NPI licensing and the logic of the syntax-semantics interface

Yusuke Kubota
NINJAL

Robert Levine
Ohio State University

April 30, 2021

Abstract

In this paper, we propose an explicit syntax-semantics interface of NPI licensing in Hybrid TLG. Hybrid TLG is a version of categorial grammar that inherits properties of both lexicalist and derivational variants of generative grammar, and it has been shown in our previous research (summarized in Kubota and Levine 2020) that it offers elegant analyses for a number of complex phenomena at the interface of syntax and semantics (especially in the domains of coordination and ellipsis) that turn out to be highly problematic for other grammatical theories. In the present paper, we extend our work to NPI licensing and report on some initial results suggesting that the flexible and systematic architecture of Hybrid TLG turns out to be successful in this domain too. Specifically, our approach captures interactions between NPI licensing (or polarity sensitivity) and other complex phenomena at the syntax-semantics interface including the scopal properties of modal auxiliaries, Gapping, and VP fronting. [151 words]

1 Introduction

The present paper has two goals, one specific, and the other somewhat more general. The specific goal is to sketch an outline of an account of NPI licensing in Hybrid Type-Logical Grammar (Hybrid TLG; Kubota and Levine 2020), a version of Type-Logical Grammar that we have been developing over the last ten years. Our approach to NPI licensing builds on the tradition of logic-based approaches to polarity marking originating in Dowty (1994) (subsequently worked out in greater detail by Bernardi 2002 and Steedman 2012; see also related work in computational linguistics such as MacCartney and Manning 2008 and Hu et al. 2019). We believe that working out an explicit syntax-semantics interface is an important task, given the consensus in the field that a purely semantic approach to NPI licensing of the sort attempted in the early literature (e.g., Ladusaw 1980) is now widely recognized as untenable (see, for example, Dowty (1994)), yet explicit and empirically robust accounts of the syntactic licensing of NPIs in a compositional system that has a logical backbone are surprisingly few in number in the current literature (Vasishth (2004) is one rare exception).

Our broader goal is to assess the viability of the architecture of the syntax-semantics interface of Hybrid TLG, which, as will become clear in the ensuing discussion, inherits properties of both lexicalist theories of syntax and certain aspects of derivational theories. We have argued in our previous work that the mapping relationship between form and meaning embodied in Hybrid TLG enables elegant analyses of a number of recalcitrant empirical phenomena, especially in the domains of coordination and ellipsis. However, as we have acknowledged in Kubota and Levine (2020) (see the discussion at the beginning of Chapter 8), the question still remains as to exactly how our approach relates to previous lexicalist approaches and to what extent the additional flexibility entertained by introducing a 'movement-like' operation in the grammar is indispensable in analyzing empirical phenomena in natural language. In the present paper, we aim to clarify some of the motivating assumptions of our earlier work and explore its further consequences. More specifically, we argue for a somewhat more abstract view on the relationship between surface syntax and semantic interpretation than is commonly entertained in (most variants of) lexicalist syntactic theories, one in which, following Oehrle (1994), the semantic scope of operators is in principle decoupled from their surface positions. We defend this view by showing that such an architecture enables a transparent account of complex interactions between NPI licensing and other phenomena pertaining to the syntax-semantics interface, taking interactions with Gapping and VP fronting as two specific test cases.

The paper is structured as follows. We start our discussion by summarizing the key results of our analysis of English modal auxiliaries, motivating it with empirical phenomena that show systematic interactions between scope of auxiliaries and other types of scopal expressions (section 2). In section 3, we turn to the issue of NPI licensing, formalizing a version of polarity-marking approach pioneered in Dowty (1994). The account here builds on and supersedes our own earlier proposal (Kubota and Levine 2019, 2020) in simplifying the theoretical toolkit somewhat (replacing the earlier ternary-valued *pol* feature with a more standard binary valued feature) as well as extending its empirical scope to cases involving downward entailing operators other than negation such as universal quantifiers. Based on the analyses of modal auxiliaries in section 2 and basic NPI licensing in section 3, the rest of the paper discusses more complex cases of NPI licensing: interactions between NPI licensing and the scopal properties of modal auxiliaries and negation (section 4) and interactions between polarity licensing and other scope-sensitive phenomena (Gapping and VP fronting; section 5). To the best of our knowledge, principled explanations are currently lacking for these phenomena in the literature of lexicalist theories of syntax. We thus take the relative ease by which Hybrid TLG offers systematic accounts of these phenomena to provide evidence in favor of the somewhat abstract architecture of syntax-semantics interface that it embodies. Finally, the present paper is not meant to offer a comprehensive analysis of the English auxiliary system, but many aspects of the syntax and semantics of polarity licensing involving auxiliaries crucially rely on the basic analysis of the auxiliary system. For this reason, we sketch a treatment of the auxiliary *do* in the Appendix, an issue that has historically played an important role in the discussion on the English auxiliary system in the syntactic literature. Our analysis demonstrates that the logic-based approach we advocate in this paper offers a novel perspective on the treatment of the auxiliary *do*, which at least serves as a viable alternative to extant analyses of this phenomenon in both the transformational and nontransformational syntax literature.

2 Higher-order modals: why and how

2.1 Why higher-order?

Critical to our approach is the somewhat non-standard analysis (within the tradition of lexicalist syntactic theories) of modals and VP negation as 'higher-order' scopal operators explored and defended in Kubota and Levine (2016, 2019, 2020). We thus start our discussion with a brief review of this analysis, together with the key empirical considerations that motivate it. Our discussion in this section is meant to serve two purposes. First, the analysis of modal auxiliaries plays an important role when we consider more complex phenomena in the later part of the paper involving NPI licensing (and its interactions with other syntactic phenomena). Second, our analysis of modals highlights some of the key properties of the underlying syntax-semantics interface of Hybrid TLG that set it apart from related lexicalist theories. The discussion in this section is thus meant to highlight some of the key similarities and differences between Hybrid TLG and previous lexicalist theories of the more familiar sort. In the course of our discussion, we will try to present the technical setup as transparently as possible, but space precludes giving a complete exposition. Readers are thus encouraged to consult Kubota and Levine (2020) for full details.

In lexicalist theories of syntax such as HPSG and categorial grammar, there is a long tradition of treating modal auxiliaries as expressions that take a VP as an argument to return another VP, starting from the seminal work by Gazdar et al. (1982) in GPSG (as well as early work in Montague Grammar in the formal semantics literature; cf. Bach (1980a)). Previous work in categorial grammar generally follows this tradition (see, e.g., Steedman 1996; Morrill 1994; but also Morrill and Valentín 2017 for an implementation of Kubota and Levine's (2016) analysis in the Displacement Calculus). Such an analysis can be expressed in Hybrid TLG by positing the following type of lexical entries for modal auxiliaries (here, $\mathrm{VP}_{\mathit{fin}}$ and $\mathrm{VP}_{\mathit{bse}}$ are abbreviations of $\mathrm{NP}\backslash\mathrm{S}_{\mathit{fin}}$ and $\mathrm{NP}\backslash\mathrm{S}_{\mathit{bse}}$):[1]

(1) $\mathsf{can}; \lambda P \lambda x.\Diamond P(x); \mathrm{VP}_{\mathit{fin}}/\mathrm{VP}_{\mathit{bse}}$

As in (1), we adopt a notation in which linguistic expressions are written as tuples of prosodic form (written in sans-serif), semantic interpretation (in which constants are written in **roman bold**), and syntactic category. The lexical entry in (1) essentially says that *can* is a verb that takes a base form VP to return a finite VP.

The derivation of a complete sentence containing an auxiliary verb is then straightforward, involving only the 'function application' rules for the familiar forward and backward slashes from the Lambek calculus:

(2)
$$\dfrac{\dfrac{\mathsf{can}; \lambda P \lambda x.\Diamond P(x); \mathrm{VP}_{\mathit{fin}}/\mathrm{VP}_{\mathit{bse}} \quad \mathsf{swim}; \mathbf{swim}; \mathrm{VP}_{\mathit{bse}}}{\mathsf{can} \circ \mathsf{swim}; \lambda x.\Diamond \mathbf{swim}(x); \mathrm{VP}_{\mathit{fin}}}{/\mathrm{E}} \quad \mathsf{john}; \mathbf{j}; \mathrm{NP}}{\mathsf{john} \circ \mathsf{can} \circ \mathsf{swim}; \Diamond \mathbf{swim}(\mathbf{j}); \mathrm{S}_{\mathit{fin}}}{\backslash\mathrm{E}}$$

Derivations are binary trees that show the history of proof about how items in the lexicon are combined with one another to form larger linguistic expressions. These trees are unlike phrase-structure trees in that they are *not* meant to represent constituent structure. To underscore this point, we write derivations in such a way that the left-to-right order of nodes doesn't correspond to word order—for example, at the last step of (2), the subject *John* is placed on the right of the VP (or, more precisely, $\mathrm{NP}\backslash\mathrm{S}_{\mathit{fin}}$) *can swim*. Word order is instead explicitly represented in the prosodic form of the derived expression (the $\backslash$E rule specifies that john is placed on the left of $\mathsf{can} \circ \mathsf{swim}$ in the derived sign).

Just as the transformational analysis of auxiliary inversion and affix hopping (Chomsky 1957) was once thought to be one of the most convincing arguments for the operation

[1]The features *fin* and *bse* here should be thought of as the (analogues of) 'VFORM' features (in G/HPSG terms) that mark finite and base forms of verbs respectively. This ensures that modals can only combine with base forms of verbs and after the modal is combined with the verb, the result is finite, and no other modal can stack on top of the resultant VP. Note that we frame the analysis in terms of features, but while this use of the notion 'feature' has precedent in type-logical versions of categorial grammar (see, e.g., Bayer and Johnson 1995), it raises certain analytic issues that in practice have led researchers to treat the distinction between, e.g., $\mathrm{S}_{\mathit{fin}}$ and $\mathrm{S}_{\mathit{bse}}$ by distinguishing subtypes of a supertype S (see, e.g., Morrill 1994; Pogodalla and Pompigne 2012). In the interests of expository simplicity we will nonetheless continue to treat the difference between such categories in terms of the more familiar notion of syntactic feature values.

of syntactic transformation, a movement-free analysis of auxiliaries whose essence is embodied in (2) has been taken in the literature of lexicalist syntactic theories to be one of the most successful demonstrations for the viability of movement-free syntactic theory (see, e.g., Blevins and Sag 2013 for a lucid discussion of this point from a contemporary perspective; note also the highly critical evaluation of affix hopping from a mathematical perspective in Pullum (2011)).[2] While we generally feel sympathetic to the tenet of 'surface-oriented' approaches to syntax, it is worth emphasizing that the question of whether this type of analysis is adequate for this specific case (i.e. the syntax and semantics of modal auxiliaries in English) ultimately needs to be judged against empirical evidence.

For the purpose of evaluating the empirical adequacy of the VP/VP type analysis of auxiliaries, two types of interactions between auxiliaries and scopal expressions are important, though they seem to have largely escaped attention in the recent discussions of English auxiliary verbs in the PSG tradition. First, auxiliaries are known to enter into scopal interactions with quantifiers in the subject position, as illustrated by the following example:

(3) The Board's decision means that every student can vote.

This sentence has a reading in which the modal *can* outscopes the subject position quantifier *every student*—on this reading, the sentence is making a claim about studentship and the right to vote, not about the (voting) rights of individuals who just happen to be students in the actual world. In the earlier GPSG and categorial grammar analyses, this type of scope relation was treated by lexically lifting the type of the subject position (from type e to $(e \to t) \to t$ semantically); see for example Gazdar et al. (1985), following earlier work by Emmon Bach (e.g., Bach (1980b)) on the possibility of modal auxiliaries taking semantic scope over subject position NPs.[3] In later variants of PSG, such as Pollard and Sag (1994), which assumed a quite different approach to the syntax-semantics interface, the interactions with quantifiers for raising verbs (including auxiliaries like that exemplified in (3)) was recognized as a major outstanding issue, for which Pollard and Yoo (1998) later developed a solution based on an extension of the quantifier storage inheritance mechanism in Pollard and Sag's (1994) formulation. Treatments of auxiliary verb syntax in still later variants of HPSG mostly leave out explicit semantics (for example, Sag et al. (2019), the final culmination of this line of work in the literature, mentions the quantifier/auxiliary interaction, but does not work out an explicit compositional semantics dealing with this issue).

Another interesting (and important) challenge for this VP/VP analysis of auxiliaries comes from the case of scope anomaly in Gapping, originally noted by Dick Oehrle and Muffy Siegel (Oehrle 1971; Siegel 1984, 1987; Oehrle 1987) but overlooked in much of the literature in lexicalist syntax until recently. The issue is that in certain examples of Gapping such as the following, modal auxiliaries in the first conjunct can scope over the

[2] Note also that in the tradition of lexicalist syntax, non-movement analyses of a number of local dependency phenomena have been proposed crosslinguistically, including passive, control, raising and complex predicate phenomena in various languages; see Müller et al. (2020) for an up-to-date summary of the HPSG treatments of the major constructions.

[3] Specifically, Gazdar et al. (1985) took auxiliaries to originate in the lexicon as operators over propositions containing generalized quantifiers as arguments of VP. This treatment, which seems to have been anticipated in Gazdar et al. (1982, 598), captures the wide scope readings of modals over quantifiers, but, as in the earlier analysis, Gazdar et al. (1985) does not contain an explicit discussion of the wide scope readings of the quantified expressions; presumably an appropriate meaning postulate or some comparable device would have been assumed.

whole conjoined sentence.[4,5]

(4) a. John shouldn't eat steak and Mary ∅ just (eat) pizza.
  b. Kim wouldn't play bingo or Sandy ∅ sit at home all evening.

It should intuitively be clear that the wide scope readings for modals in this type of examples are difficult (if not totally impossible) to derive in 'surface-oriented' approaches that exclusively adopt versions of the VP/VP analysis such as the one illustrated above.[6]

2.2 How the analysis is implemented

The two types of scopal interactions exhibited by modal auxiliaries mentioned above introduce substantial complications to the VP/VP analysis—where the augmented version of the analysis loses much of its initial simplicity—and for this reason, we departed from this long-cherished tradition in Kubota and Levine (2016, 2020), arguing for a more abstract type of analysis pioneered in Siegel (1984) (see also Oehrle (1987)) that is in a sense closer to the analysis of modal auxiliaries in movement-based theories of syntax. Specifically, Kubota and Levine (2016) posit the following type of lexical entries for modal auxiliaries in English (where $\mathbf{id}_{et} = \lambda P_{et}.P$):[7]

(5) $\lambda\sigma.\sigma(\mathsf{can});\ \lambda\mathscr{F}.\Diamond\mathscr{F}(\mathbf{id}_{et});\ \mathrm{S}_{\mathit{fin}}{\upharpoonright}(\mathrm{S}_{\mathit{fin}}{\upharpoonright}(\mathrm{VP}_{\mathit{fin}}/\mathrm{VP}_{\mathit{bse}}))$

With (5), we now have the analysis in (6):

(6)
$$
\dfrac{\dfrac{\dfrac{\begin{array}{c}\lambda\sigma.\sigma(\mathsf{can});\\ \lambda\mathscr{F}.\Diamond\mathscr{F}(\mathbf{id}_{et});\\ \mathrm{S}_{\mathit{fin}}{\upharpoonright}(\mathrm{S}_{\mathit{fin}}{\upharpoonright}(\mathrm{VP}_{\mathit{fin}}/\mathrm{VP}_{\mathit{bse}}))\end{array} \quad \textcircled{1}\!\to \dfrac{\dfrac{\begin{array}{c}\mathsf{john};\\ \mathbf{j};\mathrm{NP}\end{array} \quad \dfrac{[\varphi_1; f; \mathrm{VP}_{\mathit{fin}}/\mathrm{VP}_{\mathit{bse}}]^1 \quad \mathsf{swim};\ \mathbf{swim};\ \mathrm{VP}_{\mathit{bse}}}{\varphi_1 \circ \mathsf{swim};\ f(\mathbf{swim});\ \mathrm{VP}_{\mathit{fin}}}/\mathrm{E}}{\mathsf{john} \circ \varphi_1 \circ \mathsf{swim};\ f(\mathbf{swim})(\mathbf{j});\ \mathrm{S}_{\mathit{fin}}}\backslash\mathrm{E}}{\begin{array}{c}\lambda\varphi_1.\mathsf{john} \circ \varphi_1 \circ \mathsf{swim};\\ \lambda f.f(\mathbf{swim})(\mathbf{j}); \mathrm{S}_{\mathit{fin}}{\upharpoonright}(\mathrm{VP}_{\mathit{fin}}/\mathrm{VP}_{\mathit{bse}})\end{array}}{\upharpoonright}\mathrm{I}^1}{\textcircled{2}\!\to\ \lambda\sigma[\sigma(\mathsf{can})](\lambda\varphi_1.\mathsf{john} \circ \varphi_1 \circ \mathsf{swim});\ \Diamond\mathbf{swim}(\mathbf{j});\ \mathrm{S}_{\mathit{fin}}}{\upharpoonright}\mathrm{E}}{\lambda\varphi_1[\mathsf{john} \circ \varphi_1 \circ \mathsf{swim}](\mathsf{can});\ \Diamond\mathbf{swim}(\mathbf{j});\ \mathrm{S}_{\mathit{fin}}}}{\mathsf{john} \circ \mathsf{can} \circ \mathsf{swim};\ \Diamond\mathbf{swim}(\mathbf{j});\ \mathrm{S}_{\mathit{fin}}}
$$

[4]Similar scope anomaly is observed with quantificational determiners as well, as was first noted by McCawley (1993):

(i) No dog eats Whiskas or ∅ cat ∅ Alpo.

See Kubota and Levine (2016, 2020) for an explicit analysis of examples such as (i) that extends the approach summarized in the main text.

[5]Whitman (2010) notes some apparent counterexamples to the widely entertained view that such scope anomaly is restricted to Gapping:

(i) [They might have escaped] and [she didn't notice].

However, the source of the wide scope for the modal in examples of the sort in (i) may be some variant of modal subordination (all of Whitman's examples involve the conjunction word *and* instead of disjunction *or*). We leave further exploration of this issue for future research.

[6]For a more through discussion on this point, see Kubota and Levine (2020, 45–46). See Park (2019); Park et al. (2019) for the most detailed attempt to accommodate examples such as (4) in HPSG. Kubota and Levine (2020) contains some brief comments on Park et al.'s work.

[7]Carpenter's (1997, 245) analysis of the English auxiliary employing the quantificational type constructor (⇑) from Moortgat (1990, 1996) is motivated by interactions with subject position quantifiers of the sort exemplified by (3), and is among the important precursors of our approach.

This derivation involves some analytic techniques that go beyond the simple string-adjacent linguistic composition employed in (2). To give the reader some rough guiding intuition first, what's going on here is that the auxiliary verb scopes over the entire sentence 'at LF', as it were, but it (or, more precisely, its pronounceable string component) lowers into the preverbal auxiliary position in the surface string via an application of β-reduction in the prosodic component analogous to 'quantifying-in' in Montague Grammar. The use of a lambda calculus for representing the prosodic information of linguistic expressions and the formal modelling of quantifying-in is one of the innovations in Oehrle's watershed 1994 paper.

Let us now look at the formal aspects of the derivation more closely. Focusing on the part of the derivation before the $\upharpoonright\mathrm{I}^1$ step (labeled ① in (6)), we can see that it is identical to (2) except that the real auxiliary *can* is replaced by a hypothetical lower-order one (which has prosodic variable φ_1 as its pronunciation). Then, at step ① we see the effect of this 'hypothetical' reasoning. Specifically, at this step the hypothesis is *withdrawn* so that we obtain the 'real' conclusion, as it were, based on the hypothetical conclusion derived so far. Intuitively, from the above hypothetical proof (entertaining the existence of $\mathrm{VP}_{fin}/\mathrm{VP}_{bse}$), we are only entitled to draw a weaker conclusion that what we really have is not a full-fledged S_{fin} but $\mathrm{S}_{fin}\upharpoonright(\mathrm{VP}_{fin}/\mathrm{VP}_{bse})$, that is, an incomplete S_{fin} that is still missing $\mathrm{VP}_{fin}/\mathrm{VP}_{bse}$ in the middle. Corresponding to this syntactic category indicating the type of the missing expression, the semantics and prosody specify the 'placeholders' for the missing item via the lambda-bound variables f (for the semantics) and φ_1 (for the prosody).

At the next step ②, the higher-order entry for the auxiliary in (5) combines with the 'gapped' sentence obtained in ①. The formal operation involved at this step is very simple: we just perform function application in both the semantic and prosodic components, in accordance with the syntactic type specifications: the gapped sentence of type $\mathrm{S}_{fin}\upharpoonright(\mathrm{VP}_{fin}/\mathrm{VP}_{bse})$ is given as an argument to the higher-order function of type $\mathrm{S}_{fin}\upharpoonright(\mathrm{S}_{fin}\upharpoonright(\mathrm{VP}_{fin}/\mathrm{VP}_{bse}))$ to return S_{fin}, per the proof theory for HTLG laid out in Kubota and Levine (2020, section 2.3). Note that in the semantic component, the proposition $\mathbf{swim}(\mathbf{j})$ is obtained by feeding the lexically built-in identity function (of type $et \to et$) to the gapped sentence, and the modal operator $\Diamond$ in the semantic specification of the auxiliary simply scopes over this proposition. The prosodic component is similarly straightforward, except that it involves a couple of β-reduction simplifications unpacked stepwise via the dotted lines in (6).[8] Note in particular that the auxiliary is lexically assigned a higher-order functional prosody of type $(\mathsf{st}\to\mathsf{st})\to\mathsf{st}$ (with st the type for strings), so that it takes a $\mathsf{st}\to\mathsf{st}$ function of the gapped sentence to return a string representation for the whole sentence in which the string component of the auxiliary (i.e., the string can) ends up in the surface preverbal position.

The analysis of modal auxiliaries illustrated above thus resolves the apparent mismatch between the surface position of the modal auxiliary and its semantic scope by a 'movement-like' mechanism. This is formalized via a set of inference rules of the logical connective $\upharpoonright$ that is designed to deal with discontinuity (or, non-adjacent dependency) in natural language.[9] This same mechanism of lambda abstraction in the prosodic component is used

[8]Since the prosodic terms are equivalent, these steps are shown here just for the purpose of exposition. These reduction steps will be omitted in the derivations in what follows.

[9]For this very reason, there is a danger of overgeneration. In particular, the entry in (5) raises the obvious issue of how the locality of scoping follows (e.g., why *I insisted that John should not be accused of the crime* does not have the reading 'I didn't insist that John should be accused of the crime'). See Kubota and Levine (2020, Chapter 9, section 9.2.2) for more discussion on this issue and a solution for the overgeneration problem. In the rest of this paper, we suppress discussion of such questions to keep the

in the QR-like analysis of quantifier scope, as well as the more complex types of scopal dependency (including Barker's (2007) parasitic scope) in Hybrid TLG. See Kubota and Levine (2020) for the formal definitions of the inference rules for $\upharpoonright$ as well as a detailed discussion of both the formal and empirical aspects of this analysis of scopal expressions. For the purposes of the present paper, the key thing to keep in mind is that this architecture enables an analysis of English modal auxiliaries that involves a more abstract mechanism at the syntax-semantics interface than the 'surface-oriented' VP/VP treatment that is more standard in the literature of lexicalist theories of syntax.

An interesting formal consequence of this analysis that is worth noting at this point is that the VP/VP type assignment in (1) whose analog is lexically posited in PSG approaches just falls out as a theorem, by taking the single lexical entry posited in the lexicon (5) as an axiom.[10] The proof is shown in (7).[11]

(7)

$$
\dfrac{
\begin{array}{c}\lambda\sigma.\sigma(\mathsf{can't});\\ \lambda\mathscr{F}.\neg\Diamond\mathscr{F}(\mathbf{id}_{et});\\ \mathrm{S}_{\mathit{fin}}\upharpoonright(\mathrm{S}_{\mathit{fin}}\upharpoonright(\mathrm{VP}_{\mathit{fin}}/\mathrm{VP}_{\mathit{bse}}))\end{array}
\quad
\dfrac{
\dfrac{
[\varphi_1; x; \mathrm{NP}]^1
\quad
\dfrac{[\varphi_2; g; \mathrm{VP}_{\mathit{fin}}/\mathrm{VP}_{\mathit{bse}}]^2 \quad [\varphi_3; f; \mathrm{VP}_{\mathit{bse}}]^3}{\varphi_2 \circ \varphi_3;\ g(f);\ \mathrm{VP}_{\mathit{fin}}}/\mathrm{E}
}{\varphi_1 \circ \varphi_2 \circ \varphi_3;\ g(f)(x);\ \mathrm{S}_{\mathit{fin}}}\backslash\mathrm{E}
}{\lambda\varphi_2.\varphi_1 \circ \varphi_2 \circ \varphi_3;\ \lambda g.g(f)(x);\ \mathrm{S}_{\mathit{fin}}\upharpoonright(\mathrm{VP}_{\mathit{fin}}/\mathrm{VP}_{\mathit{bse}})}\upharpoonright\mathrm{I}^2
}{
\dfrac{
\dfrac{\varphi_1 \circ \mathsf{can't} \circ \varphi_3;\ \neg\Diamond f(x);\ \mathrm{S}_{\mathit{fin}}}{\mathsf{can't} \circ \varphi_3;\ \lambda x.\neg\Diamond f(x);\ \mathrm{VP}_{\mathit{fin}}}\backslash\mathrm{I}^1
}{\mathsf{can't};\ \lambda f\lambda x.\neg\Diamond f(x);\ \mathrm{VP}_{\mathit{fin}}/\mathrm{VP}_{\mathit{bse}}}/\mathrm{I}^3
}\upharpoonright\mathrm{E}
$$

The proof in (7) is an instance of a family of theorems called *Slanting* that have various empirical consequences (one of which is illustrated immediately below). See Kubota and Levine (2020, section 4.5) for further discussion.

Getting back to the empirical discussion, the higher-order analysis of modals with the lexical entry in (5) immediately predicts that modals can outscope quantifiers. This is shown in (8).

(8)

$$
\dfrac{
\dfrac{
\dfrac{
\dfrac{
\dfrac{
\begin{array}{l}\mathsf{vote};\\ \mathbf{vote};\\ \mathrm{VP}_{\mathit{bse}}\end{array}
\quad
\begin{bmatrix}\varphi_1;\\ f;\\ \mathrm{VP}_{\mathit{fin}}/\mathrm{VP}_{\mathit{bse}}\end{bmatrix}^1
}{\varphi_1 \circ \mathsf{vote};\ f(\mathbf{vote});\ \mathrm{VP}_{\mathit{fin}}}
\quad
\begin{bmatrix}\varphi_2;\\ y;\\ \mathrm{NP}\end{bmatrix}^2
}{\varphi_2 \circ \varphi_1 \circ \mathsf{vote};\ f(\mathbf{vote})(y);\ \mathrm{S}_{\mathit{fin}}}
}{\begin{array}{c}\lambda\varphi_2.\varphi_2 \circ \varphi_1 \circ \mathsf{vote};\\ \lambda y.f(\mathbf{vote})(y);\mathrm{S}_{\mathit{fin}}\upharpoonright\mathrm{NP}\end{array}}
\quad
\begin{array}{l}\vdots\\ \lambda\sigma_1.\sigma_1(\mathsf{every} \circ \mathsf{student});\\ \boldsymbol{\forall}_{\mathbf{student}};\\ \mathrm{S}_{\mathit{fin}}\upharpoonright(\mathrm{S}_{\mathit{fin}}\upharpoonright\mathrm{NP})\end{array}
}{\begin{array}{c}\mathsf{every} \circ \mathsf{student} \circ \varphi_1 \circ \mathsf{vote};\\ \boldsymbol{\forall}_{\mathbf{student}}(\lambda y.f(\mathbf{vote})(y));\mathrm{S}_{\mathit{fin}}\end{array}}
}{\begin{array}{l}\lambda\varphi_1.\mathsf{every} \circ \mathsf{student} \circ \varphi_1 \circ \mathsf{vote};\\ \lambda f.\boldsymbol{\forall}_{\mathbf{student}}(\lambda y.f(\mathbf{vote})(y));\mathrm{S}_{\mathit{fin}}\upharpoonright(\mathrm{VP}_{\mathit{fin}}/\mathrm{VP}_{\mathit{bse}})\end{array}}
\quad
\begin{array}{l}\lambda\sigma_2.\sigma_2(\mathsf{can});\\ \lambda\mathscr{F}.\Diamond\mathscr{F}(\mathbf{id}_{et});\\ \mathrm{S}_{\mathit{fin}}\upharpoonright(\mathrm{S}_{\mathit{fin}}\upharpoonright(\mathrm{VP}_{\mathit{fin}}/\mathrm{VP}_{\mathit{bse}}))\end{array}
}{\mathsf{every} \circ \mathsf{student} \circ \mathsf{can} \circ \mathsf{vote};\ \Diamond\boldsymbol{\forall}_{\mathbf{student}}(\lambda y.\mathbf{vote}(y));\ \mathrm{S}_{\mathit{fin}}}
$$

The narrow scope interpretation for the modal can be obtained by simply using the 'slanted' auxiliary sign in type VP/VP in (7). This way, the scopal interaction between

discussion focused and avoid notational clutter.

[10]Note moreover that the more general ((S/(NP\S))\S)/VP type (taking a Lambek-type quantifier in the subject position) can be derived as a theorem too. This can be obtained by replacing hypothesis 1 of type NP in (7) with a hypothesis of a higher-order type S/(NP\S).

[11]For readers familiar with deductive approaches to logic, we note here that the proof in (7) is essentially parallel to the proof for the following theorem in propositional logic (or, for that matter, in linear logic): $((\varphi \to \psi \to \varrho) \to \varrho) \to \zeta \vdash (\varphi \to \psi) \to \zeta$. To see the parallel, substitute NP\S for φ, NP for ψ, S for ϱ and ζ. We thank Carl Pollard for discussions of this and related issues pertaining to the relationship between the syntactic logic of Hybrid TLG and more familiar types of logics.

the subject position quantifier and the modal falls out as a predicted consequence of the higher-order analysis of modals.

This analysis extends straightforwardly to scopal interactions between modals and Gapping. We illustrate this with the version of (4a) in which only the auxiliary is gapped in the second conjunct (for a complete analysis covering full Gapping variant of (4a), see Kubota and Levine (2020, 66–67)). The analysis for the wide-scope reading for the auxiliary in Gapping is shown in (9)–(10).

