Satisfying long-distance relationships (without tiers)

A strictly anti-local approach to phonology [*]

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Abstract

We propose a model of phonological computation based on rules characterized by a small set of parameters. We focus on rules affecting segmental features. One parameter is the characterization of a class of initiator, INR, segments from which a linear SEARCH is initiated. The TRM parameter specifies the class of segments that successfully terminate a SEARCH. A third parameter is DIR, the direction of SEARCH. Fourth, a rule contains specification of a CHANGE that maps input strings to output strings by replacing INPUT string segments with OUTPUT string segments. Fifth, the CHANGE can be subject to further CONDITIONs on TRM. The model can be understood as a theory of rule environments. We also consider possible extensions, such as various versions of nested SEARCH. Immediate results include an understanding of segment opaqueness; the reduction of adjacency requirements in rules to a particular kind of opaqueness; and a characterization of 'icy targets' in terms of set-theoretic relations to the INR, TRM and CONDITION parameters of a rule.

1 Introduction

A recent influential PhD thesis (Chandlee, 2014, p.8) characterizes the important status of locality in the phonology literature by pointing out that "non-local phenomena have not been used as evidence against the overall local nature of phonology. Rather,

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the tendency has been to recast non-local patterns in such a way that they become local". This thesis and related work with Jeff Heinz and many other scholars (e.g., Heinz, 2010, 2018; Chandlee and Heinz, 2018; Burness et al., 2021) have given us a thriving research community and a rich literature studying the computational properties of phonological systems. Chandlee refers to much of the research on locality, not only since the advent of autosegmental phonology (e.g., Goldsmith 1979), but also earlier work such as Jensen (1974) which all addresses the question of "a non-local process in which the target and trigger are separated by some number of intervening segments." As Chandlee points out, "[a] variety of proposals have been put forth for how to 'ignore' this intervening material so that the target and trigger can be treated as adjacent."

Contrary to tradition and intuition, we attempt to invert both old and new locality-driven approaches to such questions by assuming that, at base, phonological rules are actually built on unbounded search procedures, and adjacency is a 'special case'. In the course of sketching this alternative, we reject McCarthy's 1988 claim that "[t]he goal of phonology is the construction of a theory in which cross-linguistically common and well-established processes emerge from very simple combinations of the descriptive parameters of the model". We suggest that the notions of target and trigger have made it difficult to see what is essential to phonology—what is common is not necessarily a good reflection of what is fundamental.

Capturing broad consensus in the field, Chandlee states that "locality in general has long been recognized as significant to phonology, in the sense that phonological changes tend to involve adjacent segments." For us—accepting a mode of explanation developed in work such as Ohala (2003), Blevins (2004) and Hale (2007)— this tendency arises from what Hale (2003) calls the "diachronic filter", the greater likelihood of certain sound patterns to be misparsed during acquisition. Perceptual and articulatory interactions among adjacent segments are more likely than among distant ones. When such interactions are grammaticalized in the transmission/acquisition process their 'local' skewing is reflected in the data of attested phonological rules. So, given a characterization of phonological UG, we expect that the set of attested patterns will not mirror perfectly the combinatorics of the computational system, but rather also reflect the diachronic filter (see section 1.2 of Hale and Reiss 2008).

In other words, although locality/adjacency-based processes are a salient part of a phonologist's phenomenal world, it does not follow that they are structurally more basic than long-distance processes. We will argue, in fact, that locality requirements must be instantiated by relative structural complexity in rules, through the specification of more information than is needed in long-distance rules.

One might go so far as to say that the literature on segmental phonology fetishizes both assimilation and locality—all of us phonologists have had an excessive and irrational commitment to, even an obsession with, these ideas. Assimilation processes, described sometimes as copy, agreement, linking, spreading or harmony, are very common. However, we will see that assimilation is not essential to segmental phonology.

Many rules are expressed in terms of local relations between adjacent segment or segments in adjacent syllables, but locality is also not essential to segmental phonology, given the existence of long-distance rules. We propose a model for phonological computation that can express assimilation and locality, without giving these notions pride of place. Our approach also makes it unnecessary to characterize the material that can intervene between a rule's target and trigger. This is an advantage since such material sometimes does not constitute a natural class.

In the spirit of Chomsky (2000b, 8), we propose that notions like 'assimilation', 'vowel harmony', 'opaqueness' and 'transparency' in phonology are 'grammatical constructions' like 'passive' or 'relative clause' in syntax, and our job is to see beyond these "taxonomic artifacts":

> The central problem [is] to find general properties of rule systems that can be attributed to the faculty of language itself, in the hope that the residue will prove to be more simple and uniform. [In the 1980's], these efforts crystallized in an approach to language that was a much more radical departure from the tradition than earlier generative grammar had been. This "Principles and Parameters" approach, as it has been called, rejected the concept of rule and grammatical construction entirely: there are no rules for forming relative clauses in Hindi, verb phrases in Swahili, passives in Japanese, and so on. The familiar grammatical constructions are taken to be taxonomic artifacts, useful for informal description perhaps but with no theoretical standing. They have something like the status of "terrestrial mammal" or "household pet". And the rules are decomposed into general principles of the faculty of language, which interact to yield the properties of expressions.

This paper can be read as an attempt to understand the nature of phonological rule environments, abstracting away from 'taxonomic artifacts' like assimilation and locality. Assimilation involves a relationship between the trigger and target of a rule, and so is intimately related to the notion of phonological rule environments. Locality is also related to the idea of phonological environments, for example, whether segmental interactions can happen "at a distance". In order to focus on this narrow topic we will mostly abstract away from the issue of how changes inside segments are effected—how does the phonology turn an /l/ into an [n], for example. The work on intrasegmental changes in Reiss (2021) is complementary to this paper, since it tries to ignore "environmental" issues and focus narrowly on how rules change segments, using only two operations, set subtraction and unification (building on Bale et al. 2014; Bale and Reiss 2018; Bale et al. 2020). Taken together, the two papers constitute a new (and obviously incomplete) attempt to model phonological computation.

