistemic modals (absent overt relitivizers; see §3.3) on the basis of their own information, and disagree with sentences with matrix epistemic modals on the basis of their own information as well. Embedded epistemic modals appear to always be interpreted relative to the embedding subject, i.e. in the sentence *John believes that it might be raining*, the embedded *might*-claim must be interpreted as a claim about John's doxastic state.[17] In this section, I simply note that the account put forward in this paper is amenable to an account of embedded epistemic modals in the style of Anand & Hacquard (2013). Such an account easily captures the shift from speaker-orientedness to subject-orientedness in embedded epistemic modals, and delivers a denotation for sentences containing embedded epistemic modals that does not vary with the choice of epistemic conversational background relative to which the full sentence is interpreted, deriving an ordinary Stalnakerian update effect for sentences with embedded epistemic modals.

The basic machinery that I take from Anand & Hacquard (2013), building on work by Yalcin (2007), is that attitude verbs shift the epistemic conversational background to the value of the attitudinal state specified by the verb, as held by the subject of the verb:[18]

(59) Where $\textsc{dox}_{x,w}$ is x's doxastic state in w (adapted from Anand & Hacquard 2013 ex. 34a)
a. $[\![\text{believe CP}]\!]^i = \lambda x.\lambda w.[\forall w' : w' \in \textsc{dox}_{x,w}][\![\text{CP}]\!]^{\textsc{dox}_{x,w}}(w') = 1$

[17] Egan et al. (2005) discuss cases in which this generalization appears not to hold. See Stephenson (2007) §4.5 for a rebuttal.

[18] Anand & Hacquard (2013) are interested in explaining why some verbs allow for embedded epistemic modals, and others don't. They propose that the space of attitude verbs that allow epistemic modals are the 'representational' (Bolinger 1968), which include verbs like *think, say,* and *discover*, as opposed to 'non-representational' attitudes, which include verbs like *wish, want,* and *demand*. They account for this distinction by saying that non-representational attitudes shift the epistemic conversational background to the empty set, which, in conjunction with the assumption that epistemic modals require a non-empty domain of quantification, delivers the distribution of epistemic modals in embedded contexts.

b. $[\![\text{John believe CP}]\!]^i = \lambda w.[\forall w' : w' \in \textsc{dox}_{j,w}][\![\text{CP}]\!]^{dox_{j,w}}(w') = 1$

c. $[\![\text{John believe might-}p]\!]^i = \lambda w.[\forall w' : w' \in \textsc{dox}_{j,w}][\exists w'' : w'' \in \textsc{dox}_{j,w}]p(w'') = 1$
$= \lambda w.[\exists w' : w' \in \textsc{dox}_{j,w}]p(w') = 1$

d. $[\![\text{John believe must-}p]\!]^i = \lambda w.[\forall w' : w' \in \textsc{dox}_{j,w}][\forall w'' : w'' \in \textsc{dox}_{j,w}]p(w'') = 1$
$= \lambda w.[\forall w' : w' \in \textsc{dox}_{j,w}]p(w') = 1$

In other words: a sentence of the form 'John believes might-p' denotes the set of all worlds in which John's beliefs are compatible with p; a sentence of the form 'John believes must-p' denotes the set of all worlds in which John's beliefs entail p. The simple quantificational semantics is implemented in the above, but an extension to the ordering semantics is trivial. What is important about either case is that the proposition denoted by such a sentence does not vary with the contextually-determined epistemic conversational background: because the attitude verb shifts the value of that parameter, its starting value has no impact on the outcome of the compositional semantic process. As such, the NSF predicts that assertions of sentences with embedded epistemic modals should not instantiate special updates; they instantiate standard intersective updates, proposing that the context set be intersected with the set of all worlds in which the subject bears the relevant attitude toward the prejacent.

3.3 Overt relativization

I turn now to a brief sketch of how overt relativizers, such as *given what John believes*, could be analyzed. It's tempting to say that these relativizing adverbials are simply context shifters, ensuring that the clause they modify is interpreted with respect to the conversational background they suggest. Such a view would look like the following:

(60) Relativizers as context-shifters (to be rejected):
$[\![\text{Given what John believes, CP}]\!]^i = \lambda w.[\![\text{CP}]\!]^{\textsc{dox}_{j,w}}(w)$

However, this can't be the story, because it predicts that relativizing adverbials should have an affect only on the interpretation of sentences whose denotation varies with the shifted conversational background. But in fact, relativizing adverbials alter the interpretation of sentences that are completely unaffected by the choice of epistemic conversational background:

(61) Given what John believes, Paul is dead (though *we* all know that he faked his death and is vacationing in the Bahamas)

This sentence shows that relativizing adverbials work in a way parallel to clause-embedding verbs: the sentence the adverbial modified here is claimed to be true only in John's belief-worlds at the world of evaluation, not in the world of evaluation itself. I propose that they can be treated in much the same way as Anand & Hacquard's (2013) treatment of attitude verbs:

(62) $[\![\text{Given what John believes, CP}]\!]^i = \lambda w.[\forall w' : w' \in \textsc{dox}_{j,w}][\![\text{CP}]\!]^{\textsc{dox}_{j,w}}(w') = 1$

On this sketch, relativizing adverbials and attitude embedding are two different syntactic/information structural strategies for arriving at the same denotation; as such, the same update potential is derived for overtly relativized epistemic modals and for embedded epistemic modals.