(9)

$$\dfrac{\dfrac{\dfrac{\begin{array}{l}\text{john};\\ \mathbf{j}; \mathrm{NP}\end{array} \quad \dfrac{\begin{bmatrix}\varphi_1;\\ f; \mathrm{VP}_{\mathit{fin}}/\mathrm{VP}_{bse}\end{bmatrix}^1 \quad \begin{array}{l}\text{eat} \circ \text{steak};\\ \mathbf{eat}(\mathbf{s}); \mathrm{VP}_{bse}\end{array}}{\begin{array}{l}\varphi_1 \circ \text{eat} \circ \text{steak};\\ f(\mathbf{eat}(\mathbf{s})); \mathrm{VP}_{\mathit{fin}}\end{array}}/\mathrm{E}}{\begin{array}{l}\text{john} \circ \varphi_1 \circ \text{eat} \circ \text{steak};\\ f(\mathbf{eat}(\mathbf{s}))(\mathbf{j}); \mathrm{S}_{\mathit{fin}}\end{array}}\backslash\mathrm{E}}{\begin{array}{l}\lambda\varphi_1.\text{john} \circ \varphi_1 \circ \text{eat} \circ \text{steak};\\ \lambda f.f(\mathbf{eat}(\mathbf{s}))(\mathbf{j}); \mathrm{S}_{\mathit{fin}}\upharpoonright(\mathrm{VP}_{\mathit{fin}}/\mathrm{VP}_{bse})\end{array}}{\upharpoonright}\mathrm{I}^1 \quad \dfrac{\begin{array}{l}\lambda\sigma_2\lambda\sigma_1\lambda\varphi.\sigma_1(\varphi) \circ\\ \quad \text{and} \circ \sigma_2(\boldsymbol{\epsilon});\\ \lambda\mathcal{F}_2\lambda\mathcal{F}_1.\mathcal{F}_1 \sqcap \mathcal{F}_2;\\ (\mathrm{S}_\alpha\upharpoonright X)\upharpoonright(\mathrm{S}_\alpha\upharpoonright X)\upharpoonright(\mathrm{S}_\alpha\upharpoonright X)\end{array} \quad \begin{array}{l}\vdots\\ \lambda\varphi_2.\text{mary} \circ \varphi_2 \circ\\ \quad \text{eat} \circ \text{pizza};\\ \lambda g.g(\mathbf{eat}(\mathbf{p}))(\mathbf{m});\\ \mathrm{S}_{\mathit{fin}}\upharpoonright(\mathrm{VP}_{\mathit{fin}}/\mathrm{VP}_{bse})\end{array}}{\begin{array}{l}\lambda\sigma_1\lambda\varphi_0.\sigma_1(\varphi_0) \circ \text{and} \circ\\ \quad \text{mary} \circ \boldsymbol{\epsilon} \circ \text{eat} \circ \text{pizza};\\ \lambda\mathcal{F}_1.\mathcal{F}_1 \sqcap \lambda g.g(\mathbf{eat}(\mathbf{p}))(\mathbf{m});\\ (\mathrm{S}_{\mathit{fin}}\upharpoonright(\mathrm{VP}_{\mathit{fin}}/\mathrm{VP}_{bse}))\upharpoonright(\mathrm{S}_{\mathit{fin}}\upharpoonright(\mathrm{VP}_{bse}/\mathrm{VP}_{bse}))\end{array}}{\upharpoonright}\mathrm{E}}{\begin{array}{c}\lambda\varphi_0.\text{john} \circ \varphi_0 \circ \text{eat} \circ \text{steak} \circ \text{and} \circ \text{mary} \circ \boldsymbol{\epsilon} \circ \text{eat} \circ \text{pizza};\\ \lambda f.f(\mathbf{eat}(\mathbf{s}))(\mathbf{j}) \sqcap \lambda g.g(\mathbf{eat}(\mathbf{p}))(\mathbf{m}); \mathrm{S}_{\mathit{fin}}\upharpoonright(\mathrm{VP}_{\mathit{fin}}/\mathrm{VP}_{bse})\end{array}}{\upharpoonright}\mathrm{E}$$

(10)

$$\dfrac{\begin{array}{l}\lambda\sigma_0.\sigma_0(\text{can't});\\ \lambda\mathscr{F}.\neg\Diamond\mathscr{F}(\mathbf{id}_{et});\\ \mathrm{S}_{\mathit{fin}}\upharpoonright(\mathrm{S}_{\mathit{fin}}\upharpoonright(\mathrm{VP}_{\mathit{fin}}/\mathrm{VP}_{bse}))\end{array} \quad \begin{array}{l}\qquad\qquad\vdots\\ \lambda\varphi_0.\text{john} \circ \varphi_0 \circ \text{eat} \circ \text{steak} \circ \text{and} \circ \text{mary} \circ \boldsymbol{\epsilon} \circ \text{eat} \circ \text{pizza};\\ \lambda f.f(\mathbf{eat}(\mathbf{s}))(\mathbf{j}) \sqcap \lambda g.g(\mathbf{eat}(\mathbf{p}))(\mathbf{m}); \mathrm{S}_{\mathit{fin}}\upharpoonright(\mathrm{VP}_{\mathit{fin}}/\mathrm{VP}_{bse})\end{array}}{\text{john} \circ \text{can't} \circ \text{eat} \circ \text{steak} \circ \text{and} \circ \text{mary} \circ \boldsymbol{\epsilon} \circ \text{eat} \circ \text{pizza}; \neg\Diamond[\mathbf{eat}(\mathbf{s})(\mathbf{j}) \wedge \mathbf{eat}(\mathbf{p})(\mathbf{m})]; \mathrm{S}_{\mathit{fin}}}{\upharpoonright}\mathrm{E}$$

The key point here is that the (auxiliary-)gapping can be analyzed as a conjunction of two clauses missing an auxiliary, which can be derived in Hybrid TLG as an expression of type $\mathrm{S}\upharpoonright(\mathrm{VP}/\mathrm{VP})$. After the gapping-specific conjunction conjoins two such 'gapped' S's (which is lexically specified to fill in the gap in the second conjunct with an empty string and inherits the gap of the first conjunct to the larger expression) the whole conjoined expression is of the right type to be given as an argument to the higher-order modal auxiliary, as in (10). Since the auxiliary takes the conjoined sentence as an argument rather than vice versa, the wide-scope reading for the modal auxiliary is obtained in the above analysis.

For the narrow-scope reading for the auxiliary, again, the slanted entry for the auxiliary is employed. Note that by replacing the higher-order entry for the auxiliary in (10) with the slanted entry derived in (7), the function-argument relation between the gapped sentence and the auxiliary is reversed, and we obtain the reading in which the modal meaning is distributed to each conjunct.

As we have shown above, the higher-order analysis of modal auxiliaries in Hybrid TLG straightforwardly captures their scopal properties in examples in which they interact with subject position quantifiers and in Gapping sentences. Moreover, this analysis can be thought of as a proper generalization of the VP/VP analysis familiar in the tradition of lexicalist theories of syntax in that a lexical sign for the auxiliary that corresponds to a straightforward implementation of this VP/VP analysis (as well as its extension that subcategorizes for a subject position quantifier) falls out as a theorem in the deductive system without any extra assumptions.

3 NPI licensing

NPI licensing—first identified as an important phenomenon in Klima (1964)—has been an important topic in semantics research since the watershed contributions of Ladusaw (1980).

However, despite the abundance of study on the precise semantic conditions governing the distribution of NPIs, there has been much less research following up the quite different strategy, advocated in Dowty (1994), of 'making certain semantic distinctions visible in the syntax', i.e., as syntactic properties that deductive rules can be made sensitive to (but see Bernardi (2002) and Steedman (2012) for some approaches that attempt to extend Dowty's work). Formulating an explicit account of the syntax-semantics interface of NPI licensing is important, since it is well-known in the literature that NPI licensing interacts with—and in several respects hinges on—other phenomena pertaining to the syntax and semantics of natural language in ways that often make nontrivial differences in terms of comparison of competing approaches (see, e.g., de Swart (1998), Richter and Soehn (2006), Israel (2011), Levine (2013), Liu et al. (2019)). For this reason, we formulate an explicit account of NPI licensing in Hybrid TLG in the rest of this paper. Our account builds on our own earlier proposal in Kubota and Levine (2019, 2020), which focused on the interactions between polarity sensitivity and the scopal interactions with negation exhibited by modal auxiliaries in English. The new proposal we formulate below improves on our earlier account in two respects: simplifying the theoretical setup somewhat and extending its empirical coverage by offering an explicit account of basic NPI licensing facts by downward-entailing operators other than negation.[12]

The proofs we present in this section depend in no way on the inflection-class marking of clauses and the functors bases on them, such as VP, VP/NP and so on. In the interest of expository clarity, we therefore suppress annotation for *f(in), b(se)* and other morphosyntactic specifications. However, we reintroduce this annotation in the following sections, where interactions between polarity marking and the syntax of auxiliaries become central to our discussion.

3.1 Basic cases with negation

In formulating an account of NPI licensing, we loosely adopt Dowty's (1994) idea of encoding polarity sensitivity via a binary syntactic feature *pol*±. The key idea is to control the distribution of NPIs in terms of this binary feature. We assume that NPIs are licensed in environments marked *pol*− and that clauses marked *pol*− (i.e. S_{-}) cannot be standalone sentences.[13] Such expressions can appear in complete derivations only as arguments of NPI licensors which reverse the polarity specification of its argument.

We now demonstrate that the *pol* subtype specifications outlined above are sufficient to account for the distribution of standard NPI expressions, as illustrated by the following minimal pair:

(11) a. Mary didn't say anything.
b. *Mary said anything.

To account for this contrast, we make the following assumptions:

(i) All standalone sentences inhabit the type S_{+}.

(ii) Lexical verbs are underspecified for the polarity marking of the root S category.

(iii) NPIs are functions that return *pol*− expressions as output.

[12] Due to length considerations, we do not attempt here a full-blown account of NPI licensing. In particular, we do not provide an account of the hierarchy of strength among NPIs noted by Zwarts (1998). One possible approach, which we will not attempt here, would be to further elaborate the syntactic encoding of polarity information.

[13] In what follows, we omit the feature name *pol*. Thus, S_{-} is an abbreviation of S_{pol-}

(iv) NPI licensors are functions that can take *pol*− expressions as arguments.

The set of lexical entries in (12) embody the above four assumptions (where $\boldsymbol{\exists}_{\mathbf{th}} = \lambda P.\exists x.\mathbf{Thing}(x) \wedge P(x)$):

(12) a. $\mathsf{say};\ \mathbf{say};\ (\mathrm{NP}\backslash \mathrm{S}_\alpha)/\mathrm{NP}$

b. $\lambda\sigma.\sigma(\mathsf{anything});\ \boldsymbol{\exists}_{\mathbf{th}};\ \mathrm{S}_{-}\upharpoonright(\mathrm{S}_{-}\upharpoonright\mathrm{NP})$

c. $\lambda\sigma_2.\sigma_2(\mathsf{didn't});\ \lambda\mathscr{G}.\neg\mathscr{G}(\mathbf{id});\ \mathrm{S}_{+}\upharpoonright(\mathrm{S}_{-}\upharpoonright(\mathrm{VP}_{-}/\mathrm{VP}_{-}))$

(11a) is then licensed as in (13).

(13)

$$\dfrac{\dfrac{\text{②}\ \dfrac{\dfrac{\text{①}\ \dfrac{\dfrac{\dfrac{\dfrac{\begin{array}{l}\mathsf{say};\\ \mathbf{say};\\ \mathrm{VP}_\alpha/\mathrm{NP}\end{array} \quad \begin{bmatrix}\varphi_1;\\ x;\\ \mathrm{NP}\end{bmatrix}^1}{\begin{array}{l}\mathsf{say}\circ\varphi_1;\\ \mathbf{say}(x);\mathrm{VP}_\alpha\end{array}} \quad \begin{bmatrix}\varphi_2;\\ f;\\ \mathrm{VP}_\alpha/\mathrm{VP}_\alpha\end{bmatrix}^2}{\varphi_2\circ\mathsf{say}\circ\varphi_1;\ f(\mathbf{say}(x));\ \mathrm{VP}_\alpha} \quad \begin{array}{l}\mathsf{mary};\\ \mathbf{m};\\ \mathrm{NP}\end{array}}{\begin{array}{c}\mathsf{mary}\circ\varphi_2\circ\mathsf{say}\circ\varphi_1;\\ f(\mathbf{say}(x))(\mathbf{m});\mathrm{S}_\alpha\end{array}}}{\begin{array}{l}\lambda\varphi_1.\mathsf{mary}\circ\varphi_2\circ\mathsf{say}\circ\varphi_1;\\ \lambda x.f(\mathbf{say}(x))(\mathbf{m});\mathrm{S}_\alpha\upharpoonright\mathrm{NP}\end{array}} \quad \begin{array}{l}\lambda\sigma_1.\sigma_1(\mathsf{anything});\\ \boldsymbol{\exists}_{\mathbf{th}};\\ \mathrm{S}_{-}\upharpoonright(\mathrm{S}_{-}\upharpoonright\mathrm{NP})\end{array}}{\begin{array}{c}\mathsf{mary}\circ\varphi_2\circ\mathsf{say}\circ\mathsf{anything};\\ \boldsymbol{\exists}_{\mathbf{th}}(\lambda x.f(\mathbf{say}(x))(\mathbf{m}));\mathrm{S}_{-}\end{array}}}{\begin{array}{l}\lambda\varphi_2.\mathsf{mary}\circ\varphi_2\circ\mathsf{say}\circ\mathsf{anything};\\ \lambda f.\boldsymbol{\exists}_{\mathbf{th}}(\lambda x.f(\mathbf{say}(x))(\mathbf{m}));\mathrm{S}_{-}\upharpoonright(\mathrm{VP}_{-}/\mathrm{VP}_{-})\end{array}} \quad \begin{array}{l}\lambda\sigma_2.\sigma_2(\mathsf{didn't});\\ \lambda\mathscr{G}.\neg\mathscr{G}(\mathbf{id});\\ \mathrm{S}_{+}\upharpoonright(\mathrm{S}_{-}\upharpoonright(\mathrm{VP}_{-}/\mathrm{VP}_{-}))\end{array}}{\mathsf{mary}\circ\mathsf{didn't}\circ\mathsf{say}\circ\mathsf{anything};\ \neg\boldsymbol{\exists}_{\mathbf{th}}(\lambda x.\mathbf{say}(x)(\mathbf{m}));\ \mathrm{S}_{+}}}{}$$

This derivation illustrates the essence of our strategy for controlling the distribution of NPIs: at the step labelled ①, the introduction of the NPI forces the result category to be rooted in S_{-}, which must revert to S_{+} in order for the sign to appear as a standalone clause. This is indeed achieved by the licensing negation at step ②.

Note that there is a close relation between the syntactic composition of NPI licensing and the semantic scope relation between the NPI and its licensor. In particular, the fact that the NPI is licensed when it falls under the scope of the licensor corresponds to selection of an argument containing an NPI—and which thereby inhabits a *pol*− subtype—by a licensor, which composes with this argument to yield a sign that has a *pol*+ specification. For this reason, we do not have a derivation for (11a) in which *anything* outscopes negation. Such a derivation would require *didn't* to compose into the proof before *anything*, yielding S_{+}, which is ineligible as an argument of *anything*.

The derivation of the failed sentence (11b) goes as follows:

(14)

$$\dfrac{\dfrac{\begin{array}{c}\vdots\\ \mathsf{mary}\circ\mathsf{said}\circ\varphi_1;\ \mathbf{say}(x)(\mathbf{m});\ \mathrm{S}_\alpha\end{array}}{\lambda\varphi_1.\mathsf{mary}\circ\mathsf{said}\circ\varphi_1;\ \lambda x.\mathbf{say}(x)(\mathbf{m});\ \mathrm{S}_\alpha\upharpoonright\mathrm{NP}} \quad \begin{array}{l}\lambda\sigma_1.\sigma_1(\mathsf{anything});\\ \boldsymbol{\exists}_{\mathbf{th}};\mathrm{S}_{-}\upharpoonright(\mathrm{S}_{-}\upharpoonright\mathrm{NP})\end{array}}{\mathsf{mary}\circ\mathsf{said}\circ\mathsf{anything};\ \boldsymbol{\exists}_{\mathbf{th}}(\lambda x.\mathbf{say}(x)(\mathbf{m}));\ \mathrm{S}_{-}}$$

Since root sentences must have positive polarity, the final proof line of (14) corresponds to an ill-formed sign as a standalone sentence.

Sentences with multiple NPIs in licensing contexts, such as (15), are also readily derivable in our approach.

(15) Mary didn't say anything to anyone.

The derivation for (15) goes as in (16).

(16)

$$
\dfrac{
\dfrac{
\dfrac{
\dfrac{
\dfrac{
\dfrac{
\begin{array}{c}\vdots\\ \textsf{mary} \circ \varphi_1 \circ \textsf{say} \circ \\ \varphi_0 \circ \textsf{to} \circ \varphi_3;\\ g(\mathbf{say}(x)(w))(\mathbf{m}); \mathrm{S}_{-}\end{array}
}{
\begin{array}{l}\lambda\varphi_0.\textsf{mary} \circ \varphi_1 \circ \\ \textsf{say} \circ \varphi_0 \circ \textsf{to} \circ \varphi_3;\\ \lambda x.g(\mathbf{say}(x)(w))(\mathbf{m});\\ \mathrm{S}_{-}{\restriction}\mathrm{NP}\end{array}
}
\quad
\begin{array}{l}\lambda\sigma_1.\sigma_1(\textsf{anything});\\ \boldsymbol{\exists}_{\mathbf{th}};\\ \mathrm{S}_{-}{\restriction}(\mathrm{S}_{-}{\restriction}\mathrm{NP})\end{array}
}{
\begin{array}{c}\textsf{mary} \circ \varphi_1 \circ \textsf{say} \circ \textsf{anything} \circ \textsf{to} \circ \varphi_3;\\ \boldsymbol{\exists}_{\mathbf{th}}(\lambda x.g(\mathbf{say}(x)(w))(\mathbf{m})); \mathrm{S}_{-}\end{array}
}
}{
\begin{array}{l}\lambda\varphi_3.\textsf{mary} \circ \varphi_1 \circ \textsf{say} \circ \textsf{anything} \circ \textsf{to} \circ \varphi_3;\\ \lambda w.\boldsymbol{\exists}_{\mathbf{th}}(\lambda x.g(\mathbf{say}(x)(w))(\mathbf{m})); \mathrm{S}_{-}{\restriction}\mathrm{NP}\end{array}
}
\quad
\begin{array}{l}\lambda\sigma_2.\sigma_2(\textsf{anyone});\\ \boldsymbol{\exists}_{\mathbf{pers}};\\ \mathrm{S}_{-}{\restriction}(\mathrm{S}_{-}{\restriction}\mathrm{NP})\end{array}
}{
\begin{array}{c}\textsf{mary} \circ \varphi_1 \circ \textsf{say} \circ \textsf{anything} \circ \textsf{to} \circ \textsf{anyone};\\ \boldsymbol{\exists}_{\mathbf{pers}}(\lambda w.\boldsymbol{\exists}_{\mathbf{th}}(\lambda x.g(\mathbf{say}(x)(w))(\mathbf{m}))); \mathrm{S}_{-}\end{array}
}
}{
\begin{array}{l}\lambda\varphi_1.\textsf{mary} \circ \varphi_1 \circ \textsf{say} \circ \textsf{anything} \circ \textsf{to} \circ \textsf{anyone};\\ \lambda g.\boldsymbol{\exists}_{\mathbf{pers}}(\lambda w.\boldsymbol{\exists}_{\mathbf{th}}(\lambda x.g(\mathbf{say}(x)(w))(\mathbf{m}))); \mathrm{S}_{-}{\restriction}(\mathrm{VP}_{-}/\mathrm{VP}_{-})\end{array}
}
\quad
\begin{array}{l}\lambda\sigma_2.\sigma_2(\textsf{didn't});\\ \lambda\mathscr{G}.\neg\mathscr{G}(\mathbf{id});\\ \mathrm{S}_{+}{\restriction}(\mathrm{S}_{-}{\restriction}(\mathrm{VP}_{-}/\mathrm{VP}_{-}))\end{array}
}{
\textsf{mary} \circ \textsf{didn't} \circ \textsf{say} \circ \textsf{anything} \circ \textsf{to} \circ \textsf{anyone};\ \neg\boldsymbol{\exists}_{\mathbf{pers}}(\lambda w.\boldsymbol{\exists}_{\mathbf{th}}(\lambda x.\mathbf{Say}(x)(w)(\mathbf{m})));\ \mathrm{S}_{+}
}
$$

This derivation illustrates multiple NPI stacking under a single licensor. Note in particular here that an NPI can scope over an expression that itself contains an NPI since it requires its argument to be *pol*$-$.

3.2 Conditionals and determiners

The case of conditionals is particularly instructive, because it highlights the role of polarity value underspecification in our analysis. We have the following facts to account for.

(17) a. If John met Mary, he must have {some/*any} impression of her.
   b. If John knows anything (about logic), he will immediately point out the flaw in the plan.

The pattern in (17) shows that the antecedent clause of a conditional sentence is an NPI licensing environment, but the consequent clause isn't. We therefore posit the following lexical entry for *if*:[14]

(18) if; $\lambda p\lambda q.p \to q$; $\mathrm{S}_{\beta}/\mathrm{S}_{+}/\mathrm{S}_{\alpha}$

Then examples such as (17b) that involve NPIs in the antecedent clause are licensed as follows:

(19)

$$
\dfrac{
\textsf{if};\ \lambda p\lambda q.p \to q;\ \mathrm{S}_{\beta}/\mathrm{S}_{+}/\mathrm{S}_{\alpha}
\quad
\begin{array}{c}\vdots\\ \textsf{john} \circ \textsf{knows} \circ \textsf{anything};\ \boldsymbol{\exists}_{\mathbf{th}}(\lambda x.\mathbf{know}(x)(\mathbf{j}));\ \mathrm{S}_{-}\end{array}
}{
\textsf{if} \circ \textsf{john} \circ \textsf{knows} \circ \textsf{anything};\ \lambda q.\boldsymbol{\exists}_{\mathbf{th}}(\lambda x.\mathbf{know}(x)(\mathbf{j})) \to q;\ \mathrm{S}_{\beta}/\mathrm{S}_{+}
}
$$

Examples such as (17a) (with *any*) are ruled out by the *pol*$+$ specification on the consequent clause. *He must have any impression of her* is specified as S_{-}, which is incompatible with the polarity requirement in (18) that the conditional imposes on the consequent clause.

This approach extends straightforwardly to cases of quantificational determiners. For example, *every* is downward entailing in both its restrictor and scope:

[14] The symbol $\to$ in (18) should be taken to be a placeholder for whatever is the right semantics for the meaning of *if* in English. It is not meant to be a claim that the semantics of conditional sentences in natural language is identical to material implication in logic.

(20) a. Every spy who John said anything to got worried.

b. *Every spy who John talked to paid any attention to it.

We posit the following lexical entry for *every* (where $\alpha, \beta \in \{+, -\}$) to account for the pattern in (20):[15]

(21) $\mathsf{every};\ \mathbf{\forall};\ \mathrm{S}_\alpha \upharpoonleft (\mathrm{S}_+ \upharpoonleft \mathrm{NP}) \upharpoonleft \mathrm{N}_\beta$

The following derivation illustrates how the quantified NP in (20a) is derived in the category $\mathrm{S}_\alpha \upharpoonleft (\mathrm{S}_+ \upharpoonleft \mathrm{NP})$ (here, $\alpha \in \{+, -\}$):

(22)

$$\begin{bmatrix} \varphi_0; \\ x; \mathrm{NP} \end{bmatrix}^0 \quad \begin{bmatrix} \varphi_1; \\ w; \mathrm{NP} \end{bmatrix}^1$$

$$\vdots \qquad \vdots$$

$$\frac{\mathsf{john} \circ \mathsf{said} \circ \varphi_0 \circ \mathsf{to} \circ \varphi_1;\ \mathbf{say}(x)(w); \mathrm{S}_\alpha}{\lambda \varphi_0.\mathsf{john} \circ \mathsf{said} \circ \varphi_0 \circ \mathsf{to} \circ \varphi_1;\ \lambda x.\mathbf{say}(x)(w); \mathrm{S}_\alpha \upharpoonleft \mathrm{NP}}$$

$$\frac{\lambda \varphi_0.\mathsf{john} \circ \mathsf{said} \circ \varphi_0 \circ \mathsf{to} \circ \varphi_1;\ \lambda x.\mathbf{say}(x)(w); \mathrm{S}_\alpha \upharpoonleft \mathrm{NP} \qquad \lambda \sigma_0.\sigma_0(\mathsf{anything});\ \mathbf{\exists_{th}};\ \mathrm{S}_- \upharpoonleft (\mathrm{S}_- \upharpoonleft \mathrm{NP})}{\mathsf{john} \circ \mathsf{said} \circ \mathsf{anything} \circ \mathsf{to} \circ \varphi_1;\ \mathbf{\exists_{th}}(\lambda x.\mathbf{say}(x)(w)(\mathbf{j})); \mathrm{S}_-}$$

$$\frac{\mathsf{john} \circ \mathsf{said} \circ \mathsf{anything} \circ \mathsf{to} \circ \varphi_1;\ \mathbf{\exists_{th}}(\lambda x.\mathbf{say}(x)(w)(\mathbf{j})); \mathrm{S}_-}{\lambda \varphi_1.\mathsf{john} \circ \mathsf{said} \circ \mathsf{anything} \circ \mathsf{to} \circ \varphi_1;\ \lambda w.\mathbf{\exists_{th}}(\lambda x.\mathbf{say}(x)(w)(\mathbf{j})); \mathrm{S}_- \upharpoonleft \mathrm{NP}}$$

$$① \rightarrow \frac{\lambda \varphi_1.\mathsf{john} \circ \mathsf{said} \circ \mathsf{anything} \circ \mathsf{to} \circ \varphi_1;\ \lambda w.\mathbf{\exists_{th}}(\lambda x.\mathbf{say}(x)(w)(\mathbf{j})); \mathrm{S}_- \upharpoonleft \mathrm{NP} \qquad \lambda \sigma_1.\mathsf{who} \circ \sigma_1(\boldsymbol{\epsilon});\ \mathbf{and}_{et};\ (\mathrm{N}_\beta \backslash \mathrm{N}_\alpha) \upharpoonleft (\mathrm{S}_\alpha \upharpoonleft \mathrm{NP})}{\mathsf{who} \circ \mathsf{john} \circ \mathsf{said} \circ \mathsf{anything} \circ \mathsf{to} \circ \boldsymbol{\epsilon};\ \mathbf{and}_{et}(\lambda w.\mathbf{\exists_{th}}(\lambda x.\mathbf{say}(x)(w)(\mathbf{j}))); \mathrm{N}_\beta \backslash \mathrm{N}_-}$$

$$\frac{\mathsf{who} \circ \mathsf{john} \circ \mathsf{said} \circ \mathsf{anything} \circ \mathsf{to} \circ \boldsymbol{\epsilon};\ \mathbf{and}_{et}(\lambda w.\mathbf{\exists_{th}}(\lambda x.\mathbf{say}(x)(w)(\mathbf{j}))); \mathrm{N}_\beta \backslash \mathrm{N}_- \qquad \mathsf{spy};\ \mathbf{spy};\ \mathrm{N}_\gamma}{\mathsf{spy} \circ \mathsf{who} \circ \mathsf{john} \circ \mathsf{said} \circ \mathsf{anything} \circ \mathsf{to} \circ \boldsymbol{\epsilon};\ \mathbf{and}_{et}(\lambda w.\mathbf{\exists_{th}}(\lambda x.\mathbf{say}(x)(w)(\mathbf{j})))(\mathbf{spy}); \mathrm{N}_-}$$

$$\frac{\mathsf{spy} \circ \mathsf{who} \circ \mathsf{john} \circ \mathsf{said} \circ \mathsf{anything} \circ \mathsf{to} \circ \boldsymbol{\epsilon};\ \mathbf{and}_{et}(\lambda w.\mathbf{\exists_{th}}(\lambda x.\mathbf{say}(x)(w)(\mathbf{j})))(\mathbf{spy}); \mathrm{N}_- \qquad \lambda \varphi_2 \lambda \sigma_2.\sigma_2(\mathsf{every} \circ \varphi_2);\ \mathbf{\forall};\ \mathrm{S}_\alpha \upharpoonleft (\mathrm{S}_+ \upharpoonleft \mathrm{NP}) \upharpoonleft \mathrm{N}_\beta}{\mathsf{every} \circ \mathsf{spy} \circ \mathsf{who} \circ \mathsf{john} \circ \mathsf{said} \circ \mathsf{anything} \circ \mathsf{to} \circ \boldsymbol{\epsilon};\ \mathbf{\forall}(\mathbf{and}_{et}(\lambda w.\mathbf{\exists_{th}}(\lambda x.\mathbf{say}(x)(w)(\mathbf{j})))(\mathbf{spy})); \mathrm{S}_\alpha \upharpoonleft (\mathrm{S}_+ \upharpoonleft \mathrm{NP})}$$

The critical step in this derivation is the passing of the *pol*− value from the relative clause to its nominal argument (mediated at step ①), as per the lexical entry for the relativizer. The transmission of this negative polarity value to N gives the universal access to the negative value introduced by *anything.* Since *every* specifies its restrictor to be compatible with an NPI (*pol* β, with $\beta \in \{+, -\}$), the two expressions can combine to yield a well-formed $\mathrm{S}_\alpha \upharpoonleft (\mathrm{S}_+ \upharpoonleft \mathrm{NP})$.

In the case of ill-formed examples such as (20b), the composition required is blocked by the type mismatch between the functor and the argument. The type of the quantified subject, $\mathrm{S}_\alpha \upharpoonleft (\mathrm{S}_+ \upharpoonleft \mathrm{NP})$, makes it incompatible with its $\mathrm{S}_- \upharpoonleft \mathrm{NP}$ argument which contains the NPI *any.* And if we construct the proof so that *every spy* scopes in first and then *anything*, the result will be a root clause of type S_-. The lexical descriptions of universals and the NPI indefinite *any* thus collaborate in ruling out (20b).

Polarity properties of other determiners can be captured similarly. The following entry for *no* illustrates the point, where the *pol*− specification on both the restrictor and the scope captures the fact that both are NPI licensing environments:[16,17]

(23) $\mathsf{no};\ \neg\mathbf{\exists};\ \mathrm{S}_\alpha \upharpoonleft (\mathrm{S}_- \upharpoonleft \mathrm{NP}) \upharpoonleft \mathrm{N}_-$

[15] The polarity underspecification of the output S in (21) is critical to an account of a seemingly problematic intervention effect associated with the interaction of negation and universal quantification; we defer discussion of this phenomenon to section 3.4 below.

[16] The PPI status of *some* is somewhat complicated by the non-specific variant of indefinite determiners that is in general compatible in both NPI and PPI environments. For this reason, we leave aside *some* and indefinites here.

[17] We actually assume a somewhat more complex type for *no* for capturing its split scope effect (Penka 2011) explicitly. See (47) in section 4 below.

