Copies from "Standard Set Theory"?
A Remark on Chomsky, Gallego & Ott (2019)

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1. *Introduction*

Replacement of trace theory by the "copy theory of movement" (Chomsky 1993:34f.) has been one of the most consequential and intensively debated aspects of minimalist syntax (Corver and Nunes 2007; Takahashi 2010a; 2010b).[1] The main specifically "minimalist"[2] idea behind this shift has been that core syntactic "computation" observes the so-called "Inclusiveness Condition," which "precludes the introduction of extraneous objects – for instance, traces and the bar-levels of X-bar Theory and other labels" (Chomsky, Gallego and Ott 2019 [CGO]:237).

In further radicalization of minimalist reasoning, the copy theory itself has begun being scrutinized for redundancies. Copies are taken to arise where the binary operation Merge applies to constituents γ_1 and γ_2 with either γ_2 being a subconstituent of γ_1 or vice versa.[3] This variant of Merge is also referred to as "Internal Merge" (IM), designed to capture (the essence of) and supersede the earlier operation "Move(-α)" (Chomsky 2005:12f.; 2008:140). Here it is of particular interest from the radical minimalist perspective to be able to tell what is meant by "arise." For example, under the "Copy+Merge theory of movement," IM ("Move")

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[1] I cite selected examplary work from the vast body of literature without in any way implying exhaustiveness.

[2] Scare quotes ("...") – not formally distinguished from direct quotes – are used throughout to flag terminology that would require critical discussion and thorough motivation, sidestepped here in the interest of conciseness.

[3] Langendoen (2003:309) proposes to distinguish the respective cases as Merge($\gamma_1/\gamma_2,\gamma_2$) and Merge($\gamma_2/\gamma_1,\gamma_1$). If in addition the "Extension Condition" (Chomsky 1993:23) or the "No Tampering Condition" (Chomsky 2007:8) holds, the root node of the containing constituent is a ("global") root in the technical sense of being undominated.

involves an independent (sub)operation Copy (Nunes 2004:89). By contrast, CGO (p.246) – following a line of thinking initiated by Bobaljik (1995) – assume that "no COPY operation is necessary given that copies are simply a by-product of IM (on standard set-theoretic assumptions)."

At this point, however, an even more fundamental question must be asked, namely, what say "standard set-theory" should have in these matters. Let us begin investigating this question by looking at example (1). Assume for (1a) that $a = \gamma_1$ and $\{ a, b \} = \gamma_2$. Then Merge(γ_1,γ_2) yields γ_3, depicted in (1b).

(1) a. $\{ a, b \}$

b. $\{ a, \{ a, b \} \}$

According to CGO, Merge(γ_1,γ_2) has "produced" two copies of *a*. Now, an elementary "materialistic" intuition about copies – verifiable by anyone familiar with a photocopy machine – is that if you have two copies you have two things. You can, for example, destroy the original copy and keep the one the machine produced, or vice versa. This notion of copy, as I will argue in Section 2, is clearly at odds with what set theorists would standardly assume about the two instances of *a* in (1b). Likewise, it does not fully fit into CGO's own exposition and agenda (Section 3). Section 4 concludes with some informal remarks about the bigger picture.

2. *Set Theory and Copies*

From the perspective of set theory (for less technical introductions, see Bagaria 2019; Halmos 1974), the question as to whether the two instances of *a* in (1b) count as one or two things can be decided fairly straightforwardly by employing the "support function" (sp) (Barwise

1975:24), which effectively removes all non-outermost brackets from sets.[4] The result of applying this to γ_3 in (1b) is shown in (2a). This is contrasted with a structure not based on IM in (2b).

(2) a. sp({ *a*, { *a*, *b* } }) = { *a*, *b* }

b. sp({ *a*, { *b*, *c* } }) = { *a*, *b*, *c* }

If *a* qua membership in the set { *a*, { *a*, *b* } } were different from *a* qua membership in the set { *a*, *b* }, the two different "things" should be preserved in sp({ *a*, { *a*, *b* } }) the same way the difference between *a*, *b*, and *c* is preserved in sp({ *a*, { *b*, *c* } }). But they aren't. According to set theory there is only one thing, called "*a*," and *a* happens to be member of two sets, γ_2 and γ_3.[5] Collapsing the "inner walls" of γ_3 by applying sp amounts to computing { *a* } $\cup$ { *a*, *b* },

[4] The definition of "sp" is given in (i):

(i) a. $\mathrm{sp}(p) = \{p\}$ b. $\mathrm{sp}(a) = \bigcup_{x \in a} \mathrm{sp}(x)$

This crucially presupposes the existence of "urelements" (Barwise 1975:1.1), a rather "natural" assumption in the context of syntactic structures "projected from" lexical formatives, "roots," or whatever else constitutes "terminals." Variable *p* ranges over urelements, *a* over sets, and *x* over the union of the sets of urelements and sets (Barwise 1975:10).

