

Negative comparison, or how to be judgmental and ignorant with scalar alternatives*

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1 Introduction

This paper is primarily about two negative comparative-modified numeral (CMN) expressions: *no more than n* and *not more than n*. These expressions look extremely similar. Naively speaking, they also carry the same non-strict comparison meaning, *less than or equal to n*. However, as noted in Nouwen (2008) (who cites Jespersen 1949, 1966, who in turns credits Stoffel 1894), they differ in interesting ways, as shown below: *no more than n* yields an exact meaning (EX) but *not more than n* does not (NO-EX). And, in addition to this, *no more than n* give rise to a speaker evaluative meaning (EVAL) whereas *not more than n* gives rise to a speaker ignorance effect (IG).

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|-----|--|-----|---|
| (1) | Jo found no more than 10 marbles. | (2) | Jo found not more than 10 marbles. |
| | a. \rightarrow She found $\neg > 10$ ($= \leq 10$). | | a. \rightarrow She found $\neg > 10$ ($= \leq 10$). |
| | b. \rightsquigarrow She found exactly 10. (EX) | | b. $\not\rightarrow$ She found exactly 10. (NO-EX) |
| | c. \rightsquigarrow Speaker thinks this is few. (EVAL) | | c. \rightsquigarrow Speaker not sure how many. (IG) |

Similar patterns can be verified, *mutatis mutandis*, for *no less than 10* and *not less than 10*.

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|-----|---|-----|---|
| (3) | Jo found no less than 10 marbles. | (4) | Jo found not less than 10 marbles. |
| | a. \rightarrow She found $\neg < 10$ ($= \geq 10$). | | a. \rightarrow She found $\neg < 10$ ($= \geq 10$). |
| | b. \rightsquigarrow She found exactly 10. (EX) | | b. $\not\rightarrow$ She found exactly 10. (NO-EX) |
| | c. \rightsquigarrow Speaker thinks this is many. (EVAL) | | c. \rightsquigarrow Speaker not sure how many. (IG) |

The existing literature proposes three different analyses related to these phenomena: a solution based on Horn (1972) (considered and dismissed by Nouwen 2008); a solution based on Horn (1972) + Fox and Hackl (2006)'s Universal Density of Measurement (Nouwen 2008's ultimate proposal); and a solution based on Horn (1972) + Mayr (2013)'s Modifier Alternatives (Mayr 2013). All these analyses are motivated by further data to do with bare numerals (BNs) and/or superlative-modified numerals (SMNs). However, none can explain how a negative CMN can yield *either* EX *or* NO-EX. Nor do they offer an explanation for IG or EVAL.

In this paper I propose a new analysis based on Horn (1972): Horn (1972) + *Negation* Alternatives. This analysis takes into account the data for BNs, CMNs, and SMNs also. And it offers a solution for both EX and NO-EX, as well as for IG and EVAL.

The plan is as follows: In Section 2 we review Nouwen (2008)'s proposal for EX based on Horn (1972), and the reasons why he dismisses it, based on Krifka (1999) and Fox and Hackl (2006). In Section 3 we review Nouwen (2008)'s proposal for EX based on Horn (1972) + Fox and Hackl (2006)'s Universal Density of Measurement hypothesis, and reasons to dismiss it based on Mayr (2013). In Section 4 we review Mayr (2013)'s proposal for NO-EX based on Horn (1972) + his own Modifier Alternatives hypothesis, and reasons to dismiss it based on Cummins et al. (2012). Finally, in Section 5 we will discuss a new proposal based on Mihoc (2021)'s defense of original Horn (1972) view for CMNs and SMNs and an enrichment of this view with *Negation* Alternatives. Finally, in Section 6 we conclude and highlight some issues for future research.

For concision, we will always discuss just *more than* (and *at least*). However, the analysis extends, *mutatis mutandis*, to *less than* (and *at most*) also.

*I would like to thank Gennaro Chierchia and Roger Schwarzschild for helpful discussions at earlier stages of this work. Any errors are my own.

2 Nouwen (2008), using Horn (1972)

Nouwen (2008) focuses on the EX pattern of negative CMNs, repeated below:

- (1) Jo found **no more than 10** marbles.
 a. \rightarrow She found $\neg > 10$ ($= \leq 10$).
 b. \rightsquigarrow She found exactly 10. (EX)

He connects this to a similar well-known EX pattern in positive bare numerals (BNs). Naively speaking, a positive BN such as *10* below is compatible with situations that we might describe as ‘at least 10’. However, in practice it is understood as ‘exactly 10’.

