The logic of the English auxiliary system

Yusuke Kubota[1] and Robert Levine[2]

[1] National Institute for Japanese Language and Linguistics
[2] Ohio State University

Abstract. This paper proposes an analysis of the English auxiliary system in Hybrid TLG. Our proposal differs from related approaches in lexicalist syntactic theories (such as HPSG and earlier variants of categorial grammar) in taking auxiliaries to be higher-order operators that syntactically (and not just semantically) scope over the local clauses in which they appear. We formulate an analysis of the familiar NI(C)E properties of auxiliaries in this approach. An advantage for the higher-order analysis comes from the fact that it offers a straightforward solution for a long-standing puzzle for earlier lexicalist approaches pertaining to the distribution of the unstressed *do*.

Keywords: auxiliary verb, Type-Logical Grammar, *do* insertion, higher-order auxiliary, categorial grammar, NICE properties

1 The NICE properties and unstressed *do*

Auxiliaries are commonly introduced in introductory syntax courses as members of a natural class whose distributional characteristics are captured by their occurrence in three supposedly quite independent constructions—inversion, sentential negation and VP ellipsis and one morphological form—NEG contraction.

(1) a. John {will/should} buy the book.
  b. John {will/should} not buy the book. (cf. *John buys not the book.)
  c. {Will/Should} John buy the book? (cf. *Buys John the book?)
  d. Who will buy the book? – John {will/should}. (cf. *John buys.)
  e. John {won't/shouldn't} buy the book. (cf. *John buysn't the book.)

Any syntactic theory should provide an explicit (and coherent) analysis of these so-called 'NICE' properties (<u>N</u>egation, <u>I</u>nversion, <u>C</u>ontraction and <u>E</u>llipsis). The distribution of the unstressed form of *do* is especially important in this connection as it has played a non-negligible role in the history of generative grammar. As is well-known, unstressed *do* (notated as *dŏ* in what follows) appears in all the NICE environments but not in simple declarative sentences:

(2) a. *John {dĭd/dŏes} buy the book.
  b. John {dĭd/dŏes} not buy the book.
  c. {Dĭd/Dŏes} John buy the book?

d. Who {bought/buys} the book? – John {dĭd/dŏes}.

e. John {dĭdn't/dŏesn't} buy the book.

On the one hand, in the early history of transformational generative grammar (starting with Chomsky (1957)), the analysis of the otherwise puzzling patterns in (2) via the so-called *do* insertion transformation was regarded as one of the most successful applications of transformational analysis to the grammar of English. On the other hand, the somewhat peculiar distributional restriction on *do* exemplified in (2a), where, unlike other auxiliaries, it is banned from nonnegative declarative environments, has long remained problematic in nontransformational treatments of English auxiliaries, a point emphasized in Sag et al. (2019). In fact, Sag et al. (2019) take the '*do* insertion' paradigm in (2) to be one of the major pieces of evidence supporting their construction-based analysis of English auxiliaries (involving a 'slight' reorganization of the role that the AUX feature plays in the overall system), which departs from the strictly lexical analysis pioneered in Gazdar et al. (1982) that has since been widely assumed as the standard analysis in the lexicalist tradition.

Given the prominent role that facts about English auxiliaries have played in the history of generative grammar, the scarcity of literature on this issue in categorial grammar research is rather surprising. In particular, to our knowledge, there is as yet no single explicit account of the well-known *do* insertion facts in the categorial grammar literature. There are of course sporadic accounts of some specific aspects of auxiliary syntax (and semantics), such as the important pioneering work by Bach (1980, 1983), which provides the basis for an explicit semantic account of subject position quantifiers in sentences with modal auxiliaries in lexicalist syntactic theories (more on this point in section 2), the analysis of VP ellipsis by Morrill and Merenciano (1996) and Jäger (2005) in the 90s, and the analysis of the 'anomalous scope' patterns that modal auxiliaries display in Gapping by Kubota and Levine (2012, 2016), to mention just a few. However, oddly enough, the NICE properties and *do* insertion facts—which have been considered to be one of the key touchstones for contemporary syntactic theory in the generative tradition—seem to have completely escaped the attention of categorial grammarians to date. This is perhaps due to the implicit assumption that at least the core of the PSG analyses of auxiliaries will more or less straightforwardly carry over to categorial grammar, given the many common theoretical assumptions that the two approaches share at the fundamental level (see for example Kubota (2021) in this connection).

But the premise of this implicit assumption is threatened when it comes to the treatment of 'challenging' facts, for which different approaches tend to resort to idiosyncratic properties of their own. This is exactly what we see in the treatment of *do* insertion in the most recent incarnation of the analysis of English auxiliaries in HPSG by Sag et al. (2019). So far as we can tell, the elaborate constructional analysis they offer is by no means straightforwardly translatable to any variant of categorial grammar. It is for this reason that we take up the old issue of NICE properties and *do* insertion facts in the present paper. In particular, we aim to shed a new light on this problem by formulating an analysis in Type-Logical Grammar. Our starting point is the ‘higher-order’ analysis of modal auxiliaries whose key idea is due to Siegel (1984) and which was explicitly formalized in a type-logical setup in Kubota and Levine (2012). This represents a departure from the traditional VP/VP analysis in the lexicalist syntax tradition, by entertaining a movement-like operation in the analysis of modal auxiliaries. We formulate an explicit account of the core syntactic properties of auxiliary verbs in English (including the NIE of the NICE properties). While formulated at a more abstract level, our approach directly builds on the lexicalist approach in identifying the commonality of the NIE constructions as phenomena that target the VP/VP lexical signs of auxiliaries. The key claim of the present paper is that there is a direct empirical payoff for entertaining this more abstract perspective on the syntax of auxiliaries, and that the evidence comes from *do* insertion. Unlike the phrase structure-based or constructional setup, in an inference-based (or deductive) system like ours, operations that target VP/VP signs can themselves be the target of still higher-order operations. This enables us to entertain a more abstract view on *do* support than a construction-based encoding of the sort proposed by Sag et al. (2019): by seeing *do* insertion as a ‘last resort’ inference strategy, as it were, we can capture the key insight of the classical transformational account in a way that completely does away with the ad-hoc structure manipulation operations inherent to the latter.

2 Modals as scope-taking operators

Our approach to modal auxiliaries is heavily influenced by Oehrle’s (1994) foundational work on quantifier scope. Oehrle’s key insight involves utilizing the lambda calculus for characterizing the prosodic component of linguistic expressions, which enables him to model Montague’s quantifying-in via lambda abstraction in the prosodic component. We implement this analysis with the non-directional implicational connective $\upharpoonright$ in Hybrid Type-Logical Grammar (Hybrid TLG).[3] In this approach, (3) is analyzed as in (4).

(3) John read every book.

(4)
$$\dfrac{\dfrac{\dfrac{\dfrac{\mathsf{read};\ \mathbf{read};\ (\mathrm{NP\backslash S})/\mathrm{NP} \quad [\varphi_1; x; \mathrm{NP}]^1}{\mathsf{read} \bullet \varphi_1;\ \mathbf{read}(x);\ \mathrm{NP\backslash S}} \quad \mathsf{john};\ \mathbf{j};\ \mathrm{NP}}{\mathsf{john} \bullet \mathsf{read} \bullet \varphi_1;\ \mathbf{read}(x)(\mathbf{j});\ \mathrm{S}}}{\lambda\varphi_1.\mathsf{john} \bullet \mathsf{read} \bullet \varphi_1;\ \lambda x.\mathbf{read}(x)(\mathbf{j});\ \mathrm{S}\upharpoonright\mathrm{NP}}{\upharpoonright}\mathrm{I}^1 \quad \begin{array}{c}\vdots\\ \lambda\sigma_1.\sigma_1(\mathsf{every} \bullet \mathsf{book});\\ \mathbf{\forall_{book}};\mathrm{S}\upharpoonright(\mathrm{S}\upharpoonright\mathrm{NP})\end{array}}{\mathsf{john} \bullet \mathsf{read} \bullet \mathsf{every} \bullet \mathsf{book};\ \mathbf{\forall_{book}}(\lambda x.\mathbf{read}(x)(\mathbf{j}));\ \mathrm{S}}$$

The crucial innovation in Oehrle’s approach is that abstraction on a prosodic variable makes it possible to separate the surface position in which a quantifier appears and its semantic scope: at the last step in (4), the quantifier sign applies to the S$\upharpoonright$NP constituent that is its (semantic) scope, but its prosodic

[3] Appendix A below contains a brief overview of Hybrid TLG. See Kubota and Levine (2020, chapter 2 and Appendix A) for a more detailed exposition.

contribution, or, 'string support' for the higher order prosodic specification $\lambda\sigma_1.\sigma_1(\mathsf{every} \bullet \mathsf{book})$, ends up in a position corresponding to the prosodic variable φ_1 in the scope constituent.