The most radical aspect of our proposal is that environments are defined by a restricted set of relationships among elements of a phonological representation, but that these relationships are not only structural (based on precedence, for example)—they also involve logical statements based on the scope of specifications that can differentiate situations like (1ab):

(1) a. Search for the first element e that has properties p and q; if you find e do C

b. Search for the first element e that has property p; if you find e and e also has property q do C

To illustrate the difference, consider these two sets of instructions:

(2) a. Run down the road til you see a red brick house; when you find a red brick house, ring the doorbell.

b. Run down the road til you see a red house; if that red house is made of brick, ring the doorbell.

Let's assume there are a fair number of red brick houses on the road, as well as some other red houses that are made of painted wood. In scenario (a), you will always end up ringing a doorbell, because you will get to a red brick house sooner or later. In scenario (b), you will on average run less far than in scenario (a), since you will sometimes stop at a red wooden house that you happen to come to before any red brick house. When you stop at a red wooden house, you will not ring the doorbell. So, the two 'algorithms' can lead to different outcomes.

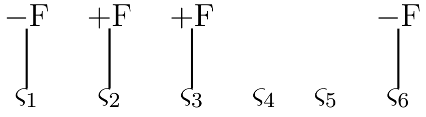
A potential objection to our discussion thus far is that we are sneaking locality into our model by virtue of the nature of searching for the *first* segment in a string that fulfills a condition, or the linear arrangement of houses along a road. In some sense, this objection is the key to our system: the ordered nature of phonological representations (abstracting, as we do here from factors like syllable structure) makes apparent locality effects fall out for free from the parameters of our Search-based rules. To give a sense of where we are headed, we propose in Section 12 a set theoretic characterization of so-called 'icy targets' (Jurgec, 2011).

2 A note on tiers

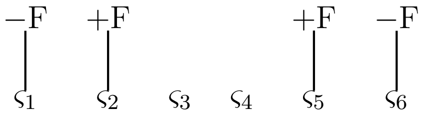
Chandlee notes that one proposal that allows phonologists to "recast non-local patterns in such a way that they become local" is to make use of distinct tiers in phonological structures. Consider the representations in (3):

(3) Segments with a tier for feature F

a.



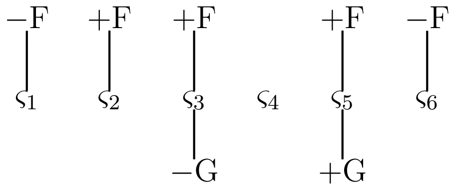
b.



In (3a), segments 2 and 3 are adjacent, and segments 1 and 6 are not adjacent. The +F values associated with segments 2 and 3 are also adjacent, but the -F values associated with segments 1 and 6 are not adjacent. In (3b), the +F values on segments 2 and 5 are adjacent, or local to each other, despite the fact that the segments themselves are separated by segments 3 and 4, because on the F-tier, nothing intervenes between the two +F values. The idea is that, in addition to the ordering among segments on the 'timing tier', there is an independent ordering on each tier, so that in (3b), the +F associated with segment 2 immediately precedes the +F associated with segment 5.

The model we propose in this paper denies the sufficiency of this approach to locality. The logic of our reasoning is this: projecting features into tiers as in (3) does indeed allow for the expression of syntagmatic (sequential) relations between segments that are mediated on another tier—we can say that in (3b) segments 2 and 5 are "adjacent on the F-tier". However, phonology requires reference to segment-internal (paradigmatic) featural relations as well, involving conditionals. In other words, syntagmatic relations among phonological units (segments, features, etc) make reference to paradigmatic relations, too. For example, phonological conditions might need to reflect adjacency of +F specifications that are linked to segments that are also linked to +G, as in the three tier representation in (4).

(4) Segments with a tier for feature F and a tier for feature G



We claim that phonological processes can require relations between segments like segment 2 and segment 5 in (4), relations that do not just involve adjacency on an individual tier—Sanskrit in section 12 provides an example. Informally, but reflecting the direction we will take, we might say that "a +F segment like 2 looks for the first +F segment to its right that is specified +G, and thus it looks beyond segments 3 and 4 and finds segment 5". Once we allow for such relations among segments, the graphical and conceptual utility of tiers evaporates (maybe completely—we can't decide that question here). We thus propose a model without tiers.[1] It is important to note, of course, that our logical model is more powerful (in the 'bad' sense) than tier-based models, and thus it needs to be justified empirically. This paper sketches the model and hints at some of the empirical justification that we have already uncovered, such as Leduc's 2021 account of 'icy targets' in Karajá.[2]

3 Where is not always part of what

Before proceeding to illustration of the proposed rule system, let's clarify the claim that assimilation is not essential to phonology. Consider a simple characterization of a phonological process in this semi-formal notation:

(5) Simple rule I: e → ẽ / ___ m,n,ŋ

Such a formulation expresses the idea that when a token of the segment *e* occurs directly before a nasal segment *n*, the vowel is realized with nasalization, as *ẽ*. It is tempting (and common) to understand the effect of this rule in terms of assimilation, some kind of copy or agreement process between the consonant and the vowel: "the vowel *e* looks at the segment to its immediate right, and if it finds +NASAL on that segment it *copies* that feature."