Note that, on the NSF account of the update potential of bare epistemic modals claims, their non-intersective update potentials fall out of interaction between their speaker-oriented epistemicity and the speaker-oriented epistemicity of the NSF account of assertion. By the same stroke, we explain why relativizing or embedding them restores classical intersective update: both are ways of shifting the value of the conversational background away from the speaker-oriented one determined by the context of utterance, thereby eliminating the interaction between the epistemicity of the modals and the epistemicity of assertion.

3.4 A probabilistic implementation

There has been increasing interest in probabilistic accounts of epistemic modals as an alternative to accounts of modals as quantifiers over worlds: see Swanson (2006, 2011, 2015), Yalcin (2010, 2012), Lassiter (2011, 2015, 2016, 2017a, 2017b), Moss (2015, 2018), Rudin (2016, 2018), Santorio & Romoli (2017), Charlow (2020) a.o. I won't rehearse arguments given in favor of probabilistic accounts here; in this section, I give a brief sketch of how the NSF could be extended to implement a probabilistic semantics for modals.

The above are a hetereogenous set of accounts; there are as many styles of probabilistic accounts of epistemic modals as there are variants on probabilistic tools for representing states of uncertainty (see Halpern 2003 for an overview). In the sketch I present here, I will assume what I take to be the simplest reasonable probabilistic implementation of an epistemic state: a finitely additive probability measure.

(63) $\mu : \wp(W) \to [0,1]$ is a member of D_μ
   a. Realism: $\mu(W) = 1$
   b. Finite additivity: $[\forall p, q \subseteq W : p \cap q = \varnothing]\mu(p) + \mu(q) = \mu(p \cup q)$

A finitely additive probability measure is a function that assigns a probability between 0 and 1 to every subset of W, such that the probability of any two disjoint propositions is the sum of their individual probabilities, and such that the total probability of the cells in any partition of W sums to 1.

I assume that an epistemic state is a probability measure: i.e., that $D_i = D_\mu$. This applies equally to contexts, as we maintain the assumption that $D_c = D_i$. Note that a probability measure expresses strictly more information than a set of words: for any probability measure, we can talk about the set of all worlds to which it assigns non-zero probability. So this is simply a richer version of the conception of epistemic states in §2.1: we can define a homomorphism relating elements of D_μ to elements of D_{st}.

I assume the following notion of refinement:

(64) Refinement (probabilistic version):
A measure μ' is a refinement of a state μ ($\mu' \leq_r \mu$) iff $[\forall p : \mu(p) = 1 \vee \mu(p) = 0]\mu(p) = \mu'(p)$

That is to say, monotonic information update cannot make true propositions false or false propositions true, but it can alter the probabilities of propositions to which intermediate credence is assigned.

On a probabilistic account of epistemic modals making use of finitely additive probability measures, epistemic modals are taken to have truth conditions expressed in terms of the value that the contextually determined epistemic conversational background maps their prejacent to (for discussion, see Swanson 2006, Yalcin 2010, and especially Lassiter 2017b). Here, for the sake of simplicity, I'll assume that *probably* requires that its prejacent be assigned a value greater than or equal to .5, and that *might* requires that its prejacent be assigned a value greater than a contextually determined threshhold θ (see Rudin 2016 for discussion of contextual effects on the strength of *might*-claims). Nothing crucial rests on the particular values assumed.

(65) a. $[\![\text{might}]\!]^i = \lambda p.i(p) > \theta$
b. $[\![\text{probably}]\!]^i = \lambda p.i(p) \geq .5$
c. $[\![\text{must}]\!]^i = \lambda p.i(p) \geq 1 - \theta$

A *might*-claim is true iff, relative to the contextually-determined epistemic conversational background, its prejacent is assigned a probability greater than a contextually-determined threshhold value θ. As the dual of *might*, a *must*-claim is true iff its prejacent is assigned a probability of at least 1-θ.

It's very easy to see what the meta-intensionalized denotations of sentences containing bare epistemic modals will be:[19]

(66) a. $MI(\text{might-}p) = \{i : i(p) > \theta\}$
b. $MI(\text{probably-}p) = \{i : i(p) \geq .5\}$
c. $MI(\text{must-}p) = \{i : i(p) \geq 1 - \theta\}$

The only thing that remains to complete this sketch of an implementation of a simple probabilist semantics is to derive a notion of update. The form of probabilistic update that is most likely familiar to most readers is Bayesian conditioning, but here I'll make use of Jeffrey's (1992) generalization of it. Classical Bayesian conditioning defines update only in terms of the observation that a proposition is true, or the observation that a proposition is false. Jeffrey conditioning generalizes this to updating on the observation that a proposition has a particular numerical value anywhere in the unit interval:

(67) Jeffrey conditioning:
For any proposition p, probability measure μ, and real number $n \in [0,1]$:
Jeff(μ, p, n) is a probability measure such that for all q,
Jeff$(\mu, p, n)(q) = n\mu(q \mid p) + (1 - n)\mu(q \mid \neg p)$

This definition is parasitic on a notion of conditional probability:

(68) Conditional probability:
For any propositions p, q, probability measure μ, $\mu(p \mid q) = \mu(p \cap q)\mu(q)$

[19] Note that meta-intensionalization only manipulates the parameterization of interpretation with respect to the epistemic conversational background i. All other aspects of interpretational parameterization (notationally suppressed for readability), including the setting of a contextual threshhold value θ, are held constant.