3.3 *Doubt*

The behavior of the verb *doubt*, which is also treated in the literature as an NPI licensor, contrasts with that of logical operators such as conditionals and quantifiers in ways that seem to point to a somewhat different treatment.[18] To see the difference between *doubt* and other licensors, note that the logical operators from the previous section such as conditionals and the universal quantifier only affect the licensing properties of the syntactically local environments. Thus, as is well known in the literature (the observation goes back to Ladusaw (1980)), when a negation occurs in the antecedent of a conditional sentence or in the restrictor of a universal quantifier, the scope of the negation is semantically an upward entailing context but NPIs are nevertheless licensed:

(24) a. If John doesn't know anything about logic, somebody has to help him.

b. Every spy who had never said anything to anyone was able to escape.

As emphasized by Dowty (1994), this shows that NPI licensing cannot be determined purely on the basis of the entailment properties of the linguistic expressions and that the information about the local syntactic licensor is crucial. This is correctly captured in our fragment since NPIs such as conditionals and universal quantifiers require the polarity value of the licensing environment to be underspecified, rather than imposing the stronger condition of *pol* $-$. Thus, as in the following derivation (with α a variable over morphosyntactic subtypes), negation can first take scope inside the antecedent clause of a conditional licensing an NPI and marking the clause as S_+, and this *pol* $+$ marked S can then serve as an antecedent clause of a conditional sentence.

(25)

$$
\dfrac{
\dfrac{
\dfrac{
\dfrac{
\dfrac{
\dfrac{
\dfrac{
\dfrac{
\begin{array}{l}\mathsf{know};\\ \mathbf{know};\\ \mathrm{VP}_-/\mathrm{NP}\end{array}
\quad
\begin{bmatrix}\varphi_1;\\ x;\\ \mathrm{NP}\end{bmatrix}^1
}{
\begin{array}{l}\mathsf{know}\circ\varphi_1;\\ \mathbf{know}(x);\mathrm{VP}_-\end{array}
}
\quad
\begin{bmatrix}\varphi_2;\\ f;\\ \mathrm{VP}_-/\mathrm{VP}_-\end{bmatrix}^2
}{
\varphi_2\circ\mathsf{know}\circ\varphi_1;\ f(\mathbf{know}(x));\ \mathrm{VP}_-
}
\quad
\begin{array}{l}\mathsf{john};\\ \mathbf{j};\\ \mathrm{NP}\end{array}
}{
\begin{array}{l}\mathsf{john}\circ\varphi_2\circ\mathsf{know}\circ\varphi_1;\\ f(\mathbf{know}(x))(\mathbf{j});\mathrm{S}_-\end{array}
}
}{
\begin{array}{l}\lambda\varphi_1.\mathsf{john}\circ\varphi_2\circ\mathsf{know}\circ\varphi_1;\\ \lambda x.f(\mathbf{know}(x))(\mathbf{j});\mathrm{S}_-\upharpoonright\mathrm{NP}\end{array}
}{\upharpoonright\mathrm{I}^1}
\quad
\begin{array}{l}\lambda\sigma.\sigma(\mathsf{anything});\\ \boldsymbol{\exists}_{\mathbf{th}};\\ \mathrm{S}_-\upharpoonright(\mathrm{S}_-\upharpoonright\mathrm{NP})\end{array}
}{
\begin{array}{l}\mathsf{john}\circ\varphi_2\circ\mathsf{know}\circ\mathsf{anything};\\ \boldsymbol{\exists}_{\mathbf{th}}(\lambda x.f(\mathbf{know}(x))(\mathbf{j}));\mathrm{S}_-\end{array}
}
}{
\begin{array}{l}\lambda\varphi_2.\mathsf{john}\circ\varphi_2\circ\mathsf{know}\circ\mathsf{anything};\\ \lambda f.\boldsymbol{\exists}_{\mathbf{th}}(\lambda x.f(\mathbf{know}(x))(\mathbf{j}));\mathrm{S}_-\upharpoonright(\mathrm{VP}_-/\mathrm{VP}_-)\end{array}
}{\upharpoonright\mathrm{I}^2}
\quad
\begin{array}{l}\lambda\sigma.\sigma(\mathsf{d\breve{o}esn't});\\ \lambda\mathscr{F}.\neg\mathscr{F}(\mathbf{id}_{et});\\ \mathrm{S}_+\upharpoonright\\ \quad(\mathrm{S}_-\upharpoonright(\mathrm{VP}_-/\mathrm{VP}_-))\end{array}
}{
\mathsf{john}\circ\mathsf{d\breve{o}esn't}\circ\mathsf{know}\circ\mathsf{anything};\ \neg\boldsymbol{\exists}_{\mathbf{th}}(\lambda x.\mathbf{know}(x)(\mathbf{j}));\ \mathrm{S}_+
}
\quad
\begin{array}{l}\mathsf{if};\\ \lambda p\lambda q.\\ \quad p\to q;\\ \mathrm{S}_\beta/\mathrm{S}_+/\mathrm{S}_\alpha\end{array}
}{
\mathsf{if}\circ\mathsf{john}\circ\mathsf{d\breve{o}esn't}\circ\mathsf{know}\circ\mathsf{anything};\ \lambda q.\neg\boldsymbol{\exists}_{\mathbf{th}}(\lambda x.\mathbf{know}(x)(\mathbf{j}))\to q;\ \mathrm{S}_\beta/\mathrm{S}_+
}
$$

In view of this general pattern of NPI licensing, the behavior of the verb *doubt* appears at first sight to be somewhat anomalous in one respect. As noted by Bernardi (2002), *doubt* is an NPI licensor, but its licensing property is flipped when the verb is directly negated by a syntactically local negation:[19]

[18] The verb *reject* seems to behave in a similar way as *doubt*.

[19] One may question the judgment on (26a) with *someone*, on the basis of the fact that examples such as (i) can be readily found.

(i) The skeptics—myself included—doubted that something as weird as "Guardians of the Galaxy" could be a hit in the mainstream (`https://www.newspressnow.com/life/st_joe_live/the_shuffle/the-shuffle-guardian-greatness/article_7cb22029-d9d7-54cb-8b88-c4eb931637ec.`

(26) a. John doubted that {anyone/*someone} left.

b. John didn't doubt that {*anyone/someone} left.

The flipping property of *doubt* in (26b) is unexpected since in all other cases a higher downward entailing operator does not affect the licensing property of a lower operator.

In the present system of NPI licensing we can capture the different properties of *doubt* and other licensors simply via a slightly different lexical specification in the polarity feature. Specifically, unlike other licensors that specify a fixed or underspecified value for the *pol* feature for the licensing environment, *doubt* has the following specification in the lexicon:

(27) $\textsf{doubt};\ \mathbf{doubt};\ \mathrm{VP}_{\mathbf{rev}(\alpha)}/\mathrm{S}_{\alpha}$

where $\mathbf{rev}(+) = -$ and $\mathbf{rev}(-) = +$. We then have the following derivation for (26b):[20]

(28)

$$
\text{②}\!\rightarrow\dfrac{
 \dfrac{
  \dfrac{
   \text{①}\!\rightarrow\dfrac{
    \dfrac{\begin{array}{l}\textsf{left};\\ \mathbf{leave};\mathrm{VP}_-\end{array} \quad \begin{bmatrix}\varphi_1;\\ y;\mathrm{NP}\end{bmatrix}^1}{\varphi_1\circ\textsf{left};\ \mathbf{leave}(y);\ \mathrm{S}_-}
   }{\lambda\varphi_1.\varphi_1\circ\textsf{left};\ \lambda y.\mathbf{leave}(y);\ \mathrm{S}_-{\upharpoonright}\mathrm{NP}}
   \quad
   \begin{array}{l}\lambda\sigma.\sigma(\textsf{anyone});\\ \exists_{\mathbf{pers}};\\ \mathrm{S}_-{\upharpoonright}(\mathrm{S}_-{\upharpoonright}\mathrm{NP})\end{array}
  }{\textsf{anyone}\circ\textsf{left};\ \exists_{\mathbf{pers}}(\lambda y.\mathbf{leave}(y));\ \mathrm{S}_-}
  \quad
  \begin{array}{l}\textsf{doubt};\\ \mathbf{doubt};\\ \mathrm{VP}_{\mathbf{rev}(\alpha)}/\mathrm{S}_\alpha\end{array}
 }{\textsf{doubt}\circ\textsf{anyone}\circ\textsf{left};\ \mathbf{doubt}(\exists_{\mathbf{pers}}(\lambda y.\mathbf{leave}(y)));\ \mathrm{VP}_+}
 \quad
 \begin{array}{l}\vdots\\ \textsf{didn't};\\ \lambda P\lambda y.\neg P(y);\\ \mathrm{VP}_+/\mathrm{VP}_-\end{array}
}{\mathbf{FAIL}}
$$

In order to satisfy the polarity requirement of *anyone* at ①, hypothetical reasoning must yield an S_- sign, which *doubt* will then map to VP_+ at ②, making it ineligible as an argument for *didn't.*

Compare this outcome with the derivation for the same sentence with (non-specific) *someone*:

(29)

$$
\dfrac{
 \dfrac{
  \text{①}\!\rightarrow\dfrac{
   \begin{array}{l}\textsf{doubt};\\ \mathbf{doubt};\\ \mathrm{VP}_{\mathbf{rev}(\alpha)}/\mathrm{S}_\alpha\end{array}
   \quad
   \begin{array}{l}\vdots\\ \textsf{someone}\circ\textsf{left};\\ \exists_{\mathbf{pers}}(\lambda x.\mathbf{leave}(x));\mathrm{S}_+\end{array}
  }{\begin{array}{l}\textsf{doubt}\circ\textsf{someone}\circ\textsf{left};\\ \mathbf{doubt}(\exists_{\mathbf{pers}}(\lambda x.\mathbf{leave}(x)));\mathrm{VP}_-\end{array}}
  \quad
  \begin{array}{l}\vdots\\ \textsf{didn't};\\ \lambda P\lambda y.\neg P(y);\\ \mathrm{VP}_+/\mathrm{VP}_-\end{array}
 }{\textsf{didn't}\circ\textsf{doubt}\circ\textsf{someone}\circ\textsf{left};\ \lambda y.\neg\mathbf{doubt}(\exists_{\mathbf{pers}}(\lambda x.\mathbf{leave}(x)))(y);\ \mathrm{VP}_+}
 \quad
 \begin{array}{l}\textsf{john};\\ \mathbf{j};\\ \mathrm{NP}\end{array}
}{\textsf{john}\circ\textsf{didn't}\circ\textsf{doubt}\circ\textsf{someone}\circ\textsf{left};\ \neg\mathbf{doubt}(\exists_{\mathbf{pers}}(\lambda x.\mathbf{leave}(x)))(\mathbf{j});\ \mathrm{S}_+}
$$

Note in this connection that if *didn't* were not included in the proof, the result would be an S_- sign. Thus, the ungrammaticality of (26a) with *someone* (on the non-specific reading) is also correctly predicted.

On this analysis, we predict the goodness of (30).

`html`)

But note that the indefinite in (i) is a specific indefinite; in contrast, a true nonspecific indefinite such as *someone or other* cannot appear in the scope of *doubt*:

(ii) I had always doubted that {*someone or other/anyone} would notice the glaring plot inconsistency in the third act.

This pattern suggests that the specific indefinite in (i) requires a different syntactic treatment from the nonspecific version in (ii), with the latter alone having the status of a PPI.

[20] For the sake of perspicuousness we use the lower order version of *didn't* here, but note that this does not affect the outcome of the proof. With the higher-order (12c), the derivation still fails since (12c) requires the 'gap' VP/VP category to have *pol*− specification.

(30) Mary doubted that John was guilty or that anything he had done suggested complicity in the crime.

Given the underspecification of *pol* values for the root S in lexical entries for verbs, we can derive the first conjunct of the clausal complement to *doubt* as S_-. This clause will then be conjoinable with the second, whose S_- value is enforced by composition with *anything*, and *doubt* then takes this coordinate S_- clause to VP_+.

There is an interesting contrast between the behavior of *doubt* on the one hand and *regret* on the other. *Regret* is an NPI licensor, rather than a polarity reversing operator like *doubt*; that is, it has the type $\mathrm{VP}_\alpha/\mathrm{S}_-$. This can be seen from the fact that *regret* licenses *any* regardless of whether it is under the scope of negation (examples like (31) can be readily found by Google search):

(31) a. I've come to regret saying anything about that.
b. I don't regret saying anything about that.

In the case of (31a), *regret* will inhabit the type $\mathrm{VP}_+/\mathrm{S}_-$, whereas (31b) is derived by instantiating the type of *regret* as $\mathrm{VP}_-/\mathrm{S}_-$, requiring an outer polarity licensor such as *didn't* to flip the type of the final VP to VP_+.

3.4 NPI licensing intervention by universal quantifiers

As noted by Bernardi (2002, 75), universal quantifiers exhibit an intervention effect on NPI licensing. Thus, on the $\mathbf{doubt} > \boldsymbol{\forall} > \boldsymbol{\exists}$ scoping, (32a) is unacceptable, even though the licensor *doubt* c-commands *any*. Intervention is closely related to the relative scope of operators. Thus, if the NPI indefinite *any* outscopes the universal (that is, on the $\mathbf{doubt} > \boldsymbol{\exists} > \boldsymbol{\forall}$ reading), the sentence is acceptable. This 'suspension' of intervention is most clearly noticeable in examples like (32b), which is structurally identical to (32a), but with the indefinite in focus.

(32) a. John doubts every boy reads any book.
b. John doubts that every boy read ANY (particular) book.

Bernardi notes that Dowty's (1994) account fails to capture the NPI licensing intervention effect in (32a). Our approach overcomes this difficulty and correctly predicts the NPI intervention effect and its suspension in (32). The only derivation possible for (32a) in our approach is the following, which assigns the reading in which the indefinite outscopes the structurally higher universal ($\mathbf{doubt} > \boldsymbol{\exists} > \boldsymbol{\forall}$):

(33)

$$
\dfrac{
\begin{array}{l}\text{doubt};\\ \mathbf{doubt};\\ \mathrm{VP}_{\mathbf{rev}(\alpha)}/\mathrm{S}_\alpha\end{array}
\quad
\dfrac{
\dfrac{
\dfrac{
\dfrac{
\dfrac{
\dfrac{
\dfrac{
\begin{array}{l}\text{read};\\ \mathbf{read};\\ \mathrm{TV}_+\end{array}
\quad
\begin{bmatrix}\varphi_1;\\ x;\\ \mathrm{NP}\end{bmatrix}^1
}{
\begin{array}{l}\text{read} \circ \varphi_1;\\ \mathbf{read}(x);\mathrm{VP}_+\end{array}
}
\quad
\begin{bmatrix}\varphi_2;\\ y;\\ \mathrm{NP}\end{bmatrix}^2
}{
\begin{array}{c}\varphi_2 \circ \text{read} \circ \varphi_1;\\ \mathbf{read}(x)(y);\mathrm{S}_+\end{array}
}
}{
\begin{array}{l}\lambda\varphi_2.\varphi_2 \circ \text{read} \circ \varphi_1;\\ \lambda y.\mathbf{read}(x)(y);\mathrm{S}_+{\upharpoonright}\mathrm{NP}\end{array}
}{\upharpoonright}\mathrm{I}^1
\quad
\begin{array}{l}\vdots\\ \lambda\sigma_1.\\ \quad\sigma_1(\text{every} \circ \text{boy});\\ \boldsymbol{\forall}_{\mathbf{boy}};\\ \mathrm{S}_\alpha{\upharpoonright}(\mathrm{S}_+{\upharpoonright}\mathrm{NP})\end{array}
}{
\text{every} \circ \text{boy} \circ \text{read} \circ \varphi_1;\ \boldsymbol{\forall}_{\mathbf{boy}}(\lambda y.\mathbf{read}(x)(y));\ \mathrm{S}_\alpha
}
}{
\begin{array}{l}\lambda\varphi_1.\text{every} \circ \text{boy} \circ \text{read} \circ \varphi_1;\\ \lambda x.\boldsymbol{\forall}_{\mathbf{boy}}(\lambda y.\mathbf{read}(x)(y));\mathrm{S}_\alpha{\upharpoonright}\mathrm{NP}\end{array}
}{\upharpoonright}\mathrm{I}^2
\quad
\begin{array}{l}\vdots\\ \lambda\sigma_2.\\ \quad\sigma_2(\text{any} \circ \text{book});\\ \boldsymbol{\exists}_{\mathbf{book}};\\ \mathrm{S}_-{\upharpoonright}(\mathrm{S}_-{\upharpoonright}\mathrm{NP})\end{array}
}{
\text{every} \circ \text{boy} \circ \text{read} \circ \text{any} \circ \text{book};\ \boldsymbol{\exists}_{\mathbf{book}}(\lambda x.\boldsymbol{\forall}_{\mathbf{boy}}(\lambda y.\mathbf{read}(x)(y)));\ \mathrm{S}_-
}
\quad
\begin{array}{l}\text{that};\\ \lambda p.p;\\ \mathrm{S}_\alpha{\upharpoonright}\mathrm{S}_\alpha\end{array}
}{
\text{that} \circ \text{every} \circ \text{boy} \circ \text{read} \circ \text{any} \circ \text{book};\ \boldsymbol{\exists}_{\mathbf{book}}(\lambda x.\boldsymbol{\forall}_{\mathbf{boy}}(\lambda y.\mathbf{read}(x)(y)));\ \mathrm{S}_-
}
}{
\text{doubt} \circ \text{that} \circ \text{every} \circ \text{boy} \circ \text{read} \circ \text{any} \circ \text{book};\ \mathbf{doubt}(\boldsymbol{\exists}_{\mathbf{book}}(\lambda x.\boldsymbol{\forall}_{\mathbf{boy}}(\lambda y.\mathbf{read}(x)(y))));\ \mathrm{VP}_+
}
$$

Crucially, the reverse scoping order between the indefinite and the universal fails:

(34)

$$\dfrac{\dfrac{\dfrac{\dfrac{\begin{array}{c}\vdots\\ \varphi_2 \circ \mathsf{read} \circ \varphi_1;\ \mathbf{read}(x)(y);\ \mathrm{S}_-\end{array}}{\begin{array}{l}\lambda\varphi_1.\varphi_2 \circ \mathsf{read} \circ \varphi_1;\\ \lambda x.\mathbf{read}(x)(y); \mathrm{S}_-\upharpoonright\mathrm{NP}\end{array}} \quad \begin{array}{l}\lambda\sigma_2.\sigma_2(\mathsf{any} \circ \mathsf{book});\\ \exists_{\mathbf{book}};\\ \mathrm{S}_-\upharpoonright(\mathrm{S}_-\upharpoonright\mathrm{NP})\end{array}}{\varphi_2 \circ \mathsf{read} \circ \mathsf{any} \circ \mathsf{book};\ \exists_{\mathbf{book}}(\lambda x.\mathbf{read}(x)(y));\ \mathrm{S}_-}}{\lambda\varphi_2.\varphi_2 \circ \mathsf{read} \circ \mathsf{any} \circ \mathsf{book};\ \lambda y.\exists_{\mathbf{book}}(\lambda x.\mathbf{read}(x)(y));\ \mathrm{S}_-\upharpoonright\mathrm{NP}} \quad \begin{array}{l}\lambda\sigma_1.\sigma_1(\mathsf{every} \circ \mathsf{boy});\\ \forall_{\mathbf{boy}};\\ \mathrm{S}_\alpha\upharpoonright(\mathrm{S}_+\upharpoonright\mathrm{NP})\end{array}}{\textbf{FAIL}}$$

Here, the narrow scope for the indefinite *any* forces the sentence to be marked S_-. But then, the universal quantifier *every* can't scope over such a sentence since it specifies its scope to be S_+. Thus, our account correctly predicts that on the $\mathbf{doubt} > \forall > \exists$ surface scope reading, (32a) is ill-formed due to the intervention of NPI licensing by *every*.

4 The semantic interaction of negation with modal auxiliaries

We have shown above that the syntactic marking of polarity information by the binary *pol* feature suffices to capture the basic NPI licensing properties of both logical operators (such as conditionals and universal quantifiers) and lexical verbs such as *doubt* and *reject*. In the rest of this paper, we consider wider ramifications of this analysis of NPI licensing, focusing on phenomena involving modal auxiliaries in English. We start this discussion with the semantic interaction of negation with modal auxiliaries in this section.

Modal auxiliaries tend to have fixed scope relations with VP level negation, as illustrated by the following examples:

(35)
a. John should not criticize Mary. $(\Box\neg\mathbf{criticize}(\mathbf{m})(\mathbf{j}))$
b. John need not criticize Mary. $(\neg\Box\,\mathbf{criticize}(\mathbf{m})(\mathbf{j}))$
c. John may not criticize Mary. $(\Diamond\neg\mathbf{criticize}(\mathbf{m})(\mathbf{j}), \neg\Diamond\,\mathbf{criticize}(\mathbf{m})(\mathbf{j}))$

Iatridou and Zeijlstra (2013) summarize the pattern as in the following table, and account for the observed scoping patterns in terms of the polarity sensitivity of different types of modal auxiliaries.

(36)

	PPI modals	Neutral modals	NPI modals
Universal	*must*, *should*, *ought to*, *to be to*	*have to*, *need to*	*need*
Existential	—	*can*, *may*	—

Kubota and Levine (2019, 2020) recast the reconstruction-based analysis by Iatridou and Zeijlstra (2013) by controlling the scopal properties of modal and negation operators in the type logic of Hybrid TLG. Here, we reformulate our earlier analysis that made use of a three-way distinction in the value of the *pol* feature in a simpler setup introduced in the previous section that employs a more standard binary *pol* feature. Specifically, we posit the following lexical entries for modal auxiliaries in English:

(37)
a. $\lambda\sigma.\sigma(\mathsf{should});\ \lambda\mathscr{F}.\Box\mathscr{F}(\mathbf{id}_{et});\ \mathrm{S}_{f,+}\upharpoonright(\mathrm{S}_{f,+}\upharpoonright(\mathrm{VP}_{f,+}/\mathrm{VP}_{b,+}))$
b. $\lambda\sigma.\sigma(\mathsf{need});\ \lambda\mathscr{F}.\Box\mathscr{F}(\mathbf{id}_{et});\ \mathrm{S}_{f,-}\upharpoonright(\mathrm{S}_{f,-}\upharpoonright(\mathrm{VP}_{f,-}/\mathrm{VP}_{b,-}))$
c. $\lambda\sigma.\sigma(\mathsf{can});\ \lambda\mathscr{F}.\Diamond\mathscr{F}(\mathbf{id}_{et});\ \mathrm{S}_{f,\alpha}\upharpoonright(\mathrm{S}_{f,\alpha}\upharpoonright(\mathrm{VP}_{f,\alpha}/\mathrm{VP}_{b,\alpha}))$ (where $\alpha \in \{+,-\}$)
d. $\lambda\sigma.\sigma(\mathsf{not});\ \lambda\mathscr{F}.\neg\mathscr{F}(\mathbf{id}_{et});\ \mathrm{S}_{\gamma,+}\upharpoonright(\mathrm{S}_{\gamma,-}\upharpoonright(\mathrm{VP}_{b,-}/\mathrm{VP}_{b,-}))$

In what follows, we demonstrate in detail the way the feature specifications in (37) interact to yield the correct scoping relations summarized in (36), but the key idea is simple. The positive polarity auxiliary *should* takes scope at the S_+ level; thus, it never scopes below a clausemate negation. By contrast, the negative polarity auxiliary *need* takes scope at the S_- level, that is, below negation in sentences that have overt morphological negation or in the scope of other NPI licensors (e.g., *if you need open the door, ...*). The negation morpheme has the function of turning a negative polarity environment S_- to a positive-polarity environment S_+ (which can serve as a standalone sentence), just like the negative auxiliary *didn't* already introduced.

We start with the derivation for the PPI auxiliary *should*.

(38)

$$
\begin{prooftree}
\AxiomC{$\lambda\sigma.\sigma(\mathsf{should}); \lambda\mathscr{F}.\Box\mathscr{F}(\mathbf{id}_{et}); \mathrm{S}_{f,+}\restriction(\mathrm{S}_{f,+}\restriction(\mathrm{VP}_{f,+}/\mathrm{VP}_{b,+}))$}
\AxiomC{$\mathsf{john}; \mathbf{j}; \mathrm{NP}$}
\AxiomC{$\begin{bmatrix}\varphi_3;\\ g;\\ \mathrm{VP}_{f,+}/\mathrm{VP}_{b,+}\end{bmatrix}^3$}
\AxiomC{$\lambda\sigma.\sigma(\mathsf{not}); \lambda\mathscr{F}.\neg\mathscr{F}(\mathbf{id}_{et}); \mathrm{S}_{\gamma,+}\restriction(\mathrm{S}_{\gamma,-}\restriction(\mathrm{VP}_{b,-}/\mathrm{VP}_{b,-}))$}
\AxiomC{$\begin{bmatrix}\varphi_2;\\ x;\\ \mathrm{NP}\end{bmatrix}^2$}
\AxiomC{$\begin{bmatrix}\varphi_1;\\ f;\\ \mathrm{VP}_{b,-}/\mathrm{VP}_{b,-}\end{bmatrix}^1$}
\AxiomC{$\mathsf{testify}; \mathbf{testify}; \mathrm{VP}_{b,-}$}
\BinaryInfC{$\varphi_1 \circ \mathsf{testify}; f(\mathbf{testify}); \mathrm{VP}_{b,-}$}
\BinaryInfC{$\varphi_2 \circ \varphi_1 \circ \mathsf{testify}; f(\mathbf{testify})(x); \mathrm{S}_{b,-}$}
\RightLabel{$\restriction\mathrm{I}^1$}
\UnaryInfC{$\lambda\varphi_1.\varphi_2 \circ \varphi_1 \circ \mathsf{testify}; \lambda f.f(\mathbf{testify})(x); \mathrm{S}_{b,-}\restriction(\mathrm{VP}_{b,-}/\mathrm{VP}_{b,-})$}
\BinaryInfC{$\varphi_2 \circ \mathsf{not} \circ \mathsf{testify}; \neg\mathbf{testify}(x); \mathrm{S}_{b,+}$}
\RightLabel{$\backslash\mathrm{I}^2$}
\UnaryInfC{$\mathsf{not} \circ \mathsf{testify}; \lambda x.\neg\mathbf{testify}(x); \mathrm{VP}_{b,+}$}
\BinaryInfC{$\varphi_3 \circ \mathsf{not} \circ \mathsf{testify}; g(\lambda x.\neg\mathbf{testify}(x)); \mathrm{VP}_{f,+}$}
\BinaryInfC{$\mathsf{john} \circ \varphi_3 \circ \mathsf{not} \circ \mathsf{testify}; g(\lambda x.\neg\mathbf{testify}(x))(\mathbf{j}); \mathrm{S}_{f,+}$}
\RightLabel{$\restriction\mathrm{I}^3$}
\UnaryInfC{$\lambda\varphi_3.\mathsf{john} \circ \varphi_3 \circ \mathsf{not} \circ \mathsf{testify}; \lambda g.g(\lambda x.\neg\mathbf{testify}(x))(\mathbf{j}); \mathrm{S}_{f,+}\restriction(\mathrm{VP}_{f,+}/\mathrm{VP}_{b,+})$}
\BinaryInfC{$\mathsf{john} \circ \mathsf{should} \circ \mathsf{not} \circ \mathsf{testify}; \Box\neg\mathbf{testify}(\mathbf{j}); \mathrm{S}_{f,+}$}
\end{prooftree}
$$

In this derivation, the negation morpheme first takes scope via hypothetical reasoning for a VP/VP expression, via the 'movement-like' mechanism introduced in the previous section. After this, the PPI modal takes scope via the same mechanism. The scopal relation between these operators corresponds to the order in which they are introduced in the derivation, in the same way that the structure of LF transparently reflects the scopal relation between operators in approaches that recognize the level of LF as a component of grammar. An alternative scoping relation in which the negation scopes over the PPI modal is blocked due to the polarity specifications of these items. The only way to obtain that interpretation is to switch the order in which *should* and *not* are introduced in the derivation in (38). However, the derivation would crash at the point where the negation morpheme is introduced. Given the lexical polarity marking, *should* marks the S that contains it to be *pol* $+$. But this conflicts with the lexical specification of the negation morpheme which requires its scope to be marked *pol* $-$. Thus, it is correctly guaranteed that we get only the $\Box > \neg$ scopal relation.

Turning now to the NPI modals, the polarity markings for these auxiliaries impose exactly the opposite restriction in terms of the relative scope relation with negation. The derivation is given in (39).

(39)
$$
\dfrac{
 \dfrac{
  \dfrac{
   \dfrac{
    \dfrac{
     \dfrac{
      \dfrac{\begin{array}{l}\mathsf{testify};\\ \mathbf{testify};\\ \mathrm{VP}_{b,-}\end{array} \quad \begin{bmatrix}\varphi_1;\\ f;\\ \mathrm{VP}_{b,-}/\mathrm{VP}_{b,-}\end{bmatrix}^1}
      {\begin{array}{l}\varphi_1\circ\mathsf{testify};\\ f(\mathbf{testify});\mathrm{VP}_{b,-}\end{array}}
      \quad \begin{bmatrix}\varphi_2;\\ h;\\ \mathrm{VP}_{f,-}/\mathrm{VP}_{b,-}\end{bmatrix}^2}
     {\begin{array}{l}\varphi_2\circ\varphi_1\circ\mathsf{testify};\\ h(f(\mathbf{testify}));\mathrm{VP}_{f,-}\end{array}}
     \quad \begin{array}{l}\mathsf{john};\\ \mathbf{j};\\ \mathrm{NP}\end{array}}
    {\begin{array}{c}\mathsf{john}\circ\varphi_2\circ\varphi_1\circ\mathsf{testify};\\ h(f(\mathbf{testify}))(\mathbf{j});\mathrm{S}_{f,-}\end{array}}\,{\upharpoonright}\mathrm{I}^2}
   {\begin{array}{l}\lambda\varphi_2.\mathsf{john}\circ\varphi_2\circ\varphi_1\circ\mathsf{testify};\\ \lambda h.h(f(\mathbf{testify}))(\mathbf{j});\mathrm{S}_{f,-}{\upharpoonright}(\mathrm{VP}_{f,-}/\mathrm{VP}_{b,-})\end{array}}
   \quad \begin{array}{l}\lambda\sigma.\sigma(\mathsf{need});\\ \lambda\mathscr{G}.\Box\mathscr{G}(\mathbf{id}_{et});\\ \mathrm{S}_{f,-}{\upharpoonright}(\mathrm{S}_{f,-}{\upharpoonright}\\ \quad(\mathrm{VP}_{f,-}/\mathrm{VP}_{b,-}))\end{array}}
  {\dfrac{\mathsf{john}\circ\mathsf{need}\circ\varphi_1\circ\mathsf{testify};\ \Box f(\mathbf{testify})(\mathbf{j});\ \mathrm{S}_{f,-}}{\begin{array}{l}\lambda\varphi_1.\mathsf{john}\circ\mathsf{need}\circ\varphi_1\circ\mathsf{testify};\\ \lambda f.\Box f(\mathbf{testify})(\mathbf{j});\mathrm{S}_{f,-}{\upharpoonright}(\mathrm{VP}_{b,-}/\mathrm{VP}_{b,-})\end{array}}\,{\upharpoonright}\mathrm{I}^1}
  \quad \begin{array}{l}\lambda\sigma.\sigma(\mathsf{not});\\ \lambda\mathscr{G}.\neg\mathscr{G}(\mathbf{id}_{et});\\ \mathrm{S}_{f,+}{\upharpoonright}(\mathrm{S}_{f,-}{\upharpoonright}\\ \quad(\mathrm{VP}_{b,-}/\mathrm{VP}_{b,-}))\end{array}}
 {\mathsf{john}\circ\mathsf{need}\circ\mathsf{not}\circ\mathsf{testify};\ \neg\Box\mathbf{testify}(\mathbf{j});\ \mathrm{S}_{f,+}}
$$

Here, the order in which the modal and the negation are introduced in the derivation is opposite from (38), and, correspondingly, the semantic relation between these operators is reversed. Here again, the opposite scoping relation is blocked via the polarity specifications of the modal and negation. Suppose the negation morpheme were introduced in the derivation first as in (38). Then, the sentence would be marked as $pol\,+$ after negation takes scope, which would no longer be able to serve as an argument to the NPI modal *need* which requires its argument to be marked as $pol\,-$.