- (5) Jo found **10** marbles.
 a. \rightarrow She found ≥ 10 .
 b. \rightsquigarrow She found exactly 10. (EX)

The challenge, as he sees it, is as follows:

- (6) **Challenge:**
 Find a theory that derives EX for both positive BNs and negative CMNs.

Nouwen notes that such a theory is already offered by Horn (1972):

Focusing on the EX pattern in positive BNs, Horn argues that it is an implicature. In particular, he proposes the following: BNs entail a lower-bounded, ‘at least’ meaning. However, they belong to natural numerical scales, for example, $\langle \dots, 9, 10, 11, \dots \rangle$. For this reason, an utterance of *10* naturally activates alternatives based on its scalemates—scalar alternatives (SA). When factored into Gricean reasoning, these give rise to negative implicatures about any non-entailed alternatives, creating an upper bound. The entailment and the implicature(s) together drive the EX pattern in *n*.

- (7) Jo found **10** marbles.
 a. \rightarrow Jo found ≥ 10 . (truth conditions)
 b. $\{ \dots, \text{Jo found } 9 (\geq 9), \text{Jo found } 11 (\geq 11), \dots \}$ (Horn-SA)
 $\rightsquigarrow \neg$ Jo found 11 (≥ 11), ... (SA-implicatures)
 \Rightarrow Jo found exactly 10. (EX; ✓)

Analogously, Nouwen argues that the EX pattern in negative comparatives is an implicature also, and arises in the exact same way: *no more than 10* belongs to the natural scale $\langle \dots, \text{no more than } 9, \text{no more than } 10, \text{no more than } 11, \dots \rangle$. Thus, an utterance of *no more than 10* naturally activates alternatives based on its scalemates; this gives rise to negative inferences about the non-entailed SA; and this captures the EX pattern in *no more than n*.

- (8) Jo found **no more than 10** marbles.
 a. \rightarrow Jo found $\neg > 10$ ($= \leq 10$). (truth conditions)
 b. $\{ \dots, \text{Jo found no more than } 9, \text{Jo found no more than } 11, \dots \}$ (Horn-SA)
 $\rightsquigarrow \neg$ Jo found no more than 9, ... (SA-implicatures)
 \Rightarrow Jo found exactly 10. (EX; ✓)

This proposal captures the challenge above. However, Nouwen (2008) notes with Krifka (1999) and Fox and Hackl (2006) that it also makes the unwelcome prediction that positive CMNs should give rise to an EX meaning also, whereas in fact they don’t:

- (9) Jo found **more than 10** marbles.
 a. \rightarrow Jo found > 10 . (truth conditions)
 b. $\{ \dots, \text{Jo found more than } 9, \text{Jo found more than } 11, \dots \}$ (Horn-SA)
 $\rightsquigarrow \neg$ Jo found more than 11 (SA-implicatures)
 \Rightarrow Jo found exactly 11. (EX; ✗)

Because of this, Nouwen ends up dismissing this view.

Additionally, we point out another even more immediate issue with this view: By analogous reasoning, it predicts EX for *not more than n* also. However, as we saw at the very outset, *not more than n* does not have EX.

3 Nouwen (2008), using Horn (1972) + Universal Density of Measurement

Nouwen (2008) focuses again on the EX pattern of negative CMNs, repeated again below:

- (1) Jo found **no more than 10** marbles.
- a. \rightarrow She found $\neg > 10$ ($= \leq 10$).
 - b. \rightsquigarrow She found exactly 10. (EX)

However, this time he connects it not just to the EX pattern of positive BNs but also to the NO-EX pattern of positive CMNs, repeated below:

- (9) Jo found **more than 10** marbles.
- a. \rightarrow She found > 10 marbles.
 - b. \nrightarrow She found exactly 11.

The challenge, as he sees it this time, is as follows:

- (10) **Challenge:**
Find a theory of EX in negative CMNs that also captures EX in positive BNs and NO-EX in positive CMNs.