[5] People who like concrete examples could imagine that *a* is Artemis and that (1b) describes the current organizational structure of the scientific board for the preservation of Keynesianism in Greece (SBPKG). Imagine further that the statutes of the SBPKG stipulate that it consist of exactly one leader and a two person subcommittee, the advisory committee for the SBPKG, i.e., ACSBPKG. Then (1b) says that currently Artemis is leader of the SBPKG and happens, at the same time, to be member of the ACSBPKG, together with Basilius. At a meeting of the SBPKG, Artemis will be present only once. Noone would expect (or allow) any copy or double of Artemis to show up. Whether she has one or two votes is another matter.

and $\{ a, a, b \} = \{ a, b \}$.[6] This latter equivalence, of course, follows from the fundamental set-theoretic "Axiom of Extension" (Suppes 1972:21):

(3) $\forall x \, [\, x \in A \leftrightarrow x \in B \,] \rightarrow A = B$

Thus, contrary to what CGO (p.246) suggest, "standard set theory" does not support the interpretation of IM as depicted in (1) as involving copies of *a*, as long as one sticks to a "standard" notion of copy. Instead, it favors adopting the perspective of "multidominance" (Citko 2011), which we return to momentarily.

In order to introduce a set-theoretically sound counterpart of copying, two strategies suggest themselves immediately. One either uses a diacritic (or functor) to distinguish *a* from its copy. Or, alternatively, one replaces standard sets by "multisets."[7] This would lead (*mutatis mutandis*) to the counterparts of (2a) in (4a) and (4b), respectively.

(4) a. $\mathrm{sp}(\{ a, \{ a_{\mathrm{C}}, b \} \}) = \{ a, a_{\mathrm{C}}, b \}$

b. $\mathrm{sp}(\{_{\mathrm{M}}\, a, \{_{\mathrm{M}}\, a, b \} \}) = \{_{\mathrm{M}}\, a, a, b \}$

However, (4a) comes close to reintroducing trace theory in that it violates the Inclusiveness Condition. And (4b) is not "standard set theory."

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[6] Guimarães (2000: section 4) relies on this fundamental set-theoretic property in his analysis of "Self-Merge," where, applied to example (1), $\mathrm{Merge}(\gamma_1,\gamma_1) = \{ a \}$.

[7] The definition by Troelstra (1992:2) reads: "A *multiset* over *A* is a mapping $f : A \rightarrow \mathbb{N}$, where $f(a) = n$ means that *a* occurs with multiplicity *n*. If $f(a) = 0$, *a* is not an element of *f* {that is to say, *a* occurs with multiplicity 0)." Thus, assuming $A = \{ a, b, c \}$, the multiset in (4b) is given by: $f_{(4b)} = \{ \langle a,2\rangle, \langle b,1\rangle, \langle c,0\rangle \}$.

3. *Discontinuous Objects and Multidominance*

One might dismiss what has been said so far and claim that CGO have been misinterpreted. For the latter to be made plausible, clearer statements about copies (and sets) would have to be offered as evidence. Unfortunately, however, this is not an easy task. About their counterpart of (1b), CGO (p.237) state "that there is still only one, discontinuous object Y [= *a*, addition by author] in K [= γ_3, addition by author], not two distinct objects." Yet, if we are dealing with discontinuous objects in the case of "copies" resulting from IM, the further expectation may be formed that these entities can be operated on as units. Is it possible, for example, to "move" or delete them together in one and the same step? (5) offers schematic instances of "transformations" that would lead to a definite yes here.