This idea of the target "looking" at its environment is the intuitive foundation of the SEARCH algorithms that will serve as the basis of our approach to phonological rule systems. In contrast, the idea of *copying* a feature from the environment will turn out not to play an important role in our model, since there are many rules where the idea of copying appears to be irrelevant, as in (6):

(6) Simple rule II: e → i / ___ m,n,ŋ

Here it looks like the rule's effect is that "the vowel *e* looks at the segment to its immediate right, and if it finds +NASAL on that segment, the vowel becomes +HIGH."

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[1] Obviously, this is not the place to reject the vast literature on tonal phonology that relies on tiers, but perhaps the arguments we present can ultimately be drawn upon for new models of tone, especially in cases where tone interacts with segmental features, where there are cross-tier effects. We should also note that the possibility of proposing ad hoc tiers that combine features, e.g., a {F,G} tier or a {+F,+G} tier is irrelevant to our discussion in (4) because there we don't have relations on a {+F,+G} tier but rather relations between a +F segment and a +F,+G segment.

[2] The mere fact that rules do make some reference to classes of segments defined by conjunctions of features, such as vowels that are +HIGH and −BACK, tells us that simultaneous reference to multiple tiers is necessary. The existence of processes like the following tell us that so-called target-trigger relations can reference multi-feature conditions: a mid vowel copies the roundness of the following vowel, if that vowel is high. All we need to complete our demonstration of the pattern illustrated in (4) is a case like the previous one, but with a non-high vowel intervening between target and trigger. We will construct structurally parallel cases below, and suggest where they might be found in nature.

One might insist that the source of the +HIGH specification is the following nasal, but that reasoning won't always work. For example, the idea of assimilation or copy or agreement is clearly irrelevant in case no segment follows the vowel, at the end of a word:

(7) Simple rule III: e → i / __%

An expression like (7) means something along these lines: "the vowel *e* looks to its immediate right, if it finds no segment there, the vowel becomes +HIGH."

Our conclusion from examples (5-7), all of which correspond to pedestrian processes attested in human languages, is that the change to the 'target' vowel has no *necessary* relationship to the environment, for example, whatever follows that target segment.[3]

What happens—raise or nasalize a vowel, say—sometimes reflects *where* it happens—after a high vowel or before a nasal, or whatever—but this is not a necessary property of rules. What happens, defined, in terms of features, can be completely independent of where it happens, in terms of features (or syllable structure or other factors). It is crucial to maintain the logical independence of the *whatness* and the *whereness* if we are to better understand phonological computation, so let's codify the insight as a principle:

(8) Principle 1: Whereness is not a necessary part of whatness, and vice versa

In other words, the sets of properties defining a rule's environment and its structural change can be disjoint—for example, vowel raising before a nasal, as discussed above, and many other processes, like obstruent devoicing in codas, all exemplify Principle 1. These examples may be phonetically natural, even if they cannot be motivated featurally, but the existence of 'crazy' or unnatural rules (Bach and Harms, 1972; Buckley, 2000) also supports Principle 1.

We are now ready to acknowledge past fetishistic errors, and to develop more healthy and measured perspectives on assimilation and locality. The present work builds on Mailhot and Reiss (2007), a paper in which every language is treated as

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[3] We are *not* claiming that there is no phonetic 'basis' to the rules in question. For example, the nasalization rule ($e \rightarrow \tilde{e}$) reflects a common pattern of assimilation, and the raising rule ($e \rightarrow i$) reflects the fact that nasal formants can obscure the regular vowel formants that indicate vowel height. However, these relations are not encoded in the synchronic grammars in question; these diachronically relevant phonetic factors are not part of the synchronic 'substance-free' phonology.

This is a good place to further explicate our rejection of McCarthy's claim about the goal of a phonological theory mentioned above. Let's imagine there are only two possible rules, the vowel nasalization rule in (5) and the vowel raising rule in (6), and each language has only one of them. Say you look at 1000 languages and 993 have "/e/ gets nasalized before a nasal"; and the other 7 have "/e/ becomes [i] before a nasal". You then need a theory that accounts for both types of language, and so you need a theory where "copy/spread/assimilate" is not essential and "change-a-feature" is essential. The formal model should not reflect the 993 vs. 7 statistics. At the right level of abstraction "change a feature in some environment" is the true cross-linguistic phenomenon.

manifesting processes based on SEARCH and COPY procedures. In that paper, the fetishization of COPY was enabled by the narrow focus on so-called feature-filling harmony processes. Consideration of a broader range of phenomena in work like Samuels (2011) and Shen (2016) has made it clear that the focus on COPY in Mailhot and Reiss (2007) was misguided. Here we develop these new insights, and aim to defetishize COPY as a phonological notion and replace it with more general operations that simply derive agreement/harmony/copying processes as special cases. This conception of the irrelevance of COPY in phonological rules is distinct from the use of Greek letter variables (α-notation), a topic we return to below in section 4.3.

That same 2007 paper succeeded somewhat in defetishizing locality, as did superficially similar work from the same period—e.g., a book entitled *Locality* (Nevins, 2010). However, that work suffers from the narrowness of the phenomena it examines, cases of feature-filling vowel harmony. Following up on the Mailhot and Reiss paper, we attempt to make the notion of locality irrelevant. Assuming that our approach is correct, phonologists will still be concerned with locality—to describe what interacts with what—but phonological computations themselves will not make any use of an explicit locality metric. A more pompous formulation may actually add clarity: locality will continue to play a role in the epistemic structure of phonology-the-discipline, but locality has no role in phonological computation or the ontology of the natural object which is the human phonology faculty.