Nouwen notes that such a theory is already offered by Fox and Hackl (2006):

Focusing on the NO-EX pattern of positive CMNs, Fox and Hackl propose adding a clause to the original Horn (1972) view, as follows:

- (11) **The Universal Density of Measurement (UDM):**
Measurement scales needed for natural language semantics are *always* dense. (p. 542)
That is, for any n and $n + \varepsilon$ there is a degree $n + \delta$ such that $n < n + \delta < n + \varepsilon$.

Fox and Hackl show that UDM does not affect Horn's original proposal for the EX pattern of positive BNs.

- (12) Jo found **10** marbles.
- a. \rightarrow Jo found ≥ 10 (truth conditions)
 - b. $\{ \text{Jo found } n (\geq n) \mid n \in (-\infty, 10) \cup (10, +\infty) \dots \}$ (UDM-SA)
For all degrees $n > 10$: $\rightsquigarrow \neg$ Jo found $n (\geq n)$ marbles. (UDM-SA-implicatures)
 \Rightarrow Jo found exactly 10. (EX: ✓)

However, UDM does help capture the NO-EX of positive CMNs: Given an infinitely dense scale, the scalar implicatures of an unembedded CM utterance would contradict the assertion, as can be seen by examining closely the implications of the assertion and of the UDM Horn-SA below.

- (13) Jo found **more than 10** marbles.
- a. \rightarrow Jo found > 10 (truth conditions)
 - b. $\{ \text{Jo found more than } n \mid n \in (-\infty, 10) \cup (10, +\infty) \dots \}$ (UDM-SA)
For all degrees $n_i > 10$: $\rightsquigarrow \neg$ Jo found more than n_i marbles. (UDM-SA-implicatures)
 \Rightarrow Contradiction: If Jo found more than 10, that means, that she found $10 + \varepsilon$, which means she found more than $(10 + \frac{\varepsilon}{2})$. Thus, there is a degree $n_i > 10$ s.t. Jo found more than n_i marbles—this degree is $(10 + \frac{\varepsilon}{2})$.

Nouwen notes that UDM derives EX for non-strict comparison meanings and NO-EX for strict comparison meanings more generally. That is, by the same reasoning that derives EX for positive

BNs and NO-EX for positive CMNs, we also obtain EX for negative CMNs, as can be verified by simply replacing $\geq n$ in the illustration for BNs with $\neg > n (= \leq n)$.

This proposal captures the challenge above. However, as acknowledged by Fox and Hackl (2006) (fn. 4 on p. 540), and in fact also by Nouwen (2008) (p. 282ff.), it also makes the unwelcome prediction—by reasoning completely analogous to the one for positive BNs or negative CMNs, that positive SMNs should have an EX meaning also; however, as known since Krifka (1999), they don't.

- (14) Jo found **at least 10** marbles.
- a. \rightarrow Jo found ≥ 10 (truth conditions)
 - b. $\{ \text{Jo found } \geq n \mid n \in (-\infty, 10) \cup (10, +\infty) \dots \}$ (UDM-SA)
 - For all degrees $n > 10$: $\rightsquigarrow \neg$ Jo found $\geq n$ marbles. (UDM-SA-implicatures)
 - \Rightarrow Jo found exactly 10 (EX: \times)

Nouwen (2008), like Fox and Hackl (2006), cites Geurts and Nouwen (2007) for observations that positive SMNs convey speaker ignorance (IG), and suggest that, for reasons related to that, these predictions might not be applicable. However, Mayr (2013) points out that UDM *generally* makes opposite SA-implicature predictions for CMNs and SMNs, whereas *generally* CMNs and SMNs are in fact identical. In short, Mayr (2013) suggests that UDM is missing something.

Additionally, we note again another even more immediate issue with this view: By analogous reasoning, it predicts EX for *not more than n* also, but this is a meaning it does not have.

One final potential issue with this view is conceptual: Whereas on the original Horn (1972)-view the numerical scale could be \mathbb{N} , \mathbb{Q} , \mathbb{R} , or even just subsets thereof, according to context, on the UDM view we are forced to believe that it is always \mathbb{R} . This runs counter to the intuition that the granularity of Horn scales is in fact very much contextual: When we count marbles, we usually have in mind \mathbb{N}^+ . When we measure weights (as in Fox and Hackl's original example), we might consider \mathbb{Q}^+ . And, when we measure temperatures, we might consider \mathbb{R} . However, even for the latter two we rarely go beyond a couple of decimal places.