Thus, the polarity markings on the auxiliaries and negation in (37) correctly account for the scopal interactions between different types of modal auxiliaries and negation in English. There are moreover some immediate consequences that follow from the lexical entries posited in (37) in relation to the slanting lemma discussed at the end of section 2. Note first that the negation morpheme can be slanted to the $\mathrm{VP}_{b,+}/\mathrm{VP}_{b,-}$ category as follows:

(40)
$$
\dfrac{\dfrac{\dfrac{
 \dfrac{
  \dfrac{
   \dfrac{\begin{bmatrix}\varphi_4;\\ g;\mathrm{VP}_{b,-}\end{bmatrix}^4 \quad \begin{bmatrix}\varphi_2;\\ h;\mathrm{VP}_{b,-}/\mathrm{VP}_{b,-}\end{bmatrix}^2}{\varphi_2\circ\varphi_4;\ h(g);\ \mathrm{VP}_{b,-}}
   \quad \begin{bmatrix}\varphi_3;\\ x;\\ \mathrm{NP}\end{bmatrix}^3}
  {\varphi_3\circ\varphi_2\circ\varphi_4;\ h(g)(x);\ \mathrm{S}_{b,-}}\,{\upharpoonright}\mathrm{I}^2}
 {\lambda\varphi_2.\varphi_3\circ\varphi_2\circ\varphi_4;\ \lambda h.h(g)(x);\ \mathrm{S}_{b,-}{\upharpoonright}(\mathrm{VP}_{b,-}/\mathrm{VP}_{b,-})}
 \quad \begin{array}{l}\lambda\sigma.\sigma(\mathsf{not});\\ \lambda\mathscr{F}.\neg\mathscr{F}(\mathbf{id}_{et});\\ \mathrm{S}_{\gamma,+}{\upharpoonright}(\mathrm{S}_{\gamma,-}{\upharpoonright}(\mathrm{VP}_{b,-}/\mathrm{VP}_{b,-}))\end{array}}
{\varphi_3\circ\mathsf{not}\circ\varphi_4;\ \neg g(x);\ \mathrm{S}_{b,+}}\,\backslash\mathrm{I}^3}
{\mathsf{not}\circ\varphi_4;\ \lambda x.\neg g(x);\ \mathrm{VP}_{b,+}}\,/\mathrm{I}^4}
{\mathsf{not};\ \lambda g\lambda x.\neg g(x);\ \mathrm{VP}_{b,+}/\mathrm{VP}_{b,-}}
$$

This should make sense, given that the function of the negation morpheme is to turn the polarity value from negative to positive without any consequence for the VFORM feature.

Slanting of the PPI auxiliary *should* yields the following sign:

(41)
$$
\dfrac{\dfrac{\dfrac{
 \dfrac{
  \dfrac{
   \dfrac{\begin{bmatrix}\varphi_4;\\ g;\mathrm{VP}_{b,+}\end{bmatrix}^4 \quad \begin{bmatrix}\varphi_2;\\ h;\mathrm{VP}_{f,+}/\mathrm{VP}_{b,+}\end{bmatrix}^2}{\varphi_2\circ\varphi_4;\ h(g);\ \mathrm{VP}_{f,+}}
   \quad \begin{bmatrix}\varphi_3;\\ x;\\ \mathrm{NP}\end{bmatrix}^3}
  {\varphi_3\circ\varphi_2\circ\varphi_4;\ h(g)(x);\ \mathrm{S}_{f,+}}\,{\upharpoonright}\mathrm{I}^2}
 {\lambda\varphi_2.\varphi_3\circ\varphi_2\circ\varphi_4;\ \lambda h.h(g)(x);\ \mathrm{S}_{f,+}{\upharpoonright}(\mathrm{VP}_{f,+}/\mathrm{VP}_{b,+})}
 \quad \begin{array}{l}\lambda\sigma.\sigma(\mathsf{should});\\ \lambda\mathscr{G}.\Box\mathscr{G}(\mathbf{id}_{et});\\ \mathrm{S}_{f,+}{\upharpoonright}(\mathrm{S}_{f,+}{\upharpoonright}(\mathrm{VP}_{f,+}/\mathrm{VP}_{b,+}))\end{array}}
{\varphi_3\circ\mathsf{should}\circ\varphi_4;\ \Box g(x);\ \mathrm{S}_{f,+}}\,\backslash\mathrm{I}^3}
{\mathsf{should}\circ\varphi_4;\ \lambda x.\Box g(x);\ \mathrm{VP}_{f,+}}\,/\mathrm{I}^4}
{\mathsf{should};\ \lambda g\lambda x.\Box g(x);\ \mathrm{VP}_{f,+}/\mathrm{VP}_{b,+}}
$$

Note that the slanted entries of *should* and *not* in (41) and (40) directly compose with each other via function composition (or an equivalent theorem in the Lambek calculus) to yield the following sign that reflects the correct scopal relation between these items:

(42)

$$\cfrac{\cfrac{\vdots}{\textsf{should};\ \lambda f\lambda x.\Box f(x);\ \mathrm{VP}_{f,+}/\mathrm{VP}_{b,+}} \qquad \cfrac{\cfrac{\vdots}{\textsf{not};\ \lambda g\lambda x.\neg g(x);\ \mathrm{VP}_{b,+}/\mathrm{VP}_{b,-}} \quad [\varphi_4; g; \mathrm{VP}_{b,-}]^4}{\textsf{not}\circ\varphi_4;\ \lambda x.\neg g(x);\ \mathrm{VP}_{b,+}}}{\cfrac{\textsf{should}\circ\textsf{not}\circ\varphi_4;\ \lambda x.\Box\neg g(x);\ \mathrm{VP}_{f,+}}{\textsf{should}\circ\textsf{not};\ \lambda g\lambda x.\Box\neg g(x);\ \mathrm{VP}_{f,+}/\mathrm{VP}_{b,-}}\,/\mathrm{I}^4}$$

With the NPI modal *need*, we obtain a somewhat different result. Slanting the auxiliary *need* by itself yields the following sign:

(43)

$$\cfrac{\cfrac{\cfrac{\cfrac{\begin{bmatrix}\varphi_4;\\ g;\mathrm{VP}_{b,-}\end{bmatrix}^4 \quad \begin{bmatrix}\varphi_2;\\ h;\mathrm{VP}_{f,-}/\mathrm{VP}_{b,-}\end{bmatrix}^2}{\varphi_2\circ\varphi_4;\ h(g);\ \mathrm{VP}_{f,-}} \quad \begin{bmatrix}\varphi_3;\\ x;\\ \mathrm{NP}\end{bmatrix}^3}{\varphi_3\circ\varphi_2\circ\varphi_4;\ h(g)(x);\ \mathrm{S}_{f,-}}}{\lambda\varphi_2.\varphi_3\circ\varphi_2\circ\varphi_4;\ \lambda h.h(g)(x);\ \mathrm{S}_{f,-}\upharpoonright(\mathrm{VP}_{f,-}/\mathrm{VP}_{b,-})}\upharpoonright\mathrm{I}^2 \quad \begin{matrix}\lambda\sigma.\sigma(\textsf{need});\\ \lambda\mathscr{G}.\Box\mathscr{G}(\mathbf{id}_{et});\\ \mathrm{S}_{f,-}\upharpoonright(\mathrm{S}_{f,-}\upharpoonright(\mathrm{VP}_{f,-}/\mathrm{VP}_{b,-}))\end{matrix}}{\cfrac{\cfrac{\varphi_3\circ\textsf{need}\circ\varphi_4;\ \Box g(x);\ \mathrm{S}_{f,-}}{\textsf{need}\circ\varphi_4;\ \lambda x.\Box g(x);\ \mathrm{VP}_{f,-}}\backslash\mathrm{I}^3}{\textsf{need};\ \lambda g\lambda x.\Box g(x);\ \mathrm{VP}_{f,-}/\mathrm{VP}_{b,-}}/\mathrm{I}^4}$$

This entry can be used in a *pol* − environment (such as the antecedent clause of a conditional sentence), but unlike the slanted PPI modal *should*, it cannot directly combine with the slanted negation morpheme due to feature mismatch (negated VPs are VP_+, but *need* requires its complement to be VP_-). This is actually the desired consequence, since if the two were able to combine, then it would incorrectly be predicted that *need not* would have the same modal-outscoping interpretation as *should not*.

This, however, does not mean that the negated NPI modal *need not* cannot be associated with a lower-order VP/VP category. In fact, such an assignment is derivable as a case of slanting via the following proof:

(44)

$$\cfrac{\cfrac{\cfrac{\cfrac{\cfrac{\cfrac{\cfrac{\begin{bmatrix}\varphi_4;\\ g;\\ \mathrm{VP}_{b,-}\end{bmatrix}^4 \quad \begin{bmatrix}\varphi_1;\\ f;\\ \mathrm{VP}_{b,-}/\mathrm{VP}_{b,-}\end{bmatrix}^1}{\varphi_1\circ\varphi_4;\ f(g);\ \mathrm{VP}_{b,-}} \quad \begin{bmatrix}\varphi_2;\\ h;\\ \mathrm{VP}_{f,-}/\mathrm{VP}_{b,-}\end{bmatrix}^2}{\varphi_2\circ\varphi_1\circ\varphi_4;\ h(f(g));\ \mathrm{VP}_{f,-}} \quad \begin{bmatrix}\varphi_3;\\ x;\\ \mathrm{NP}\end{bmatrix}^3}{\varphi_3\circ\varphi_2\circ\varphi_1\circ\varphi_4;\ h(f(g))(x);\ \mathrm{S}_{f,-}}}{\begin{matrix}\lambda\varphi_2.\varphi_3\circ\varphi_2\circ\varphi_1\circ\varphi_4;\\ \lambda h.h(f(g))(x);\mathrm{S}_{f,-}\upharpoonright(\mathrm{VP}_{f,-}/\mathrm{VP}_{b,-})\end{matrix}}\upharpoonright\mathrm{I}^2 \quad \begin{matrix}\lambda\sigma.\sigma(\textsf{need});\\ \lambda\mathscr{G}.\Box\mathscr{G}(\mathbf{id}_{et});\\ \mathrm{S}_{f,-}\upharpoonright(\mathrm{S}_{f,-}\upharpoonright\\ (\mathrm{VP}_{f,-}/\mathrm{VP}_{b,-}))\end{matrix}}{\varphi_3\circ\textsf{need}\circ\varphi_1\circ\varphi_4;\ \Box f(g)(x);\ \mathrm{S}_{f,-}}}{\begin{matrix}\lambda\varphi_1.\varphi_3\circ\textsf{need}\circ\varphi_1\circ\varphi_4;\\ \lambda f.\Box f(g)(x);\mathrm{S}_{f,-}\upharpoonright(\mathrm{VP}_{b,-}/\mathrm{VP}_{b,-})\end{matrix}}\upharpoonright\mathrm{I}^1 \quad \begin{matrix}\lambda\sigma.\sigma(\textsf{not});\\ \lambda\mathscr{G}.\neg\mathscr{G}(\mathbf{id}_{et});\\ \mathrm{S}_{f,+}\upharpoonright(\mathrm{S}_{f,-}\upharpoonright\\ (\mathrm{VP}_{b,-}/\mathrm{VP}_{b,-}))\end{matrix}}{\cfrac{\cfrac{\varphi_3\circ\textsf{need}\circ\textsf{not}\circ\varphi_4;\ \neg\Box g(x);\ \mathrm{S}_{f,+}}{\textsf{need}\circ\textsf{not}\circ\varphi_4;\ \lambda x.\neg\Box g(x);\ \mathrm{VP}_{f,+}}\upharpoonright\mathrm{I}^3}{\textsf{need}\circ\textsf{not};\ \lambda g\lambda x.\neg\Box g(x);\ \mathrm{VP}_{f,+}/\mathrm{VP}_{b,-}}\upharpoonright\mathrm{I}^4}$$

Note in particular that the right negation-outscoping interpretation is assigned to this derived linguistic sign corresponding to the string *need not*.

Note that under our approach, Barker's (2018) interpretation of NPIs as signaling narrow scope interpretation emerges as an epiphenomenon of the lexical specifications for NPIs. For example, in (45), *anything* obligatorily scopes below negation.

(45) John doesn't want anything

In order to license the indefinite wide-scope interpretation, it would be necessary to have *didn't* compose into the derivation first, giving rise to an $\mathrm{S}_+ \upharpoonright \mathrm{NP}$. But since the type assigned to *anything* is $\mathrm{S}_- \upharpoonright (\mathrm{S}_- \upharpoonright \mathrm{NP})$, the type mismatch with an $\mathrm{S}_+ \upharpoonright \mathrm{NP}$ argument means that it will not be possible to introduce this NPI at a later step.

One interesting outcome of this is the possibility of inverse scoping for the NPI modal *need* in examples such as (46).

(46) With this last minute bailout, the company need fire no employee.

The scopal order here is $\neg > \Box > \exists$, which follows from the higher order sign we posit in Kubota and Levine (2016, 2020) for the negative quantifier *no* and its related forms, as per (47), which show how inverse NPI licensing proceeds directly under the analysis outlined to this point (with $\alpha \in \{-,+\}$ and $\gamma \in \{b, f\}$, and where $\mathrm{Det} = \mathrm{S}_{\delta,\beta} \upharpoonright (\mathrm{S}_{\delta,\beta} \upharpoonright \mathrm{NP}) \upharpoonright \mathrm{N}$):[21]

(47)

$$\frac{\mathsf{fire};\ \mathbf{fire};\ \mathrm{TV}_{b,-} \qquad [\varphi_1;\ x;\ \mathrm{NP}]^1}{\mathsf{fire} \circ \varphi_1;\ \mathbf{fire}(x);\ \mathrm{VP}_{b,-}}\ /\mathrm{E}$$

$$\frac{[\varphi_2;\ f;\ \mathrm{VP}_{b,-}/\mathrm{VP}_{b,-}]^2 \qquad \mathsf{fire} \circ \varphi_1;\ \mathbf{fire}(x);\ \mathrm{VP}_{b,-}}{\varphi_2 \circ \mathsf{fire} \circ \varphi_1;\ f(\mathbf{fire}(x));\ \mathrm{VP}_{b,-}}\ /\mathrm{E}$$

$$\frac{\mathsf{the} \circ \mathsf{company};\ \iota(\mathbf{c});\ \mathrm{NP} \qquad \varphi_2 \circ \mathsf{fire} \circ \varphi_1;\ f(\mathbf{fire}(x));\ \mathrm{VP}_{b,-}}{\mathsf{the} \circ \mathsf{company} \circ \varphi_2 \circ \mathsf{fire} \circ \varphi_1;\ f(\mathbf{fire}(x))(\iota(\mathbf{c}));\ \mathrm{S}_{b,-}}\ \backslash\mathrm{E}$$

$$\frac{\mathsf{the} \circ \mathsf{company} \circ \varphi_2 \circ \mathsf{fire} \circ \varphi_1;\ f(\mathbf{fire}(x))(\iota(\mathbf{c}));\ \mathrm{S}_{b,-}}{\lambda\varphi_1.\mathsf{the} \circ \mathsf{company} \circ \varphi_2 \circ \mathsf{fire} \circ \varphi_1;\ \lambda x.f(\mathbf{fire}(x))(\iota(\mathbf{c}));\ \mathrm{S}_{b,-} \upharpoonright \mathrm{NP}}\ \upharpoonright\mathrm{I}^1$$

$$\frac{[\tau;\ \mathscr{F};\ \mathrm{Det}]^3 \qquad \mathsf{employee};\ \mathbf{emp};\ \mathrm{N}}{\tau(\mathsf{employee});\ \mathscr{F}(\mathbf{emp});\ \mathrm{S}_{b,-} \upharpoonright (\mathrm{S}_{b,-} \upharpoonright \mathrm{NP})}\ \upharpoonright\mathrm{E}$$

$$\frac{\tau(\mathsf{employee});\ \mathscr{F}(\mathbf{emp});\ \mathrm{S}_{b,-} \upharpoonright (\mathrm{S}_{b,-} \upharpoonright \mathrm{NP}) \qquad \lambda\varphi_1.\mathsf{the} \circ \mathsf{company} \circ \varphi_2 \circ \mathsf{fire} \circ \varphi_1;\ \lambda x.f(\mathbf{fire}(x))(\iota(\mathbf{c}));\ \mathrm{S}_{b,-} \upharpoonright \mathrm{NP}}{\tau(\mathsf{employee})(\lambda\varphi_1.\mathsf{the} \circ \mathsf{company} \circ \varphi_2 \circ \mathsf{fire} \circ \varphi_1);\ \mathscr{F}(\mathbf{emp})(\lambda x.f(\mathbf{fire}(x))(\iota(\mathbf{c})));\ \mathrm{S}_{b,-}}\ \upharpoonright\mathrm{E}$$

$$\frac{\tau(\mathsf{employee})(\lambda\varphi_1.\mathsf{the} \circ \mathsf{company} \circ \varphi_2 \circ \mathsf{fire} \circ \varphi_1);\ \mathscr{F}(\mathbf{emp})(\lambda x.f(\mathbf{fire}(x))(\iota(\mathbf{c})));\ \mathrm{S}_{b,-}}{\lambda\varphi_2.\tau(\mathsf{employee})(\lambda\varphi_1.\mathsf{the} \circ \mathsf{company} \circ \varphi_2 \circ \mathsf{fire} \circ \varphi_1);\ \lambda f.\mathscr{F}(\mathbf{emp})(\lambda x.f(\mathbf{fire}(x))(\iota(\mathbf{c})));\ \mathrm{S}_{b,-} \upharpoonright (\mathrm{VP}_{b,-}/\mathrm{VP}_{b,-})}\ \upharpoonright\mathrm{I}^2$$

$$\frac{\lambda\sigma.\sigma(\mathsf{need});\ \lambda\mathscr{G}.\Box_{\mathbf{need}}\mathscr{G}(\mathbf{id}_{et});\ \mathrm{S}_{f,-} \upharpoonright (\mathrm{S}_{b,-} \upharpoonright (\mathrm{VP}_{b,-}/\mathrm{VP}_{b,-})) \qquad \lambda\varphi_2.\tau(\mathsf{employee})(\lambda\varphi_1.\mathsf{the} \circ \mathsf{company} \circ \varphi_2 \circ \mathsf{fire} \circ \varphi_1);\ \lambda f.\mathscr{F}(\mathbf{emp})(\lambda x.f(\mathbf{fire}(x))(\iota(\mathbf{c})));\ \mathrm{S}_{b,-} \upharpoonright (\mathrm{VP}_{b,-}/\mathrm{VP}_{b,-})}{\tau(\mathsf{employee})(\lambda\varphi_1.\mathsf{the} \circ \mathsf{company} \circ \mathsf{need} \circ \mathsf{fire} \circ \varphi_1);\ \Box_{\mathbf{need}}\mathscr{F}(\mathbf{emp})(\lambda x.\mathbf{fire}(x)(\iota(\mathbf{c})));\ \mathrm{S}_{f,-}}\ \upharpoonright\mathrm{E}$$

$$\frac{\tau(\mathsf{employee})(\lambda\varphi_1.\mathsf{the} \circ \mathsf{company} \circ \mathsf{need} \circ \mathsf{fire} \circ \varphi_1);\ \Box_{\mathbf{need}}\mathscr{F}(\mathbf{emp})(\lambda x.\mathbf{fire}(x)(\iota(\mathbf{c})));\ \mathrm{S}_{f,-}}{\lambda\tau.\tau(\mathsf{employee})(\lambda\varphi_1.\mathsf{the} \circ \mathsf{company} \circ \mathsf{need} \circ \mathsf{fire} \circ \varphi_1);\ \lambda\mathscr{F}.\Box_{\mathbf{need}}\mathscr{F}(\mathbf{emp})(\lambda x.\mathbf{fire}(x)(\iota(\mathbf{c})));\ \mathrm{S}_{f,-} \upharpoonright (\mathrm{S}_{b,-} \upharpoonright (\mathrm{S}_{b,-} \upharpoonright \mathrm{NP}) \upharpoonright \mathrm{N})}\ \upharpoonright\mathrm{I}^3$$

$$\frac{\lambda\rho.\rho(\lambda\varphi\lambda\sigma.\sigma(\mathsf{no} \circ \varphi));\ \lambda\mathscr{P}.\neg\mathscr{P}(\mathbf{\exists});\ \mathrm{S}_{\gamma,+} \upharpoonright (\mathrm{S}_{\gamma,\alpha} \upharpoonright \mathrm{Det}) \qquad \lambda\tau.\tau(\mathsf{employee})(\lambda\varphi_1.\mathsf{the} \circ \mathsf{company} \circ \mathsf{need} \circ \mathsf{fire} \circ \varphi_1);\ \lambda\mathscr{F}.\Box_{\mathbf{need}}\mathscr{F}(\mathbf{emp})(\lambda x.\mathbf{fire}(x)(\iota(\mathbf{c})));\ \mathrm{S}_{f,-} \upharpoonright (\mathrm{S}_{b,-} \upharpoonright (\mathrm{S}_{b,-} \upharpoonright \mathrm{NP}) \upharpoonright \mathrm{N})}{\mathsf{the} \circ \mathsf{company} \circ \mathsf{need} \circ \mathsf{fire} \circ \mathsf{no} \circ \mathsf{employee};\ \neg\Box_{\mathbf{need}}\mathbf{\exists}_{\mathbf{emp}}(\lambda x.\mathbf{fire}(x)(\iota(\mathbf{c})));\ \mathrm{S}_{f,+}}\ \upharpoonright\mathrm{E}$$

A potential difficulty with this treatment of inverse NPI licensing is that it does not in itself explain why other common items such as *any* fail to appear in the same environments that license *need*:

(48) John has (*ever) worried (*at any time) about nothing.

[21] An interesting consequence of the calculus of Hybrid TLG is that, *no* can be slanted down to its lower order GQ form of type $\mathrm{S} \upharpoonright (\mathrm{S} \upharpoonright \mathrm{NP}) \upharpoonright \mathrm{N}$, as in the following derivation (from which it further follows that it can be slanted down to Lambek-type quantifier entries in subject, object positions, etc.):

(i)

$$\frac{[\tau;\ \mathscr{P};\ \mathrm{Det}]^1 \qquad [\varphi;\ P;\ \mathrm{N}]^2}{\tau(\varphi);\ \mathscr{P}(P);\ \mathrm{S} \upharpoonright (\mathrm{S} \upharpoonright \mathrm{NP})}\ \upharpoonright\mathrm{E}$$

$$\frac{\tau(\varphi);\ \mathscr{P}(P);\ \mathrm{S} \upharpoonright (\mathrm{S} \upharpoonright \mathrm{NP}) \qquad [\sigma;\ Q;\ \mathrm{S} \upharpoonright \mathrm{NP}]^3}{\tau(\varphi)(\sigma);\ \mathscr{P}(P)(Q);\ \mathrm{S}}\ \upharpoonright\mathrm{E}$$

$$\frac{\tau(\varphi)(\sigma);\ \mathscr{P}(P)(Q);\ \mathrm{S}}{\lambda\tau.\tau(\varphi)(\sigma);\ \lambda\mathscr{P}.\mathscr{P}(P)(Q);\ \mathrm{S} \upharpoonright \mathrm{Det}}\ \upharpoonright\mathrm{I}^1$$

$$\frac{\lambda\rho.\rho(\lambda\varphi\lambda\sigma.\sigma(\mathsf{no} \circ \varphi));\ \lambda\mathscr{P}.\neg\mathscr{P}(\mathbf{\exists});\ \mathrm{S} \upharpoonright (\mathrm{S} \upharpoonright \mathrm{Det}) \qquad \lambda\tau.\tau(\varphi)(\sigma);\ \lambda\mathscr{P}.\mathscr{P}(P)(Q);\ \mathrm{S} \upharpoonright \mathrm{Det}}{\sigma(\mathsf{no} \circ \varphi);\ \lambda\mathscr{P}.\neg\mathscr{P}(P)(Q)(\mathbf{\exists});\ \mathrm{S}}\ \upharpoonright\mathrm{E}$$

$$\frac{\sigma(\mathsf{no} \circ \varphi);\ \lambda\mathscr{P}.\neg\mathscr{P}(P)(Q)(\mathbf{\exists});\ \mathrm{S}}{\lambda\sigma.\sigma(\mathsf{no} \circ \varphi);\ \lambda Q.\neg\mathbf{\exists}(P)(Q);\ \mathrm{S} \upharpoonright (\mathrm{S} \upharpoonright \mathrm{NP})}\ \upharpoonright\mathrm{I}^3$$

$$\frac{\lambda\sigma.\sigma(\mathsf{no} \circ \varphi);\ \lambda Q.\neg\mathbf{\exists}(P)(Q);\ \mathrm{S} \upharpoonright (\mathrm{S} \upharpoonright \mathrm{NP})}{\lambda\varphi\lambda\sigma.\sigma(\mathsf{no} \circ \varphi);\ \lambda P\lambda Q.\neg\mathbf{\exists}(P)(Q);\ \mathrm{S} \upharpoonright (\mathrm{S} \upharpoonright \mathrm{NP}) \upharpoonright \mathrm{N}}\ \upharpoonright\mathrm{I}^2$$

Nothing in our syntactic treatment blocks the ill-formed options in (48). In our view, this is exactly as it should be; we believe, following de Swart (1998), that the factors affecting the acceptability of inverse scoping (as vs. its grammaticality) are complex and diverse, and in the long run will turn out to hinge on functional factors, as in de Swart's analysis, that it would be a mistake to attempt to build into syntactic combinatorics.

To summarize the discussion in this section, we first showed that by encoding polarity markings on PPI and NPI modal auxiliaries, their scopal relations with negation can be properly regulated. In the latter part of the present section, we demonstrated that this more elaborate analysis that incorporates polarity sensitivity still retains the derivability relation from the higher-order entry to the lower-order entry discussed in the previous section. Importantly, the 'slanted' versions of the auxiliary entries fully retain their scopal properties encoded in the original higher-order entries. This is an encouraging result, since it attests to the systematicity of the logic underlying the type-logical syntax assumed here. In the next section, we demonstrate some further payoff of formalizing an analysis of NPI licensing within an explicit syntactic theory that has a logical underpinning, by discussing interactions between polarity sensitivity and other phenomena (specifically, VP fronting and Gapping) that pertain to the syntax-semantics interface.

5 Consequences of the analysis

The advantage of 'making certain semantic distinctions visible in the syntax' (Dowty 1994) is that it makes it possible to account for interactions between polarity-sensitive phenomena and other syntactic phenomena explicitly. In some cases—such as the NPI licensing patterns in Gapping discussed in section 5.1—we obtain an effect that is equivalent to a strictly semantic analysis of NPI licensing based on the notion of downward entailment. In other cases, of the sort we survey in sections 5.2 and 5.3, the semantic approach does not appear to offer a clear basis for the patterns observed, whereas the facts emerge straightforwardly on the syntactic approach we take, a point we amplify in the discussion in those sections.

5.1 NPI licensing across Gapping conjuncts

We start with the observation that under this analysis, we automatically get the well-formedness of (49).

(49) John can't live in Boston and Mary live anywhere else.

This sentence is well-formed on both the modal wide-scope reading and the narrow-scope reading.

(50) illustrates the derivation for the wide-scope reading for the modal.[22]

[22]Here, for the sake of exposition, we simply assume that the covert variable y in the denotation of *anywhere else* is a free variable. Since it enters into variable binding interpretations under the scope of quantifiers (Culicover and Jackendoff 1995), in a more proper analysis it should be treated on a par with overt pronouns.