This analysis, as Oehrle demonstrates, captures scope ambiguity effortlessly, without resort to any special mechanisms. (5b) shows how the inverse scope reading for (5a) is obtained in this approach.

(5) a. Some student read every book.

b.

$$\dfrac{\dfrac{\dfrac{\dfrac{\vdots}{\begin{array}{l}\varphi_1 \bullet \mathsf{read} \bullet \varphi_2;\\ \mathbf{read}(u)(v); \mathrm{S}\end{array}}}{\begin{array}{l}\lambda\varphi_1.\varphi_1 \bullet \mathsf{read} \bullet \varphi_2;\\ \lambda v.\mathbf{read}(u)(v); \mathrm{S}{\upharpoonright}\mathrm{NP}\end{array}} \quad \begin{array}{l}\vdots\\ \lambda\sigma_1.\sigma_1(\mathsf{some} \bullet \mathsf{student});\\ \boldsymbol{\exists}_{\mathbf{student}}; \mathrm{S}{\upharpoonright}(\mathrm{S}{\upharpoonright}\mathrm{NP})\end{array}}{\begin{array}{l}\mathsf{some} \bullet \mathsf{student} \bullet \mathsf{read} \bullet \varphi_2;\\ \boldsymbol{\exists}_{\mathbf{student}}(\lambda v.\mathbf{read}(u)(v)); \mathrm{S}\end{array}}}{\begin{array}{l}\lambda\varphi_2.\mathsf{some} \bullet \mathsf{student} \bullet \mathsf{read} \bullet \varphi_2;\\ \lambda u.\boldsymbol{\exists}_{\mathbf{student}}(\lambda v.\mathbf{read}(u)(v)); \mathrm{S}{\upharpoonright}\mathrm{NP}\end{array}} \quad \begin{array}{l}\vdots\\ \lambda\sigma_2.\sigma_2(\mathsf{every} \bullet \mathsf{book});\\ \boldsymbol{\forall}_{\mathbf{book}}; \mathrm{S}{\upharpoonright}(\mathrm{S}{\upharpoonright}\mathrm{NP})\end{array}}{\mathsf{some} \bullet \mathsf{student} \bullet \mathsf{read} \bullet \mathsf{every} \bullet \mathsf{book};\ \boldsymbol{\forall}_{\mathbf{book}}(\lambda u.\boldsymbol{\exists}_{\mathbf{student}}(\lambda v.\mathbf{read}(u)(v))); \mathrm{S}}$$

The order in which the two GQs compose into the proof determines the scopal ordering; an alternative derivation whose only difference from (5b) is the introduction of the universal before the existential will yield a second reading for (5a) corresponding to surface scope ($\exists > \forall$).

It is not only multiple tokens of GQs that create scope ambiguities, however. Modals interact with GQs in much the same way:

(6) Every student can vote.

(6) has two subtly—but critically—different readings. On one reading, where the universal scopes widely, (6) says that every individual who happens to be a student (in the actual world) has the right or ability to vote. On the other reading, the sentence does not refer to students in the actual world, but instead merely makes a statement about a possible situation: whoever happens to be a student in that situation has the right or ability to vote. The following formulas disambiguate these two readings:

(7) a. $\boldsymbol{\forall}_{\mathbf{student}}(\lambda y.\Diamond\mathbf{vote}(y))$

b. $\Diamond\boldsymbol{\forall}_{\mathbf{student}}(\lambda y.\mathbf{vote}(y))$

In the face of ambiguous data such as (6), it seems natural to extend Oehrle's treatment of GQs to modals as well.[4] This is in fact straightforward, by assum-

[4] The wide scope reading for the modal in sentences like (6) has long been known to pose a challenge for the VP/VP analysis in lexicalist approaches. For example, Gazdar et al. (1985), noting this difficulty, wind up positing a version of the modal *may* with the semantics ($\lambda Q\lambda\mathscr{P}.\Diamond\,\mathscr{P}(Q)$ in an extensionalized fragment) that directly subcategorizes for a GQ-type expression as the subject following Bach (1980, 1983). It is unclear how the modal narrow scope reading is obtained in their approach.

ing that modals are GQ-like expressions, except that they scope over S with a VP/VP functor (instead of an NP) withdrawn (here, $\mathsf{id}_{et} = \lambda P_{et}.P$):[5]

(8) $\lambda\sigma.\sigma(\mathsf{can});\ \lambda\mathscr{F}.\Diamond\mathscr{F}(\mathsf{id}_{et});\ \mathrm{S}_{\mathit{fin}}\upharpoonright(\mathrm{S}_{\mathit{fin}}\upharpoonright(\mathrm{VP}_{\mathit{fin}}/\mathrm{VP}_{\mathit{bse}}))$

With this specification, we straightforwardly obtain (9) for the modal wide-scope reading for (6) (for how the narrow scope reading (7a) is obtained, see below):

(9)
$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\mathsf{vote};\ \mathbf{vote};\ \mathrm{VP}_b \quad [\varphi_1;\ f;\ \mathrm{VP}_f/\mathrm{VP}_b]^1}{\varphi_1 \bullet \mathsf{vote};\ f(\mathbf{vote});\ \mathrm{VP}_f} \quad [\varphi_2;\ y;\ \mathrm{NP}]^2}{\varphi_2 \bullet \varphi_1 \bullet \mathsf{vote};\ f(\mathbf{vote})(y);\ \mathrm{S}_f}\upharpoonright\mathrm{I}^2}{\lambda\varphi_2.\varphi_2 \bullet \varphi_1 \bullet \mathsf{vote};\ \lambda y.f(\mathbf{vote})(y);\ \mathrm{S}_f\upharpoonright\mathrm{NP}} \quad \begin{matrix}\vdots \\ \lambda\sigma_1.\sigma_1(\mathsf{every} \bullet \mathsf{student});\\ \mathbf{\forall_{student}};\ \mathrm{S}_f\upharpoonright(\mathrm{S}_f\upharpoonright\mathrm{NP})\end{matrix}}{\dfrac{\mathsf{every} \bullet \mathsf{student} \bullet \varphi_1 \bullet \mathsf{vote};\ \mathbf{\forall_{student}}(\lambda y.f(\mathbf{vote})(y));\ \mathrm{S}_f}{\lambda\varphi_1.\mathsf{every} \bullet \mathsf{student} \bullet \varphi_1 \bullet \mathsf{vote};\ \lambda f.\mathbf{\forall_{student}}(\lambda y.f(\mathbf{vote})(y));\ \mathrm{S}_f\upharpoonright(\mathrm{VP}_f/\mathrm{VP}_b)}\upharpoonright\mathrm{I}^1} \quad \begin{matrix}\lambda\sigma_2.\sigma_2(\mathsf{can});\\ \lambda\mathscr{F}.\Diamond\mathscr{F}(\mathsf{id}_{et});\\ \mathrm{S}_f\upharpoonright(\mathrm{S}_f\upharpoonright(\mathrm{VP}_f/\mathrm{VP}_b))\end{matrix}}{\mathsf{every} \bullet \mathsf{student} \bullet \mathsf{can} \bullet \mathsf{vote};\ \Diamond\mathbf{\forall_{student}}(\lambda y.\mathbf{vote}(y));\ \mathrm{S}_f}$$

The key point here is that the quantifier is introduced in the derivation before the modal auxiliary, entailing its narrow scope.

Before proceeding further, it is important to recognize a potential overgeneration problem that arises with the scope-operator analysis of modals along the lines of (8). Unless appropriate constraints are imposed, the present analysis has the danger of predicting readings for modals (and related expressions such as VP negation) in which they scope out of their local clauses. This is clearly impossible in English. For example, the following sentence does not have a reading paraphrasable as something like 'it should be the case that John thought Ann is to buy/is buying the car':

(10) John thought Ann should buy the car.

In order to prevent this type of overgeneration and restrict the scope of modal auxiliaries to the local clause in which they occur, we can employ a clause-level indexing mechanism of the sort proposed by Pogodalla and Pompigne (2012) for a slightly different purpose.[6] A full description of the indexing convention is

[5] The features *fin* and *bse* here (abbreviated as *f* and *b* below) should be thought of as the (analogues of) 'VFORM' features (in G/HPSG terms) that mark finite and base forms of verbs respectively. This ensures that modals can only combine with base forms of verbs and after the modal is combined with the verb, the result is finite, and no other modal can stack on top of the resultant VP.