(5) a. [... [*a* [*b a*]]] ⇒ [... *a* ... *a* ... [_ [*b* _]]]

b. [... [*a* [*b a*]]] ⇒ [... [~~*a*~~ [*b* ~~*a*~~]]]

CGO do not go into such direct ramifications of their remark, and there is no evidence that any such operations on discontinuous objects exist. Instead, differential treatment of "copies" is exactly the point of binding/reconstruction-theoretic arguments for the copy theory of movement (Chomsky 1993; Takahashi 2010a; 2010b), and treatments of remnant movement (Thiersch 2017) as well as challenges to freezing such as "chain interleaving" (Collins 1994) would seem to suggest the same disunity.[8]

To clarify things further, we have to understand the following passage from CGO (p.241): "Suppose X is raised from within P [a "phase," addition by author] by IM. If syntactic objects are defined as sets of occurrences, it follows that P subsequently no longer contains X, since it does not contain the set of X's occurrences." Applied to the simple case in (1), we could

[8] CGO's discussion of phases, Spell-Out, Transfer, and (in)accessibility of constituents (pp.240-242) is relevant for determining their perspective here.

assume that γ_2 in (1b) counts as phasal. Also, adopting the definition of "(positions of) occurrences" by Collins and Stabler (2016:51),[9] we arrive at $\langle\gamma_3,a\rangle$ and $\langle\gamma_3,\gamma_2,a\rangle$ as the two occurrences of *a* in (1b). The resulting "syntactic object" (SO) is depicted in (6a) and the syntactic object corresponding to γ_2 in (6b).

(6) a. $\{ \langle\{ a, \{ a, b \} \}, \{ a, b \}, a\rangle, \langle\{ a, \{ a, b \} \}, a\rangle \}$

b. $\{ \langle\{ a, b \}\rangle \}$

The SO corresponding to γ_2, $SO_{(6b)}$, does not "contain" $SO_{(6a)}$, the discontinous SO constituted by the two occurences of *a*, because there is a member of $SO_{(6a)}$, in which $\{ a, b \}$ does not precede *a*.

All of this is very well, of course, but the transition from the world of constituents as sets, where "dominance" is established by recursively checking membership, to the world of (potentially discontinuous) "syntactic objects," where "dominance" is established by persistent precedence in a set of lists of sets constituting "syntactic objects," is something of a leap. Still, all would be quite in order, were it not the case that CGO (p.246) had dismissed the interpretation of "copies" in terms of "multidominance" – the assumption that irrespective of notation, (1b) says there is only one constituent γ_1 [= *a*], which is immediately dominated by multiple items, namely, γ_2 [= $\{ a, b \}$] and γ_3 [= $\{ a, \{ a, b \}\}$] – on the grounds that "complex graph-theoretic objects are not defined by simplest MERGE." However, the "simplicity" of Merge itself, as characterized in (7) (p.246), appears to be compromised by the shift to SOs.

[9] "Consider a syntactic object SO = $\{ S_1, \{ S_1, S_2 \} \}$. We say that S_1 occurs twice in SO. The position of an occurrence is given by a "path" from SO to the particular occurrence. A path is a sequence of syntactic objects $\langle SO_1, SO_2, ..., SO_n\rangle$ where for every adjacent pair SO_i, SO_{i+1} of objects in the path, $SO_{i+1} \in SO_i$ (i.e., SO_{i+1} is immediately contained in SO_i)." Collins and Stabler (l.c.) also discuss open questions about and shortcomings of alternative definitions.

(7) "MERGE [...] applies to two objects X and Y, yielding a new one, K = {X,Y}."

CGO employ the terms "object" and "syntactic object" quasi-synonymously,[10] so if indeed "syntactic objects are defined as sets of occurrences" (p.241), the definition of Merge would seem to have to be revised and "lifted" to sets of sequences of constituents. And, what's more, we have arrived at structures that approximate the "immediate constituency" (IC) / "immediate dominance" (ID) relations defined by Gärtner (2002:3.3) to capture "multidominance" in a minimalist derivational system. In its simplest (IC-based) form, that system would render (1b) as { $\langle a,\langle a,b\rangle\rangle$, $\langle b,\langle a,b\rangle\rangle$, $\langle\langle a,\langle a,\langle a,b\rangle\rangle\rangle$, $\langle\langle a,b\rangle,\langle a,\langle a,b\rangle\rangle\rangle$ } (Gärtner 2002:147f.). The same kinds of redundancy that emerge in (6a) show up here as well. They serve the keeping track of "occurrences." It therefore seems that CGO's misgivings about "complex graph-theoretic objects" are at least premature. As long as no complete (derivational) system is presented that properly defines and "stitches together" lower and higher "constructs" (lexical items, constituents, occurrences, syntactic objects, chains etc.) and operations that manipulate them, matters of "complexity" are hard to assess.