4 Mayr (2013), using Horn (1972) + Modifier Alternatives

Insofar as our patterns are concerned, Mayr (2013)'s focus is on *not more than n*, repeated below:

- (2) Jo found **not more than 10** marbles.
- a. \rightarrow She found $\neg > 10 (= \leq) 10$ marbles.
 - b. $\not\rightsquigarrow$ She found exactly 10. (NO-EX)

As mentioned just now, he connects this to the fact that, with respect to the SA-implicatures expected from Horn (1972), (*not-*)CMNs are generally like (*not-*)SMNs—neither give rise to EX, neither in a positive nor in a negative context (among others).

- (15) Jo found **more than 10 / at least 10** marbles.
- a. \rightarrow She found $> 10 / \geq 10$.
 - b. $\not\rightsquigarrow$ She found exactly 11 / 10. (NO-EX)

- (16) Jo didn't find **more than 10 / at least 10** marbles.¹
- a. \rightarrow She found $\leq 10 / < 10$.
 - b. $\not\rightsquigarrow$ She found more than 9 / at least 9. (NO-EX)

The challenge, as he sees it, is as follows:

- (17) **Challenge:**
Find a theory of NO-EX in negative CMNs that also captures EX in positive BNs and NO-EX in positive or negative CMNs or SMNs.

¹SMNs under negation are actually degraded. See Geurts and Nouwen (2007), Cohen and Krifka (2014), Spector (2014, 2015), Mihoc (2021), Mihoc and Davidson (2021). This is however not crucial here.

Mayr (2013) notes that no such theory is available, and goes on propose one himself. In particular, he proposes adding a different clause to the original Horn (1972) view, as follows:

(18) **Modifier Alternatives (MA):**

In modified numerals, SA are derived not just by replacing the numeral with another numeral, but also the comparative/superlative modifier with another modifier of the same type. That is, *more than 10* belongs to a scale of the form $\langle \dots, \text{more than } 9, \text{more than } 10, \text{more than } 11, \dots, \text{less than } 9, \text{less than } 10, \text{less than } 11, \dots \rangle$.

Since BNs do not have any modifier, Mayr's MA hypothesis does not affect Horn (1972)'s correct prediction of EX for BNs. However, as stated above, it seems to leave unchanged Horn's incorrect prediction of EX for CMNs and SMNs also.

(19) Jo found **more than 10** marbles.

- a. \rightarrow Jo found > 10 . (truth conditions)
- b. $\{ \text{more than } n \mid n \in S \} \cup \{ \text{less than } n \mid n \in S \}$ (MA-SA)
- $\rightsquigarrow \neg$ Jo found more than 11, ... (MA-SA-implicatures)
- \Rightarrow Jo found exactly 11. (EX; ✗)

Mayr notes this happens because the traditional Gricean implicature calculation mechanism assumed by Horn (1972) negates only those alternatives compatible with the assertion that *entail* the assertion; but, of the newly added SA, the only ones compatible with the assertion, namely, the ones from the set $\{ \text{less than } 11, \dots \}$, are logically *independent* with respect to the assertion.

Mayr proposes thus to modify the implicature calculation mechanism to negate any alternative compatible with the assertion that is *not entailed* by the assertion. As desired, this does make a difference. However, the result is not NO-EX but rather contradiction:

(20) Jo found **more than 10** marbles.

- a. \rightarrow Jo found > 10 . (truth conditions)
- b. $\{ \text{more than } n \mid n \in S \} \cup \{ \text{less than } n \mid n \in S \}$ (MA-SA)
- $\rightsquigarrow \neg$ Jo found more than 11, ..., \neg Jo found less than 12, ... (MA-SA-implicatures')
- \Rightarrow Contradiction. For example, if Jo found more than 10 but not more than 11, that must mean that she found less than 12, but that would contradict the MA-SA-implicature' that \neg Jo found less than 12. And so on. (\perp ; ✗)

Mayr comments that the reason why the result we obtained was contradictory is because the MA-SA are *symmetric*—excluding one automatically includes another. He argues that this calls for an implicature calculation mechanism that will exclude only those among the non-entailed alternatives whose exclusion does not automatically lead to the inclusion of other non-entailed alternatives. He notes that such a mechanism is already provided by Fox (2007)'s contradiction-free *silent-only* exhaustivity operator O.² With this addition in place, implicature calculation is vacuous. He argues that *this* is the reason for NO-EX.