(50) a.

$$\dfrac{\mathsf{live};\ \mathbf{live};\ \mathrm{VP}_{b,\alpha}/\mathrm{PP} \qquad \begin{bmatrix}\varphi_2;\\ x;\\ \mathrm{PP}\end{bmatrix}^2}{\mathsf{live} \circ \varphi_2;\ \mathbf{live}(x);\ \mathrm{VP}_{b,\alpha}}$$

$$\dfrac{\mathsf{live} \circ \varphi_2;\ \mathbf{live}(x);\ \mathrm{VP}_{b,\alpha} \qquad \begin{bmatrix}\varphi_3;\\ f;\\ \mathrm{VP}_{f,\gamma}/\mathrm{VP}_{b,\beta}\end{bmatrix}^3}{\varphi_3 \circ \mathsf{live} \circ \varphi_2;\ f(\mathbf{live}(x));\ \mathrm{VP}_{f,\gamma}}$$

$$\dfrac{\varphi_3 \circ \mathsf{live} \circ \varphi_2;\ f(\mathbf{live}(x));\ \mathrm{VP}_{f,\gamma} \qquad \mathsf{mary};\ \mathbf{m};\ \mathrm{NP}}{\mathsf{mary} \circ \varphi_3 \circ \mathsf{live} \circ \varphi_2;\ f(\mathbf{live}(x));\ \mathrm{S}_{f,\gamma}}$$

$$\dfrac{\mathsf{mary} \circ \varphi_3 \circ \mathsf{live} \circ \varphi_2;\ f(\mathbf{live}(x));\ \mathrm{S}_{f,\gamma}}{\lambda\varphi_2.\mathsf{mary} \circ \varphi_3 \circ \mathsf{live} \circ \varphi_2;\ \lambda x.f(\mathbf{live}(x))(\mathbf{m});\ \mathrm{S}_{f,\gamma}{\upharpoonright}\mathrm{PP}}{\upharpoonright}\mathrm{I}^2$$

$$\dfrac{\lambda\varphi_2.\mathsf{mary} \circ \varphi_3 \circ \mathsf{live} \circ \varphi_2;\ \lambda x.f(\mathbf{live}(x))(\mathbf{m});\ \mathrm{S}_{f,\gamma}{\upharpoonright}\mathrm{PP} \qquad \mathsf{anywhere} \circ \mathsf{else};\ \mathbf{\exists}(\lambda w.\mathbf{place}(w) \wedge w \neq y);\ \mathrm{S}_{\gamma,-}{\upharpoonright}(\mathrm{S}_{\gamma,-}{\upharpoonright}\mathrm{PP})}{\mathsf{mary} \circ \varphi_3 \circ \mathsf{live} \circ \mathsf{anywhere} \circ \mathsf{else};\ \mathbf{\exists}(\lambda w.\mathbf{place}(w) \wedge w \neq y)(\lambda x.f(\mathbf{live}(x))(\mathbf{m}));\ \mathrm{S}_{f,-}}$$

$$\dfrac{\mathsf{mary} \circ \varphi_3 \circ \mathsf{live} \circ \mathsf{anywhere} \circ \mathsf{else};\ \mathbf{\exists}(\lambda w.\mathbf{place}(w) \wedge w \neq y)(\lambda x.f(\mathbf{live}(x))(\mathbf{m}));\ \mathrm{S}_{f,-}}{\lambda\varphi_3.\mathsf{mary} \circ \varphi_3 \circ \mathsf{live} \circ \mathsf{anywhere} \circ \mathsf{else};\ \lambda f.\mathbf{\exists}(\lambda w.\mathbf{place}(w) \wedge w \neq y)(\lambda x.f(\mathbf{live}(x))(\mathbf{m}));\ \mathrm{S}_{f,-}{\upharpoonright}(\mathrm{VP}_{f,-}/\mathrm{VP}_{b,\beta})}{\upharpoonright}\mathrm{I}^3$$

b.

$$\dfrac{\mathsf{can't};\ \lambda\mathscr{G}.\neg\Diamond\mathscr{G}(\mathbf{id}_{et});\ \mathrm{S}_{f,+}{\upharpoonright}(\mathrm{S}_{f,-}{\upharpoonright}(\mathrm{VP}_{f,-}/\mathrm{VP}_{b,-})) \qquad \begin{array}{l}\vdots\\ \lambda\varphi_3.\mathsf{john} \circ \varphi_3 \circ \mathsf{live} \circ \mathsf{in} \circ \mathsf{boson} \circ \\ \quad \mathsf{and} \circ \mathsf{mary} \circ \varphi_3 \circ \mathsf{live} \circ \mathsf{anywhere} \circ \mathsf{else};\\ \lambda f.[f(\mathbf{live}(\mathbf{b}))(\mathbf{j}) \wedge \\ \quad [\mathbf{\exists}(\lambda w.\mathbf{place}(w) \wedge w \neq y)(\lambda x.f(\mathbf{live}(x))(\mathbf{m}))]];\\ \mathrm{S}_{f,-}{\upharpoonright}(\mathrm{VP}_{f,-}/\mathrm{VP}_{b,\beta})\end{array}}{\begin{array}{l}\mathsf{john} \circ \mathsf{can't} \circ \mathsf{live} \circ \mathsf{in} \circ \mathsf{boson} \circ \mathsf{and} \circ \mathsf{mary} \circ \mathsf{live} \circ \mathsf{anywhere} \circ \mathsf{else};\\ \neg\Diamond[\mathbf{live}(\mathbf{b})(\mathbf{j}) \wedge \mathbf{\exists}(\lambda w.\mathbf{place}(w) \wedge w \neq y)(\lambda x.\mathbf{live}(x)(\mathbf{m}))];\ \mathrm{S}_{f,+}\end{array}}$$

The point here is that the gapped conjuncts can be derived in type $\mathrm{S}_{f,-}{\upharpoonright}(\mathrm{VP}_{f,-}/\mathrm{VP}_{b,\beta})$ as in (50a), which is an NPI licensing environment, and then, the auxiliary *can't* can scope over the whole conjoined gapped sentence to complete the derivation as usual.

For the narrow scope reading for the modal, note that there is an alternative derivation for the gapped conjunct in a slightly different type $\mathrm{S}_{f,+}{\upharpoonright}(\mathrm{VP}_{f,+}/\mathrm{VP}_{b,-})$ as in (51). This allows for the conjoined gapped sentence to take the slanted auxiliary in type $\mathrm{VP}_{f,+}/\mathrm{VP}_{b,-}$ as an argument to complete the derivation.

(51) a.

$$\dfrac{\mathsf{live};\ \mathbf{live};\ \mathrm{VP}_{b,\alpha}/\mathrm{PP} \qquad \begin{bmatrix}\varphi_2;\\ x;\\ \mathrm{PP}\end{bmatrix}^2}{\mathsf{live} \circ \varphi_2;\ \mathbf{live}(x);\ \mathrm{VP}_{b,\alpha}}$$

$$\dfrac{\mathsf{live} \circ \varphi_2;\ \mathbf{live}(x);\ \mathrm{VP}_{b,\alpha} \qquad \begin{bmatrix}\varphi_3;\\ y;\\ \mathrm{NP}\end{bmatrix}^3}{\varphi_3 \circ \mathsf{live} \circ \varphi_2;\ \mathbf{live}(x)(y);\ \mathrm{S}_{b,\alpha}}$$

$$\dfrac{\varphi_3 \circ \mathsf{live} \circ \varphi_2;\ \mathbf{live}(x)(y);\ \mathrm{S}_{b,\alpha}}{\lambda\varphi_2.\varphi_3 \circ \mathsf{live} \circ \varphi_2;\ \lambda x.\mathbf{live}(x)(y);\ \mathrm{S}_{b,\alpha}{\upharpoonright}\mathrm{PP}}{\upharpoonright}\mathrm{I}^2$$

$$\dfrac{\lambda\varphi_2.\varphi_3 \circ \mathsf{live} \circ \varphi_2;\ \lambda x.\mathbf{live}(x)(y);\ \mathrm{S}_{b,\alpha}{\upharpoonright}\mathrm{PP} \qquad \mathsf{anywhere} \circ \mathsf{else};\ \mathbf{\exists}(\lambda w.\mathbf{place}(w) \wedge w \neq z);\ \mathrm{S}_{\gamma,-}{\upharpoonright}(\mathrm{S}_{\gamma,-}{\upharpoonright}\mathrm{PP})}{\varphi_3 \circ \mathsf{live} \circ \mathsf{anywhere} \circ \mathsf{else};\ \mathbf{\exists}(\lambda w.\mathbf{place}(w) \wedge w \neq z)(\lambda x.\mathbf{live}(x)(y));\ \mathrm{S}_{b,-}}$$

$$\dfrac{\varphi_3 \circ \mathsf{live} \circ \mathsf{anywhere} \circ \mathsf{else};\ \mathbf{\exists}(\lambda w.\mathbf{place}(w) \wedge w \neq z)(\lambda x.\mathbf{live}(x)(y));\ \mathrm{S}_{b,-}}{\mathsf{live} \circ \mathsf{anywhere} \circ \mathsf{else};\ \lambda y.\mathbf{\exists}(\lambda w.\mathbf{place}(w) \wedge w \neq z)(\lambda x.\mathbf{live}(x)(y));\ \mathrm{VP}_{b,-}}\backslash\mathrm{I}^3$$

$$\dfrac{\mathsf{live} \circ \mathsf{anywhere} \circ \mathsf{else};\ \lambda y.\mathbf{\exists}(\lambda w.\mathbf{place}(w) \wedge w \neq z)(\lambda x.\mathbf{live}(x)(y));\ \mathrm{VP}_{b,-} \qquad \begin{bmatrix}\varphi_4;\\ f;\\ \mathrm{VP}_{f,+}/\mathrm{VP}_{b,-}\end{bmatrix}^4}{\varphi_4 \circ \mathsf{live} \circ \mathsf{anywhere} \circ \mathsf{else};\ f(\lambda y.\mathbf{\exists}(\lambda w.\mathbf{place}(w) \wedge w \neq z)(\lambda x.\mathbf{live}(x)(y)));\ \mathrm{VP}_{f,+}}$$

$$\dfrac{\varphi_4 \circ \mathsf{live} \circ \mathsf{anywhere} \circ \mathsf{else};\ f(\lambda y.\mathbf{\exists}(\lambda w.\mathbf{place}(w) \wedge w \neq z)(\lambda x.\mathbf{live}(x)(y)));\ \mathrm{VP}_{f,+} \qquad \mathsf{mary};\ \mathbf{m};\ \mathrm{NP}}{\mathsf{mary} \circ \varphi_4 \circ \mathsf{live} \circ \mathsf{anywhere} \circ \mathsf{else};\ f(\lambda y.\mathbf{\exists}(\lambda w.\mathbf{place}(w) \wedge w \neq z)(\lambda x.\mathbf{live}(x)(y)))(\mathbf{m});\ \mathrm{S}_{f,+}}$$

$$\dfrac{\mathsf{mary} \circ \varphi_4 \circ \mathsf{live} \circ \mathsf{anywhere} \circ \mathsf{else};\ f(\lambda y.\mathbf{\exists}(\lambda w.\mathbf{place}(w) \wedge w \neq z)(\lambda x.\mathbf{live}(x)(y)))(\mathbf{m});\ \mathrm{S}_{f,+}}{\lambda\varphi_4.\mathsf{mary} \circ \varphi_4 \circ \mathsf{live} \circ \mathsf{anywhere} \circ \mathsf{else};\ \lambda f.f(\lambda y.\mathbf{\exists}(\lambda w.\mathbf{place}(w) \wedge w \neq z)(\lambda x.\mathbf{live}(x)(y)))(\mathbf{m});\ \mathrm{S}_{f,+}{\upharpoonright}(\mathrm{VP}_{f,+}/\mathrm{VP}_{b,-})}{\upharpoonright}\mathrm{I}^4$$

b.

$$\dfrac{\begin{array}{ll} & \vdots \\ \textsf{can't}; & \lambda\varphi_3.\textsf{john} \circ \varphi_3 \circ \textsf{live} \circ \textsf{in} \circ \textsf{boson} \circ \\ & \quad \textsf{and} \circ \textsf{mary} \circ \varphi_3 \circ \textsf{live} \circ \textsf{anywhere} \circ \textsf{else}; \\ \lambda P \lambda y.\neg\Diamond P(y); & \lambda f[f(\mathbf{live}(\mathbf{b}))(\mathbf{j}) \wedge f(\lambda y.\exists(\lambda w.\mathbf{place}(w) \wedge w \neq z)(\lambda x.\mathbf{live}(x)(y)))(\mathbf{m})]; \\ \mathrm{VP}_{f,+}/\mathrm{VP}_{b,-} & \mathrm{S}_{f,+}\upharpoonright(\mathrm{VP}_{f,+}/\mathrm{VP}_{b,-}) \end{array}}{\begin{array}{c}\textsf{john} \circ \textsf{can't} \circ \textsf{live} \circ \textsf{in} \circ \textsf{boson} \circ \textsf{and} \circ \textsf{mary} \circ \textsf{live} \circ \textsf{anywhere} \circ \textsf{else}; \\ \neg\Diamond\mathbf{live}(\mathbf{b})(\mathbf{j}) \wedge \neg\Diamond\exists(\lambda w.\mathbf{place}(w) \wedge w \neq z)(\lambda x.\mathbf{Live}(x)(\mathbf{m})); \mathrm{S}_{f,+}\end{array}}$$

We note in passing that the analysis in Puthawala (2018), which extends the treatment of wide-scope modality in Gapping in Kubota and Levine (2016, 2020) to Stripping, yields the possibility of NPIs as the Stripped apparent remnant, with subject or object interpretation predictably ambiguous:

(52) Mary won't go to the movies with John, or anyone else.

5.2 Gapping and topicalization

Potter et al. (2017) note an interesting correlation between the scope of modal auxiliaries and topicalization in Gapping. According to them, when the conjoined clauses of Gapping independently host topicalization, the distributive, narrow-scope reading is forced for a gapped auxiliary verb. By contrast, if there is a single shared topicalized phrase corresponding to 'movement traces' in the two clauses in an ATB-manner, we obtain the opposite scoping pattern in which the gapped auxiliary scopes over the conjunction. We reproduce in (53) the relevant data:

(53) a. Cavier, James can't order and chilli, Mary. $(\neg P \wedge \neg Q)$

b. To Mary, James didn't give the cupcakes or Bill the chocolates. $(\neg(P \vee Q))$

This pattern may at first sight seem puzzling, but in fact, it (or, at least the core part of this generalization) already follows from our account of auxiliary scope and topicalization. Specifically, our approach predicts that only the distributive reading is available in (53a) whereas (53b) allows for the wide-scope reading for the modal auxiliary. The unavailability of the distributive reading for (53b) is also predicted once we spell out some intuitively plausible assumptions about topic marking.

The key factor that forces the distributive scope reading for examples like (53a) (i.e. Gapping sentences involving two clauses that independently host topicalization) is that topicalization takes place over S_+'s—a strictly syntactic assumption (and one that receives independent motivation from the facts about scopal properties of fronted VPs discussed in the next section). That is, since the two clauses have topicalized phrases and what's missing is a VP, we have a conjunction of $\mathrm{S}_+\upharpoonright\mathrm{TV}_+$. The first conjunct can be derived as follows:

(54)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\begin{bmatrix}\varphi_1; \\ P; \mathrm{TV}_{f,+}\end{bmatrix}^1 \quad \begin{bmatrix}\varphi_2; \\ x; \mathrm{NP}\end{bmatrix}^2}{\varphi_1 \circ \varphi_2;\ P(x);\ \mathrm{VP}_{b,+}} \quad \begin{array}{l}\textsf{mary}; \\ \mathbf{m}; \\ \mathrm{NP}\end{array}}{\textsf{mary} \circ \varphi_1 \circ \varphi_2;\ P(x)(\mathbf{m});\ \mathrm{S}_{f,+}}}{\lambda\varphi_2.\textsf{mary} \circ \varphi_1 \circ \varphi_2;\ \lambda x.P(x)(\mathbf{m});\ \mathrm{S}_{f,+}\upharpoonright\mathrm{NP}}\upharpoonright\mathrm{I}^2 \quad \dfrac{\begin{array}{l}\lambda\varphi\lambda\sigma.\varphi \circ \sigma(\epsilon); \\ \lambda\mathcal{P}\lambda\mathcal{C}.\mathcal{C}(\mathcal{P}); \\ \mathrm{S}_{f,+}\upharpoonright(\mathrm{S}_{f,+}\upharpoonright X)\upharpoonright X\end{array} \quad \begin{array}{l}\textsf{chilli}; \\ \mathbf{ch}; \\ \mathrm{NP}\end{array}}{\begin{array}{l}\lambda\sigma.\textsf{chilli} \circ \sigma(\epsilon); \\ \lambda\mathscr{C}.\mathscr{C}(\mathbf{ch}); \mathrm{S}_{f,+}\upharpoonright(\mathrm{S}_{f,+}\upharpoonright\mathrm{NP})\end{array}}}{\textsf{chilli} \circ \textsf{mary} \circ \varphi_1 \circ \epsilon;\ P(\mathbf{ch})(\mathbf{m});\ \mathrm{S}_{f,+}}}{\lambda\varphi_1.\textsf{chilli} \circ \textsf{mary} \circ \varphi_1 \circ \epsilon;\ \lambda P.P(\mathbf{ch})(\mathbf{m});\ \mathrm{S}_{f,+}\upharpoonright\mathrm{TV}_{f,+}}$$

A sign with parallel form will be derived for the first conjunct, and via the Gapping conjunction introduced in section 2 (augmented with polarity specifications), we obtain the following:

(55) $\lambda\varphi.\mathsf{cavier} \circ \mathsf{james} \circ \varphi \circ \mathsf{and} \circ \mathsf{chilli} \circ \mathsf{mary};\ \lambda R.R(\mathbf{cv})(\mathbf{j}) \wedge R(\mathbf{ch})(\mathbf{m});\ \mathrm{S}_{+}\upharpoonright\mathrm{TV}_{+}$

This gapped sentence combines with the VP *can't order*, which is derived as follows:

(56)

$$\dfrac{\dfrac{\dfrac{\mathsf{order};\ \mathbf{order};\ \mathrm{TV}_{b,\alpha} \quad [\varphi_3; u; \mathrm{NP}]^3}{\mathsf{order} \circ \varphi_3;\ \mathbf{order}(u);\ \mathrm{VP}_{b,\alpha}} \quad \begin{matrix} \vdots \\ \mathsf{can't}; \\ \lambda P\lambda y.\neg\Diamond\, P(y); \mathrm{VP}_{f,+}/\mathrm{VP}_{b,-} \end{matrix}}{\mathsf{can't} \circ \mathsf{order} \circ \varphi_3;\ \lambda y.\mathbf{order}(u)(y);\ \mathrm{VP}_{f,+}}}{\mathsf{can't} \circ \mathsf{order};\ \lambda u\lambda y.\neg\Diamond\mathbf{order}(u)(y);\ \mathrm{TV}_{f,+}}$$

The derivation completes by giving (56) as an argument to the gapped sentence derived in (54)–(55), and this yields the distributive scope reading:

(57)

$$\dfrac{\begin{matrix} \vdots \\ \lambda\varphi.\mathsf{cavier} \circ \mathsf{james} \circ \varphi \circ \mathsf{and} \circ \mathsf{chilli} \circ \mathsf{mary}; \\ \lambda R.R(\mathbf{cv})(\mathbf{j}) \wedge R(\mathbf{ch})(\mathbf{m}); \mathrm{S}_{f,+}\upharpoonright\mathrm{TV}_{f,+} \end{matrix} \qquad \begin{matrix} \vdots \\ \mathsf{can't} \circ \mathsf{order}; \\ \lambda u\lambda y.\neg\Diamond\mathbf{order}(u)(y); \mathrm{TV}_{f,+} \end{matrix}}{\mathsf{cavier} \circ \mathsf{james} \circ \mathsf{can't} \circ \mathsf{order} \circ \mathsf{and} \circ \mathsf{chilli} \circ \mathsf{mary};\ \neg\Diamond\mathbf{order}(\mathbf{cv})(\mathbf{j}) \wedge \neg\Diamond\mathbf{order}(\mathbf{ch})(\mathbf{m});\ \mathrm{S}_{f,+}}$$

In order to derive the auxiliary wide scope reading, the auxiliary *can* and the negation morpheme need to scope over the whole conjoined clause. But this is impossible due to the mismatch of the polarity values on S for the conjoined clauses (S_{+}) and the scope of the auxiliary and the negation morpheme. That is, in order to derive the $\neg\Diamond(P \wedge Q)$ reading for (53a), the NPI modal *can* first needs to scope over conjunction, over which the negation morpheme takes scope. But this forces conjunction to take place at the level of S_{-}, and this will conflict with the S_{+} specification of sentences containing topicalized phrases. Thus, the derivation does not go through, and it is correctly predicted that (53a) does not induce the wide scope reading for the gapped auxiliary.

In contrast to examples like (53a), examples like (53b) that host a single topicalized phrase linked to both clauses license the wide scope reading for the gapped auxiliary. In order to derive (53b), we need an analysis of multiple filler-gap dependency, which is a somewhat tricky issue in Type-Logical Grammar. But since the exact details of the treatment of multiple gap phenomena don't really matter for our purposes, here we simply assume that multiple filler-gap dependency is licensed by a variant of the topicalization operator that looks like the following:

(58) $\lambda\varphi\lambda\sigma.\varphi \circ \sigma(\boldsymbol{\epsilon})(\boldsymbol{\epsilon});\ \lambda\mathcal{U}\lambda\mathcal{R}.\mathcal{R}(\mathcal{U})(\mathcal{U});\ \mathrm{S}_{f,+}\upharpoonright(\mathrm{S}_{f,+}\upharpoonright X\upharpoonright X)\upharpoonright X$

The topicalization operator in (58) establishes the filler-gap relation between a single filler and multiple gaps at the same time.

The wide-scope reading for the auxiliary for (53b) can then be derived in the following manner. We first derive the two conjuncts in category $\mathrm{S}_{-}\upharpoonright(\mathrm{VP}_{-}/\mathrm{PP}/\mathrm{NP})$ as follows:

(59)

$$\dfrac{\dfrac{\dfrac{\dfrac{[\varphi_1; P; \mathrm{VP}_{f,-}/\mathrm{PP}_{to}/\mathrm{NP}]^1 \quad \mathsf{the} \circ \mathsf{chocolates};\ \iota(\mathbf{cho});\ \mathrm{NP}}{\varphi_1 \circ \mathsf{the} \circ \mathsf{chocolates};\ P(\iota(\mathbf{cho}));\ \mathrm{VP}_{f,-}/\mathrm{PP}_{to}} \quad \begin{bmatrix} \varphi_2; \\ w; \mathrm{PP}_{to} \end{bmatrix}^2}{\varphi_1 \circ \mathsf{the} \circ \mathsf{chocolates} \circ \varphi_2;\ P(\iota(\mathbf{cho}))(w);\ \mathrm{VP}_{f,-}} \quad \begin{matrix} \mathsf{bill}; \\ \mathbf{b}; \mathrm{NP} \end{matrix}}{\mathsf{bill} \circ \varphi_1 \circ \mathsf{the} \circ \mathsf{chocolates} \circ \varphi_2;\ P(\iota(\mathbf{cho}))(w)(\mathbf{b});\ \mathrm{S}_{f,-}}}{\lambda\varphi_1.\mathsf{bill} \circ \varphi_1 \circ \mathsf{the} \circ \mathsf{chocolates} \circ \varphi_2;\ \lambda P.P(\iota(\mathbf{cho}))(w)(\mathbf{b});\ \mathrm{S}_{f,-}\upharpoonright(\mathrm{VP}_{f,-}/\mathrm{PP}_{to}/\mathrm{NP})}$$

Note that the ditransitive verb gap (for Gapping) is explicitly bound in (59) (as reflected in the syntactic category), but the derived linguistic sign also contains an unbound gap of type PP corresponding to the topicalized PP (which is introduced later in the derivation). The two gapped sentences $\mathrm{S}_{f,-}\upharpoonright(\mathrm{VP}_{f,-}/\mathrm{PP}/\mathrm{NP})$ derived along the lines of (59) are then taken as arguments by the Gapping operator, yielding (60).

(60) $\lambda\varphi.\mathsf{james} \circ \varphi \circ \mathsf{the} \circ \mathsf{cupcakes} \circ \varphi_5 \circ \mathsf{or} \circ \mathsf{bill} \circ \boldsymbol{\epsilon} \circ \mathsf{the} \circ \mathsf{chocolates} \circ \varphi_2;$
$\lambda P.P(\iota(\mathbf{cpk}))(v)(\mathbf{j}) \vee P(\iota(\mathbf{cho}))(w)(\mathbf{b}); \mathrm{S}_{f,-}\upharpoonright(\mathrm{VP}_{f,-}/\mathrm{PP}_{to}/\mathrm{NP})$

Then, a derived ditransitive verb of type ($\mathrm{VP}_-/\mathrm{PP}/\mathrm{NP}$), which itself contains an auxiliary verb-type gap $\mathrm{VP}_-/\mathrm{VP}_-$, is given as an argument to this conjoined gapped S, as in (61).

(61)

$$\dfrac{\dfrac{\dfrac{\left[\begin{array}{l}\varphi_0;\\ f;\\ \mathrm{VP}_{f,-}/\mathrm{VP}_{b,-}\end{array}\right]^0 \quad \begin{array}{l}\mathsf{give};\\ \mathbf{give};\\ \mathrm{VP}_{b,-}/\mathrm{PP}_{to}/\mathrm{NP}\end{array}}{\varphi_0 \circ \mathsf{give};\ f(\mathbf{give});\ \mathrm{VP}_{f,-}/\mathrm{PP}_{to}/\mathrm{NP}}\mathrm{FC} \quad \begin{array}{l}\vdots\\ \lambda\varphi.\mathsf{james} \circ \varphi \circ \mathsf{the} \circ \mathsf{cupcakes} \circ \varphi_5 \circ\\ \quad \mathsf{or} \circ \mathsf{bill} \circ \boldsymbol{\epsilon} \circ \mathsf{the} \circ \mathsf{chocolates} \circ \varphi_2;\\ \lambda P.P(\iota(\mathbf{cpk}))(v)(\mathbf{j}) \vee P(\iota(\mathbf{cho}))(w)(\mathbf{b});\\ \mathrm{S}_{f,-}\upharpoonright(\mathrm{VP}_{f,-}/\mathrm{PP}_{to}/\mathrm{NP})\end{array}}{\begin{array}{l}\mathsf{james} \circ \varphi_0 \circ \mathsf{give} \circ \mathsf{the} \circ \mathsf{cupcakes} \circ \varphi_5 \circ \mathsf{or} \circ \mathsf{bill} \circ \boldsymbol{\epsilon} \circ \mathsf{the} \circ \mathsf{chocolates} \circ \varphi_2;\\ f(\mathbf{give})(\iota(\mathbf{cpk}))(v)(\mathbf{j}) \vee f(\mathbf{give})(\iota(\mathbf{cho}))(w)(\mathbf{b}); \mathrm{S}_{f,-}\end{array}}}{\begin{array}{l}\lambda\varphi_0.\mathsf{james} \circ \varphi_0 \circ \mathsf{give} \circ \mathsf{the} \circ \mathsf{cupcakes} \circ \varphi_5 \circ \mathsf{or} \circ \mathsf{bill} \circ \boldsymbol{\epsilon} \circ \mathsf{the} \circ \mathsf{chocolates} \circ \varphi_2;\\ \lambda f.f(\mathbf{give})(\iota(\mathbf{cpk}))(v)(\mathbf{j}) \vee f(\mathbf{give})(\iota(\mathbf{cho}))(w)(\mathbf{b}); \mathrm{S}_{f,-}\upharpoonright(\mathrm{VP}_{f,-}/\mathrm{VP}_{b,-})\end{array}}$$

Finally, the auxiliary takes scope in the usual manner and then the ATB topicalized PP combines with the whole sentence by the double topicalization operator defined in (58).

(62)

$$\dfrac{\dfrac{\begin{array}{l}\lambda\varphi\lambda\sigma.\\ \quad \varphi \circ \sigma(\boldsymbol{\epsilon})(\boldsymbol{\epsilon});\\ \lambda\mathcal{U}\lambda\mathcal{R}.\mathcal{R}(\mathcal{U})(\mathcal{U});\\ \mathrm{S}_{f,+}\upharpoonright(\mathrm{S}_{f,+}\upharpoonright X\upharpoonright X)\upharpoonright X\end{array} \quad \begin{array}{l}\mathsf{to} \circ \mathsf{mary};\\ \mathbf{m};\\ \mathrm{PP}_{to}\end{array}}{\begin{array}{l}\lambda\sigma.\mathsf{to} \circ \mathsf{mary} \circ \sigma(\boldsymbol{\epsilon})(\boldsymbol{\epsilon});\\ \lambda R.R(\mathbf{m})(\mathbf{m});\\ \mathrm{S}_{f,+}\upharpoonright(\mathrm{S}_{f,+}\upharpoonright\mathrm{PP}_{to}\upharpoonright\mathrm{PP}_{to})\end{array}} \quad \dfrac{\dfrac{\begin{array}{l}\lambda\sigma_2.\sigma_2(\mathsf{didn't});\\ \lambda\mathscr{G}.\neg\mathscr{G}(\mathbf{id});\\ \mathrm{S}_{f,+}\upharpoonright(\mathrm{S}_{f,-}\upharpoonright(\mathrm{VP}_{f,-}/\mathrm{VP}_{b,-}))\end{array} \quad \begin{array}{l}\vdots\\ \lambda\varphi_0.\mathsf{james} \circ \varphi_0 \circ \mathsf{give} \circ\\ \quad \mathsf{the} \circ \mathsf{cupcakes} \circ \varphi_5 \circ\\ \quad \mathsf{or} \circ \mathsf{bill} \circ \boldsymbol{\epsilon} \circ \mathsf{the} \circ \mathsf{chocolates} \circ \varphi_2;\\ \lambda f.f(\mathbf{give})(\iota(\mathbf{cpk}))(v)(\mathbf{j})\\ \quad \vee f(\mathbf{give})(\iota(\mathbf{cho}))(w)(\mathbf{b});\\ \mathrm{S}_{f,-}\upharpoonright(\mathrm{VP}_{f,-}/\mathrm{VP}_{b,-})\end{array}}{\begin{array}{l}\mathsf{james} \circ \mathsf{didn't} \circ \mathsf{give} \circ \mathsf{the} \circ \mathsf{cupcakes} \circ \varphi_5 \circ\\ \quad \mathsf{or} \circ \mathsf{bill} \circ \boldsymbol{\epsilon} \circ \mathsf{the} \circ \mathsf{chocolates} \circ \varphi_2;\\ \neg[\mathbf{give}(\iota(\mathbf{cpk}))(v)(\mathbf{j}) \vee \mathbf{give}(\iota(\mathbf{cho}))(w)(\mathbf{b})]; \mathrm{S}_{f,+}\end{array}}}{\begin{array}{l}\lambda\varphi_5\lambda\varphi_2.\mathsf{james} \circ \mathsf{didn't} \circ \mathsf{give} \circ \mathsf{the} \circ \mathsf{cupcakes} \circ \varphi_5 \circ\\ \quad \mathsf{or} \circ \mathsf{bill} \circ \boldsymbol{\epsilon} \circ \mathsf{the} \circ \mathsf{chocolates} \circ \varphi_2;\\ \lambda v\lambda w.\neg[\mathbf{give}(\iota(\mathbf{cpk}))(v)(\mathbf{j}) \vee \mathbf{give}(\iota(\mathbf{cho}))(w)(\mathbf{b})];\\ \mathrm{S}_{f,+}\upharpoonright\mathrm{PP}_{to}\upharpoonright\mathrm{PP}_{to}\end{array}}}{\begin{array}{l}\mathsf{to} \circ \mathsf{mary} \circ \mathsf{james} \circ \mathsf{didn't} \circ \mathsf{give} \circ \mathsf{the} \circ \mathsf{cupcakes} \circ \boldsymbol{\epsilon} \circ \mathsf{or} \circ \mathsf{bill} \circ \boldsymbol{\epsilon} \circ \mathsf{the} \circ \mathsf{chocolates} \circ \boldsymbol{\epsilon};\\ \neg[\mathbf{give}(\iota(\mathbf{cpk}))(\mathbf{m})(\mathbf{j}) \vee \mathbf{give}(\iota(\mathbf{cho}))(\mathbf{m})(\mathbf{b})]; \mathrm{S}_{f,+}\end{array}}$$

The key point in this derivation is that the two clauses are conjoined before topicalization takes place. The auxiliary and the negation morpheme then scopes over this conjoined S_-, yielding the auxiliary wide-scope reading.