4. *Syntax and Set Theory*

Let us, in concluding, return to the issue of determining the status of set theory within a minimalist syntactic architecture. Even the most cursory glance at the standard literature on set theory (Bagaria 2019; Halmos 1974; Suppes 1972) reveals that there is a vast gap between the work set theory is supposed to do in core syntax à la CGO and what it "really does" otherwise. As far as I can see, the main point of talking of sets in the first place is that they

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[10] Elsewhere, Merge is explicitly characterized as either applying to two "syntactic objects" (Chomsky 1995:208; 2000:101; 2008:137), two "objects" (Chomsky 2005:11; Chomsky 2009:25; Chomsky 2013:40), two "elements" (Chomsky 2004:108), or two "structures" (Chomsky 2007:5).

constitute the simplest – because unordered – "product" of (binary) "puttings together" called "Merge." Quite strikingly, Chomsky (2000:116) does not even invoke set membership (via "$\in$") in the definition of "immediate containment" but "constructs" the latter from "Merge products." If so, contentful appeal to set theory would seem to be eliminable from syntax. To mark this, "Merge products" could be called "M-sets" so as not to raise any expectations about additional properties of such objects. And indeed, Chomsky (1995:351,fn.7) alerts us to the power of set theory, which threatens of drastically undermine any genuinely minimalist agenda:[11]

> Note that considerations of this nature can be invoked only within a fairly disciplined minimalist approach. Thus, with sufficiently rich formal devices (say, set theory), counterparts to any object (nodes, bars, indices, etc.) can readily be constructed from features. There is no essential difference, then, between admitting new kinds of objects and allowing richer use of formal devices; we assume that these (basically equivalent) options are permitted only when forced by empirical properties of language.

But, in addition to awareness of the complexity of formal tools, formal (meta-)grammarians have to constantly monitor their use of such tools wrt two things, (i) meta- vs. object-level, and (ii) notation vs. content. As for (i), let us agree that characterizing "I-language" in one form or another involves presenting a formal system, whether in terms of a "static grammar" or a "dynamic computational system" (e.g., "C_{HL}"). Here one has to ask whether set theory is used in talking about that system (meta-level) or taken to be part of that system (object-level).[12] As for (ii), one must reflect upon whether one simply adopts set-theoretic notation as

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[11] A perhaps less felicitously phrased passage on the obscureness of the relation between set theory and (structural aspects of) natural language syntax (Chomsky 2012:91) can be understood as conveying the same "warning." For a not so charitable interpretation, see Behme (2015).

[12] As, for example, the system of propositional logic is usually taken to form part of the system of prediate logic.

suggestive or "handy," or whether one commits to the meaning of that notation as determined by set theory.[13] The latter strategy is the trickier one, as, first of all, there are many varieties of set theory (Ferreirós 2007), and, secondly, one may wish not to make any "deeper" commitments to the "axioms" that regulate that meaning. My above indicated impression that CGO-style contentful appeal to set theory can be eliminated from syntax, means that I would advise against the latter strategy unless and until the overall architecture of the enterprise in question is laid out in full.

5. ***References***

Bagaria, Joan. 2019. "Set Theory." in *Stanford Encyclopedia of Philosophy*, edited by Edward Zalta. Stanford, CA.

Barwise, Jon. 1975. *Admissible Sets and Structures. An Approach to Definability Theory*. Heidelberg: Springer.

Behme, Christina. 2015. "Is the Ontology of Biolinguistics Coherent?" *Language Sciences* 15:32-42.

Bobaljik, Jonathan. 1995. "In Terms of Merge." *MIT Working Papers in Linguistics* 27:41-64.

Chomsky, Noam. 1993. "A Minimalist Program for Linguistic Theory." Pp. 1-52 in *The View from Building 20*, edited by Ken Hale and Samuel Keyser. Cambridge MA: MIT Press.

—. 1995. "Categories and Transformations." Pp. 219-394 in *The Minimalist Program*. Cambridge, MA: MIT Press.

—. 2000. "Minimalist Inquiries: The Framework." Pp. 89-155 in *Step By Step*, edited by Roger Martin, David Michaels, and Juan Uriagereka. Cambridge MA: MIT Press.