(21) Jo found **more than 10** marbles.

- a. \rightarrow Jo found > 10 . (truth conditions)
- b. $\{ \text{more than } n \mid n \in S \} \cup \{ \text{less than } n \mid n \in S \}$ (MA-SA)
- \rightsquigarrow — (MA-SA-implicatures'')
- \Rightarrow Jo found > 10 . (NO-EX; ✓)

Mayr comments that the same assumptions derive NO-EX in negative CMNs or SMNs also. As his main focus is elsewhere, he doesn't actually demonstrate this, but to see it we just need to retrace our steps, only this time in a negative context. Let's again suppose that the implicature calculation mechanism excludes all those alternatives compatible with the assertion that are not entailed by it. As in the positive context, this leads to contradiction.

²The transition from a pragmatic view of implicature calculation to a grammatical view of implicature calculation is not crucial here.

- (22) Jo didn't find **more than 10** marbles.
- a. $\rightarrow \neg$ Jo found > 10 . (truth conditions)
 - b. $\{\neg \text{more than } n \mid n \in S\} \cup \{\neg \text{less than } n \mid n \in S\}$ (MA-SA)
 $\rightsquigarrow \neg\neg$ Jo found more than 9, \dots , $\neg\neg$ Jo found less than 10, \dots (MA-SA-implicatures')
 \Rightarrow Contradiction. For example, if \neg Jo found more than 10 but $\neg\neg$ Jo found more than 9, that must mean that \neg Jo found less than 10, but that would contradict the MA-SA-implicature' that $\neg\neg$ Jo found less than 10. And so on. (\perp ; \times)

However, if we again assume that the implicature calculation mechanism only excludes those among the non-entailed alternatives whose exclusion does not automatically lead to the inclusion of other non-entailed alternatives, we again obtain a vacuous result, which again, as desired, provides an explanation for NO EX.

- (23) Jo didn't find **more than 10** marbles.
- a. $\rightarrow \neg$ Jo found > 10 . (truth conditions)
 - b. $\{\neg \text{more than } n \mid n \in S\} \cup \{\neg \text{less than } n \mid n \in S\}$ (MA-SA)
 \rightsquigarrow — (MA-SA-implicatures'')
 $\Rightarrow \neg$ Jo found > 10 . (NO-EX; \checkmark)

This proposal captures the challenge that Mayr set out to meet. It moreover provides a fairly principled explanation for why, although BNs, CMNs, and SMNs all entail one bound, in a plain positive context only BNs also implicate a second bound that gives rise to their signature EX meaning. However, I will argue that this proposal is perhaps less innocuous than it might appear. That is because it does not just prevent CMNs and SMNs from acquiring a second bound that would give rise to EX—it prevents them from acquiring *any* kind of second bound. This makes them conceptually quite different from BNs. More relevantly, however, this is a difference that does not in fact correspond to intuition: When we make a plain CMN or SMN assertion, we do not really have in mind the full interval denoted by the assertion, but usually just a portion thereof, within contextual limits. For example, *Jo found more than 10 marbles* does not really mean any number in $(10, +\infty)$ but rather some number between 11 and 14, or between 11 and 19, corresponding to an implicature of the form *... but not more than 15 / 20*. An example from Spector (2014:42) makes the same point—in a context where grades are attributed on the basis of the number of problems solved, with 0-5 yielding a C, 6-8 yielding a B, and 9- yielding an A, *John solved more than 5 SA-implicates that he didn't solve more than 9, so he gets a B*. A series of experiments by Cummins et al. (2012) guides to the same conclusion: Although they do not SA-implicate a second bound that would lead to an EX meaning, CMNs and SMNs do implicate *some* second bound, and are thus in this respect not fundamentally different from BNs.

Additionally, once again, we point out another even more immediate issue with this proposal: By analogous reasoning, it predicts *no more than n* to have NO-EX, contrary to what we saw at the beginning.

As before, one final potential issue with this view is conceptual: As discussed above, the MA addition to Horn (1972) results in a set of SA that is no longer totally ordered by monotonicity. As Mayr himself notes, this contradicts the traditional understanding, also reinforced in Matsumoto (1995), that Horn-sets are fundamentally sets ordered by monotonicity.