One issue needs to be commented on before closing this section. Our analysis does not rule out the distributive scope reading for sentences like (53b) in the combinatoric component of syntax. To see this point, note that Gapping conjunction can take place at the level of $\mathrm{S}_+\upharpoonright(\mathrm{VP}_+/\mathrm{PP}/\mathrm{NP})$ as well (instead of at the level of $\mathrm{S}_-\upharpoonright(\mathrm{VP}_-/\mathrm{PP}/\mathrm{NP})$ as in (60)), which then yields the distributive reading just as in the derivation for the 'separate topicalization' example (53a). After this, ATB topicalization of the PP *to Mary* out of the conjoined clauses can take place to complete the derivation:[23]

[23] It should be pointed out that in the abbreviated part of the derivation above ①, $\mathrm{VP}_{f,+}/\mathrm{PP}/\mathrm{NP}$ is hypothesized instead of $\mathrm{VP}_{f,-}/\mathrm{PP}/\mathrm{NP}$, so that we obtain $\mathrm{S}_{f,+}\upharpoonright(\mathrm{VP}_{f,+}/\mathrm{PP}/\mathrm{NP})$ at step ①.

(63)

$$
\dfrac{
\dfrac{
\begin{array}{l}\lambda\varphi\lambda\sigma.\\ \quad \varphi \circ \sigma(\epsilon)(\epsilon);\\ \lambda\mathcal{U}\lambda\mathcal{R}.\mathcal{R}(\mathcal{U})(\mathcal{U});\\ \mathrm{S}_{f,+}\upharpoonright(\mathrm{S}_{f,+}\upharpoonright X\upharpoonright X)\upharpoonright X\end{array}
\quad
\begin{array}{l}\mathsf{to} \circ \mathsf{mary};\\ \mathbf{m};\\ \mathrm{PP}_{to}\end{array}
}{
\begin{array}{l}\lambda\sigma.\mathsf{to} \circ \mathsf{mary} \circ \sigma(\epsilon)(\epsilon);\\ \lambda R.R(\mathbf{m})(\mathbf{m});\\ \mathrm{S}_{f,+}\upharpoonright(\mathrm{S}_{f,+}\upharpoonright\mathrm{PP}_{to}\upharpoonright\mathrm{PP}_{to})\end{array}
}
\quad
\dfrac{
\dfrac{
\begin{array}{c}\vdots\\ \begin{array}{l}\mathsf{didn't} \circ \mathsf{give};\\ \lambda u\lambda z.\neg\mathbf{give}(u)(z);\\ \mathrm{VP}_{f,+}/\mathrm{PP}/\mathrm{NP}\end{array}\end{array}
\quad
\begin{array}{c}\vdots\\ \begin{array}{l}\lambda\varphi_1.\mathsf{james} \circ \varphi_1 \circ \mathsf{the} \circ \mathsf{cupcakes} \circ \varphi_2 \circ\\ \quad \mathsf{or} \circ \mathsf{bill} \circ \epsilon \circ \mathsf{the} \circ \mathsf{chocolates} \circ \varphi_3;\\ \lambda T.T(\iota(\mathbf{cpks}))(w)(\mathbf{j}) \vee T(\iota(\mathbf{cho}))(v)(\mathbf{b});\\ \mathrm{S}_{f,+}\upharpoonright(\mathrm{VP}_{f,+}/\mathrm{PP}/\mathrm{NP})\end{array}\end{array}
}{
\begin{array}{l}\mathsf{james} \circ \mathsf{didn't} \circ \mathsf{give} \circ \mathsf{the} \circ \mathsf{cupcakes} \circ \varphi_2 \circ\\ \quad \mathsf{or} \circ \mathsf{bill} \circ \epsilon \circ \mathsf{the} \circ \mathsf{chocolates} \circ \varphi_3;\\ \neg\mathbf{give}(\iota(\mathbf{cpks}))(w)(\mathbf{j}) \vee \neg\mathbf{give}(\iota(\mathbf{cho}))(v)(\mathbf{b}); \mathrm{S}_{f,+}\end{array}
}{\textcircled{1}\!\to}
}{
\begin{array}{l}\lambda\varphi_2\lambda\varphi_3.\mathsf{james} \circ \mathsf{didn't} \circ \mathsf{give} \circ \mathsf{the} \circ \mathsf{cupcakes} \circ \varphi_2 \circ\\ \quad \mathsf{or} \circ \mathsf{bill} \circ \epsilon \circ \mathsf{the} \circ \mathsf{chocolates} \circ \varphi_3;\\ \lambda w\lambda v.\neg\mathbf{give}(\iota(\mathbf{cpks}))(w)(\mathbf{j}) \vee \neg\mathbf{give}(\iota(\mathbf{cho}))(v)(\mathbf{b});\\ \mathrm{S}_{f,+}\upharpoonright\mathrm{PP}_{to}\upharpoonright\mathrm{PP}_{to}\end{array}
}
}{
\begin{array}{l}\mathsf{to} \circ \mathsf{mary} \circ \mathsf{james} \circ \mathsf{didn't} \circ \mathsf{give} \circ \mathsf{the} \circ \mathsf{cupcakes} \circ \epsilon \circ \mathsf{or} \circ \mathsf{bill} \circ \epsilon \circ \mathsf{the} \circ \mathsf{chocolates} \circ \epsilon;\\ \neg\mathbf{give}(\iota(\mathbf{cpks}))(\mathbf{m})(\mathbf{j}) \vee \neg\mathbf{give}(\iota(\mathbf{cho}))(\mathbf{m})(\mathbf{b}); \mathrm{S}_{f,+}\end{array}
}
$$

As shown in (63) this yields the distributive reading for (53b). But there is an independent explanation for the unavailability of the distributive reading in examples such as (53b).[24] To see this point, note that the distributive reading is associated with a distinct prosodic tune in which the subject receives focal stress, and the Gapping remnant is both stressed and pronounced with high pitch. Assuming that this focal stress on the subject is realized by syntactically realizing the subject NP as a contrastive topic, it immediately follows that (53b) cannot have an additional topicalized phrase, given the independent constraint that English does not allow multiple topicalization. Thus, we take it that the unavailability of the distributive reading for (53b) follows from constraints pertaining to the prosody-syntax-semantics interface, rather than from the combinatoric system of (narrow) syntax alone.

To conclude, the syntactic polarity marking of the sort pioneered in Dowty's 1994 work, together with independently justified assumptions about topicalization, fully accounts for the complex interactions between the scope-taking possibilities of modal auxiliaries in Gapping sentences involving topicalization noted by Potter et al. (2017).

5.3 VP fronting

As noted by Kubota and Levine (2019), a particularly interesting consequence of the analysis presented above is that it predicts, with minimum additional assumptions, that negation contained in extracted VP constituents will necessarily scope below the modal auxiliary. This is illustrated in (64):

(64) Not vote, John can. $(= \Diamond\neg\mathbf{vote}(\mathbf{j}))$

The restriction of examples of this sort to the narrow-scope modal interpretation implies, under the analysis presented earlier, that the NPI interpretation of *can* is unavailable. But there is no apparent basis for this restriction on the interpretation of sentences such as

[24] For readers who are not convinced by the interface-based account of the unacceptability of (53b) we offered in the main text, we'd like to note that ruling out (53b) in the narrow syntax would also be possible without too much extra stipulation. In order to rule out the derivation in (63), it would suffice to simply stipulate that S_+ level conjunction disallows topicalization out of it. Such a constraint would be trivial to implement in versions of Type-Logical Grammar that are equipped with mechanisms for dealing with syntactic island constraints in the combinatoric component of syntax such as Morrill (2010). (Technically, this would be a constraint that would disallow withdrawing a new variable in an island-marked syntactic context; note that in (63), Gapping would still be allowed since 'extraction' for Gapping has already taken place before the conjunction takes place, as reflected in the syntactic category of the conjoined S's.) In the present paper, we will not try to settle the issue of which of the two alternatives is more plausible.

(64) based on semantically defined scopal environments involving downward or upward entailment, veridicality (or its lack), or relevant presuppositions introduced by VP fronting. For example, it does not appear to be the case that negated material in fronted position fails to scope over NPIs within its extraction domain:

(65) It's to none of THOSE people that I'd ever say anything about the work of the Secret Committee, I'll tell you THAT much.

(66) a. To none of those people do I remember John ever saying a single word.
b. *I remember John ever saying a single word to none of those people.

The contrast in (66) is particularly telling; given the failure of inverse NPI licensing in (66b), the relatively far better status of (66a) can only be accounted for by the possibility of the fronted negative operator. There does not then appear to be any basis for an argument that the narrow scope of negation in (64) is independendently accounted for by the inability of negation to scope wide in extracted position.

Instead, we suggest that such examples find their explanation on the basis of the syntactic possibilities determined by specification for polarity values. In particular, all we need to do to account for the pattern in (64) is assume that the topicalization operator, as vs. certain other extraction licensors, requires its scope to be S_{+}, as in the following lexical entry:

(67) $\lambda\varphi\lambda\sigma.\varphi \circ \sigma(\epsilon);\ \lambda\mathcal{P}\lambda\mathcal{C}.\mathcal{C}(\mathcal{P});\ \mathrm{S}_{f,+}{\upharpoonleft}(\mathrm{S}_{f,+}{\upharpoonleft}X){\upharpoonleft}X$

This restriction is intuitively plausible, since it essentially amounts to the claim that only S-typed signs that can independently stand alone can host topicalization. Independent motivation for this (entirely syntactic) assumption comes from the interactions between topicalization and Gapping discussed in the preceding section 5.2.

The derivation for (64) then goes as follows:

(68)

$$\dfrac{\dfrac{\dfrac{\begin{array}{c}\vdots\\ \lambda\varphi_1.\mathsf{john}\circ\varphi_1\circ\varphi_0;\\ \lambda f.f(Q)(\mathbf{j});\\ \mathrm{S}_{f,+}{\upharpoonleft}(\mathrm{VP}_{f,+}/\mathrm{VP}_{b,+})\end{array}\quad \begin{array}{c}\lambda\sigma_1.\sigma_1(\mathsf{can});\\ \lambda\mathscr{F}.\Diamond\mathscr{F}(\mathbf{id});\\ \mathrm{S}_{f,\alpha}{\upharpoonleft}(\mathrm{S}_{f,\alpha}{\upharpoonleft}(\mathrm{VP}_{f,\alpha}/\mathrm{VP}_{b,\alpha}))\end{array}}{\mathsf{john}\circ\mathsf{can}\circ\varphi_0;\ \Diamond Q(\mathbf{j});\ \mathrm{S}_{f,+}}}{\lambda\varphi_0.\mathsf{john}\circ\mathsf{can}\circ\varphi_0;\ \lambda Q.\Diamond Q(\mathbf{j});\ \mathrm{S}_{f,+}{\upharpoonleft}\mathrm{VP}_{b,+}} \quad \dfrac{\dfrac{\begin{array}{c}\mathsf{not};\\ \lambda P\lambda y.\neg P(y);\\ \mathrm{VP}_{b,+}/\mathrm{VP}_{b,-}\end{array}\quad \begin{array}{c}\mathsf{vote};\\ \mathbf{vote};\\ \mathrm{VP}_{b,-}\end{array}}{\begin{array}{c}\mathsf{not}\circ\mathsf{vote};\\ \lambda y.\neg\mathbf{vote}(y);\ \mathrm{VP}_{b,+}\end{array}}\quad \begin{array}{c}\lambda\varphi\lambda\sigma.\varphi\circ\sigma(\epsilon);\\ \lambda\mathcal{P}\lambda\mathcal{C}.\mathcal{C}(\mathcal{P});\\ \mathrm{S}_{f,+}{\upharpoonleft}(\mathrm{S}_{f,+}{\upharpoonleft}X){\upharpoonleft}X\end{array}}{\begin{array}{c}\lambda\sigma.\mathsf{not}\circ\mathsf{vote}\circ\sigma(\epsilon);\\ \lambda\mathscr{C}.\mathscr{C}(\lambda y.\neg\mathbf{vote}(y));\ \mathrm{S}_{f,+}{\upharpoonleft}(\mathrm{S}_{f,+}{\upharpoonleft}\mathrm{VP}_{b,+})\end{array}}}{\mathsf{not}\circ\mathsf{vote}\circ\mathsf{john}\circ\mathsf{can}\circ\epsilon;\ \Diamond\neg\mathbf{vote}(\mathbf{j});\ \mathrm{S}_{f,+}}$$

The other reading is blocked due to feature conflict in the polarity specification. We refer the reader to Kubota and Levine (2019) for details (the different implementations of the polarity feature in the current proposal and our earlier account in Kubota and Levine (2019) is orthogonal to the point under discussion), but the crucial factor is the lexical specification of the NPI version of *can*, which has the syntactic category $\mathrm{S}_{-}{\upharpoonleft}(\mathrm{S}_{-}{\upharpoonleft}(\mathrm{VP}_{-}/\mathrm{VP}_{-}))$. This means that a sentence containing this modal can only project S_{-}, which cannot host topicalization since the topicalization operator requires the host sentence to be S_{+}. Thus, the $\neg\Diamond$ reading is correctly blocked for (64).

6 In place of a conclusion...

So, how does this all relate to other, more familiar approaches to modal auxiliaries, polarity licensing and the syntax-semantics interface? In what sense is it different from earlier accounts and which aspects of the latter (if any) does it inherit? Given that categorial grammar belongs to the family of lexicalist approaches to syntax (in the broader sense), one important (and obvious) alternative to compare is the lexicalist approach that directly inherits the GPSG and earlier categorial grammar approaches of the sort pursued in detail in the HPSG literature by Kim and Sag (2002) (whose core idea is retained essentially in its original form in the most recent version of the analysis of English auxiliaries in this tradition published as Sag et al. (2019)). We find it fitting to give a brief comparison with this line of analysis at the end of this paper, since, after all, we started our discussion with the well-respected tradition of the VP/VP analysis of English auxiliaries.

Kim and Sag (2002) propose to account for the scopal properties of modal auxiliaries by positing different phrase structural relations between the auxiliary and negation along the following lines:

(69) a. $[_{\mathrm{VP}}$ modal $[_{\mathrm{VP}}$ *not* $[_{\mathrm{VP}} \ldots]]]$

b. $[_{\mathrm{VP}}$ modal *not* $[_{\mathrm{VP}} \ldots]]$

According to Kim and Sag, (69a) induces the modal-outscoping semantic interpretation (which respects surface syntax) whereas (69b) induces the negation-outscoping semantic interpretation via the lexically encoded specification for the special type of auxiliary verb entries that take negation as an extra argument.

In Kubota and Levine (2020, Chapter 9, section 9.4), we have noted some issues which in our view seem problematic for this type of analysis. Instead of reproducing these arguments, we would now like to emphasize a different aspect of the comparison that was perhaps not brought out fully clearly there: from a certain perspective, the 'abstract syntax'-type analysis we have proposed in the preceding sections is actually not so different from the lexicalist analysis of the sort represented by Kim and Sag's (2002) approach.

To see this point, note first that the independently slantable auxiliaries and negation in our approach (cf. the discussion at the end of section 4) correspond to the phrase structural configuration in (69a). In both approaches, the modal outscopes the negation in this case. The parallel here should be easy to see: the slanted VP/VP entries of the auxiliary and negation in our approach each head finite and base-form VPs in a way completely analogous to the phrase-structural configuration in (69a) licensed by the lexically posited entries for the auxiliary and negation in a lexicalist approach.

For the negation-outscoping cases, the structure in (69b) corresponds to the auxiliary-negation pair in our approach where the independently slanted versions of these items (i.e., *need* in VP/VP and *not* in VP/VP) are *not* composable with each other. This makes sense, since in Kim and Sag's (2002) phrase structural approach too, there is something special about the auxiliary lexical entries in these cases (that is, they take the negation morpheme as an additional argument) which make them somewhat deviant from the simple VP/VP lexical entries of the sort traditionally assumed in the GPSG and categorial grammar literature. But interestingly, in both our approach and Kim and Sag's (2002) approach, the combination of the modal and negation (such as *need not*) has the traditional combinatorial property of VP/VP expressions—in the latter, the auxiliary entry looks the same with other auxiliaries after taking negation as an extra argument; in the former, the sequence of strings *need not* is derivable in the VP/VP category via slanting, as demonstrated in section 4.

Thus, when seen at a suitably abstract level, there is a striking degree of similarity between the two approaches. We find this convergence to be a highly illuminating consequence—two independent proposals for the same set of complex analytic issues in the grammar of English arrive at results that embody essentially the same key analytic insight. By emphasizing the similarity, we of course do not mean to deny the possibility that there may still remain some genuine analytic or empirical differences between the two approaches—the tension between the 'concrete' vs. 'abstract' approaches to syntax does exist in the current literature. But however this tension is to be resolved, there is something in common to the two that should be retained in any successful analysis of phenomena in this empirical domain.

Acknowledgment

This work was supported by the NINJAL collaborative research project 'Cross-linguistic Studies of Japanese Prosody and Grammar'.

References

Bach, Emmon. 1980a. Tenses and aspects as functions on verb-phrases. In C. Rohrer, ed., *Time, Tense, and Quantifiers*, 19–37. Tuebingen: Niemeyer.

Bach, Emmon W. 1980b. In defense of passive. *Linguistics and Philosophy* 3(3):297–341.

Barker, Chris. 2007. Parasitic scope. *Linguistics and Philosophy* 30(4):407–444.

Barker, Chris. 2018. Negative polarity as scope marking. *Linguistics and Philosophy* 41(5):483–510.

Bayer, Samuel and Mark Johnson. 1995. Features and agreement. In *Proceedings of ACL 33*, 70–76. Boston: Association for Computational Linguistics.

Bernardi, Raffaella. 2002. *Reasoning with Polarity in Categorial Type Logic*. Ph.D. thesis, University of Utrecht.

Blevins, James P. and Ivan A. Sag. 2013. Phrase structure grammar. In M. den Dikken, ed., *The Cambridge Handbook of Generative Syntax*, 202–225. Cambridge: Cambridge University Press.

Carpenter, Bob. 1997. *Type-Logical Semantics*. Cambridge, MA: MIT Press.

Chomsky, Noam. 1957. *Syntactic Structures*. The Hague: Mouton.

Culicover, Peter W. and Ray Jackendoff. 1995. Something else for the binding theory. *Linguistic Inquiry* 26(2):249–275.

de Swart, Henriëtte. 1998. Licensing of negative polarity items under inverse scope. *Lingua* 105:175–200.

Dowty, David. 1994. The role of negative polarity and concord marking in natural language reasoning. In M. Harvey and L. Santelmann, eds., *Proceedings from Semantics and Linguistic Theory IV*, 114–144. Ithaca, NY: Cornell University.

Gazdar, Gerald, Ewan Klein, Geoffrey K. Pullum, and Ivan A. Sag. 1985. *Generalized Phrase Structure Grammar*. Cambridge, MA: Harvard University Press.

Gazdar, Gerald, Geoffrey Pullum, and Ivan Sag. 1982. Auxiliaries and related phenomena in a restrictive theory of grammar. *Language* 58:591–638.

Hu, Hai, Qi Chen, and Larry Moss. 2019. Natural language inference with monotonicity. In *Proceedings of the 13th International Conference on Computational Semantics: Short Papers*, 8–15. Gothenburg, Sweden: Association for Computational Linguistics.

Iatridou, Sabine and Hedde Zeijlstra. 2013. Negation, polarity and deontic modals. *Linguistic Inquiry* 44:529–568.

Israel, Michael. 2011. *The Grammar of Polarity: Pragmatics, Sensitivity, and the Logic of Scales*. Cambridge, UK: Cambridge University Press.

Kim, Jong-Bok and Ivan Sag. 2002. Negation without head movement. *Natural Language and Linguistic Theory* 20:339–412.

Klima, Edward. 1964. Negation in English. In J. A. Fodor and J. J. Katz, eds., *The Structure of Language: Readings in the Philosophy of Language*, 246–323. Englewood Cliffs, New Jersey: Prentice-Hall.

Kubota, Yusuke and Robert Levine. 2016. Gapping as hypothetical reasoning. *Natural Language and Linguistic Theory* 34(1):107–156.

Kubota, Yusuke and Robert Levine. 2017. Pseudogapping as pseudo-VP ellipsis. *Linguistic Inquiry* 48(2):213–257.

Kubota, Yusuke and Robert Levine. 2019. Modal auxiliaries and negation: A type-logical account. In R. Iemhoff, M. Moortgat, and R. de Queiroz, eds., *Logic, Language, Information, and Computation: 26th International Workshop, WoLLIC 2019, Utrecht, The Netherlands, July 2-5, 2019 Proceedings*, 415–432. Heidelberg: Springer.

Kubota, Yusuke and Robert Levine. 2020. *Type-Logical Syntax*. Cambridge, MA: MIT Press. Available Open Access at `https://direct.mit.edu/books/book/4931/Type-Logical-Syntax`.

Ladusaw, William. 1980. *Polarity Sensitivity as Inherent Scope Relations*. New York: Garland.

Levine, Robert. 2013. The modal need VP gap (non)anomaly. In E. Csipak, R. Eckardt, M. Liu, and M. Sailer, eds., *Beyond 'Any' and 'Ever': New Perspectives on Negative Polarity Sensitivity*, 241–265. Berlin: de Gruyter.

Liu, Mingya, Peter König, and Jutta L. Mueller. 2019. Novel ERP evidence for processing differences between negative and positive polarity items in German. *Frontiers in Psychology* 10:376:1–14. `https://doi.org/10.3389/fpsyg.2019.00376`.

MacCartney, Bill and Christopher D. Manning. 2008. Modeling semantic containment and exclusion in natural language inference. In *Proceedings of the 22nd International Conference on Computational Linguistics (COLING 2008)*, 521–528. Manchester, UK: COLING 2008 Organizing Committee.

McCawley, James D. 1993. Gapping with shared operators. In D. A. Peterson, ed., *Berkeley Linguistics Society*, 245–253. Berkeley: University of California.

Moortgat, Michael. 1990. The quantification calculus. In H. Hendriks and M. Moortgat, eds., *Theory of Flexible Interpretation: Esprit DYANA Deliverable R.1.2.A*. Institute for Language, Logic and Information, University of Amsterdam.

Moortgat, Michael. 1996. Generalized quantification and discontinuous type constructors. In H. Bunt and A. van Horck, eds., *Discontinuous Constituency*, No. 6 in Natural Language Processing, 181–207. Berlin, New York: de Gruyter.

Morrill, Glyn. 1994. *Type Logical Grammar: Categorial Logic of Signs*. Dordrecht: Kluwer.

Morrill, Glyn. 2010. *Categorial Grammar: Logical Syntax, Semantics, and Processing*. Oxford: Oxford University Press.

Morrill, Glyn and Oriol Valentín. 2017. A reply to Kubota and Levine on gapping. *Natural Language and Linguistic Theory* 35(1):257–270.

Müller, Stefan, Anne Abeillé, Robert D. Borsley, and Jean-Pierre Koenig, eds. 2020. *Head-Driven Phrase Structure Grammar: The Handbook*. Berlin: Language Science Press.

Oehrle, Richard T. 1971. On Gapping. Unpublished Manuscript, MIT.

Oehrle, Richard T. 1987. Boolean properties in the analysis of gapping. In G. J. Huck and A. E. Ojeda, eds., *Syntax and Semantics: Discontinuous Constituency*, vol. 20, 203–240. New York: Academic Press.

Oehrle, Richard T. 1994. Term-labeled categorial type systems. *Linguistics and Philosophy*

17(6):633–678.

Park, Sang-Hee. 2019. *Gapping: A Constraint-Based Syntax-Semantics Interface*. Ph.D. thesis, State University of New York at Buffalo.

Park, Sang-Hee, Jean-Pierre Koenig, and Rui P. Chaves. 2019. A semantic underspecification-based analysis of scope ambiguities in Gapping. In M. T. Espinal, E. Castroviejo, M. Leonetti, and L. McNally, eds., *Proceedings of Sinn und Bedeutung 23*, vol. 2, 237–251.

Penka, Doris. 2011. *Negative Indefinites*. Oxford: Oxford University Press.

Pogodalla, Sylvain and Florent Pompigne. 2012. Controlling extraction in Abstract Categorial Grammars. In P. de Groote and M.-J. Nederhof, eds., *Formal Grammar 2010/2011*, 162–177. Heidelberg: Springer.

Pollard, Carl J. and Ivan A. Sag. 1994. *Head-Driven Phrase Structure Grammar*. Studies in Contemporary Linguistics. Chicago and London: University of Chicago Press.

Pollard, Carl J. and Eun Jung Yoo. 1998. A unified theory of scope for quantifiers and *wh*-phrases. *Journal of Linguistics* 34:415–445.

Potter, David, Michael Frazier, and Masaya Yoshida. 2017. A two-source hypothesis for Gapping. *Natural Language and Linguistic Theory* 35:1123–1160.

Pullum, Geoffrey K. 2011. On the mathematical foundations of *Syntactic Structures*. *Journal of Logic, Language and Information* 20:277–296.

Puthawala, Daniel. 2018. Stripping isn't so mysterious, or anomalous scope, either. In A. Foret, G. Kobele, and S. Pogodalla, eds., *Formal Grammar 2018*, 102–120. PUBLISHER.

Richter, Frank and Jan-Philipp Soehn. 2006. Braucht niemanden zu scheren: A survey of NPI licensing in German. In S. Müller, ed., *Proceedings of the 13th International Conference on Head-Driven Phrase Structure Grammar*, 421–440. Stanford: CSLI.

Sag, Ivan, Rui P. Chaves, Anne Abeillé, Bruno Estigarribia, Frank Van Eynde, Dan Flickinger, Paul Kay, Laura Michaelis, Stefan Müller, Geoffrey Pullum, and Thomas Wasow. 2019. Lessons from the English auxiliary system. *Journal of Linguistics* 56:87–155.

Siegel, Muffy A. 1987. Compositionality, case, and the scope of auxiliaries. *Linguistics and Philosophy* 10(1):53–75.

Siegel, Muffy E. A. 1984. Gapping and interpretation. *Linguistic Inquiry* 15(3):523–530.

Steedman, Mark. 1996. *Surface Structure and Interpretation*. Cambridge, MA: MIT Press.

Steedman, Mark. 2012. *Taking Scope*. Cambridge, MA: MIT Press.

Vasishth, Shravan. 2004. Work order, negation, and negative polarity in hindi. *Research on Language and Computation* 2:127–146.

Whitman, Neal. 2010. Deriving wide-scoping operators in an associative lambek categorial grammar. *LSA Annual Meeting Extended Abstracts* 1:12–1–5.

Zwarts, Frans. 1998. Three types of polarity. In F. Hamm and E. Hinrichs, eds., *Plurality and Quantification*, 177–238. Dordrecht: Kluwer.

A The NICE properties revisited

A paper on English auxiliaries will not be complete without addressing the NICE properties. Since this topic is largely orthogonal to the issues pertaining to polarity and semantic scope we have focused on in the main body of the paper, we discuss it in this Appendix.

Auxiliaries are commonly introduced to students in introductory syntax courses as members of a natural class whose distributional characteristics are captured by their occurrence in three supposedly quite independent constructions—inversion, sentential negation and VP Ellipsis (VPE) and one morphological form—NEG contraction.

(70) a. John {will/should/can} buy the book.
b. John {will/should/can} not buy the book. (cf. *John buys not the book.)
c. {Will/Should/Can} John buy the book? (cf. *Buys John the book?)
d. Who {will/should/can} buy the book? – John {will/should/can}. (cf. *John buys.)
e. John {won't/shouldn't/can't} buy the book. (cf. *John buysn't the book.)

Any syntactic theory should provide an explicit (and coherent) analysis of the NICE properties. The distribution of the unstressed form of *do* is especially important in this connection as it has played a non-negligible role in the history of (both transformational and nontransformational variants of) generative grammar. As is well-known, unstressed *do* appears in the NICE environments but not in simple declarative sentences:

(71) a. *John {dĭd/dŏes} buy the book.
b. John {dĭd/dŏes} not buy the book.
c. {Dĭd/Dŏes} John buy the book?
d. Who {bought/buys} the book? – John {dĭd/dŏes}.
e. John {dĭdn't/dŏesn't} buy the book.

On the one hand, in the early history of transformational generative grammar, the analysis of the otherwise puzzling patterns in (71) via the the so-called *do* insertion transformation was regarded as one of the most successful applications of transformational analysis to the grammar of English. On the other hand, the somewhat peculiar distributional restriction on *do* exemplified in (71a), where, unlike other auxiliaries, it is banned from non-negative declarative environments, has long remained problematic in nontransformational treatments of English auxiliaries, a point emphasized in Sag et al. (2019). In fact, Sag et al. (2019) take the '*do* insertion' paradigm in (71) to be one of the major pieces of evidence supporting their construction-based analysis of English auxiliaries (involving a 'slight' reorganization of the role that the AUX feature plays in the overall system), which departs from the strictly lexical analysis pioneered in Gazdar et al. (1982) that has since been widely assumed as the standard analysis in the lexicalist tradition.

From the discussion in the main text, it should be clear that our approach is neither transformational nor nontransformational. But then, how does it handle the well-known NICE properties and the '*do* support' facts? We address this question here and sketch an analysis of the relevant facts. Here too, our approach builds on and integrates the insights of both of the two traditions in an (at least in our view) novel way. For the basic analysis of the NI(C)E properties, we build on the lexicalist approach in identifying the commonality of these constructions as phenomena that target the VP/VP lexical signs of auxiliaries. But unlike the phrase structure-based or constructional setup, in an inference-based (or deductive) system like ours, operations that target VP/VP signs can themselves be the target of still higher-order operations. This enables us to entertain a more abstract view on *do* support than a construction-based encoding of the sort proposed by Sag et al. (2019): by seeing *do* insertion as a 'last resort' inference strategy, as it were, we can capture the key insight of the classical transformational account in a way that completely does away with the ad-hoc structure manipulation operations inherent to the latter.

A.1 An operator-based analysis of basic NIE properties

In the ensuing discussion, we set aside contraction, since this phenomenon is arguably lexical in nature, and should thus be handled by idiosyncratic lexical rules (or equivalent devices) of some sort. For the analyses of the other three phenomena, we generally follow the tradition of nontransformational approaches in taking these phenomena to target auxiliary signs of the form VP/VP (i.e. the 'lowered' lexical entries in our setup).

As noted in Kubota and Levine (2017, 2020), VP ellipsis lends itself to a particularly natural treatment in this approach, with the following type of operator triggering ellipsis:

(72) $\lambda\varphi.\varphi$; $\lambda\mathscr{F}.\mathscr{F}(P)$; $\mathrm{VP}_{f,\beta}\upharpoonright(\mathrm{VP}_{f,\beta}/\mathrm{VP}_{b,\alpha})$

The effect of this operator is to map a functor seeking a VP complement and yielding a VP into a stand-alone VP, with the semantics of the original modal operator applied to a free variable P whose value is supplied contextually. The following derivation illustrates the workings of the VP ellipsis operator in (72). Here, the meaning of the antecedent VP *sing* is supplied as the value of the free variable P in (72).