Informally speaking, Jeffrey conditioning sets the value of a proposition p to some real number in the unit interval, and then renormalizes the probabilities the resulting measure assigns to all other propositions so that the proportional distribution of probabilities over propositions remains the same, modulo the new probability for p.

Jeffrey conditioning-based update potentials for two of the three metaintensionalized denotations in (66) are given below:

(69) a. Update Potential for *probably-p*:
For any c,
$$c[\text{probably-}p] = \begin{cases} c \text{ if } c(p) \geq .5 \\ \textsc{Jeff}(c, p, .5) \text{ otherwise} \end{cases}$$

b. Update Potential for *must-p* (probabilist version):
For any c, θ,
$$c[\text{must-}p] = \begin{cases} c \text{ if } c(p) \geq 1 - \theta \\ \textsc{Jeff}(c, p, 1 - \theta) \text{ otherwise} \end{cases}$$

To unpack these update potentials, first note that any c such that $c(p) = 0$ is not s-compatible for any s of the form *must-p* or *probably-p*, by the definition of refinement in (64). Any c that already assigns a value in the indicated range to p should be left alone; if its value for p were to be lowered, that would violate s-conservativity. Finally, in order for the update function to be p-commensurate, any s-compatible c that assigns a value beneath the target range to p has the value it assigns to p raised to the bottom of the target range, and the rest of the probabilities it assigns renormalized.

In the case of *might*, there is no minimum value in the target range for the value of p, as, due to the fact that for any real numbers n, n', there is another real number n'' such that $n < n'' < n$, there is no unique real number that is the smallest real number greater than θ. I assume, following Rudin (2016), that the context supplies a probability grain g, a set of threshhold values constituting a uniform partition of the unit interval into equivalence classes, capturing the grain size of distinctions in probability relevant to the conversation. Though there isn't a unique $n \in [0,1]$ such that $n > \theta$, there *is* a unique $n \in g$ such that $n > \theta$. Call that unique n $n_{g,\theta}$ This allows us to derive the following update potential for *might-p*:

(70) Update Potential for *might-p* (probabilist version):
$$\text{For any } c, \theta, g,\ c[\text{might-}p] = \begin{cases} c \text{ if } c(p) > \theta \\ \textsc{Jeff}(c, p, n_{g,\theta}) \text{ otherwise} \end{cases}$$

This concludes this paper's sketch of a probabilistic extension of the NSF; a more robust investigation of a probabilist proposal is outside the scope of this paper.

4 Conclusion

This paper adds its voice to a growing literature questioning the soundness of the conclusions that motivated the move from solipsistic contextualism about epistemic modals to the family of approaches discussed in the introduction. This growing literature includes work by

Phillips & Mandelkern (2020), who show that 'eavesdropper' effects, argued to be a desideratum for the semantics of epistemic modals, obtain for all sentences, regardless of their context-sensitivity. It also includes Kroll & Rysling (2019), who show that the QUD-sensitivity of truth value judgments are likewise not restricted to such elements (cf. Beddor & Egan 2018, who argue that the QUD-sensitivity of truth value judgments of modal sentences are evidence for a relativistic semantics for epistemic modals). Taken together, these papers suggest that much of the motivation for non-contextualist treatments of epistemic modals actually reflects the manipulability of truth-value judgment tasks in general, rather than particular properties of epistemic modals.

This paper adds to that literature not in the domain of experimental investigation of the properties of truth value judgment tasks, but rather in the domain of theorizing about the formalization of assertive update. The moral, however, is the same: what has been taken to represent a property that motivates an exceptional treatment of epistemic modals with respect to 'ordinary' declarative sentences may actually fall out of a more mechanism that applies to all declarative sentences equally.

This paper provides a proof of concept that the disagreement problem can be solved by tinkering with the formalization of assertive update, rather than by tinkering with the semantics of epistemic modals. I do not argue that this paper's proposal is strictly superior on empirical grounds to proposals that solve the disagreement problem within the semantics of epistemic modals; however, that an illocutionary solution is possible does defang arguments that the disagreement problem is fatal to solipsistic contextualism, and the fact that the problem can be dissolved by simply changing the camera angle on the theory of assertion, without altering the fundamental intuitions, should help assure us that the problem isn't just being explained away. I take the proposal here to be particularly explanatorily attractive in that their very epistemicity itself is what explains the exceptional behavior of bare epistemic modals.

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