5 This paper, using Horn (1972) + Negation Alternatives

5.1 Looking back, looking forward

We have seen three views of negative CMNs. Each tried to capture either EX or NO-EX in negative CMNs. Each also proposed a solution that took into account additional relevant facts about BNs and SMNs. However, each turned out to suffer from significant issues. Also, none provided an explanation for how negative CMNs can be either EX or NO-EX, or give rise to IG or EVAL.

The goal of this section is to provide a fourth view—hopefully broader and more complete.

Unlike the previous literature, we will aim to capture not just the EX or the NO EX pattern of negative comparison, but rather *all* the facts from the outset, repeated below:

- | | |
|--|---|
| (1) Jo found no more than 10 marbles. | (2) Jo found not more than 10 marbles. |
| a. \rightarrow She found $\neg > 10$ ($= \leq 10$). | a. \rightarrow She found $\neg > 10$ ($= \leq 10$). |
| b. \rightsquigarrow She found exactly 10. (EX) | b. $\not\rightsquigarrow$ She found exactly 10. (NO-EX) |
| c. \rightsquigarrow Speaker thinks this is few. (EVAL) | c. \rightsquigarrow Speaker not sure how many. (IG) |

Like the previous literature, we will also keep in mind all the facts uncovered before us, as well as the one we discussed at the end of the previous section, all repeated in summary below:

- (24) Jo found **10 / more than 10 / at least 10** marbles.
- a. \rightarrow Jo found $\geq 10 / > 10 / \geq 10$.
- b. $\rightsquigarrow \neg$ Jo found $\geq 11 / \neg$ Jo found, e.g., $> 15 / \neg$ Jo found, e.g., ≥ 15 (SA-implicatures)

We will additionally consider two further facts:

First, positive CMNs and SMNs give rise to IG.³ Mayr and Meyer (2014) illustrate this with the example below:

- (25) A: How many kids do you have? B: ??More than 3. / ??At least 3.

As I argue in Mihoc (2020, 2021), this suggests that the reason why positive CMNs and SMNs do not give rise to EX, even as they do give rise to SA-implicatures, is because EX would clash with IG—an effect which in positive contexts comes from *subdomain* alternatives (DA), but which unfortunately we will not be able to discuss in any further detail here. If this is on the right track, this suggests that we may safely assume traditional SA-implicatures in CMNs and SMNs, just as in BNs—the differences with respect to SA-implicatures already have a principled independent source.

Second, negative BNs are predicted by Horn (1972) to give rise to EX, as shown below. However, as discussed, for example, by Spector (2013:279-80), they don't.

- (26) Peter didn't solve 10 problems.
- a. $\rightarrow \neg$ He solved 10 (≥ 10).
- b. $\not\rightsquigarrow \neg \neg$ He solved 9 (≥ 9).
 \Rightarrow He solved exactly 9. (EX; ✗)

As I argue in Mihoc (2020, 2021), this suggests that NO-EX was never a good reason to assume a fundamental split between SA-implicatures in CMNs and SMNs vs. BNs.

All in all, these facts point away from UDM or MA and back to original Horn (1972).

As a result, the challenge, as I see it, stands as follows:

- (27) **Challenge:**
 Find a theory of negative CMNs that maintains Horn (1972)'s original story about EX while also adding to it a story for NO-EX, IG, and EVAL.

In what follows we will try to meet this challenge.

5.2 Deriving EX

I follow Nouwen (2008)'s first proposal in assuming that, as in BNs, EX in *no*-CMNs comes from Horn (1972)-SA.

³The literature originally admitted this effect only for SMNs. That is because, as discussed with introspective and experimental evidence in Geurts and Nouwen (2007), unlike SMNs, CMNs can accommodate specificity. See, however, Mihoc (2020, 2021) for a way to reconcile this with a generalization of IG in both. See also Westera and Brasoveanu (2014) or Cremers et al. (2021) for experimental evidence of IG in both.