It should now be apparent that our critique of tiers in the previous section is intimately related to Principle 1. We can, opportunistically, appeal to, say, the spread of +NASAL on the NASAL tier in the rule that nasalizes a vowel before a nasal consonant in (5); and then ignore tiers in the rule that raises a vowel before a nasal in (6). We (pretty much all of us phonologists) do this in one way or another because we fail to live by Principle 1, and we succumb to the delusion that the nasalization rule is somehow a better reflection of the essence of phonology than the raising rule. We all have to get over this. The nasalization rule and the raising rule are identical in form—they both change a feature value (for either HIGH or NASAL) on a segment (that meets certain conditions, like "is /e/") when that segment appears before another segment that meet other conditions (like "is a nasal"). Accepting Principle 1 is what it means to believe that phonology is Substance Free, at least in the sense intended by these authors (see Hale and Reiss 2000, 2008; Reiss 2017; Bale and Reiss 2018; Volenec and Reiss 2020, 2022; Leduc et al. Forthcoming and related work).

4 Toy grammar basics

We now turn to properties of our rule system. In order to help maintain Principle 1, we abjure the use of phonetic symbols and use simple geometric shape symbols.[4]

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[4] A reader suggested that our shapes look 'jocular', but experience has shown that the logic of our system is much easier to follow using these symbols instead of defining dummy phonological

4.1 Atomic segments

In our simplest toy system rules map strings consisting of symbols from an inventory of three "segments" ■, ● and ▲ to other strings consisting of the members of the same symbol set. For now, let's express rules informally, in plain English and something like traditional rule notation:[5]

(9) Rule a:

- A square becomes a circle before a triangle.
- ■ → ● / ___▲
- Sample mapping: ■ ■▲ ■ ● ■ ▲ ⇝ ■ ● ▲ ■ ● ● ▲

This example is sufficient to illustrate Principle 1. The specification of the change—what: "replace a square with a circle"—is independent of the specification of the environment—where: "before a triangle". We belabor this point because it is so important.

4.2 Features and partial descriptions in rules

We now expand our system of symbols. In our new toy system rules apply to strings built from an inventory of six "segments": ■, ●, ▲, □, ○ and △. In rules, classes of symbols can be referenced as follows. We use two colors, B(lack) and W(hite) to refer to the classes consisting of the first three symbols, and the last three symbols, respectively (e.g., B denotes any Black symbol). We also use the symbols ▨, ◍ and ◭ to denote any square, any circle and any triangle, respectively. Finally, we use a symbol SHAPE to refer to any member of the set of six symbols.[6]

Again, let's express rules informally, in plain English and something like traditional rule notation:

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segments, like, say, x as {+F,−G} and y as {+F,−G,−H}. The latter approach requires readers to refer back to the specifications for x and y-type symbols. In contrast, when we discuss features that constitute shape symbols, a quick glance is sufficient to tell us that a black square has the properties 'black' and 'square'. The reader can thus focus on the logic.

[5] Additional details need to be specified, like whether rules apply by iterating, left-to-right or right-to-left through the input string, or else globally (simultaneously throughout a string)—we'll address this issue below. The squiggly arrow '⇝' should be read as 'leads to', meaning that the form on the lefthand side, which may be either a UR or a subsequent form in the derivation, comes out via the application of one or more rules as the form on the righthand side, which is a later stage of the derivation, and is potentially the SR.

[6] So, there is no such thing as a gray triangle in the set of grammars we are describing—there is no such thing as a triangle that is neither B or W. If there were, we would be using underspecified representations. Instead, in our system, a gray triangle, or the symbol B, or the symbol SHAPE, can occur in rules as *partial descriptions* that intensionally define certain subsets (the natural classes) of the six shape symbols. We do use underspecified representations in further developments of this work.

(10) Rule b:

- A white square becomes a black circle before a white triangle.
- □ → ● / __ △
- Sample mapping: □ □△ ■ ● □ ▲ ⇝ □ ● △ ■ ● □ ▲

(11) Rule c:

- A black square becomes a white circle after a triangle.
- ■ → ○ / ◭ __
- Sample mapping: □ ■▲ ■ ● △ ■ ⇝ □ ■▲ ○ ● △ ○

(12) Rule d:

- A square becomes a circle (of the same color) after a black triangle.
- ◧ → ◐ / ▲ __
- Sample mapping: △ ■▲ □ ● ▲ ■ ⇝ △ ■▲ ○ ● ▲ ●

Rule c (11) changes a black square to a white circle after *any* triangle, regardless of its color. Rule d (12) changes a square of either color to a circle (maintaining its color) after a black triangle.

The crucial point here is that there is still no reason to give up Principle 1. Just because we have a larger inventory of segments, and just because we see that segments are no longer unanalyzable wholes, it does not follow that Principle 1 should be abandoned. We have already seen, with examples like (6,7) that maintenance of Principle 1, even in a system of symbols that can be decomposed into features, appears to be justified—whatness and whereness are logically independent.

4.3 Greek variables are another story

Now, there is a well known way to force a connection between the change and the environment in phonology, namely the use of Greek letter variables. Indulge our lack of formality here and let us introduce C to denote a feature for color which can take the value B or W, and let α be a variable whose domain is the set {B,W}. So, "having the color Black" is denoted by BC, and "having the color white" is denoted by "WC". We can use Greek letter variables to express a dependency between the value of a feature in the environment and a feature in the rule change:

(13) Rule e:

- A square takes on the color of a following triangle
- ◧ → αC / __(◭,αC)
- Sample mapping: □ ■△ ■ ● □ ▲ ⇝ □ □△ ■ ● ■ ▲

Here we clearly do have a connection between whatness and whereness—change the color of a square to match that of an immediately following triangle—but this is due to the newly introduced mechanism of a Greek letter variable. Principle 1 stands if we discount Greek letter variables, which is what we will continue to do for now.[7]

5 Simple Search-based rules

In this section, we propose a new syntax for rules modelled on our "run down the road" examples in (1,2) above. We start with simple rule structures and develop more complicated formats as we proceed. Our rule environments can be stated as static conditions that representations must meet, but we find it more enlightening to present the rules as containing a Search procedure. If this Search procedure terminates successfully, if its demands are satisfied, the structural Change rewrites the relevant part of an input string. An important issue as we proceed is the possibility of information flow between the Search and Change parts of a rule. Examples will clarify.