[6] Another, and perhaps more standard, approach for dealing with this type of issue in the TLG literature is to employ certain types of 'modality' operators in type logic. See, e.g., Moortgat (2011) and Morrill (2010) and references cited therein. A reviewer notes that it may be possible to simplify certain aspects of our index-based approach (which is essentially nothing more than a bookkeeping device) by recasting the relevant aspects of the analysis by using modality operators. We leave it for future work to investigate this issue in detail.

given in section 9.2.2 of Kubota and Levine (2020). We illustrate its key points briefly in what follows. With the indexing restrictions made explicit, we have the following lexicon:

(11) a. $\lambda\sigma.\sigma(\mathsf{should}); \lambda\mathscr{G}.\Box\mathscr{G}(\mathsf{id}_{et}); \mathrm{S}^n_f{\upharpoonright}(\mathrm{S}^n_f{\upharpoonright}(\mathrm{VP}^n_f/\mathrm{VP}^n_b)$

b. $\mathsf{thought}; \mathbf{think}; \mathrm{VP}^{n+1}_f|\mathrm{S}^n_f$

The explicit indexing on the S and VP/VP categories in (11a) ensures that modals take scope directly over the clauses that are 'projections' of the VP/VP gaps that they bind, guaranteeing the clause-boundedness of the scope of these operators, as we now show.

A failed derivation for (10) is given in (12).

(12)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\begin{array}{c}\vdots\\ \mathsf{buy} \bullet \mathsf{the} \bullet \mathsf{car}; \\ \mathbf{buy}(\iota(\mathbf{car})); \mathrm{VP}^1_b\end{array} \quad \left[\begin{array}{l}\varphi_1; \\ f; \\ \mathrm{VP}^1_f/\mathrm{VP}^1_b\end{array}\right]^1}{\begin{array}{c}\varphi_1 \bullet \mathsf{buy} \bullet \mathsf{the} \bullet \mathsf{car}; \\ f(\mathbf{buy}(\iota(\mathbf{car}))); \mathrm{VP}^1_b\end{array}} \quad \begin{array}{l}\mathsf{ann}; \\ \mathbf{a}; \\ \mathrm{NP}\end{array}}{\begin{array}{c}\mathsf{ann} \bullet \varphi_1 \bullet \mathsf{buy} \bullet \mathsf{the} \bullet \mathsf{car}; \\ f(\mathbf{buy}(\iota(\mathbf{car})))(\mathbf{a}); \mathrm{S}^1_f\end{array}} \quad \begin{array}{l}\mathsf{thought}; \\ \mathbf{think}; \\ \mathrm{VP}^{n+1}_f|\mathrm{S}^n_f\end{array}}{\begin{array}{c}\mathsf{thought} \bullet \mathsf{ann} \bullet \varphi_1 \bullet \mathsf{buy} \bullet \mathsf{the} \bullet \mathsf{car}; \\ \mathbf{think}(f(\mathbf{buy}(\iota(\mathbf{car})))(\mathbf{a})); \mathrm{VP}^2_f\end{array}} \quad \begin{array}{l}\mathsf{john}; \\ \mathbf{j}; \\ \mathrm{NP}\end{array}}{\begin{array}{c}\mathsf{john} \bullet \mathsf{thought} \bullet \mathsf{ann} \bullet \varphi_1 \bullet \mathsf{buy} \bullet \mathsf{the} \bullet \mathsf{car}; \\ \mathbf{think}(f(\mathbf{buy}(\iota(\mathbf{car})))(\mathbf{a}))(\mathbf{j}); \mathrm{S}^2_f\end{array}}{\upharpoonright}\mathrm{I}^1}{\begin{array}{l}\lambda\varphi_1.\mathsf{john} \bullet \mathsf{thought} \bullet \mathsf{ann} \bullet \varphi_1 \bullet \mathsf{buy} \bullet \mathsf{the} \bullet \mathsf{car}; \\ \lambda f.\mathbf{think}(f(\mathbf{buy}(\iota(\mathbf{car})))(\mathbf{a}))(\mathbf{j}); \mathrm{S}^2_f|(\mathrm{VP}^1_f/\mathrm{VP}^1_b)\end{array} \quad \begin{array}{l}\lambda\sigma.\sigma(\mathsf{should}); \\ \lambda\mathscr{G}.\Box\mathscr{G}(\mathsf{id}_{et}); \\ \mathrm{S}^n_f{\upharpoonright}(\mathrm{S}^n_f{\upharpoonright}(\mathrm{VP}^n_f/\mathrm{VP}^n_b))\end{array}}{\mathbf{FAIL}}$$

Here, the withdrawn VP/VP (from the embedded clause) carries the index 1 but this doesn't match the index value 2 on the S. Since the modal operator explicitly requires these values to match with each other, the derivation fails at the step at which the modal is introduced. We assume this clause-level indexing mechanism throughout, but omit the indices in the interest of minimizing notational clutter.

A strong indication that the higher-order treatment of modals presented above is on the right track comes from the fact that just such an analysis seems to be required independently in order to account for the seemingly anomalous scope of modals in examples such as (13):

(13) Mrs J can't live in Boston and Mr J in LA! $(\neg\Diamond > \wedge)$

See Kubota and Levine (2016, 2020) for detailed discussion of how the kind of higher order description in (8) provides a natural account of such examples, which display (on one of their readings) an unusual scoping pattern in which the negative modal scopes over the conjunction.

Importantly, in such examples, as well as simpler examples such as (6), modals can also scope narrowly with respect to the other scopal operator (in

(6), the quantifier; in (13), the conjunction). In the present setup, this falls out as a straightforward consequence of the higher-order lexical entry in (8). That is, a VP/VP entry for a modal auxiliary which essentially corresponds to (the simpler version of) the lexical entries for modal auxiliaries in lexicalist theories of syntax falls out as a theorem from (8), as in the following proof:

(14)

$$\dfrac{\dfrac{\dfrac{\begin{array}{c}\lambda\sigma.\sigma(\textsf{can't});\\ \lambda\mathscr{F}.\neg\Diamond\mathscr{F}(\textsf{id}_{et});\\ \mathrm{S}_f{\upharpoonright}(\mathrm{S}_f{\upharpoonright}(\mathrm{VP}_f/\mathrm{VP}_b))\end{array} \quad \dfrac{\dfrac{[\varphi_1; x; \mathrm{NP}]^1 \quad \dfrac{[\varphi_2; g; \mathrm{VP}_f/\mathrm{VP}_b]^2 \quad [\varphi_3; f; \mathrm{VP}_b]^3}{\varphi_2 \bullet \varphi_3;\ g(f);\ \mathrm{VP}_f}/\mathrm{E}}{\varphi_1 \bullet \varphi_2 \bullet \varphi_3;\ g(f)(x);\ \mathrm{S}_f}\backslash\mathrm{E}}{\lambda\varphi_2.\varphi_1 \bullet \varphi_2 \bullet \varphi_3;\ \lambda g.g(f)(x);\ \mathrm{S}_f{\upharpoonright}(\mathrm{VP}_f/\mathrm{VP}_b)}{\upharpoonright}\mathrm{I}^2}{\varphi_1 \bullet \textsf{can't} \bullet \varphi_3;\ \neg\Diamond f(x);\ \mathrm{S}_f}{\upharpoonright}\mathrm{E}}{\textsf{can't} \bullet \varphi_3;\ \lambda x.\neg\Diamond f(x);\ \mathrm{VP}_f}\backslash\mathrm{I}^1}{\textsf{can't};\ \lambda f\lambda x.\neg\Diamond f(x);\ \mathrm{VP}_f/\mathrm{VP}_b}/\mathrm{I}^3$$

Using the VP/VP entry for the modal derived in (14), the narrow scope readings for the modal for both (8) and (13) follow straightforwardly.

Beyond its wide empirical reach illustrated above, our more abstract treatment of the syntax-semantics interface of modals can be seen as a unification of what have been viewed as two very distinct, competing analyses. A common treatment of modals in the Principles and Parameters approach proposed in the period following Pollock (1989) is to take them as originating under their own functional head and moving to Spec of TP (see, for example, Iatridou and Zeijlstra (2013), Harwood (2014) and Radford (2018, 241) for discussion of this general line of analysis). This treatment of modals as operators raised into higher positions to scope over propositional content offers—at least in principle—a strategy for solving the puzzles posed by (6) and (13), in conjunction with certain ancillary assumptions (e.g., Johnson (2000) on Gapping, but see Kubota and Levine (2016) for a critique). On the other hand, the lexicalist treatment of modals as verbs combining with VP complements has been empirically successful in capturing morphosyntactic dependencies involving auxiliaries since its introduction in Gazdar (1982) and Gazdar et al. (1982),[7] but encounters the serious hurdles noted above. On our analysis, it is exactly these two types of seemingly rival analyses which fall out of the single lexical entry in (8).