More specifically, I follow my previous proposal in Mihoc (2021) in assuming that CMNs have the truth conditions below and activate the SA below.⁴

$$(28) \quad \begin{array}{l} \text{a. } \llbracket \text{more/less than } 3 \text{ P Q} \rrbracket = \max(\lambda d_d . \exists x[|x| = d \wedge P(x) \wedge Q(x)] \in \overbrace{\llbracket \text{much/little} \rrbracket}^{\{4,5,\dots\}/\{\dots,1,2\}}(3)) \\ \text{b. } \{\max(\lambda d . \exists x[|x| = d \wedge P(x) \wedge Q(x)] \in \llbracket \text{much/little} \rrbracket(m) \mid m \in S\} \quad (\text{Horn-SA}) \end{array}$$

Second, I follow Nouwen (2008) in assuming that *no/not*-CMNs both entail simply the negation of a CMN. Also, Nouwen’s first explicit proposal, following Horn (1972), and my own implicit proposal in Mihoc (2021), in assuming that, just like CMNs, *no/not*-CMNs activate SA obtained by replacing the numeral with a traditional Horn (1972)-style scalemate.

$$(29) \quad \begin{array}{l} \text{a. } \llbracket \text{no / not more than } n \text{ P Q} \rrbracket = \neg(\llbracket \text{more than } n \text{ P Q} \rrbracket) \\ \text{b. } \{\neg \max(\lambda d . \exists x[|x| = d \wedge P(x) \wedge Q(x)] \in \llbracket \text{much/little} \rrbracket(m) \mid m \in S\} \quad (\text{Horn-SA}) \end{array}$$

Third, I follow my own proposal in Mihoc (2021), following Chierchia (2013), in assuming that the SA of CMNs, just like those of BNs or SMNs, are factored into meaning via the contradiction-based silent exhaustivity operator *O*:

$$(30) \quad \begin{array}{l} \llbracket O \rrbracket (C_{\langle\langle s,t \rangle, t \rangle}, P_{\langle s,t \rangle}, w_s) \quad (\text{Mihoc 2021, following Chierchia 2013}) \\ \text{a. } \text{true iff } p(w) \wedge \forall q \in C[q(w) \rightarrow p \subseteq q] \quad (\text{assertion}) \\ \text{i.e., iff the prejacent is true and any alternative to it that is true is entailed by it, i.e.,} \\ \text{any alternative not entailed by it is false} \end{array}$$

Modulo the details of the implementation, my proposal for EX is then identical to that from Nouwen based on Horn: The source of EX in *no*-CMNs lies with Horn-SA—specifically, the non-entailed SA, which are used by *O* to give rise to a scalar implicature.

$$(31) \quad \begin{array}{l} \llbracket O_{SA}(\text{Jo found no more than 10 marbles}) \rrbracket \\ = \neg(11 \vee \dots) \wedge \neg\neg(10 \vee \dots) \\ = (10 \vee \dots) \wedge \neg(11 \vee \dots) \\ = 10 \quad (\text{EX; } \checkmark) \end{array}$$

So far we have explained how *no*-CMNs get EX. But how do they get EVAL?

5.3 Deriving EVAL

I follow my previous proposal for SMNs in Mihoc (2021), itself following a proposal by Crnič (2011, 2012) in minimizers, in assuming that EVAL in CMNs comes from the fact that their SA can be exploited via a secondary implicature calculation mechanism also—the silent exhaustivity operator (E)ven.

E is typically defined as entailing the prejacent and presupposing that the prejacent is contextually less likely than all of its alternatives (Crnič 2011, 2012, Chierchia 2013).⁵ In Mihoc (2021) I argue that non-end-of-scale items require a revision to this definition—E merely presupposes that the prejacent is less likely than any alternative it entails. I adopt this modification here also.

$$(32) \quad \begin{array}{l} \llbracket E \rrbracket (C_{\langle\langle s,t \rangle, t \rangle}, P_{\langle s,t \rangle}, w_s) \quad (\text{Mihoc 2021’s modification of Crnič 2012, Chierchia 2013}) \\ \text{a. } \text{true iff } p(w) \quad (\text{assertion}) \text{ i.e., iff the prejacent is true} \\ \text{b. } \text{defined iff } \forall q \in C[p \subseteq q \rightarrow p \prec_c q] \quad (\text{scalar presupposition}) \\ \text{i.e., iff the prejacent is contextually less likely than any of the alternatives it entails} \end{array}$$

I also assume with Crnič (2012) that E in fact always uses the prejacent and the alternatives in an exact sense, as if strengthened at some level via *O*—an idea that will become concrete shortly.

My proposal for EVAL then is identical to my proposal in Mihoc (2021) for SMNs: The source

⁴I also assume they activate subdomain alternatives (DA). However, they are not relevant here, so I put them aside.