(73)

$$\dfrac{\begin{matrix}\textsf{john};\\ \mathbf{j}; \mathrm{NP}\end{matrix} \quad \dfrac{\begin{matrix}\textsf{can};\\ \lambda P\lambda x.\Diamond P(x);\\ \mathrm{VP}_{f,+}/\mathrm{VP}_{b,+}\end{matrix} \quad \begin{matrix}\textsf{sing};\\ \mathbf{sing};\\ \mathrm{VP}_{b,+}\end{matrix}}{\textsf{can} \circ \textsf{sing}; \lambda x.\Diamond\mathbf{sing}(x); \mathrm{VP}_{f,+}}/\mathrm{E}}{\textsf{john} \circ \textsf{can} \circ \textsf{sing}; \Diamond\mathbf{sing}(\mathbf{j}); \mathrm{S}_{f,+}}\backslash\mathrm{E} \qquad \dfrac{\begin{matrix}\textsf{bill};\\ \mathbf{b};\\ \mathrm{NP}\end{matrix} \quad \dfrac{\begin{matrix}\lambda\varphi.\varphi;\\ \lambda\mathscr{F}.\mathscr{F}(\mathbf{sing});\\ \mathrm{VP}_{f,\beta}\upharpoonright(\mathrm{VP}_{f,\beta}/\mathrm{VP}_{b,\alpha})\end{matrix} \quad \begin{matrix}\textsf{can't};\\ \lambda P\lambda x.\neg\Diamond P(x);\\ \mathrm{VP}_{f,+}/\mathrm{VP}_{b,-}\end{matrix}}{\textsf{can't}; \lambda x.\neg\Diamond\mathbf{sing}(x); \mathrm{VP}_{f,+}}\upharpoonright\mathrm{E}}{\textsf{bill} \circ \textsf{can't}; \neg\Diamond\mathbf{sing}(\mathbf{b}); \mathrm{S}_{f,+}}\backslash\mathrm{E}$$

Inversion can similarly be treated by an operation that targets VP/VP expressions. For reasons that will become clear below (see the discussion of interaction cases below in section A.2), we separate the inversion operator and the semantic operator that introduces the polar question meaning.[25] The inversion operator can be formulated as follows:

(74) $\lambda\varphi.\varphi$; $\lambda g\lambda y\lambda R.g(R)(y)$; $(\mathrm{S}_{inv,\beta}/\mathrm{VP}_{b,\alpha}/\mathrm{NP})\upharpoonright(\mathrm{VP}_{f,\beta}/\mathrm{VP}_{b,\alpha})$

As can be seen from its syntactic type, (74) takes an auxiliary lexical sign and returns an alternative sign which combines with the subject NP first and then with the VP (seeking both arguments to its right). This results in the correct, inverted word order.[26] The interrogative semantics is supplied by the polar question operator in (75), which takes the inverted S as an argument and returns a Q_{fin}:[27]

(75) $\lambda\varphi.\varphi$; $\lambda p.\mathbf{?}p$; $\mathrm{Q}_{f,\beta}\upharpoonright\mathrm{S}_{inv,\beta}$

Here, **?** is the semantic operator that forms polar interrogative semantics on the basis of the propositional meaning of its argument.

[25]The separation of the syntactic operation of inversion and question semantics is motivated by the fact that inversion is found in contexts other than polar questions. However, we deal with the simple polar questions cases only here. Extension of this approach to other types of inversion (which often come with peculiar idiosyncrasies) is left for future work.

[26]S_{inv} categories are introduced only by the inversion operator which requires finite-form auxiliaries ($\mathrm{VP}_{f,\beta}/\mathrm{VP}_{b,\alpha}$) as their input. From this, it follows that S_{inv} expressions are always 'headed' by finite auxiliaries.

[27]We distinguish between Q_{fin} and Q_{inf}, the latter corresponding to infinitive polar questions as in *We don't know whether to say anything*, which satisfies the NPI-licensing diagnostic for interrogatives, though we will not discuss the syntax-semantics interface of such sentences.

With these two operators, polar question sentences can be derived as follows:[28]

(76)

$$\dfrac{\begin{array}{l}\lambda\varphi.\varphi;\\ \lambda p.\mathbf{?}p;\\ \mathrm{Q}_{f,\beta}\upharpoonright\mathrm{S}_{inv,\beta}\end{array} \quad \dfrac{\dfrac{\dfrac{\begin{array}{l}\lambda\varphi.\varphi;\\ \lambda g\lambda y\lambda R.g(R)(y);\\ (\mathrm{S}_{inv,\beta}/\mathrm{VP}_{b,\alpha}/\mathrm{NP})\upharpoonright(\mathrm{VP}_{f,\beta}/\mathrm{VP}_{b,\alpha})\end{array} \quad \begin{array}{l}\textsf{can};\\ \lambda P\lambda v.\Diamond P(v);\\ \mathrm{VP}_{f,+}/\mathrm{VP}_{b,+}\end{array}}{\textsf{can};\ \lambda y\lambda R.\Diamond R(y);\ \mathrm{S}_{inv,+}/\mathrm{VP}_{b,+}/\mathrm{NP}} \quad \begin{array}{l}\textsf{john};\\ \mathbf{j};\\ \mathrm{NP}\end{array}}{\textsf{can} \circ \textsf{john};\ \lambda R.\Diamond R(\mathbf{j});\ \mathrm{S}_{inv,+}/\mathrm{VP}_{b,+}} \quad \begin{array}{c}\vdots\\ \textsf{solve} \circ \textsf{this} \circ \textsf{equation};\\ \mathbf{solve}(\iota(\mathbf{equ}));\mathrm{VP}_{b,+}\end{array}}{\textsf{can} \circ \textsf{john} \circ \textsf{solve} \circ \textsf{this} \circ \textsf{equation};\ \Diamond\mathbf{solve}(\iota(\mathbf{equ}))(\mathbf{j});\ \mathrm{S}_{inv,+}}}{\textsf{can} \circ \textsf{john} \circ \textsf{solve} \circ \textsf{this} \circ \textsf{equation};\ \mathbf{?}\Diamond\mathbf{solve}(\iota(\mathbf{equ}))(\mathbf{j});\ \mathrm{Q}_{f,+}}$$

By comparing the VP ellipsis and inversion operators, we can see that they both directly target $\mathrm{VP}_f/\mathrm{VP}_b$ expressions, namely, the lowered syntactic type of auxiliary verbs. It may then appear that negation is somewhat different, since the negation morpheme (in its lowered form) simply has the $\mathrm{VP}_\alpha/\mathrm{VP}_\alpha$ syntactic type by itself (with α ranging over morphosyntactic subtypes, e.g. *fin, bse, . . .* etc.):

(77) $\textsf{not};\ \lambda P\lambda y.\neg P(y);\ \mathrm{VP}_{\alpha,+}/\mathrm{VP}_{\alpha,-}$

However, it is trivial to lift this type to an auxiliary-seeking syntactic type via a version of the 'Geach' theorem:

(78)

$$\dfrac{\dfrac{\dfrac{\dfrac{\textsf{not};\ \lambda P\lambda y.\neg P(y);\ \mathrm{VP}_{\alpha,+}/\mathrm{VP}_{\alpha,-} \quad [\varphi_1; Q; \mathrm{VP}_{b,-}]^1}{\textsf{not} \circ \varphi_1;\ \lambda y.\neg Q(y);\ \mathrm{VP}_{b,+}} \quad [\varphi_2; f; \mathrm{VP}_{f,+}/\mathrm{VP}_{b,+}]^2}{\varphi_2 \circ \textsf{not} \circ \varphi_1;\ f(\lambda y.\neg Q(y));\ \mathrm{VP}_{f,+}}\,/\mathrm{I}^1}{\varphi_2 \circ \textsf{not};\ \lambda Q.f(\lambda y.\neg Q(y));\ \mathrm{VP}_{f,+}/\mathrm{VP}_{b,-}}\,\upharpoonright\mathrm{I}^2}{\lambda\varphi_2.\varphi_2 \circ \textsf{not};\ \lambda f\lambda Q.f(\lambda y.\neg Q(y));\ (\mathrm{VP}_{f,+}/\mathrm{VP}_{b,-})\upharpoonright(\mathrm{VP}_{f,+}/\mathrm{VP}_{b,+})}$$

While somewhat roundabout, (78) assigns exactly the same semantics for sentences such as *John should not come* as when the simper form of negation in (77) is used:[29]

(79)

$$\dfrac{\begin{array}{l}\textsf{john};\\ \mathbf{j};\\ \mathrm{NP}\end{array} \quad \dfrac{\dfrac{\begin{array}{l}\textsf{should};\\ \lambda P\lambda v.\Box P(v);\\ \mathrm{VP}_{f,+}/\mathrm{VP}_{b,+}\end{array} \quad \begin{array}{c}\vdots\\ \lambda\varphi_2.\varphi_2 \circ \textsf{not};\\ \lambda f\lambda Q.f(\lambda y.\neg Q(y)); (\mathrm{VP}_{f,+}/\mathrm{VP}_{b,-})\upharpoonright(\mathrm{VP}_{f,+}/\mathrm{VP}_{b,+})\end{array}}{\textsf{should} \circ \textsf{not};\ \lambda Q\lambda v.\Box\neg Q(v);\ \mathrm{VP}_{f,+}/\mathrm{VP}_{b,-}} \quad \begin{array}{l}\textsf{come};\\ \mathbf{come};\\ \mathrm{VP}_{b,-}\end{array}}{\textsf{should} \circ \textsf{not} \circ \textsf{come};\ \lambda v.\Box\neg\mathbf{come}(v);\ \mathrm{VP}_{f,+}}}{\textsf{john} \circ \textsf{should} \circ \textsf{not} \circ \textsf{come};\ \Box\neg\mathbf{come}(\mathbf{j});\ \mathrm{S}_{f,+}}$$

The approach to NIE operators outlined above (which is essentially a type-logical implementation of the 'valence-based' approach familiar in many lexicalist theories of syntax) reveals a very suggestive commonality among their respective types: schematically, all these operators have the form $\mathrm{X}\upharpoonright(\mathrm{VP}_f/\mathrm{VP}_b)$.[30] Given this common aspect of the NIE

[28]We assume that (74) targets lexical signs (the dashed lines in (76) indicates the status of this rule as such). This assumption is supported by the fact that in certain cases we have lexical irregularities of one kind or another which are characteristic of lexical idiosyncrasies. For example, Gazdar et al. (1985) observes that inverted *shall* has a much closer relation to its deontic avatar *should* than does the uninverted version (*Shall we try the idea out and see if we like it?*), while inverted *aren't* is compatible with a first person singular subject, as in *Aren't I clever!* These facts suggest an extended period during which at least some of the inverted versions of the auxiliaries drifted away from their standard paradigmatic properties—a natural historical development on the assumption that (74) embodies a lexical operation.

[29]Technically, (79) can be normalized to a simpler proof involving the 'un-Geached' entry in (77).

[30]To account for the behavior of the aspect auxiliaries (i.e. *have* and *be*), we assume a subtype of S which we might label as *nonfin*, comprising the subtypes *bse, prog, perf* and *pass* (to include the version of auxiliary *be* which appears VP_{pass}). Thus, the current approach will need to be further refined, but it will do as a placeholder for the more detailed analysis required.

'argument structure', the distribution of unstressed *do* can be rethought in terms of the interaction between an operator whose string-support is do and which targets functors of the form $X\upharpoonright(VP_f/VP_b)$. As we will see, this treatment of *do* support has as inevitable consequence the restriction of the unstressed *do* to just the context where we find it (i.e. (71b–d)), ruling out cases such as the declarative (71a).

The analysis of '*do* support' just outlined can be implemented by positing the following higher-order operators:

(80)
a. $\lambda\sigma.\sigma(\textsf{dŏ});\ \lambda\mathscr{F}.\mathscr{F}(\lambda Q\lambda z.\mathbf{N}(Q(z)));\ X\upharpoonright(X\upharpoonright(VP_{f,\alpha}/VP_{b,\alpha}))$
b. $\lambda\sigma.\sigma(\textsf{dŏes});\ \lambda\mathscr{F}.\mathscr{F}(\lambda Q\lambda z.\mathbf{N}(Q(z)));\ X\upharpoonright(X\upharpoonright(VP_{f,\alpha}/VP_{b,\alpha}))$
c. $\lambda\sigma.\sigma(\textsf{dĭd});\ \lambda\mathscr{F}.\mathscr{F}(\lambda Q\lambda z.\mathbf{P}(Q(z)));\ X\upharpoonright(X\upharpoonright(VP_{f,\alpha}/VP_{b,\alpha}))$

Here $X \in \{VP_f,\ VP_f/VP_b,\ S_f/VP_f/NP\}$, and we take $\mathbf{N}$ and $\mathbf{P}$ to be tense operators for present and past respectively.[31] Given the higher order type assigned to unstressed *do* in (80), examples such as (81) appear to fail immdiately on the default assumption that *like the movie* is a VP, hence simply type-incompatible with the operator in (80c).

(81) Mary dĭd like the movie.

Matters are somewhat more complex than this, however; we return to the point below.

The idea here is that, unlike modal auxiliaries, English does not have full-fledged lexical auxiliaries that have tense meanings only, since tense is expressed by inflection for lexical verbs. But then, when no modal auxiliary is present, there is no way to form interrogative, negative or ellipsed sentences. The operators in (80) come to rescue, as it were, in such cases, so that we get the effect as if a 'default' auxiliary *dŏ(es)/dĭd* were fed as an argument to the NIE operators to get the derivations go through.

For the ellipsis operator in (72), the following derivation then becomes available, with the help of (80) (where $X = VP_{f,+}$).

(82)
$$\dfrac{\begin{array}{l}\textsf{mary};\\ \mathbf{m};\\ NP\end{array} \quad \dfrac{\begin{array}{l}\lambda\sigma.\sigma(\textsf{dĭd});\\ \lambda\mathscr{F}.\mathscr{F}(\lambda Q\lambda z.\mathbf{P}(Q(z)));\\ X\upharpoonright(X\upharpoonright(VP_{f,\alpha}/VP_{b,\alpha}))\end{array} \quad \begin{array}{l}\lambda\varphi.\varphi;\\ \lambda h.h(P);\\ VP_{f,\beta}\upharpoonright(VP_{f,\beta}/VP_{b,\alpha})\end{array}}{\textsf{dĭd};\ \lambda z.\mathbf{P}\,P(z);\ VP_{f,+}}}{\textsf{mary}\circ\textsf{dĭd};\ \mathbf{P}\,P(\mathbf{m});\ S_{f,+}}$$

This result allows us to license sentences such as *John liked the movie, but I don't think Mary did.*

Moving on to inversion, the fact that unstressed *do* is licensed in inversion environments directly falls out when we take $X = S_{inv}/VP_b/NP$ In (80). The proof proceeds as follows:

(83)
$$\dfrac{\begin{array}{l}\lambda\varphi.\varphi;\\ \lambda p.\mathbf{?}p;\\ Q_{fin}\upharpoonright S_{inv}\end{array} \quad \dfrac{\dfrac{\begin{array}{l}\textsf{john};\\ \mathbf{j};\\ NP\end{array} \quad \dfrac{\begin{array}{l}\lambda\sigma.\sigma(\textsf{dĭd});\\ \lambda\mathscr{F}.\mathscr{F}(\lambda Q\lambda z.\mathbf{P}(Q(z)));\\ X\upharpoonright(X\upharpoonright(VP_{f,\alpha}/VP_{b,\alpha}))\end{array} \quad \begin{array}{l}\lambda\varphi.\varphi;\\ \lambda g\lambda y\lambda R.g(R)(y);\\ (S_{inv,\beta}/VP_{b,\alpha}/NP)\upharpoonright(VP_{f,\beta}/VP_{b,\alpha})\end{array}}{\textsf{dĭd};\ \lambda y\lambda R.\mathbf{P}R(y);\ S_{inv,\alpha}/VP_{b,\alpha}/NP}}{\textsf{dĭd}\circ\textsf{john};\ \lambda R.\mathbf{P}R(\mathbf{j});\ S_{inv,\alpha}/VP_{b,\alpha}} \quad \begin{array}{l}\textsf{sing};\\ \mathbf{sing};\\ VP_{b,+}\end{array}}{\textsf{dĭd}\circ\textsf{john}\circ\textsf{sing};\ \mathbf{P}\,\mathbf{sing}(\mathbf{j});\ S_{inv,+}}}{\textsf{dĭd}\circ\textsf{john}\circ\textsf{sing};\ \mathbf{?P}\,\mathbf{sing}(\mathbf{j});\ Q_{f,+}}$$

Finally, with negation, the Geached variant derived in (78) has the right syntactic type $(VP_f/VP_b)\upharpoonright(VP_f/VP_b)$ to be given as an argument to the *do* insertion operator (80), as illustrated in the following derivation:

[31] For notational ease, we adopt a 'syncategoremmatic' treatment of tense operators (just as with modals), but nothing crucially hinges on this assumption. It is trivial to rewrite the analysis with explicit temporal variables in the translation language.

(84)

$$\dfrac{\text{john; } \mathbf{j}\text{; NP} \qquad \dfrac{\dfrac{\begin{array}{l}\lambda\sigma.\sigma(\text{dĭd});\\ \lambda\mathscr{F}.\mathscr{F}(\lambda Q\lambda z.\mathbf{P}\,(Q(z)));\\ \text{X}\upharpoonright(\text{X}\upharpoonright(\text{VP}_{f,\alpha}/\text{VP}_{b,\alpha}))\end{array} \quad \begin{array}{c}\vdots\\ \lambda\varphi_2.\varphi_2 \circ \text{not};\\ \lambda f\lambda Q.f(\lambda y.\neg Q(y));\,(\text{VP}_{f,+}/\text{VP}_{b,-})\upharpoonright(\text{VP}_{f,+}/\text{VP}_{b,+})\end{array}}{\text{dĭd} \circ \text{not; } \lambda R\lambda z.\mathbf{P}\,\neg R(z)\text{; } \text{VP}_{f,+}/\text{VP}_{b,-}} \quad \begin{array}{l}\text{sing;}\\ \mathbf{sing};\\ \text{VP}_{b,-}\end{array}}{\text{dĭd} \circ \text{not} \circ \text{sing; } \lambda z.\mathbf{P}\,\neg\mathbf{sing}(z)\text{; } \text{VP}_{f,+}}}{\text{john} \circ \text{dĭd} \circ \text{not} \circ \text{sing; } \mathbf{P}\,\neg\mathbf{sing}(\mathbf{j})\text{; } \text{S}_{f,+}}$$

One important issue that needs to be addressed at this point is that, unless properly constrained, the *do* insertion operator (80) will overgenerate wildly, totally nullifying its original motivation. To see this point, note for example that ordinary VPs can be lifted to the type $\text{VP}_f\upharpoonright(\text{VP}_f/\text{VP}_b)$ syntactically, which is exactly the same type as the ellipsis operator in (72):

(85)

$$\dfrac{\dfrac{[\varphi;\mathscr{F};\text{VP}_{f,\beta}/\text{VP}_{b,\alpha}]^1 \quad \text{sing; } \mathbf{sing}\text{; VP}_{b,\alpha}}{\varphi \circ \text{sing; } \mathscr{F}(\mathbf{sing})\text{; VP}_{f,\beta}}}{\lambda\varphi.\varphi \circ \text{sing; } \lambda\mathscr{F}.\mathscr{F}(\mathbf{sing})\text{; VP}_{f,\beta}\upharpoonright(\text{VP}_{f,\beta}/\text{VP}_{b,\alpha})}\upharpoonright\text{I}^1$$

But then, by applying (80) to (85), we would incorrectly overgenerate the declarative variant (71a). Similarly, VP adverbs such as *always* and *seriously* are of the same type $\text{VP}_\alpha/\text{VP}_\alpha$ as the negation morpheme, from which it follows that they can also be 'Geached' to the category $(\text{VP}_f/\text{VP}_b)\upharpoonright(\text{VP}_f/\text{VP}_b)$, potentially overgenerating unstressed *do* in non-negative sentences.

One way to block overgeneration of this sort is to restrict the application of the *do* insertion operator (80) inside the lexicon. This way, syntactically derived higher-order functions such as the type $\text{VP}_f\upharpoonright(\text{VP}_f/\text{VP}_b)$ overt VP derived in (85) is exempt from (80). After all, *do* support conceptually has the status of a repair strategy for operators that are specified in the lexicon to target VP_f/VP_b expressions when the target expression happens to be unavailable due to a lexical gap. It then seems reasonable to restrict the application of the *do* insertion operator to the lexicon too (we have used dashed lines in the derivations in (82)–(84) above to underscore this point). The case of negation may appear to be problematic for this move, but there is a simple solution for this: for a functional element such as the negation morpheme, it doesn't seem too controversial to assume that an additional type assignment reflecting reanalysis (which has its origin as a syntactically derived theorem along the lines of (78)) is simply posited as an alternative lexical entry. This then allows (80) to strictly target lexical operators only, and its distribution can be properly constrained to cases in which a default auxiliary needs to be 'substituted' due to a gap in the English auxiliary verb paradigm.

A.2 NIE interactions

The NIE patterns discussed and accounted for above are of course only the simplest cases of such patterns, with negation, inversion and ellipsis separately obtained by the respective operators. But these three phenomena interact with each other in a range of ways, as exemplified by the following examples:

(86)
a. (Mary renewed her passport in time, but) John {will/dĭd} not. (negation/ellipsis)
b. {Should/Dĭd} John not say {something/anything}? (inversion/negation)
c. Mary remembered to renew her passport in time. – Yes, but {should/dĭd} John? (inversion/ellipsis)
d. Mary remembered to renew her passport in time. – Yes, and {should/dĭd} John not? (inversion/ellipsis/negation)

In what follows, we first discuss how interactions of the NIE phenomena are to be accounted for, and then address some more complex interactions involving other constructions such as Gapping.

A.2.1 Basic NIE interactions

Starting with the simplest type of interactions, the interaction of negation with ellipsis such as in (86a) is straightforward. The derivation for the version with the auxiliary *do* goes as in (87).

(87)

$$
\dfrac{
  \dfrac{
    \dfrac{
      \begin{array}{l}
      \lambda\varphi_2.\varphi_2 \circ \mathsf{not};\\
      \lambda f\lambda Q.f(\lambda y.\neg Q(y));\\
      (\mathrm{VP}_{f,+}/\mathrm{VP}_{b,-})\upharpoonright(\mathrm{VP}_{f,+}/\mathrm{VP}_{b,+})
      \end{array}
      \quad
      \begin{array}{l}
      \lambda\sigma.\sigma(\mathsf{d\breve{i}d});\\
      \lambda\mathscr{F}.\mathscr{F}(\lambda R\lambda z.\mathbf{P}R(z));\\
      (\mathrm{VP}_{f,+}/\mathrm{VP}_{b,-})\upharpoonright\\
      \quad((\mathrm{VP}_{f,+}/\mathrm{VP}_{b,-})\upharpoonright(\mathrm{VP}_{f,\alpha}/\mathrm{VP}_{b,\alpha}))
      \end{array}
    }{
      \mathsf{d\breve{i}d} \circ \mathsf{not};\ \lambda Q.\lambda y.\neg\mathbf{P}Q(y);\ \mathrm{VP}_{f,+}/\mathrm{VP}_{b,-}
    }
    \quad
    \begin{array}{l}
    \lambda\varphi_1.\varphi_1;\\
    \lambda h\lambda v.h(P(v));\\
    \mathrm{VP}_{f,\gamma}\upharpoonright(\mathrm{VP}_{f,\gamma}/\mathrm{VP}_{b})
    \end{array}
  }{
    \mathsf{d\breve{i}d} \circ \mathsf{not};\ \lambda v.\neg\mathbf{P}P(v);\ \mathrm{VP}_{f,+}
  }
  \quad
  \begin{array}{l}
  \mathsf{john};\\
  \mathbf{j};\mathrm{NP}
  \end{array}
}{
  \mathsf{john} \circ \mathsf{d\breve{i}d} \circ \mathsf{not};\ \neg\mathbf{P}P(\mathbf{j});\ \mathrm{S}_{f,+}
}
$$

Note that the negated VP *did not* is derivable in the category $\mathrm{VP}_{f,+}/\mathrm{VP}_{b,-}$. From there, the derivation proceeds in the same way as in simpler derivations for VP ellipsis examples.

The interaction between inversion and negation is similarly straightforward. Specifically, licensing (86b) is simply a matter of combining an inverted version of *did* with an already negated VP. Given that the NPI containing VP *say anything* is $\mathrm{VP}_{b,-}$ and the negation morpheme can be derived in $\mathrm{VP}_{b,+}/\mathrm{VP}_{b,-}$, the derivation is straightforward, as in (88).

(88)

$$
\dfrac{
  \begin{array}{l}
  \lambda\varphi.\varphi;\\
  \lambda p.\mathbf{?}p;\\
  \mathrm{Q}_{f,\beta}\upharpoonright\mathrm{S}_{inv,\beta}
  \end{array}
  \quad
  \dfrac{
    \dfrac{
      \begin{array}{c}\vdots\\
      \begin{array}{l}
      \mathsf{not};\\
      \lambda P\lambda y.\\
      \quad\neg P(y);\\
      \mathrm{VP}_{b,+}/\mathrm{VP}_{b,-}
      \end{array}\end{array}
      \quad
      \begin{array}{c}\vdots\\
      \begin{array}{l}
      \mathsf{say} \circ \mathsf{anything};\\
      \lambda y.\exists_{\mathbf{thing}}\\
      \quad(\lambda x.\mathbf{say}\\
      \quad(x)(y));\\
      \mathrm{VP}_{b,-}
      \end{array}\end{array}
    }{
      \begin{array}{l}
      \mathsf{not} \circ \mathsf{say} \circ \mathsf{anything};\\
      \lambda y.\neg\exists_{\mathbf{thing}}(\lambda x.\mathbf{say}(x)(y));\\
      \mathrm{VP}_{b,+}
      \end{array}
    }
    \quad
    \dfrac{
      \dfrac{
        \begin{array}{l}
        \lambda\varphi_1.\varphi_1;\\
        \lambda g\lambda y\lambda R.g(R)(y);\\
        (\mathrm{S}_{inv,\beta}/\mathrm{VP}_{b,\alpha}/\mathrm{NP})\upharpoonright\\
        \quad(\mathrm{VP}_{f,\beta}/\mathrm{VP}_{b,\alpha})
        \end{array}
        \quad
        \begin{array}{l}
        \lambda\sigma.\sigma(\mathsf{d\breve{i}d});\\
        \lambda\mathscr{F}.\mathscr{F}\\
        \quad(\lambda Q\lambda z.\mathbf{P}(Q(z)));\\
        \mathrm{X}\upharpoonright(\mathrm{X}\upharpoonright(\mathrm{VP}_{f,\alpha}/\mathrm{VP}_{b,\alpha}))
        \end{array}
      }{
        \mathsf{d\breve{i}d};\ \lambda y\lambda R.\mathbf{P}R(y);\ \mathrm{S}_{inv,\beta}/\mathrm{VP}_{b,\alpha}/\mathrm{NP}
      }
      \quad
      \begin{array}{l}
      \mathsf{john};\\
      \mathbf{j};\\
      \mathrm{NP}
      \end{array}
    }{
      \mathsf{d\breve{i}d} \circ \mathsf{john};\ \lambda R.\mathbf{P}R(\mathbf{j});\ \mathrm{S}_{inv,\beta}/\mathrm{VP}_{b,\alpha}
    }
  }{
    \mathsf{d\breve{i}d} \circ \mathsf{john} \circ \mathsf{not} \circ \mathsf{say} \circ \mathsf{anything};\ \mathbf{P}\neg\exists_{\mathbf{thing}}(\lambda x.\mathbf{say}(x)(\mathbf{j}));\ \mathrm{S}_{inv,\beta}
  }
}{
  \mathsf{d\breve{i}d} \circ \mathsf{john} \circ \mathsf{not} \circ \mathsf{say} \circ \mathsf{anything};\ \mathbf{?P}\neg\exists_{\mathbf{thing}}(\lambda x.\mathbf{say}(x)(\mathbf{j}));\ \mathrm{Q}_{f,\beta}
}
$$

Finally, the interaction between VP ellipsis and inversion of the sort exemplified by (86c) requires a slight extension of the analysis of VP ellipsis. Specifically, we replace the ellipsis operator introduced above in (72) with the following somewhat more abstract entry, with σ a variable over prosodic functors of type $(\mathbf{st} \to \mathbf{st}) \to \mathbf{st}$:

(89) $\lambda\sigma.\sigma(\boldsymbol{\epsilon});\ \lambda\mathscr{F}.\mathscr{F}(P);\ (\mathrm{S}_{\gamma,\beta}\upharpoonright\mathrm{NP})\upharpoonright((\mathrm{S}_{\gamma,\beta}\upharpoonright\mathrm{NP})\upharpoonright\mathrm{VP}_{b,\alpha})$ (where $\gamma \in \{\mathit{fin}, \mathit{inv}\}$)

(89) is a generalization of (72) in that the latter can be derived as a theorem from the former, as shown in (90).

(90)

$$\dfrac{\begin{bmatrix}\varphi_1;\\ h;\mathrm{VP}_{f,\beta}/\mathrm{VP}_{b,\alpha}\end{bmatrix}^1 \quad \begin{bmatrix}\varphi_2;\\ Q;\mathrm{VP}_{b,\alpha}\end{bmatrix}^2}{\varphi_1\circ\varphi_2;\ h(Q);\ \mathrm{VP}_{f,\beta}}$$

$$\dfrac{\varphi_1\circ\varphi_2;\ h(Q);\ \mathrm{VP}_{f,\beta} \quad \begin{bmatrix}\varphi_3;\\ y;\\ \mathrm{NP}\end{bmatrix}^3}{\varphi_3\circ\varphi_1\circ\varphi_2;\ h(Q)(y);\ \mathrm{S}_{f,\beta}}$$

$$\dfrac{\varphi_3\circ\varphi_1\circ\varphi_2;\ h(Q)(y);\ \mathrm{S}_{f,\beta}}{\lambda\varphi_3.\varphi_3\circ\varphi_1\circ\varphi_2;\ \lambda y.h(Q)(y);\ \mathrm{S}_{f,\beta}\upharpoonright\mathrm{NP}}\upharpoonright\mathrm{I}^3$$

$$\dfrac{\lambda\varphi_3.\varphi_3\circ\varphi_1\circ\varphi_2;\ \lambda y.h(Q)(y);\ \mathrm{S}_{f,\beta}\upharpoonright\mathrm{NP}}{\begin{array}{l}\lambda\varphi_2\lambda\varphi_3.\varphi_3\circ\varphi_1\circ\varphi_2;\\ \lambda Q\lambda y.h(Q)(y);(\mathrm{S}_{f,\beta}\upharpoonright\mathrm{NP})\upharpoonright\mathrm{VP}_{b,\alpha}\end{array}}\upharpoonright\mathrm{I}^2$$

$$\dfrac{\begin{array}{l}\lambda\varphi_2\lambda\varphi_3.\varphi_3\circ\varphi_1\circ\varphi_2;\\ \lambda Q\lambda y.h(Q)(y);(\mathrm{S}_{f,\beta}\upharpoonright\mathrm{NP})\upharpoonright\mathrm{VP}_{b,\alpha}\end{array} \quad \begin{array}{l}\lambda\sigma.\sigma(\epsilon);\\ \lambda\mathscr{F}.\mathscr{F}(P);\\ (\mathrm{S}_{\gamma,\beta}\upharpoonright\mathrm{NP})\upharpoonright((\mathrm{S}_{\gamma,\beta}\upharpoonright\mathrm{NP})\upharpoonright\mathrm{VP}_{b,\alpha})\end{array}}{\lambda\varphi_3.\varphi_3\circ\varphi_1\circ\epsilon;\ \lambda y.h(P)(y);\ \mathrm{S}_{f,\beta}\upharpoonright\mathrm{NP}}$$

$$\dfrac{\lambda\varphi_3.\varphi_3\circ\varphi_1\circ\epsilon;\ \lambda y.h(P)(y);\ \mathrm{S}_{f,\beta}\upharpoonright\mathrm{NP} \quad \begin{bmatrix}\varphi_4;\\ v;\\ \mathrm{NP}\end{bmatrix}^4}{\varphi_4\circ\varphi_1;\ h(P)(v);\ \mathrm{S}_{f,\beta}}$$

$$\dfrac{\varphi_4\circ\varphi_1;\ h(P)(v);\ \mathrm{S}_{f,\beta}}{\varphi_1;\ \lambda v.h(P)(v);\ \mathrm{VP}_{f,\beta}}\backslash\mathrm{I}^4$$

$$\dfrac{\varphi_1;\ \lambda v.h(P)(v);\ \mathrm{VP}_{f,\beta}}{\lambda\varphi_1.\varphi_1;\ \lambda h\lambda v.h(P)(v);\ \mathrm{VP}_{f,\beta}\upharpoonright(\mathrm{VP}_{f,\beta}/\mathrm{VP}_{b,\alpha})}\upharpoonright\mathrm{I}^1$$

The point of generalizing the ellipsis operator as in (89) is that we can now obtain as a theorem an alternative version of the ellipsis operator that can apply directly to inverted auxiliaries. The proof for this theorem is almost identical to (90) just shown above, except that the directionality of the slash for the subject argument is converted from \ to / by the inversion operator (74).