3 Higher-order operator analysis of NIE properties

From the discussion above, it should be clear that our approach differs from the traditional lexicalist analyses of auxiliaries in that it takes modal auxiliaries to be higher-order operators that scope over clausal constituents. Aside from some advantages it offers in the analysis of scopal interactions with other operators

[7] Note also that the distributive interepretation of modals in Gapping is straightforward in this type of approach, whereas the high-modal analysis such as Johnson (2000) in the transformational literature struggles to obtain this reading. See Kubota and Levine (2016) Park (2019) and Potter et al. (2017) for some relevant discussion.

(such as generalized quantifiers and conjunction in Gapping, as noted above), one may rightfully wonder whether there is any payoff to this more abstract analysis of English auxiliaries. We argue in this section that additional advantage does in fact come from the analysis of the familiar syntactic properties of English auxiliaries, in particular, the somewhat puzzling distribution of the unstressed *do*, which has—as noted in section 1—proven problematic in lexicalist analyses of English auxiliaries.

In order to formulate an analysis of *do* insertion, we need an explicit analysis of (at least a subset of) NICE properties. In the rest of this paper, we set aside contraction since this phenomenon involves morphological idiosyncrasy that justifies a lexical treatment. For the other three phenomena, our analysis builds on the key idea that auxiliary verbs are syntactically operators that fill in the preverbal gap position of type VP/VP. Unlike the more traditional VP/VP analysis, the higher-order analysis of auxiliaries introduced in the previous section opens up an analytic possibility in which we can define operators that manipulate the type VP/VP gap before it gets filled in by the lexical auxiliary. Our analysis of the NIE operators crucially exploits this possibility.

3.1 Basic actions of the NIE operators

We start with inversion. In sentences with inverted auxiliaries such as polar questions, the auxiliary verb appears at the beginning of the clause rather than in the preverbal position. This word order change can simply be handled by positing the following higher-order operator that maps a $\mathrm{S}{\restriction}(\mathrm{VP}/\mathrm{VP})$ to another $\mathrm{S}{\restriction}(\mathrm{VP}/\mathrm{VP})$, which differs only in the prosodic specification. The fact that inversion has taken place is recorded in the syntactic feature *inv*, a standard technique for distinguishing inverted from non-inverted clauses in lexicalist approaches.[8,9]

(15) $\lambda\sigma\lambda\varphi.\varphi \bullet \sigma(\boldsymbol{\epsilon})$; $\lambda\mathscr{F}.\mathscr{F}$; $(\mathrm{S}_{inv}{\restriction}(\mathrm{VP}_f/\mathrm{VP}_b)){\restriction}(\mathrm{S}_f{\restriction}(\mathrm{VP}_f/\mathrm{VP}_b))$

After the inversion operator applies to $\mathrm{S}{\restriction}(\mathrm{VP}/\mathrm{VP})$, the result is passed on to the higher-order auxiliary. The latter fills in the auxiliary string in the gap position—which has been moved to the clause-initial position by the inversion operator—to complete the derivation (here, we have slightly generalized the lexical entry for the auxiliary, replacing S_f with S_α, where $\alpha \in \{\mathit{fin}, \mathit{inv}\}$).

[8] The semantics of the inversion operator is the identity function. We assume that a separate operator (of the sort assumed in Kubota and Levine (2021)) is responsible for introducing the semantics of polar questions. The separation of the syntactic operation of inversion and question semantics is motivated by the fact that inversion is found in contexts other than polar questions.

[9] $\boldsymbol{\epsilon}$ designates the empty string.

(16)

$$\dfrac{\begin{array}{l}\lambda\sigma.\sigma(\mathsf{should});\\ \lambda\mathscr{F}.\Box\mathscr{F}(\mathsf{id}_{et});\\ \mathrm{S}_\alpha\upharpoonright(\mathrm{S}_\alpha\upharpoonright(\mathrm{VP}_f/\mathrm{VP}_b))\end{array} \quad \dfrac{\begin{array}{l}\lambda\sigma\lambda\varphi.\varphi\bullet\sigma(\boldsymbol{\epsilon});\\ \lambda\mathscr{F}.\mathscr{F};\\ (\mathrm{S}_{inv}\upharpoonright(\mathrm{VP}_f/\mathrm{VP}_b))\upharpoonright(\mathrm{S}_f\upharpoonright(\mathrm{VP}_f/\mathrm{VP}_b))\end{array} \quad \begin{array}{l}\vdots\\ \lambda\varphi.\mathsf{john}\bullet\varphi\bullet\mathsf{come};\\ \lambda f.f(\mathbf{come})(\mathbf{j});\\ \mathrm{S}_f\upharpoonright(\mathrm{VP}_f/\mathrm{VP}_b)\end{array}}{\lambda\varphi.\varphi\bullet\mathsf{john}\bullet\mathsf{come};\ \lambda f.f(\mathbf{come})(\mathbf{j});\ \mathrm{S}_{inv}\upharpoonright(\mathrm{VP}_f/\mathrm{VP}_b)}}{\mathsf{should}\bullet\mathsf{john}\bullet\mathsf{come};\ \Box\mathbf{come}(\mathbf{j});\ \mathrm{S}_{inv}}$$

An important fact about auxiliary inversion is that it is a clause-bound phenomenon.

(17) *Will anybody who __ be vaccinated after arrival may visit Japan?

We utilize the clause-level indexing mechanism from the previous section to capture this fact explicitly. Specifically, (15) is actually an abbreviated version of (18), which makes it explicit that the 'auxiliary gap' that the inversion operator targets is a local one.

(18) $\lambda\sigma\lambda\varphi.\varphi\bullet\sigma(\boldsymbol{\epsilon});\ \lambda\mathscr{F}.\mathscr{F};\ (\mathrm{S}^n_{inv}\upharpoonright(\mathrm{VP}^n_f/\mathrm{VP}^n_b))\upharpoonright(\mathrm{S}^n_f\upharpoonright(\mathrm{VP}^n_f/\mathrm{VP}^n_b))$

We make the same assumption about the ellipsis and negation operators we introduce below, but continue to suppress the clause-level indexing for the sake of readability. It should be kept in mind that these indices are present in the official version of the analysis.

Moving on to ellipsis, descriptively, VP ellipsis is a phenomenon in which an auxiliary stands in for a full VP. We here assume a somewhat more elaborate analysis of VP ellipsis than the one we utilized in Kubota and Levine (2017, 2020), where the the VP ellipsis operator is defined as a higher-order operator that replaces a type VP gap by a type VP/VP gap.

(19) $\lambda\sigma\lambda\varphi.\sigma(\varphi);\ \lambda\mathscr{G}\lambda f.\mathscr{G}(f(P));\ (\mathrm{S}_f\upharpoonright(\mathrm{VP}_f/\mathrm{VP}_b))\upharpoonright(\mathrm{S}_b\upharpoonright\mathrm{VP}_b)$

The ellipsis operator supplies the contextual variable P as the meaning of the missing VP. We assume that technically P is just a free variable and that its value is contextually determined just like the referent of (free) pronouns. A sample derivation is given in (20). Just as in the case of inversion, the ellipsis operator applies first and the resultant S↾(VP/VP) expression is passed on to the higher-order auxiliary.

(20)

$$\dfrac{\begin{array}{l}\lambda\sigma.\sigma(\mathsf{should});\\ \lambda\mathscr{F}.\Box\mathscr{F}(\mathsf{id}_{et});\\ \mathrm{S}_\alpha\upharpoonright(\mathrm{S}_\alpha\upharpoonright(\mathrm{VP}_f/\mathrm{VP}_b))\end{array} \quad \dfrac{\begin{array}{l}\lambda\sigma\lambda\varphi.\sigma(\varphi);\\ \lambda\mathscr{G}\lambda f.\mathscr{G}(f(P));\\ (\mathrm{S}_f\upharpoonright(\mathrm{VP}_f/\mathrm{VP}_b))\upharpoonright(\mathrm{S}_b\upharpoonright\mathrm{VP}_b)\end{array} \quad \begin{array}{l}\vdots\\ \lambda\varphi.\mathsf{john}\bullet\varphi;\\ \lambda Q.Q(\mathbf{j});\mathrm{S}_b\upharpoonright\mathrm{VP}_b\end{array}}{\lambda\varphi.\mathsf{john}\bullet\varphi;\ \lambda f.f(P)(\mathbf{j});\ \mathrm{S}_f\upharpoonright(\mathrm{VP}_f/\mathrm{VP}_b)}}{\mathsf{john}\bullet\mathsf{should};\ \Box P(\mathbf{j});\ \mathrm{S}_f}$$

Finally, negation is listed in the lexicon as a higher-order operator similar to the modal auxiliaries as in (21) in order to capture the polarity-sensitive scopal interactions with modals (see Kubota and Levine (2021) for details), but an alternative sign that applies to a VP/VP-gapped sentence and inserts the negation morpheme right after the gap can be obtained as a theorem as in (22).