The simplest rules contain just a specification of Search and the Change that applies if the former is 'successful'. Consider the parameters of (14). A Search instance is initiated by each initiator segment INR, and is terminated by the first segment meeting the terminator conditions for TRM found in a specified direction DIR (L/R). So, each INR has a single TRM, but a given TRM can correspond to multiple INRs (see below). The Change specification replaces an INPUT, which is always an INR, with a different symbol in the OUTPUT—this is effected as a string to string mapping.[8]

(14) Rule 1

- Turn a ■ into a ○ if there is a ▲ anywhere to its right.
- Rule 1 Parameters

(where?) Search: INR: ■, TRM: ▲, DIR: R

(what?) Change: INPUT: INR, OUTPUT: ○

The information listed for each parameter is called its 'specification'. The specification

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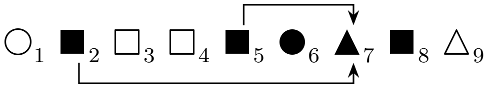
[7] Some scholars, such as McCawley (1971) consider Greek variables to be metavariables that are not part of phonology, but are just part of linguists' descriptive apparatus. Reiss (2003) argues, in contrast, that these variables must be part of grammar in order to formulate certain generalizations involving quantificational logic. Since Principle 1 is orthogonal to the status of Greek variables, we can ignore the issue here. Note that, in some rules, Greek variables occur only in the environment, as in a rule that targets a glide between identical vowels. In such a case, there is no necessary dependency between the environment and the change, consistent with Principle 1.

[8] We'll be more explicit about exactly how rules are interpreted in section 15.

of INR is ■, the specification for DIR is R, and so on.[9] Crucially, the value of the INR from the Search part of the rule is assigned as the value of the specification of the INPUT in the Change part of the rule. This is an example of the information flow mentioned above. A case of a phonological process that a toy rule like (14) models comes from the Formosan language Truku Seediq, where there is a segment that is realized as [q] if there is a uvular to its right (at any distance), and otherwise as [k] (see Shen 2016 for analysis of data provided by Lee 2009).

If we apply (14) to the string in (15), then black square INRs in positions 2 and 5 'see' the same TRM, the black triangle in position 7, as the arrows in (15) show.

(15)


It is crucial to note that in our system, this 'regressive' process in (15) shows a rightward Search (left-to-right), whereas in most of the literature, especially on harmony processes, regressive processes are conceptualized as involving right-to-left parsing of the string, with feature 'donors' looking for 'recipients'.[10]

Rule 1 (14) thus performs a mapping like (16) with this input string.

(16) $\bigcirc_1\ \blacksquare_2\ \square_3\ \square_4\ \blacksquare_5\ \bullet_6\ \blacktriangle_7\ \blacksquare_8\ \triangle_9 \rightsquigarrow \bigcirc_1\ \bigcirc_2\ \square_3\ \square_4\ \bigcirc_5\ \bullet_6\ \blacktriangle_7\ \blacksquare_8\ \triangle_9$

The black squares in positions 2 and 5 'see' the black triangle in position 7 to the right, and so those squares change to white circles. The black square in position

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[9] In (14), the specifications for INR and TRM are individual shapes, for example, "each black square is an INR". As we move to using partial descriptions in rules, think of the specification in terms of a set of properties that make a symbol an INR. In (14), that set contains just 'black' and 'square'. If another rule's specification for INR were a gray square, the set of properties would contains just 'square'. In other words, our specification of parameters can be thought of in terms of sets of properties defining classes of symbols, but we don't want to burden the notation any further, especially when we move to using actual phonological features.

[10] Given our adoption of Principle 1, this perspective doesn't work, since there may not be a 'donor' segment for the relevant feature—recall the rules that raised a vowel before a nasal in (6) or at the end of a word in (7). Following Mailhot and Reiss (2007) our 'recipient as initiator' perspective allows each Search instance to be a function, mapping from each INR to a single TRM that defines the rule environment. The standard alternative approach of mapping from 'donor/trigger' to all the 'targets' in a domain means that trigger-target mapping won't always be a function, since a given trigger can map to more than one target. In the standard approach, applied to (15), the black triangle in position 7 maps to both the preceding black squares in positions 2 and 5. So, the standard approach is both conceptually flawed (relying on feature 'donors') and less straightforward formally (by not guaranteeing unique mapping). To reiterate, the mapping from INRs to their respective TRMs is a function; the mapping from a 'trigger' to its 'targets' is not necessarily a function. For us the INR-to-TRM mapping reconceptualizes what is traditionally called a rule environment.

At first blush, the INR-TRM pairs found by our algorithm may appear to include both 'greedy' (maximal) and 'non-greedy' (minimal) parses of strings, as well as all the in-between matches, in the sense of these terms used in discussions of regular expressions. In fact, our matching is always non-greedy, as long as we remember to initiate the parse from each INR/target, and not from the TRM/trigger, and to adjust the direction as needed.

8 does not have a black triangle to its right, so it does not change. Note that as specified, this rule is a ‘long-distance’ rule—recall that we are defetishizing locality. Search proceeds from each INR until its respective TRM is found, or else the end of the word is reached (or the end of another specified domain—a matter requiring enhancements to rule syntax that we leave for the future). In other words, the input string is the same as the output string except that each black square that has a black triangle anywhere to its right is changed into a white circle. As just noted, this rule is long-distance by default—it is not limited to changing INRs that are adjacent to their respective TRMs.