(91)

$$\dfrac{\begin{array}{l}\lambda\varphi.\varphi;\\ \lambda g\lambda y\lambda R.g(R)(y);\\ (\mathrm{S}_{inv,\beta}/\mathrm{VP}_{b,\alpha}/\mathrm{NP})\upharpoonright\\ \quad(\mathrm{VP}_{f,\beta}/\mathrm{VP}_{b,\alpha})\end{array} \quad \begin{bmatrix}\varphi_1;\\ g;\\ \mathrm{VP}_{f,\beta}/\mathrm{VP}_{b,\alpha}\end{bmatrix}^1}{\varphi_1;\ \lambda z\lambda Q.g(Q)(z);\ \mathrm{S}_{inv,\beta}/\mathrm{VP}_{b,\alpha}/\mathrm{NP}}$$

$$\dfrac{\varphi_1;\ \lambda z\lambda Q.g(Q)(z);\ \mathrm{S}_{inv,\beta}/\mathrm{VP}_{b,\alpha}/\mathrm{NP} \quad \begin{bmatrix}\varphi_2;\\ v;\\ \mathrm{NP}\end{bmatrix}^2}{\varphi_1\circ\varphi_2;\ \lambda Q.g(Q)(v);\ \mathrm{S}_{inv,\beta}/\mathrm{VP}_{b,\alpha}}$$

$$\dfrac{\varphi_1\circ\varphi_2;\ \lambda Q.g(Q)(v);\ \mathrm{S}_{inv,\beta}/\mathrm{VP}_{b,\alpha} \quad \begin{bmatrix}\varphi_3;\\ R;\\ \mathrm{VP}_{b,\alpha}\end{bmatrix}^3}{\varphi_1\circ\varphi_2\circ\varphi_3;\ g(R)(v);\ \mathrm{S}_{inv,\beta}}$$

$$\dfrac{\varphi_1\circ\varphi_2\circ\varphi_3;\ g(R)(v);\ \mathrm{S}_{inv,\beta}}{\lambda\varphi_2.\varphi_1\circ\varphi_2\circ\varphi_3;\ \lambda v.g(R)(v);\ \mathrm{S}_{inv,\beta}\upharpoonright\mathrm{NP}}\upharpoonright\mathrm{I}^2$$

$$\dfrac{\lambda\varphi_2.\varphi_1\circ\varphi_2\circ\varphi_3;\ \lambda v.g(R)(v);\ \mathrm{S}_{inv,\beta}\upharpoonright\mathrm{NP}}{\begin{array}{l}\lambda\varphi_3\lambda\varphi_2.\varphi_1\circ\varphi_2\circ\varphi_3;\\ \lambda R\lambda v.g(R)(v);(\mathrm{S}_{inv,\beta}\upharpoonright\mathrm{NP})\upharpoonright\mathrm{VP}_{b,\alpha}\end{array}}\upharpoonright\mathrm{I}^3$$

$$\dfrac{\begin{array}{l}\lambda\varphi_3\lambda\varphi_2.\varphi_1\circ\varphi_2\circ\varphi_3;\\ \lambda R\lambda v.g(R)(v);(\mathrm{S}_{inv,\beta}\upharpoonright\mathrm{NP})\upharpoonright\mathrm{VP}_{b,\alpha}\end{array} \quad \begin{array}{l}\lambda\sigma.\sigma(\epsilon);\\ \lambda\mathscr{F}.\mathscr{F}(P);\\ (\mathrm{S}_{\gamma,\beta}\upharpoonright\mathrm{NP})\upharpoonright\\ \quad((\mathrm{S}_{\gamma,\beta}\upharpoonright\mathrm{NP})\upharpoonright\mathrm{VP}_{b,\alpha})\end{array}}{\lambda\varphi_2.\varphi_1\circ\varphi_2\circ\epsilon;\ \lambda v.g(P)(v);\ \mathrm{S}_{inv,\beta}\upharpoonright\mathrm{NP}}$$

$$\dfrac{\lambda\varphi_2.\varphi_1\circ\varphi_2\circ\epsilon;\ \lambda v.g(P)(v);\ \mathrm{S}_{inv,\beta}\upharpoonright\mathrm{NP} \quad \begin{bmatrix}\varphi_4;\\ w;\\ \mathrm{NP}\end{bmatrix}^4}{\varphi_1\circ\varphi_4;\ g(P)(w);\ \mathrm{S}_{inv,\beta}}$$

$$\dfrac{\varphi_1\circ\varphi_4;\ g(P)(w);\ \mathrm{S}_{inv,\beta}}{\varphi_1;\ \lambda w.g(P)(w);\ \mathrm{S}_{inv,\beta}/\mathrm{NP}}\upharpoonright\mathrm{I}^4$$

$$\dfrac{\varphi_1;\ \lambda w.g(P)(w);\ \mathrm{S}_{inv,\beta}/\mathrm{NP}}{\lambda\varphi_1.\varphi_1;\ \lambda g\lambda w.g(P)(w);\ (\mathrm{S}_{inv,\beta}/\mathrm{NP})\upharpoonright(\mathrm{VP}_{f,\beta}/\mathrm{VP}_{b,\alpha})}\upharpoonright\mathrm{I}^1$$

Then, by applying (91) to *should*, we obtain an inverted auxiliary licensing VP ellipsis with which the derivation for the version of (86c) with a lexical auxiliary is straightforward:

(92)

$$\dfrac{\dfrac{\dfrac{\begin{array}{l}\vdots\\ \lambda\varphi_1.\varphi_1;\\ \lambda g\lambda w.g(P)(w);(\mathrm{S}_{inv,\beta}/\mathrm{NP})\upharpoonright(\mathrm{VP}_{f,\beta}/\mathrm{VP}_{b,\alpha})\end{array} \quad \begin{array}{l}\mathsf{should};\\ \lambda T\lambda u.\Box T(u);\\ \mathrm{VP}_{f,+}/\mathrm{VP}_{b,+}\end{array}}{\mathsf{should};\ \lambda w.\Box P(w);\ \mathrm{S}_{inv,+}/\mathrm{NP}} \quad \begin{array}{l}\mathsf{john};\\ \mathbf{j};\\ \mathrm{NP}\end{array}}{\mathsf{should}\circ\mathsf{john};\ \Box P(\mathbf{j});\ \mathrm{S}_{inv,+}} \quad \begin{array}{l}\lambda\varphi_2.\varphi_2;\\ \lambda p.\boldsymbol{?}p;\\ \mathrm{Q}_{f,\beta}\upharpoonright\mathrm{S}_{inv,\beta}\end{array}}{\mathsf{should}\circ\mathsf{john};\ \boldsymbol{?}\Box P(\mathbf{j});\ \mathrm{Q}_{f,+}}$$

The behavior of unstressed *did* proves to mirror perfectly that of the 'standard' modals. The derivation is given in (93), where the only substantial difference from (92) is that the *do* insertion operator takes the ellipsis operator as an argument instead of the latter taking a lexical auxiliary as an argument, as in all the other unstressed *do* derivations.

(93)

$$
\dfrac{\dfrac{\dfrac{\begin{array}{l}\vdots\\ \lambda\varphi_1.\varphi_1;\\ \lambda g\lambda w.g(R)(w);\\ (\mathrm{S}_{inv,+}/\mathrm{NP})\upharpoonright(\mathrm{VP}_{f,+}/\mathrm{VP}_{b,\alpha})\end{array} \quad \begin{array}{l}\lambda\sigma.\sigma(\mathsf{d\breve{i}d});\\ \lambda\mathscr{F}.\mathscr{F}(\lambda Q\lambda z.\mathbf{P}\,Q(z));\\ \mathrm{X}\upharpoonright(\mathrm{X}\upharpoonright(\mathrm{VP}_{f,\alpha}/\mathrm{VP}_{b,\alpha}))\end{array}}{\mathsf{d\breve{i}d};\ \lambda w.\mathbf{P}R(w);\ \mathrm{S}_{inv,+}/\mathrm{NP}} \quad \begin{array}{l}\mathsf{john};\\ \mathbf{j};\\ \mathrm{NP}\end{array}}{\mathsf{d\breve{i}d}\circ\mathsf{john};\ \mathbf{P}R(\mathbf{j});\ \mathrm{S}_{inv,+}} \quad \begin{array}{l}\lambda\varphi_2.\varphi_2;\\ \lambda p.\mathbf{?}p;\\ \mathrm{Q}_{f,\beta}\upharpoonright\mathrm{S}_{inv,\beta}\end{array}}{\mathsf{d\breve{i}d}\circ\mathsf{john};\ \mathbf{?P}R(\mathbf{j});\ \mathrm{Q}_{f,+}}
$$

Finally, essentially the same proof narrative will yield (86d), with the difference that we first apply the inversion operator to license a string just as in (88), but with variables in the subject position and in the complement position of *not*. Abstraction on these will give us a sign typed $(\mathrm{S}_{inv}\upharpoonright\mathrm{NP})\upharpoonright\mathrm{VP}$, to which the generalized ellipsis operator can apply, yielding a sign seeking an NP argument.

(94)

$$
\dfrac{\begin{array}{l}\lambda\varphi.\varphi;\\ \lambda p.\mathbf{?}p;\\ \mathrm{Q}_{f,\beta}\upharpoonright\mathrm{S}_{inv,\beta}\end{array} \quad \dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\begin{array}{l}\vdots\\ \mathsf{not};\\ \lambda P\lambda y.\\ \quad\neg P(y);\\ \mathrm{VP}_{b,+}/\mathrm{VP}_{b,-}\end{array} \quad \begin{bmatrix}\varphi_0;\\ Q;\\ \mathrm{VP}_{b,-}\end{bmatrix}^0}{\begin{array}{l}\mathsf{not}\circ\varphi_0;\\ \lambda y.\neg Q(y);\mathrm{VP}_{b,+}\end{array}} \quad \dfrac{\dfrac{\begin{array}{l}\lambda\varphi_1.\varphi_1;\\ \lambda g\lambda y\lambda R.g(R)(y);\\ (\mathrm{S}_{inv,\beta}/\mathrm{VP}_{b,\alpha}/\mathrm{NP})\upharpoonright\\ \quad(\mathrm{VP}_{f,\beta}/\mathrm{VP}_{b,\alpha})\end{array} \quad \begin{array}{l}\lambda\sigma.\sigma(\mathsf{d\breve{i}d});\\ \lambda\mathscr{F}.\mathscr{F}(\lambda Q\lambda z.\\ \quad\mathbf{P}(Q(z)));\\ \mathrm{X}\upharpoonright(\mathrm{X}\upharpoonright\\ \quad(\mathrm{VP}_{b,\alpha}/\mathrm{VP}_{b,\alpha}))\end{array}}{\mathsf{d\breve{i}d};\ \lambda y\lambda R.\mathbf{P}R(y);\ \mathrm{S}_{inv,\alpha}/\mathrm{VP}_{b,\alpha}/\mathrm{NP}} \quad \begin{bmatrix}\varphi_1;\\ v;\\ \mathrm{NP}\end{bmatrix}^1}{\mathsf{d\breve{i}d}\circ\varphi_1;\ \lambda R.\mathbf{P}R(v);\ \mathrm{S}_{inv,\alpha}/\mathrm{VP}_{b,\alpha}}}{\mathsf{d\breve{i}d}\circ\varphi_1\circ\mathsf{not}\circ\varphi_0;\ \mathbf{P}\,\neg Q(v);\ \mathrm{S}_{inv,+}}}{\lambda\varphi_1.\mathsf{d\breve{i}d}\circ\varphi_1\circ\mathsf{not}\circ\varphi_0;\ \lambda v.\mathbf{P}\neg Q(v);\ \mathrm{S}_{inv,+}\upharpoonright\mathrm{NP}}\upharpoonright\mathrm{I}^1}{\begin{array}{l}\lambda\varphi_0\lambda\varphi_1.\mathsf{d\breve{i}d}\circ\varphi_1\circ\mathsf{not}\circ\varphi_0;\\ \lambda Q\lambda v.\mathbf{P}\neg Q(v);\mathrm{S}_{inv,+}\upharpoonright\mathrm{NP}\upharpoonright\mathrm{VP}_{b,-}\end{array}}\upharpoonright\mathrm{I}^0 \quad \begin{array}{l}\lambda\sigma.\sigma(\boldsymbol{\epsilon});\\ \lambda\mathscr{F}.\mathscr{F}(T);\\ (\mathrm{S}_{\gamma,\beta}\upharpoonright\mathrm{NP})\upharpoonright\\ \quad((\mathrm{S}_{\gamma,\beta}\upharpoonright\\ \quad\mathrm{NP})\upharpoonright\mathrm{VP}_{b,\alpha})\end{array}}{\lambda\varphi_1.\mathsf{d\breve{i}d}\circ\varphi_1\circ\mathsf{not}\circ\boldsymbol{\epsilon};\ \lambda v.\mathbf{P}\neg T(v);\ \mathrm{S}_{inv,+}\upharpoonright\mathrm{NP}} \quad \begin{array}{l}\mathsf{john};\\ \mathbf{j};\\ \mathrm{NP}\end{array}}{\mathsf{d\breve{i}d}\circ\mathsf{john}\circ\mathsf{not}\circ\boldsymbol{\epsilon};\ \mathbf{P}\,\neg T(\mathbf{j});\ \mathrm{S}_{inv,+}}}{\mathsf{d\breve{i}d}\circ\mathsf{john}\circ\mathsf{not}\circ\boldsymbol{\epsilon};\ \mathbf{?P}\,\neg T(\mathbf{j});\ \mathrm{Q}_{f,+}}
$$

A.2.2 More complex NIE interactions involving Gapping

To conclude our discussion of NIE interactions, here we consider somewhat more complex cases in which NIE interact with the Gapping scope patterns. In examples such as (95), the problem to be acounted for is how the *do* insertion operator given above in (80) supports the wide scope of negation over the Gapping conjunction in the Oehrle/Siegel example:

(95) John did not eat steak and Mary pizza.

Such examples follow directly from the higher order form for unstressed *dĭd* that we provided earlier. The proof runs as in (96), where we instantiate X in the unstressed *do* forms as $\mathrm{VP}_{f,-}/\mathrm{VP}_{b,-}$. The trick here is to feed a hypothetical $(\mathrm{VP}_{f,-}/\mathrm{VP}_{b,-})\upharpoonright(\mathrm{VP}_{f,-}/\mathrm{VP}_{b,-})$ derived from a $\mathrm{VP}_{b,-}/\mathrm{VP}_{b,-}$ hypothesis via Geach to the *do* insertion operator to form an auxiliary that contains a $\mathrm{VP}_{b,-}/\mathrm{VP}_{b,-}$ hypothesis as in (96a), which later gets bound by the higher-order negation after the Gapping conjunction is formed, as in (96b).

(96) a.

$$\dfrac{\begin{array}{l}\lambda\sigma_1.\sigma_1(\mathsf{d\breve{\imath}d});\\ \lambda\mathscr{F}.\mathscr{F}(\lambda R\lambda z.\mathbf{P}\,R(z));\\ (\mathrm{VP}_{f,-}/\mathrm{VP}_{b,-})\upharpoonleft\\ \quad((\mathrm{VP}_{f,-}/\mathrm{VP}_{b,-})\upharpoonleft(\mathrm{VP}_{f,-}/\mathrm{VP}_{b,-}))\end{array} \quad \dfrac{\dfrac{\dfrac{\dfrac{\begin{bmatrix}\varphi_2;\\ f;\\ \mathrm{VP}_{b,-}/\mathrm{VP}_{b,-}\end{bmatrix}^2 \quad \begin{bmatrix}\varphi_1;\\ Q;\\ \mathrm{VP}_{b,-}\end{bmatrix}^1}{\varphi_2\circ\varphi_1;\ f(Q);\ \mathrm{VP}_{b,-}} \quad \begin{bmatrix}\varphi_0;\\ g;\\ \mathrm{VP}_{f,-}/\mathrm{VP}_{b,-}\end{bmatrix}^0}{\varphi_0\circ\varphi_2\circ\varphi_1;\ g(f(Q));\ \mathrm{VP}_{f,-}}}{\varphi_0\circ\varphi_2;\ \lambda Q.g(f(Q));\ \mathrm{VP}_{f,-}/\mathrm{VP}_{b,-}}/\mathrm{I}^1}{\begin{array}{l}\lambda\varphi_0.\varphi_0\circ\varphi_2;\\ \lambda g\lambda Q.g(f(Q));(\mathrm{VP}_{f,-}/\mathrm{VP}_{b,-})\upharpoonleft(\mathrm{VP}_{f,-}/\mathrm{VP}_{b,-})\end{array}}\upharpoonleft\mathrm{I}^0}{\mathsf{d\breve{\imath}d}\circ\varphi_2;\ \lambda Q\lambda z.\mathbf{P}\,f(Q)(z);\ \mathrm{VP}_{f,-}/\mathrm{VP}_{b,-}}$$

b.

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\begin{array}{l}\vdots\\ \mathsf{d\breve{\imath}d}\circ\varphi_2;\\ \lambda Q\lambda z.\\ \quad\mathbf{P}f(Q)(z);\\ \mathrm{VP}_{f,-}/\mathrm{VP}_{b,-}\end{array} \quad \dfrac{\begin{array}{l}\mathsf{eat};\\ \mathbf{eat};\\ \mathrm{VP}_{b,-}/\mathrm{NP}\end{array} \quad \begin{bmatrix}\varphi_4;\\ x;\\ \mathrm{NP}\end{bmatrix}^4}{\begin{array}{l}\mathsf{eat}\circ\varphi_4;\\ \mathbf{eat}(x);\mathrm{VP}_{b,-}\end{array}}}{\begin{array}{l}\mathsf{d\breve{\imath}d}\circ\varphi_2\circ\mathsf{eat}\circ\varphi_4;\\ \lambda z.\mathbf{P}f(\mathbf{eat}(x))(z);\mathrm{VP}_{f,-}\end{array}}}{\begin{array}{l}\mathsf{d\breve{\imath}d}\circ\varphi_2\circ\mathsf{eat};\\ \lambda x\lambda z.\mathbf{P}f(\mathbf{eat}(x))(z);\mathrm{VP}_{f,-}/\mathrm{NP}\end{array}}/\mathrm{I}^4 \quad \begin{array}{l}\vdots\\ \lambda\varphi_5.\mathsf{john}\circ\varphi_5\circ\mathsf{steak}\circ\\ \quad\mathsf{and}\circ\mathsf{mary}\circ\mathsf{pizza};\\ \lambda P[P(\mathbf{steak})(\mathbf{j})\wedge\\ \quad P(\mathbf{pizza})(\mathbf{m})];\\ \mathrm{S}_{f,-}\upharpoonleft(\mathrm{VP}_{f,-}/\mathrm{NP})\end{array}}{\begin{array}{l}\mathsf{john}\circ\mathsf{d\breve{\imath}d}\circ\varphi_2\circ\mathsf{eat}\circ\mathsf{steak}\circ\mathsf{and}\circ\mathsf{mary}\circ\mathsf{pizza};\\ \mathbf{P}f(\mathbf{eat}(\mathbf{steak}))(\mathbf{j})\wedge\mathbf{P}f(\mathbf{eat}(\mathbf{pizza}))(\mathbf{m});\mathrm{S}_{f,-}\end{array}}}{\begin{array}{l}\lambda\varphi_2.\mathsf{john}\circ\mathsf{d\breve{\imath}d}\circ\varphi_2\circ\mathsf{eat}\circ\mathsf{steak}\circ\mathsf{and}\circ\mathsf{mary}\circ\mathsf{pizza};\\ \lambda f[\mathbf{P}f(\mathbf{eat}(\mathbf{steak}))(\mathbf{j})\wedge\mathbf{P}f(\mathbf{eat}(\mathbf{pizza}))(\mathbf{m})];\\ \mathrm{S}_{f,-}\upharpoonleft(\mathrm{VP}_{b,-}/\mathrm{VP}_{b,-})\end{array}}\upharpoonleft\mathrm{I}^2 \quad \begin{array}{l}\lambda\sigma_2.\sigma_2(\mathsf{not});\\ \lambda\mathscr{G}.\neg\mathscr{G}(\mathbf{id}_{et});\\ \mathrm{S}_{f,+}\upharpoonleft(\mathrm{S}_{f,-}\upharpoonleft(\mathrm{VP}_{b,-}/\mathrm{VP}_{b,-}))\end{array}}{\begin{array}{l}\mathsf{john}\circ\mathsf{d\breve{\imath}d}\circ\mathsf{not}\circ\mathsf{eat}\circ\mathsf{steak}\circ\mathsf{and}\circ\mathsf{mary}\circ\mathsf{pizza};\\ \neg[\mathbf{P}\,\mathbf{eat}(\mathbf{steak})(\mathbf{j})\wedge\mathbf{P}\,\mathbf{eat}(\mathbf{pizza})(\mathbf{m})];\mathrm{S}_{f,+}\end{array}}$$

We next consider a case in which an inverted auxiliary outscopes a Gapping conjunction, as in (97):

(97) Should John eat steak and Mary eat (just) pizza?

The proof is straightforward, despite the ostensible complexity. The key idea is that, with the inversion operator, the higher-order auxiliary *should* can be converted from its ordinary type $\mathrm{S}\upharpoonleft(\mathrm{S}\upharpoonleft(\mathrm{VP}/\mathrm{VP}))$ in the lexicon to a type $\mathrm{S}\upharpoonleft(\mathrm{S}\upharpoonleft(\mathrm{S}/\mathrm{VP}/\mathrm{NP}))$, which 'fills-in' a gap for an inverted auxiliary.

(98)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\begin{array}{l}\lambda\varphi.\varphi;\\ \lambda g\lambda y\lambda R.g(R)(y);\\ (\mathrm{S}_{inv,\beta}/\mathrm{VP}_{b,\alpha}/\mathrm{NP})\upharpoonleft\\ \quad(\mathrm{VP}_{f,\beta}/\mathrm{VP}_{b,\alpha})\end{array} \quad \begin{bmatrix}\varphi_0;\\ f;\\ \mathrm{VP}_{f,\beta}/\mathrm{VP}_{b,\alpha}\end{bmatrix}^0}{\varphi_0;\ \lambda y\lambda R.f(R)(y);\ \mathrm{S}_{inv,\beta}/\mathrm{VP}_{b,\alpha}/\mathrm{NP}} \quad \begin{bmatrix}\sigma_1;\\ \mathscr{G};\\ \mathrm{S}_{inv,\gamma}\upharpoonleft(\mathrm{S}_{inv,\beta}/\mathrm{VP}_{b,\alpha}/\mathrm{NP})\end{bmatrix}^1}{\sigma_1(\varphi_0);\ \mathscr{G}(\lambda y\lambda R.f(R)(y));\ \mathrm{S}_{inv,\gamma}}}{\lambda\varphi_0.\sigma_1(\varphi_0);\ \lambda f.\mathscr{G}(\lambda y\lambda R.f(R)(y));\ \mathrm{S}_{inv,\gamma}\upharpoonleft(\mathrm{VP}_{f,\beta}/\mathrm{VP}_{b,\alpha})}\upharpoonleft\mathrm{I}^0 \quad \begin{array}{l}\lambda\sigma_2.\sigma_2(\mathsf{should});\\ \lambda\mathscr{F}.\Box\mathscr{F}(\mathbf{Id});\\ \mathrm{S}_{inv,+}\upharpoonleft\\ \quad(\mathrm{S}_{inv,+}\upharpoonleft(\mathrm{VP}_{f,+}/\mathrm{VP}_{b,+}))\end{array}}{\sigma_1(\mathsf{should});\ \Box\mathscr{G}(\lambda y\lambda R.R(y));\ \mathrm{S}_{inv,+}}}{\lambda\sigma_1.\sigma_1(\mathsf{should});\ \lambda\mathscr{G}.\Box\mathscr{G}(\lambda y\lambda R.R(y));\ \mathrm{S}_{inv,+}\upharpoonleft(\mathrm{S}_{inv,+}\upharpoonleft(\mathrm{S}_{inv,+}/\mathrm{VP}_{b,+}/\mathrm{NP}))}\upharpoonleft\mathrm{I}^1$$

This is then combined with a conjunction of two type $\mathrm{S}_{f,\zeta}\upharpoonleft(\mathrm{S}_{f,\delta}/\mathrm{VP}_{b,\gamma}/\mathrm{NP})$ clauses, that is, clauses that are missing a fronted auxiliary, which can be derived as follows:

(99) a.

$$\dfrac{\dfrac{\dfrac{[\varphi_2; f; \mathrm{S}_{inv,\alpha}/\mathrm{VP}_{b,\beta}/\mathrm{NP}]^2 \quad \mathsf{john};\ \mathbf{j};\ \mathrm{NP}}{\varphi_2\circ\mathsf{john};\ f(\mathbf{j});\ \mathrm{S}_{inv,\alpha}/\mathrm{VP}_{b,\beta}} \quad \begin{array}{c}\vdots\\ \mathsf{eat}\circ\mathsf{steak};\ \mathbf{eat}(\mathbf{steak});\ \mathrm{VP}_{b,\beta}\end{array}}{\varphi_2\circ\mathsf{john}\circ\mathsf{eat}\circ\mathsf{steak};\ f(\mathbf{j})(\mathbf{eat}(\mathbf{steak}));\ \mathrm{S}_{inv,\alpha}}}{\lambda\varphi_2.\varphi_2\circ\mathsf{john}\circ\mathsf{eat}\circ\mathsf{steak};\ \lambda f.f(\mathbf{j})(\mathbf{eat}(\mathbf{steak}));\ \mathrm{S}_{inv,\alpha}\upharpoonleft(\mathrm{S}_{inv,\alpha}/\mathrm{VP}_{b,\beta}/\mathrm{NP})}\upharpoonleft\mathrm{I}^2$$

b.
$$\dfrac{\vdots}{\begin{array}{l}\lambda\varphi_6.\varphi_6 \circ \mathsf{john} \circ \mathsf{eat} \circ \mathsf{steak} \circ \mathsf{and} \circ \mathsf{mary} \circ \mathsf{eat} \circ \mathsf{pizza};\\ \lambda h.h(\mathbf{j})(\mathbf{eat}(\mathbf{steak})) \wedge h(\mathbf{m})(\mathbf{eat}(\mathbf{pizza})); \mathrm{S}_{inv,\alpha}\upharpoonright(\mathrm{S}_{inv,\alpha}/\mathrm{VP}_{b,\beta}/\mathrm{NP})\end{array}}$$

The derivation completes by apply the sign derived in (98) to the one obtained in (99), over which the interrogative operator takes scope.

(100)
$$\dfrac{\dfrac{\begin{array}{l}\vdots\\ \lambda\varphi_6.\varphi_6 \circ \mathsf{john} \circ \mathsf{eat} \circ \mathsf{steak} \circ\\ \quad \mathsf{and} \circ \mathsf{mary} \circ \mathsf{eat} \circ \mathsf{pizza};\\ \lambda h.h(\mathbf{j})(\mathbf{eat}(\mathbf{steak})) \wedge h(\mathbf{m})(\mathbf{eat}(\mathbf{pizza}));\\ \mathrm{S}_{inv,\alpha}\upharpoonright(\mathrm{S}_{inv,\alpha}/\mathrm{VP}_{b,\beta}/\mathrm{NP})\end{array} \quad \begin{array}{l}\lambda\sigma_2.\sigma_1(\mathsf{should});\\ \lambda\mathscr{G}.\Box\mathscr{G}(\lambda y\lambda R.R(y));\\ \mathrm{S}_{inv,+}\upharpoonright(\mathrm{S}_{inv,+}\upharpoonright(\mathrm{S}_{inv,+}/\mathrm{VP}_{b,+}/\mathrm{NP}))\end{array}}{\begin{array}{l}\mathsf{should} \circ \mathsf{john} \circ \mathsf{eat} \circ \mathsf{steak} \circ \mathsf{and} \circ \mathsf{mary} \circ \mathsf{eat} \circ \mathsf{pizza};\\ \Box[\mathbf{eat}(\mathbf{steak})(\mathbf{j}) \wedge \mathbf{eat}(\mathbf{pizza})(\mathbf{m})]; \mathrm{S}_{inv,+}\end{array}} \quad \begin{array}{l}\lambda\varphi.\varphi;\\ \lambda p.\mathbf{?}(p);\\ \mathrm{Q}_{f,\eta}\upharpoonright\mathrm{S}_{inv,\eta}\end{array}}{\begin{array}{l}\mathsf{should} \circ \mathsf{john} \circ \mathsf{eat} \circ \mathsf{steak} \circ \mathsf{and} \circ \mathsf{mary} \circ \mathsf{eat} \circ \mathsf{pizza};\\ \mathbf{?}\Box[\mathbf{eat}(\mathbf{steak})(\mathbf{j}) \wedge \mathbf{eat}(\mathbf{pizza})(\mathbf{m})]; \mathrm{Q}_{f,+}\end{array}}$$