(21) $\lambda\sigma.\sigma(\textsf{not});\ \lambda\mathscr{F}.\neg\mathscr{F}(\textsf{id}_{et});\ \mathrm{S}_\alpha{\upharpoonright}(\mathrm{S}_\alpha{\upharpoonright}(\mathrm{VP}_b/\mathrm{VP}_b))$

(22)

$$\dfrac{\dfrac{\dfrac{\left[\begin{array}{l}\sigma;\\ \mathscr{F};\\ \mathrm{S}_f{\upharpoonright}(\mathrm{VP}_f/\mathrm{VP}_b)\end{array}\right]^5 \quad \dfrac{\dfrac{\dfrac{\begin{array}{l}\lambda\sigma.\sigma(\textsf{not});\\ \lambda\mathscr{F}.\neg\mathscr{F}(\textsf{id}_{et});\\ \mathrm{S}_\alpha{\upharpoonright}(\mathrm{S}_\alpha{\upharpoonright}(\mathrm{VP}_b/\mathrm{VP}_b))\end{array} \quad \dfrac{\dfrac{\left[\begin{array}{l}\varphi_4;\\ x;\\ \mathrm{NP}\end{array}\right]^4 \quad \dfrac{\left[\begin{array}{l}\varphi_1;\\ f;\\ \mathrm{VP}_f/\mathrm{VP}_b\end{array}\right]^1 \quad \dfrac{\left[\begin{array}{l}\varphi_2;\\ g;\ \mathrm{VP}_b/\mathrm{VP}_b\end{array}\right]^2 \quad \left[\begin{array}{l}\varphi_3;\\ P;\ \mathrm{VP}_b\end{array}\right]^3}{\varphi_2 \bullet \varphi_3;\ g(P);\ \mathrm{VP}_b}}{\varphi_1 \bullet \varphi_2 \bullet \varphi_3;\ f(g(P));\ \mathrm{VP}_f}}{\varphi_4 \bullet \varphi_1 \bullet \varphi_2 \bullet \varphi_3;\ f(g(P))(x);\ \mathrm{S}_f}\,{\upharpoonright}\mathrm{I}^2}{\begin{array}{l}\lambda\varphi_2.\varphi_4 \bullet \varphi_1 \bullet \varphi_2 \bullet \varphi_3;\\ \lambda g.f(g(P))(x);\mathrm{S}_f{\upharpoonright}(\mathrm{VP}_b/\mathrm{VP}_b)\end{array}}}{\varphi_4 \bullet \varphi_1 \bullet \textsf{not} \bullet \varphi_3;\ \neg f(P)(x);\ \mathrm{S}_f}\,\backslash\mathrm{I}^4}{\varphi_1 \bullet \textsf{not} \bullet \varphi_3;\ \lambda x.\neg f(P)(x);\ \mathrm{VP}_f}\,/\mathrm{I}^3}{\varphi_1 \bullet \textsf{not};\ \lambda P\lambda x.\neg f(P)(x);\ \mathrm{VP}_f/\mathrm{VP}_b}}{\sigma(\varphi_1 \bullet \textsf{not});\ \mathscr{F}(\lambda P\lambda x.\neg f(P)(x));\ \mathrm{S}_f}\,{\upharpoonright}\mathrm{I}^1}{\lambda\varphi_1.\sigma(\varphi_1 \bullet \textsf{not});\ \lambda f.\mathscr{F}(\lambda P\lambda x.\neg f(P)(x));\ \mathrm{S}_f{\upharpoonright}(\mathrm{VP}_f/\mathrm{VP}_b)}\,{\upharpoonright}\mathrm{I}^5}{\begin{array}{l}\lambda\sigma\lambda\varphi_1.\sigma(\varphi_1 \bullet \textsf{not});\\ \lambda\mathscr{F}\lambda f.\mathbf{Cf}(\lambda P\lambda x.\neg f(P)(x));(\mathrm{S}_f{\upharpoonright}(\mathrm{VP}_f/\mathrm{VP}_b)){\upharpoonright}(\mathrm{S}_f{\upharpoonright}(\mathrm{VP}_f/\mathrm{VP}_b))\end{array}}$$

This is a theorem that falls out from (21) regardless of whether one wants it or not. This more complex sign in (22) turns out to play a crucial role in our analysis of the interactions of the other NIE operators in the next section.

We can use (22) for deriving a simple sentence containing negation goes as follows:

(23)

$$\dfrac{\begin{array}{l}\lambda\sigma.\sigma(\textsf{should});\\ \lambda\mathscr{F}.\Box\mathscr{F}(\textsf{id}_{et});\\ \mathrm{S}_\alpha{\upharpoonright}(\mathrm{S}_\alpha{\upharpoonright}(\mathrm{VP}_f/\mathrm{VP}_b))\end{array} \quad \dfrac{\begin{array}{c}\vdots\\ \lambda\sigma\lambda\varphi_1.\sigma(\varphi_1 \bullet \textsf{not});\\ \lambda\mathscr{F}\lambda f.\mathscr{F}(\lambda P\lambda x.\neg f(P)(x));\\ (\mathrm{S}_f{\upharpoonright}(\mathrm{VP}_f/\mathrm{VP}_b)){\upharpoonright}(\mathrm{S}_f{\upharpoonright}(\mathrm{VP}_f/\mathrm{VP}_b))\end{array} \quad \begin{array}{c}\vdots\\ \lambda\varphi.\textsf{john} \bullet \varphi \bullet \textsf{come};\\ \lambda f.f(\mathbf{come})(\mathbf{j});\\ \mathrm{S}_f{\upharpoonright}(\mathrm{VP}_f/\mathrm{VP}_b)\end{array}}{\lambda\varphi.\textsf{john} \bullet \varphi \bullet \textsf{not} \bullet \textsf{come};\ \lambda f.\neg f(\mathbf{come})(\mathbf{j});\ \mathrm{S}_f{\upharpoonright}(\mathrm{VP}_f/\mathrm{VP}_b)}}{\textsf{john} \bullet \textsf{should} \bullet \textsf{not} \bullet \textsf{come};\ \Box\neg\mathbf{come}(\mathbf{j});\ \mathrm{S}_f}$$

3.2 NIE Interactions

An important property of the analysis of the NIE operators above is that these operators interact with one another systematically to yield the right results for cases in which the relevant phenomena interact with one another. In order to facilitate discussion on this point, we introduce some abbreviatory notation first. Specifically, we write **Inv**, **Ell** and **Neg** for the three operators introduced above and **Lex** for some auxiliary lexical entry (*should* is chosen just for an illustration; $\neg$ is 'generalized negation' such that $\neg_{et\to et} = \lambda\mathscr{F}\lambda P\lambda x.\neg\mathscr{F}(P)(x)$).[10,11]

[10] The entries in (24) need the index markings of the sort discussed in section 3.1 (in order to prevent overgeneration of examples such as **Should John say Mary ∅ get the job?*), but we keep omitting these for the sake of notational transparency.

[11] Computationally inclined readers may find it unfortunate that our entries for **Inv** and **Ell** are not lexicalized—a concern that we share. We agree with the reviewer

(24) a. **Inv** $= \lambda\sigma\lambda\varphi.\varphi \bullet \sigma(\epsilon);\ \lambda\mathscr{F}.\mathscr{F};\ (\mathrm{S}_{inv}{\upharpoonright}(\mathrm{VP}_f/\mathrm{VP}_b)){\upharpoonright}(\mathrm{S}_f{\upharpoonright}(\mathrm{VP}_f/\mathrm{VP}_b))$

b. **Ell** $= \lambda\sigma\lambda\varphi.\sigma(\varphi);\ \lambda\mathscr{G}\lambda f.\mathscr{G}(f(P));\ (\mathrm{S}_f{\upharpoonright}(\mathrm{VP}_f/\mathrm{VP}_b)){\upharpoonright}(\mathrm{S}_b{\upharpoonright}\mathrm{VP}_b)$

c. **Neg** $= \lambda\sigma\lambda\varphi.\sigma(\varphi \bullet \mathsf{not});$
$\lambda\mathscr{F}\lambda g.\mathscr{F}(\neg_{et\to et}\, g);\ (\mathrm{S}_f{\upharpoonright}(\mathrm{VP}_f/\mathrm{VP}_b)){\upharpoonright}(\mathrm{S}_f{\upharpoonright}(\mathrm{VP}_f/\mathrm{VP}_b))$

d. **Lex** $= \lambda\sigma.\sigma(\mathsf{should});\ \lambda\mathscr{F}.\Box\mathscr{F}(\mathsf{id}_{et});\ \mathrm{S}_\alpha{\upharpoonright}(\mathrm{S}_\alpha{\upharpoonright}(\mathrm{VP}_f/\mathrm{VP}_b))$