[13] The syntax of set theory (as relevant here) is built from individual constants "*a*", "*b*", "*c*" etc., the two-place predicate constant "$\in$", and the "syncategorematic" expressions "{" (left bracket), "}" (right bracket, and "," (comma). And its (extensional) semantics (as relevant here) can be given in terms of a model $\langle D,I\rangle$ with D the universe of things set theory is about and I an interpretation function such that, in particular, $I(a)$, $I(b)$, I(c) etc. are members of D, and $I(\in) \subseteq D \times D$. Given that I is a function, any use of the name "*a*" refers to the same thing, I(a). This is another way of saying that "standard set theory" does not support interpretation of (1b) ({ a, { a, b } }) as involving two "copies" in the sense of two entities. To insist on the latter amounts to an appeal to a particular syntax of set theory with its subtle "excess of notation over subject matter" (Quine 1995:5).

—. 2004. "Beyond Explanatory Adequacy." Pp. 104-31 in *Structures and Beyond*, edited by Adriana Belletti. Oxford: OUP.

—. 2005. "Three Factors in Language Design." *Linguistic Inquiry* 36:1-22.

—. 2007. "Approaching UG from Below." Pp. 1-29 in *Interfaces + Recursion + Language?*, edited by Uli Sauerland and Hans-Martin Gärtner. Berlin: Mouton de Gruyter.

—. 2008. "On Phases." Pp. 133-66 in *Foundational Issues in Linguistic Theory*, edited by Robert Freidin, Carlos Otero, and Maria Luisa Zubizaretta. Cambridge MA: MIT Press.

—. 2009. "Opening Remarks." Pp. 13-43 in *Of Minds and Language. A Dialogue with Noam Chomsky in the Basque Country*, edited by Massimo Piattelli-Palmarini, Juan Uriagereka, and Pello Salaburu. Oxford: OUP.

—. 2012. *The Science of Language. Interviews with James McGilvray*. Cambridge: CUP.

—. 2013. "Problems of Projection." *Lingua* 130:33-49.

Chomsky, Noam, Ángel Gallego, and Dennis Ott. 2019. "Generative Grammar and the Faculty of Language: Insights, Questions, and Challenges." *Catalan Journal of Linguistics* (Special Issue): 229-61.

Citko, Barbara. 2011. "Multidominance." Pp. 119-42 in *The Oxford Handbook of Linguistic Minimalism*, edited by Cedric Boeckx. Oxford: OUP.

Collins, Chris. 1994. "Economy of Derivation and the Generalized Proper Binding Condition." *Linguistic Inquiry* 25:45-61.

Collins, Chris, and Edward Stabler. 2016. "A Formalization of Minimalist Syntax." *Syntax* 19:43-78.

Corver, Norbert, and Jairo Nunes. 2007. "From Trace Theory to Copy Theory." Pp. 1-9 in *The Copy Theory of Movement*, edited by Norbert Corver and Jairo Nunes. Amsterdam: John Benjamins.

Ferreirós, José. 2007. *Labyrinth of Thought. A History of Set Theory and Its Role in Modern Mathematics*. Basel: Birkhäuser.

Gärtner, Hans-Martin. 2002. *Generalized Transformations and Beyond*. Berlin: Akademie-Verlag.

Guimarães, Maximiliano. 2000. "In Defense of Vacuous Projections in Bare Phrase Structure." *University of Maryland Working Papers in Linguistics* 9:90-115.

Halmos, Paul R. 1974. *Naive Set Theory*. New York: Springer.

Langendoen, Terence. 2003. "Merge." Pp. 307-18 in *Formal Approaches to Function in Grammar*, edited by Andrew Carnie, Heidi Harley, and MaryAnn Willie. Amsterdam: John Benjamins.

Nunes, Jairo. 2004. *Linearization of Chains and Sideward Movement*. Cambridge MA: MIT Press.

Quine, Willard Van Orman. 1995. "Whitehead and the Rise of Modern Logic." Pp. 1-36 in *Selected Logic Papers*. Cambridge, MA: Harvard University Press.

Suppes, Patrick. 1972. *Axiomatic Set Theory*. New York: Dover.

Takahashi, Shoichi. 2010a. "Traces or Copies, or Both - Part I: Characterizing Movement Properties." *Language and Linguistics Compass* 4:1091-103.

—. 2010b. "Traces or Copies, or Both – Part II: Introducing Further Considerations." *Language and Linguistics Compass* 4:1104-15.

Thiersch, Craig. 2017. "Remnant Movement." in *The Wiley Blackwell Companion to Syntax*, edited by Martin Everaert and Henk van Riemsdijk. Hoboken, NJ: John Wiley & Sons.

Troelstra, Anne Sjerp. 1992. *Lectures on Linear Logic*. Stanford, CA: CSLI Publications.