Let’s apply (14) to two minimally distinct strings:

(17) Mappings from (14)

a. ■$_1$ ●$_2$ ■$_3$ ▲$_4$ ■$_5$ ▲$_6$ ■$_7$ △$_8$ $\rightsquigarrow$ ○$_1$ ●$_2$ ○$_3$ ▲$_4$ ○$_5$ ▲$_6$ ■$_7$ △$_8$
b. ■$_1$ ●$_2$ ■$_3$ △$_4$ ■$_5$ ▲$_6$ ■$_7$ △$_8$ $\rightsquigarrow$ ○$_1$ ●$_2$ ○$_3$ △$_4$ ○$_5$ ▲$_6$ ■$_7$ △$_8$

As shown in (17), every black square that has a black triangle to its right is turned into a white circle: the same shapes are changed in (a) and (b). The black squares in position 7 do not change, because there is no black triangle to the right in either (a) or (b). In (b) black squares in positions 1, 3 and 5 all share the TRM black triangle in position 6. Contrast this with (a) where the black squares in positions 1 and 3 find their TRM in position 4. As long as a black square can ‘see’ a black triangle, the Search is satisfied—locality plays no role.

6 A partial description for TRM

A slight twist on (14) is given in (18):

(18) Rule 2

- Turn a ■ into a ○ if there is a ◭ anywhere to its right.
- Rule 2 Parameters

  Search: INR: ■, TRM: ◭ , DIR: R
  Change: INPUT: INR, OUTPUT: ○

This rule provides the mapping in (19):

(19) ○$_1$ ■$_2$ □$_3$ □$_4$ ■$_5$ ●$_6$ ▲$_7$ ■$_8$ △$_9$ $\rightsquigarrow$ ○$_1$ ○$_2$ □$_3$ □$_4$ ○$_5$ ●$_6$ ▲$_7$ ○$_8$ △$_9$

The black squares in positions 2 and 5 ‘see’ the black triangle in position 7 to the right, and so those squares change to white circles. The black square in position 8 sees the white triangle in position 9, so it changes too.

Let’s apply (18) to the same two strings we used in (17):

(20) Mappings from (18)

a. ■$_1$ ●$_2$ ■$_3$ ▲$_4$ ■$_5$ ▲$_6$ ■$_7$ △$_8$ ⇝ ○$_1$ ●$_2$ ○$_3$ ▲$_4$ ○$_5$ ▲$_6$ ○$_7$ △$_8$
b. ■$_1$ ●$_2$ ■$_3$ △$_4$ ■$_5$ ▲$_6$ ■$_7$ △$_8$ ⇝ ○$_1$ ●$_2$ ○$_3$ △$_4$ ○$_5$ ▲$_6$ ○$_7$ △$_8$

As shown in (20), every black square that has any triangle to its right is turned into a white circle: again, the same shapes are changed in (a) and (b), but now the black squares in position 7 find the white triangles in position 8, and they change to circles—contrast this with the outcome in (17). This is not surprising because the specification of TRM in (18) is less specific (so more general) than that in (14)—in the new rule any triangle can serve as a TRM.

7 So what is Copy?

We have said that we want to defeteshize assimilation, which is expressed in the literature in terms of notions such as linking, spreading, copying and so on. We'll use Copy as a cover term, and consider how to express such phenomena in the current model.

Rule 2 (18) was designed to involve a mapping that makes no intuitive sense in terms of the features of shape symbols. In phonological terms, it would be hard to motivate the rule phonetically. If we look at (21) below we might imagine that the notion of Copy is somehow relevant:[11]

(21) Rule 3

- From a ▨ Search for a ▲ to the right and turn the ▨ into a ■
- Rule 3 Parameters
  Search: INR: ▨, TRM: ▲, DIR: R
  Change: INPUT: INR, OUTPUT: ■

We might want to say that the target squares assimilate to the blackness of the black triangle trigger. But Principle 1 tells us to avoid that temptation. The TRM specification and the OUTPUT specification just happen to both have the property B.[12] So, in our model, Copy is just a name for what happens when TRM and OUTPUT happens to share properties. There is no need to build the notion into a model of phonological computation, aside from the very different mechanism of Greek letter variables discussed above. Non-alpha 'linking' of features, for example, should be treated just like the introduction of any arbitrary feature. In other words, nasalization of a vowel before a nasal has the same computational properties as raising of a vowel before a nasal, as discussed above. If we adopt the model in Reiss (2021) all

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[11] If INR were □ we'd get the same result. As formulated, the rule applies vacuously to ■.

[12] Again, why such a rule might be more likely to occur than another rule that doesn't look like assimilation is a question for the study of the phonetics of sound change as discussed in detail by scholars such as Hale (2007) and Blevins (2004).

intrasegmental feature changes can be expressed as combinations of rules based on set subtraction and unification. So in that model, apparent Copy is just feature-filling (into INR) of a valued feature that happens to be found in the environment (on TRM).

8 Conditions on Change: another parameter

We now introduce another component into our rule syntax. Here is a rule that imposes an extra condition on whether the Change applies. It is important to see that this rule is different from both (14) and (18). This is like the rule that says to stop at the first red house and ring the doorbell if the house is made of brick (2b).

(22) Rule 4

- From a ■ Search for a ◮ to the right and turn the ■ into a ○ if TRM is B (▲)
- Rule 4 Parameters

  Search: INR: ■, TRM: ◮, DIR: R
  Change: INPUT: INR, OUTPUT: ○, $\boxed{\text{CONDITION: TRM: B}}$

The specification of Search is the same as in (18)—Search terminates at the first triangle of any color to the right of INR, but here, in (22), the change only applies if the terminator is B(lack). This new type of condition is placed in a box for clarity. This is not the same as terminating at a black triangle as in (14). We'll say that (14) has a 'specification' that INR be a black square, and a 'specification' that TRM be a black triangle. Similarly, (22) has a 'specification' that INR be a black square, and a 'specification' that TRM be any triangle (like (18)). However, (22) also has a CONDITION on TRM that it be black. We reserve the term CONDITION for this new element of rule syntax: the specification of the CONDITION in (22) is 'TRM is B'.