Given these abbreviatory notations, we can derive the inverted, complement-elided and negated versions of the auxiliary lexical signs as follows, via function composition of **Lex** and the three operators (here, $\circ$ denotes function composition; proofs for the theorems in (25) are omitted due to space constraints but are all straightforward):

(25) a. **Lex** $\circ$ **Inv** $= \lambda\sigma.\mathsf{should} \bullet \sigma(\epsilon);\ \lambda\mathscr{F}.\Box\mathscr{F}(\mathsf{id}_{et});\ \mathrm{S}_{inv}{\upharpoonright}(\mathrm{S}_f{\upharpoonright}(\mathrm{VP}_f/\mathrm{VP}_b))$

b. **Lex** $\circ$ **Ell** $= \lambda\sigma.\sigma(\mathsf{should});\ \lambda\mathscr{G}.\Box\mathscr{G}(P);\ \mathrm{S}_f{\upharpoonright}(\mathrm{S}_b{\upharpoonright}\mathrm{VP}_b)$

c. **Lex** $\circ$ **Neg** $= \lambda\sigma.\sigma(\mathsf{should} \bullet \mathsf{not});\ \lambda\mathscr{F}.\Box\mathscr{F}(\neg_{et\to et});\ \mathrm{S}_f{\upharpoonright}(\mathrm{S}_f{\upharpoonright}(\mathrm{VP}_f/\mathrm{VP}_b))$

One interesting consequence that immediately follows from the above analysis is that a 'slanted' version of the inverted auxiliary sign is obtained as a theorem, as in the following proof:

(26)

$$\dfrac{\dfrac{\dfrac{\begin{array}{c}\textbf{Lex} \circ \textbf{Inv}\\ \vdots \\ \lambda\sigma.\mathsf{should} \bullet \sigma(\epsilon);\\ \lambda\mathscr{F}.\Box\mathscr{F}(\mathsf{id}_{et});\\ \mathrm{S}_{inv}{\upharpoonright}(\mathrm{S}_f{\upharpoonright}(\mathrm{VP}_f/\mathrm{VP}_b))\end{array} \quad \dfrac{\dfrac{\begin{bmatrix}\varphi_3;\\ x;\mathrm{NP}\end{bmatrix}^3 \quad \dfrac{\begin{bmatrix}\varphi_1;\\ f;\mathrm{VP}_f/\mathrm{VP}_b\end{bmatrix}^1 \quad \begin{bmatrix}\varphi_2;\\ P;\mathrm{VP}_b\end{bmatrix}^2}{\varphi_1 \bullet \varphi_2;\ f(P);\ \mathrm{VP}_f}}{\varphi_3 \bullet \varphi_1 \bullet \varphi_2;\ f(P)(x);\ \mathrm{S}_f}}{\lambda\varphi_1.\varphi_3 \bullet \varphi_1 \bullet \varphi_2;\ \lambda f.F(P)(x);\ \mathrm{S}_f{\upharpoonright}(\mathrm{VP}_f/\mathrm{VP}_b)}{\upharpoonright}\mathrm{I}^1}{\mathsf{should} \bullet \varphi_3 \bullet \varphi_2;\ \Box P(x);\ \mathrm{S}_{inv}}}{\mathsf{should} \bullet \varphi_3;\ \lambda P.\Box P(x);\ \mathrm{S}_{inv}/\mathrm{VP}_b}{\upharpoonright}\mathrm{I}^2}{\mathsf{should};\ \lambda x\lambda P.\Box P(x);\ \mathrm{S}_{inv}/\mathrm{VP}_b/\mathrm{NP}}{\upharpoonright}\mathrm{I}^3$$

Thus, just as there is a close connection between the present approach and the more traditional lexicalist approach in the analysis of basic cases, (the analog of) the inverted auxiliary entry in lexicalist approaches also falls out as a theorem in the present setup.

On the present approach, the interactions of the NIE phenomena can be captured by the interactions of the three operators. Since the derivations are all straightforward, we omit them but just list the relevant composed operators that are involved in each of the NIE interactions in (27e–h).

(27) a. John will come. **Lex**

b. Will John come? **Lex** $\circ$ **Inv**

c. John will $\varnothing$. **Lex** $\circ$ **Ell**

d. John will not come. **Lex** $\circ$ **Neg**

---

who raised this issue that it may plausibly be argued that these operations (or at least **Inv**) are morphological processes rather than empty operators in syntax. See also footnote 12 in this connection.

e. Will John not come? **Lex** ∘ **Inv** ∘ **Neg**
f. John will not ∅. **Lex** ∘ **Neg** ∘ **Ell**
g. Will John? **Lex** ∘ **Inv** ∘ **Ell**
h. Will John not ∅? **Lex** ∘ **Inv** ∘ **Neg** ∘ **Ell**

In contemporary lexicalist approaches and their variants (such as Sag et al. (2019)), the NIE phenomena are typically treated via lexical operations, or in terms of constructional schemata. In such approaches, the interactions of these phenomena are captured by letting the lexical rules or constructional schemata feed into one another. Here, the idea is the same, except that in our case the interactions of the relevant operators are governed by the same logic of syntactic combinatorics (in which function composition is a theorem) that governs other aspects of syntax.[12]

4 The distribution of the unstressed *do*

4.1 *Do* insertion as a higher-order operator

With the analyses of the NIE operators in place, we are now ready to account for the distribution of unstressed *do*. As noted in section 1, the curious property of this auxiliary is that its distribution is limited to environments in which an auxiliary MUST appear. The challenge here is that if we simply posit the following lexical entry for *dŏ/dĭd* that is essentially identical in form to modal auxiliaries, then, we predict that *dŏ/dĭd* can appear in environments in which an auxiliary CAN appear, but this immediately leads to overgeneration of the declarative case (here, **P** is the past tense operator).

(28) $\mathbb{LEX} = \lambda\sigma.\sigma(\mathsf{d\breve{i}d}); \lambda\mathscr{F}.\mathbf{P}\,\mathscr{F}(\mathsf{id}_{et}); \mathrm{S}_\alpha{\upharpoonleft}(\mathrm{S}_\alpha{\upharpoonleft}(\mathrm{VP}_f/\mathrm{VP}_b))$

The problem that earlier PSG-based approaches face essentially stems from the fact that if we view *dŏ/dĭd* as a distinct lexical entry with the same syntactic properties as other auxiliaries in order to capture the parallel behaviors in the NI(C)E environments, then there is no straightforward solution for the overgeneration issue represented by (2a). Sag et al. (2019) resort to a complex solution within Construction-based HPSG that involves a substantial change to the way in which the AUX feature is used in earlier lexicalist approaches.

Our claim here is that in the logic-based setup of Type-Logical Grammar, there is a conceptually simpler solution for this problem that cannot be easily translated to a PSG setup. Specifically, we take '*do* insertion' to be mediated by an operator that takes NIE operators as arguments to produce the same effect as the 'phantom' (i.e. non-existent) lexical entry for *dŏ/dĭd* in (28) above. In other words, we posit **Do** which satisfies the following property:

[12] This of course raises the concern that we are perhaps blurring the boundary between syntax and morphology, blatantly going against the tenet of the lexicalist thesis (in the narrower sense of adhering to the 'lexical integrity principle'). We acknowledge this to be an important question, but leave further investigation to a future occasion.

(29) For $f \in \{\textbf{Neg}/\textbf{Ell}/\textbf{Inv}\}$, $\textbf{Do}(f) \equiv \mathbb{L}\text{ex} \circ f$

Given that we know what $\mathbb{L}\text{ex}$ is (see (28)), defining **Do** turns out to be straightforward:[13]

(30)
$$\begin{aligned}
\textbf{Do} &= \lambda f.\ \mathbb{L}\text{ex} \circ f \\
&= \lambda f \lambda g.\ \mathbb{L}\text{ex}(f(g)) \\
&= \lambda\rho\lambda\sigma.[\lambda\sigma_0.\sigma_0(\mathsf{did})](\rho(\sigma));\ \lambda\mathscr{G}\lambda h.[\lambda\mathscr{F}.\mathbf{P}\,\mathscr{F}(\mathsf{id}_{et})](\mathscr{G}(h));\ (S_\alpha \upharpoonright X)\upharpoonright(S_\alpha \upharpoonright (VP_f/VP_b)\upharpoonright X) \\
&= \lambda\rho\lambda\sigma.\rho(\sigma)(\mathsf{d\check{i}d});\ \lambda\mathscr{G}\lambda h.\mathbf{P}\,\mathscr{G}(h)(\mathsf{id}_{et});\ (S_\alpha \upharpoonright X)\upharpoonright(S_\alpha \upharpoonright (VP_f/VP_b)\upharpoonright X)
\end{aligned}$$

where $X \in \{S_f \upharpoonright (VP_f/VP_b),\ S_b \upharpoonright VP_b\}$

Given the definition in (30), it should be straightforward to see that we get the right result in the NIE sentences. For example, in the inversion case, we have:

(31)
$$\begin{aligned}
\textbf{Do}(\textbf{Inv}) &= \lambda f.[\mathbb{L}\text{ex} \circ f](\textbf{Inv}) \\
&= \mathbb{L}\text{ex} \circ \textbf{Inv} \\
&= \lambda\sigma.\mathsf{d\check{i}d} \bullet \sigma(\epsilon);\ \lambda\mathscr{F}.\mathbf{P}\,\mathscr{F}(\mathsf{id}_{et});\ S_{inv}\upharpoonright(S_f \upharpoonright (VP_f/VP_b))
\end{aligned}$$

The key point here is that **Do** closes off the VP/VP gap by directly APPLYING TO the NIE operators. This means that it can't work alone, from which it immediately follows that (2a) is not licensed. In this respect, the present proposal is reminiscent of the idea of 'last resort' that constitutes the underlying intuition of various formulations of *do* insertion in derivational approaches in generative grammar since Chomsky (1957). While intuitively appealing, the exact status of the *do* insertion operation has been problematic in virtually all variants of derivational approaches throughout the history of generative grammar. In the present logic-based setup, there is a conceptually simple and mathematically precise way of formalizing the operation of *do* insertion as a higher-order operator that specifically targets operators that apply to modal auxiliaries. In this connection, we would like to emphasize that 'logic' in the title of this paper should be construed broadly, not just referring to the underlying deductive system itself, but also the way in which the whole system of grammar is set up. This enables us to make sense of the systematicity inherent to the grammar of natural language in terms of the notion of logical inference. Section 4.2 discusses some further consequences of the present approach where (at least in our view) this perspective becomes important.

At this point, a reader with sound skepticism may wonder about the cognitive plausibility of positing such an abstract operator—a concern that we share. This is a complex issue, but our own current view is that this is a cost that is

[13] Note that the **Do** operator exploits the fact that the prosodic calculus in Hybrid TLG is a lambda calculus in which any higher-order operator can be defined. So far as we can tell, this cannot be implemented straightforwardly as a lexical item in related TLG approaches such as the Displacement Calculus (Morrill et al., 2011) and NL_λ (Barker and Shan, 2015), let alone CCG. One might wonder whether this operator really needs to be posited as a lexical item, as opposed to, e.g., being treated as some kind of meta-operation in the lexicon. The discussion in section 4.2 is relevant for this issue, but we refrain from investigating it in detail in this paper.

worth paying, since it makes the competence grammar more streamlined, and moreover, it has the potential of shedding new light on other aspects such as diachronic change. Specifically, the analysis we have presented above can potentially shed a new light on the ORIGIN of *do* insertion as well, but we need to review the history of English in order to address this point. Thus, in the next section we present a preliminary sketch of an account that addresses the status of *do* insertion, an issue that has received considerable attention in historical syntax. We hope to convince the reader that the logic-based approach we advocate here has something new to offer to this debate in the neighboring field of diachronic syntax as well.

4.2 The origin of *do*

In this section, we argue that the analysis of unstressed *do* presented above has a potential further advantage in offering a natural explanation for how this peculiar distribution arose in the history of English. Admittedly, our discussion is highly speculative, and it also builds heavily on work on historical syntax by other scholars, most importantly, by Anthony Warner (Warner, 1993). The point we would like to make is modest: we believe that Warner's view, originally expressed in the theoretical vocabulary of HPSG, is essentially on the right track, but that its key insight can be expressed even more transparently by taking a logic-based perspective of the sort we have advocated above.

According to Warner (1993), the development of *do* took place in two stages in the period of early Modern English. The first stage coincides with the establishment of modals as a distinct class during the second half of the 15th century and the first half of the 16th century. During this period, there was a set of systematic changes to a class of verbs including *can*, *may* and *will* in the direction that the notion 'modal auxiliary' became a coherent grammatical category (this included the loss of nonfinite forms of *can*, *may* and *will*, and the loss of nonmodal (i.e., main verb) meanings for *can*, *may* and *will*, among other changes). The auxiliary use of *do* in the Standard variety of English started to develop around the same period. Warner follows Traugott in viewing that periphrastic *do* at this stage was associated with a range of pragmatic functions pertaining to 'affirmation of speaker truthfulness' (Traugott, 1982, 257).

In an influential study that statistically demonstrated the two-stage development of the auxiliary use of *do*, Kroch (1989) observes that from the beginning of the 16th century to the period of 1550–1575, the occurrence of *do* increases in all syntactic environments, including the declarative. But after the period of 1575–1600, the development in different contexts starts to diverge. *Do* in polar questions continues to increase but the use of *do* in declarative sentences starts to decline. In negative environments, there is an initial sharp decline, followed by a steady increase (but at a lower rate than in positive polar questions). Kroch attributes this two-stage change to the loss of V2 in English within an account of parametric change in the Principles and Parameters (P&P) framework.

Warner offers a reinterpretation of Kroch's data that does not rely on the P&P assumptions (Warner's view is endorsed by Hudson (1997) as well). According to Warner, the split in the development of *do* in different syntactic environments after the period of 1575–1600 can be attributed to a process of reanalysis of the following sort. The key factor is the fact that the pragmatic function associated with *do* (which is essentially semantic focus on polarity contrast) is an inherent property of polar questions. Thus, with the steady increase of *do* in polar questions, the pragmatic function originally associated with *do* was reanalyzed as the constructional meaning of the polar interrogative itself, with *do* having only the function of contributing tense information. *Do* then becomes essentially an 'allomorph' of the tense affix whose distribution is limited to the inverted interrogative environment. The use of periphrastic *do* in the declarative then declines gradually with this reanalysis, via blocking by simple verb inflection for expressing tense. Note that this account is also consistent with the initial drop and the later recovery in negative environments; the reanalysis of *do* is triggered in the interrogative context, and then spreads to other syntactic contexts via analogy. Warner attributes the source of this reanalysis to child language learning. Due to the pragmatic function, *do* in the declarative environment was restricted to the literary style, but it was common in polar interrogatives in colloquial speech. Then, the child learning the language was most likely exposed to utterances containing the auxiliary use of *do* only in polar questions, from which s/he would infer that *do* is nothing more than a tense auxiliary.

While all this seems plausible, one point that remains unclear in Warner's account is the status of *do* as an 'allomorph' of the tense affix. This is of course a descriptively accurate characterization of *do* in present-day English, but the theoretical issue (which previous PSG proposals struggled to account for) is how exactly this descriptive generalization is to be implemented in a formally explicit theory. Warner (1993, 250, n28) claims that a lexical specification involving an implicational constraint along the lines of (32) captures this generalization:

(32) SUBCAT ⟨[−ELLIPSIS, −AUX]⟩ & *do* ⊃ semantic prominence

This says that an overt *do* carries the effect of 'semantic prominence' unless it appears in a NICE environment (SUBCAT ⟨[−ELLIPSIS, −AUX]⟩). However, this is just a restatement of the facts, and, perhaps more disturbingly, it is unclear how such a complex lexical restriction arose out of a simple process of reanalysis of the sort Warner himself advocates.

Thus, even though the general outline of Warner's reanalysis-based account is quite attractive, the exact nature of the reanalysis (in particular, the formal status of the resultant lexical entry for *do*) within the PSG-based setup he adopts is somewhat unclear. It is then interesting to see that a conceptually much simpler reformulation becomes available once we recast his account within a type-logical setup. From our perspective, Warner's account can be simply understood as a reanalysis of $\mathbb{LEX}$ by **Do**. That is, in the first phase of the two-stage development of *do*, $\mathbb{LEX}$ used to exist in the grammar of English, so its distribution was not restricted to NICE environments. But its usage was restricted to literary style in declaratives, due to the pragmatic function it was associated with. A child acquiring the language then gets exposed to occurrences of *do* only in polar questions. This much is the same as in Warner's account. But then, we essentially have a situation in which the child has to choose between two competing hypotheses, $\mathbb{LEX}$ or **Do**, but the available evidence gives him/her confidence only for the weaker hypothesis of positing **Do**. Given the lack of positive evidence for the stronger hypothesis, the child opts for the more conservative hypothesis. Crucially, unlike in Warner's original account, the formal status of **Do** is perfectly explicit here: it is an operator that mimics the effect of an auxiliary just when, despite the lack of evidence for an independent lexical auxiliary, we find an operator (**Inv**) that is looking to combine with an auxiliary—without the help of **Do**, there is no way to complete the derivation. Moreover, our reconceptualization provides a clear motivation for why the reanalysis has happened: **Do** wins over $\mathbb{LEX}$ in the acquisition context since it is the less risky hypothesis that is consistent with all the data that the child encounters.