⁵E is also assumed to presuppose the existence of some true alternative.

of EVAL in *no*-CMNs lies again with Horn (1972)-SA—just that this time it is the *entailed* SA, used by E to give rise to a scalar presupposition.⁶

$$(33) \quad \llbracket \text{ES}_A(\text{Jo found no more than 10 marbles}) \rrbracket (w)$$

b. defined iff $\forall q \in \{\text{Jo found no more than } n \mid n > 10\}$

$$\underbrace{\llbracket \text{OS}_A(\llbracket \text{Jo found no more than 10} \rrbracket)(w) \prec_c \text{OS}_A(\llbracket \text{Jo found no more than } n \rrbracket)(w) \rrbracket}_{\text{'exactly 10 is less likely than, e.g., exactly 11'} \Rightarrow \text{'that's few!'}}$$

We've addressed how *no*-CMNs get EX and EVAL. But how do *not*-CMNs get NO-EX and IG?

5.4 Deriving NO-EX

The fundamental problem with all the previous solutions for *no*-CMNs and *not*-CMNs was that they treated them as *exactly* on par. However, to derive a difference between them, we must first *assume* a difference. I argue this difference is really the *obvious* difference—we're dealing with two different negations, *no* and *not*. I propose these negations activate different SA: *No [scalar]* only activates SA based on replacement of the scalar with other scalars—a traditional scale-related SA-generation mechanism. In contrast, *not [scalar]* can also activate SA based on deletion of *not*—a larger set of SA, SA+ based on a scale+structure-related SA-generation mechanism (see Fox and Katzir 2011). If we assume this, the result of OS_A is no longer EX but rather a contradiction.

$$(34) \quad \llbracket \text{OS}_{A+}(\text{Jo found not more than 2 marbles}) \rrbracket$$

$$= \neg(3 \vee 4 \vee \dots) \wedge \underbrace{\neg\neg(2 \vee \dots) \wedge \neg\neg(1 \vee \dots) \wedge \dots}_{\text{implic's from scale-related alt's}} \wedge \underbrace{\neg(2 \vee \dots) \wedge \neg(1 \vee \dots) \wedge \dots}_{\text{implic's from structure-related alt's}} = \perp \quad \times$$

This contradictory result captures the absence of EX. But how do we capture the fact that this sentence is nevertheless acceptable and conveys IG?

5.5 Deriving IG

I follow Chierchia (2013) (who in turn follows Kratzer and Shimoyama 2002 and others) in assuming that, when exhaustification leads to contradiction, a null, speaker-oriented epistemic necessity modal, \Box_S , can be inserted at matrix level between the exhaustivity operator and its prejacent, as a last resort rescue mechanism. This straightforwardly captures IG. Interestingly, this would mean that the source for NO EX in negative contexts is actually similar to that in positive contexts—in both cases it seems to come from IG, just that there IG was assumed to come from the DA, whereas here from SA.

$$(35) \quad \llbracket \text{OS}_{A+}(\text{Jo found not more than 2 marbles}) \rrbracket = \neg(3 \vee 4 \vee \dots) \wedge$$

$$\underbrace{\neg\Box_S \neg(2 \vee \dots) \wedge \neg\Box_S \neg(1 \vee \dots) \wedge \dots}_{\text{implic's from scale-related alt's}} \wedge \underbrace{\neg\Box_S (2 \vee \dots) \wedge \neg\Box_S (1 \vee \dots) \wedge \dots}_{\text{implic's from structure-related alt's}} \Rightarrow \text{IG} \quad \checkmark$$

6 Conclusion and Outlook

In this paper we have examined a series of puzzles to do with exactness, evaluativity, and ignorance in negative CMNs. Taking into account further puzzles to do with BNs and SMNs, the existing literature has distanced CMNs from the original view from Horn (1972), but it still hasn't solved the full puzzle. I have argued that a full solution requires a return to Horn (1972)-SA—only this time enhanced with O(nly), E(ven), and Negation Alternatives.

⁶The existential presupposition would create a problem for numerals, as, e.g., for this example, it would amount to saying that, e.g., $O(\text{Jo found no more than 11 marbles}) = \text{Jo found exactly 11 marbles}$ is also true. This is an interesting problem. I will however have to leave it to future research.

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