To reiterate, the only difference between (14) and (22) is that in the former, the TRM is a black triangle and there is no condition on Change—the Change happens if Search is satisfied; whereas in the latter, TRM in the rule is a gray triangle, and there is a condition that Change applies only if the actual TRM in a given instance is black. Since any triangle terminates the Search from a black square, but only a black triangle will trigger the Change, it should be obvious that a white triangle behaves like a "blocker". For an input string ■ △ ▲ the black square will not change, since the white triangle "blocks" access to the black triangle.

9 Conditions on change yield "opaqueness"

Let's illustrate this blocking effect more thoroughly. We get the following mappings from a rule like (22):

(23) Mappings from (22)

a. ■$_1$ ●$_2$ ■$_3$ ▲$_4$ ■$_5$ ▲$_6$ ■$_7$ △$_8$ $\rightsquigarrow$ ○$_1$ ●$_2$ ○$_3$ ▲$_4$ ○$_5$ ▲$_6$ ■$_7$ △$_8$

b. ■$_1$ ●$_2$ ■$_3$ △$_4$ ■$_5$ ▲$_6$ ■$_7$ △$_8$ $\rightsquigarrow$ ■$_1$ ●$_2$ ■$_3$ △$_4$ ○$_5$ ▲$_6$ ■$_7$ △$_8$

For the inputs (23a) and (23b), (22) makes different changes. For example, in (a) the squares in positions 1 and 3 are changed to white circles—because each of these initiate a Search that is terminated by the triangle in position 4, and that triangle is black. In contrast, in (b), there is no change in positions 1 or 3—they remain as black squares, because the relevant instances of Search are terminated by the triangle in position 4, and that triangle is white.

In terms familiar to a phonologist, we might say that the white triangle in position 4 of (23b) is *opaque*, since it "blocks" a black square from 'finding' a black triangle to the right. Similarly, the white triangle in position 4 of (17b) showing mapping by (14) is *transparent*, because the black square's Search is not terminated by that white triangle.

Now it is apt to recall Principle 1. With this formalized understanding of (some uses of the terms) opaqueness and transparency, we see that these phenomena are completely independent of the 'phonetic' substance of the features involved in the rule's change. The change in both rules involves turning a black square into a white circle. The transparent/opaque behavior of the triangles in (14) and (22) has no 'featural' relation to the changes. We contend that this is a good model for real phonological patterns: so-called "opaque" segment behavior is just a reflection of the kind of conditions on Change given in (22). The apparent transparency in (14) just emerges from the lack of a CONDITION on Change and the fact that a white triangle shares some properties with the black triangle TRM. Transparency is not encoded in the grammar in any way, but is rather an epiphenomenon. The so-called opaqueness of the white triangle in (22) is similarly epiphenomenal—it arises from characterization of TRM and the further CONDITION on TRM.

10 Lateral Nasalization in Bantu

In this section we'll examine data from two minimally different sets of languages to see how the model we have developed can be applied. The Bantu languages Lamba and Bemba change a *l* to *n* when the first consonant to the left is a nasal, as shown in the Lamba data in (24) and the Bemba in (25). The *s*/*š* and vowel alternations are irrelevant for our purposes. The reciprocal suffix doesn't alternate because it contains an underlying *n*, but the applied suffix shows up as *n* only when the preceding consonant (or onset) is a nasal—we assume it contains *l* underlyingly, at least for now.

(24) Lamba verb forms

PAST	NEUTER	APPLICATIVE	RECIPROCAL	Gloss
masa	mašika	mašila	masana	'plaster'
tula	tulika	tulila	tulana	'dig'
pata	patika	patila	patana	'scold'
kaka	kačika	kačila	kakana	'tie'
ima	imika	imina	imana	'rise'
puma	pumika	pumina	pumana	'flog'
ŋaŋa	ŋaŋika	ŋaŋina	ŋaŋana	'snigger'

(25) Bemba verb forms

V	APPLICATIVE	
-fika	-fikila	'arrive'
-tuma	-tumina	'send'
-someka	-somekela	'plug'

Odden (1994) compares the Lamba and Bemba type 'nasal spreading' to that found in Kikongo and Tshiluba, where the *l* to *n* change applies even at a distance.

(26) Kikongo verb forms

INFINITIVE	APPLICATIVE	gloss
kutoota	kutootila	'harvest'
kukina	/kukinila/ ⇝ kukinina	'dance'
kukinisa	/kukinisila/ ⇝ kukinisina	'make dance'
kudumuka	/kudumukila/ ⇝ kudumukina	'jump'
kudumukisa	/kudumukisila/ ⇝ kudumukisina	'make jump'

In the Kikongo applicative for 'dance' [kukinina], we assume the INR *l* 'finds' the nasal *n* of the root and the *l* surfaces as *n*. In the form 'make dance' the *s* of the causative suffix is 'transparent' to the SEARCH, and the underlying *l* of the applicative again surfaces as *n*. In the form 'jump' there is a non-nasal root consonant, the *k*, which is transparent, so the *l* of the applicative again surfaces as *n*—the *l* of the applicative suffix can 'see' the *m*. Even when the *l* of the applicative is separated from the nasal *m* by two non-nasal consonants, as in /kudumukisila/ surfacing as [kudumukisina], the underlying *l* still is nasalized to *n*: the *l* can 'see through' both the *s* and the *k*. So, Kikongo has long-distance nasalization. In contrast, as we saw above, Lamba forms like *mašila* and Bemba forms like *somekela* suggest that those languages require syllable adjacency or some other measure of locality for the nasalization of *l* to occur.