Thus, the higher-order analysis of *do* insertion not only characterizes the distribution of *do* in present-day English, but it also potentially illuminates the process by which it arose. Of course, there is no way of directly proving or disproving the hypothesis we have entertained above, but we take it that, other things being equal, an analysis that provides a natural motivation for known facts about historical change is more preferable than one that doesn't.

5 Conclusion

In this paper, we have proposed an analysis of the English auxiliary system in Hybrid TLG. Given the lack of a detailed and systematic study of English auxiliaries in the past literature of categorial grammar, our conclusions in this paper should be of interest to practitioners of categorial grammar of various stripes (and for practitioners of other grammatical theories more generally). First, we believe that the conceptually simple analysis of *do* insertion that crucially relies on the higher-order treatment of modals and the NIE operators generally argues for the advantage of contemporary variants of TLG, all of which are equipped with machinery for dealing with this type of abstract syntactic composition in one way or another (but there are some possible subtleties here; note the point we have made in footnote 13). This then raises an interesting question for CCG. On the one hand, the higher-order analysis we have argued for is not directly implementable in CCG—to see this point, the reader is invited to think about how to reformulate the auxiliary entry in (8) (which is crucial for obtaining the 'anomalous scope' interpretation in Gapping and related phenomena) or the **Do** operator in (30) in CCG. On the other hand, an elaborate construction-based analysis of the sort recently advocated by Sag et al. (2019) is also unlikely to straightforwardly carry over to CCG. But then, the treatment of *do* insertion seems to remain one of the major open questions for CCG. We refrain from speculating about possible responses to this challenge, but instead just raise this issue explicitly here since it is (in our view) yet another variant of the long-term tension between 'surface-oriented' vs. 'abstract' syntax of the sort that various scholars have commented on since the very inception of nontransformational variants of syntax at the beginning of the 80s.

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A A brief introduction to Hybrid Type-Logical Grammar

The main text assumes familiarity with Hybrid TLG (Kubota and Levine, 2020). This appendix provides a brief, self-contained introduction to Hybrid TLG so that readers who are not familiar with it can understand the analysis in this paper. For a more detailed exposition, see Kubota and Levine (2020).

Hybrid TLG is essentially an extension of the Lambek calculus (Lambek, 1958) with one additional, non-directional mode of implication. The full set of inference rules in Hybrid TLG are given in (33).

(33)

Connective	**Introduction**	**Elimination**
$/$	$\dfrac{\begin{array}{ccc} \vdots & \underline{[\varphi;\, x;\, A]^n} & \vdots \\ \vdots & \vdots & \vdots \end{array}}{\dfrac{b \bullet \varphi;\, \mathcal{F};\, B}{b;\, \lambda x.\mathcal{F};\, B/A}\,/\mathrm{I}^n}$	$\dfrac{a;\, \mathcal{F};\, A/B \quad b;\, \mathcal{G};\, B}{a \bullet b;\, \mathcal{F}(\mathcal{G});\, A}\,/\mathrm{E}$
$\backslash$	$\dfrac{\begin{array}{ccc} \vdots & \underline{[\varphi;\, x;\, A]^n} & \vdots \\ \vdots & \vdots & \vdots \end{array}}{\dfrac{\varphi \bullet b;\, \mathcal{F};\, B}{b;\, \lambda x.\mathcal{F};\, A\backslash B}\,\backslash\mathrm{I}^n}$	$\dfrac{b;\, \mathcal{G};\, B \quad a;\, \mathcal{F};\, B\backslash A}{b \bullet a;\, \mathcal{F}(\mathcal{G});\, A}\,\backslash\mathrm{E}$
$\upharpoonright$	$\dfrac{\begin{array}{ccc} \vdots & \underline{[\varphi;\, x;\, A]^n} & \vdots \\ \vdots & \vdots & \vdots \end{array}}{\dfrac{b;\, \mathcal{F};\, B}{\lambda\varphi.b;\, \lambda x.\mathcal{F};\, B\upharpoonright A}\,\upharpoonright\mathrm{I}^n}$	$\dfrac{a;\, \mathcal{F};\, A\upharpoonright B \quad b;\, \mathcal{G};\, B}{a(b);\, \mathcal{F}(\mathcal{G});\, A}\,\upharpoonright\mathrm{E}$

The key difference between $/, \backslash$ and $\upharpoonright$ is that while the Introduction and Elimination rules for $/, \backslash$ refer to the phonological forms of the input and output strings (so that, for example, the applicability of the $/$I rule is conditioned on the presence of the phonology of the hypothesis φ on the right periphery of the phonology of the input $b \bullet \varphi$), the rules for $\upharpoonright$ is not constrained that way.[14] For reasoning involving $\upharpoonright$, the phonological terms themselves fully specify the ways in which the output phonology is constructed from the input phonologies. Specifically, for $\upharpoonright$, the phonological operations associated with the Introduction and Elimination rules mirror exactly the semantic operations for these rules: function application and λ-abstraction, respectively. We assume that the binary connective $\bullet$ in the phonological term calculus represents the string concatenation operation and that $\bullet$ is associative in both directions. For notational convenience, we implicitly assume the axiom $(\varphi_1 \bullet \varphi_2) \bullet \varphi_2 \equiv \varphi_1 \bullet (\varphi_2 \bullet \varphi_3)$ and leave out all the brackets indicating the internal constituency of complex phonological terms.

Thus, the present system without the rules for $\upharpoonright$ is equivalent to the Lambek calculus (Lambek 1958), while the system with only the rules for $\upharpoonright$ is essentially equivalent to the family of approaches known as Linear Categorial Grammar (de Groote, 2001; Muskens, 2003; Martin and Pollard, 2014), all of which are essentially direct descendants of Oehrle (1994).

The latter component, namely, the lambda abstraction in the prosodic calculus is what distinguishes Hybrid TLG from other variants of TLG that are based on the Lambek calculus (such as Moortgat (2011) and Morrill et al. (2011)). As demonstrated by Oehrle (1994), this enables a straightforward and formally explicit implementation of Montague's (1973) quantifying-in:

(34)

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathsf{read};\ \mathbf{read};\ (\mathrm{NP}\backslash\mathrm{S})/\mathrm{NP} \quad [\varphi_2; x; \mathrm{NP}]^1}{\mathsf{read} \bullet \varphi_2;\ \mathbf{read}(x);\ \mathrm{NP}\backslash\mathrm{S}} \quad \mathsf{john};\ \mathbf{j};\ \mathrm{NP}}{\mathsf{john} \bullet \mathsf{read} \bullet \varphi_2;\ \mathbf{read}(x)(\mathbf{j});\ \mathrm{S}}}{\lambda\varphi_2.\mathsf{john} \bullet \mathsf{read} \bullet \varphi_2;\ \lambda x.\mathbf{read}(x)(\mathbf{j});\ \mathrm{S}\upharpoonright\mathrm{NP}}\upharpoonright\mathrm{I}^1 \quad \begin{array}{c}\vdots \\ \lambda\sigma.\sigma(\mathsf{every} \bullet \mathsf{book}); \\ \mathbf{\forall_{book}}; \mathrm{S}\upharpoonright(\mathrm{S}\upharpoonright\mathrm{NP})\end{array}}{\mathsf{john} \bullet \mathsf{read} \bullet \mathsf{every} \bullet \mathsf{book};\ \mathbf{\forall_{book}}(\lambda x.\mathbf{read}(x)(\mathbf{j}));\ \mathrm{S}}$$

As (34) illustrates, quantifiers are entered in the lexicon in type $\mathrm{S}\upharpoonright(\mathrm{S}\upharpoonright\mathrm{NP})$, with the standard generalized quantifier meanings for their semantics and a phonology that is a higher-order function typed $(\mathbf{st} \to \mathbf{st}) \to \mathbf{st}$ (with $\mathbf{st}$ the type of strings), which 'lowers' the quantifier string in the position in the sentence (bound by the λ-operator in the phonology) corresponding to the semantic variable bound. As in Montague's quantifying-in, the order in which the quantifier combines with the sentence that it lowers into determines its scope.

In the analysis in the main text, we extend Oehrle's approach to scopal operators beyond the familiar domain of generalized quantifiers. In particular, we show how the behavior of English auxiliaries, long known to display puzzling and refractory relationships between their syntactic and semantic behavior, can be explained through a treatment which parallels the way in which the scopal interaction of generalized quantifiers is accounted for.

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[14] In this respect, the proof system of Hybrid TLG follows most closely Morrill and Solias (1993) and Morrill (1994); see Moortgat (2011) and Bernardi (2002) for an alternative formulation where sensitivity to directionality is mediated through a presumed correspondence between surface string and the form of structured antecedents in the sequent-style notation of natural deduction.