The strictly local rule of Bemba and Lamba can be formulated as follows:

(27) Bemba/Lamba Rule with local nasalization

- Turn an /l/ into a nasal if the first consonant to the left is a nasal
- Bemba/Lamba Rule (version 1)

SEARCH: INR: +Lateral, TRM: −SYLLABIC, DIR: L

Change: INPUT: INR, OUTPUT: +Nasal, CONDITION: TRM is +Nasal

Vowels are 'transparent' to the Search for the next consonant to the left, but any consonant will terminate the Search. The CONDITION on Change determines whether the /l/ becomes /n/. If that TRM is a nasal, the INR becomes a nasal.[13] In a form like *somekela*, when *l* initiates a Search the corresponding TRM is the *k*, so the *m* is not visible. Therefore, the *l* remains in the output.

For Kikongo and Tshiluba the appropriate rule would be something like this:

(28) Kikongo/Tshiluba Rule

- Turn an /l/ into a nasal if any consonant to the left is a nasal
- Rule parameters

  Search: INR: +Lateral, TRM: −Syllabic,+Nasal, DIR: L
  Change: INPUT: INR, OUTPUT: +Nasal

In discussing these two kinds of rules, Odden says that "[e]xplicit adjacency conditions are ... necessary in a complete account of intervening material". This way of thinking is typical of the literature on such topics, but very different from our approach. We ensure adjacency by making the specification on TRM *more* general (e.g., "stop at the first consonant you come to" rather than "stop at the the first nasal consonant you come to"), and we do not have to characterize intervening material overtly for long-distance rules. We don't have to worry about what segments block or don't block spreading (a.k.a. autosegmental copying) because our model has no copying! Blocking, spreading and so on are not part of the model, and the 'grammatical constructions' (in Chomsky's terms) they refer to can be derived indirectly from the specification of the parameters of each language's rules.

Another issue that arises in discussion of long-distance rules is the question of iterative rule application: do changes work their way through a string, from trigger to nearest target, and from that first target to the next target, and so on? This idea of iterativity is taken for granted in much work on vowel harmony, but the model we have developed allows for global or simultaneous rule application to account for what appears to be iterativity. Since more than one INR can find the same TRM, each relevant Change can apply in parallel. This should be apparent from the mappings we showed in (17ab), but we will illustrate here with our nasalization rules. Here we reanalyze particular cases of apparent iterative rule application, but of course, our goal is a model in which there is no iterative application at all, so each rules applies globally to its input strings—the changes apply simultaneously in the string without left-to-right or right-to-left iterativity.

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[13] We reiterate that the details of such an intrasegmental change are dealt with more fully in Reiss (2021).

Consider a hypothetical underlying form for a Bantu verb with two suffixes containing laterals, something like /map-il-il-a/. How do we expect it to surface? Well, in the Kikongo/Tshiluba system, we expect each *l* to surface as *n*, because each *l* is an INR that finds the same TRM *m*. So we predict the outcome to be *mapinina*, which in fact corresponds to attested forms. In contrast, in the Lamba/Bemba system, we expect *mapilila*, with no nasalization of *l*, which is again the right outcome.

We know that the two sets of languages behave superficially identical to each other when the verb root ends in a nasal and there is a single affix containing *l*: hypothetical /pam-il-a/ surfaces as *pamina* in both systems.

Now what do we predict will happen if we have a root ending in a nasal but with two suffixes containing laterals, something like /pam-il-il-a/? For the Kikongo/Tshiluba system, we predict *paminina*, which we get, for example in the Tshiluba form [u-d^{y}im-in^{y}-ine] from /u-d^{y}im-il^{y}-ile/ (Howard, 1972). Using our schematic form /$p_1a_2m_3$-i_4l_5-i_6l_7-a_8/ we can see that the *l* INR in position 5 will find the *m* TRM in position 3; and that the *l* INR in position 7 will also find the *m* TRM in position 3. There is no ordering needed between these two Search-instances, so there is no iterative rule application.

What about the Lamba/Bemba system? From an underlying form like /pam-il-il-a/, we expect the leftmost *l* to nasalize, because the first consonant to its left is *m*. We don't predict that the rightmost *l* will also nasalize, because it is too far away from the trigger *m*. However, Bemba presents forms with the reduplicated applicative /-ilil/ and the reversive/-ulul/ that show that both laterals do become /n/: /kom-ilil-a/ ⇝ *komenena* 'totally lock away', /som-ulul-a/ ⇝ *somonona* 'unplug'. Does this mean that we do need iterative rule application for the Lamba/Bemba system?

There is a simple solution. As an initiator, INR, *l* starts a Search for a TRM which must be a non-lateral consonant. The rule's Change transforms the /l/ to become +Nasal.[14] For expository purposes, we can ignore the change to the feature Lateral here. The rule in (27) for Bemba and Lamba must be revised to something like this:

(29) Bemba/Lamba Rule (final version)

- Turn an /l/ into a nasal if the first −Lateral consonant to the left is +Nasal
- Rule parameters

  Search: INR: +Lateral, TRM: −Syllabic, −Lateral, DIR: L

  Change: INPUT: INR, OUTPUT: +Nasal, CONDITION: TRM is +Nasal

With an underlying form like /$p_1a_2m_3$-i_4L_5-i_6L_7-a_8/, the /L/ INRs in position 5 and

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[14] We have been avoiding the nitty-gritty mechanisms of the Change part of rules—again, we refer to Reiss (2021) for a more thorough treatment.

7 will both find the TRM *m* in position 3, and both will receive the value +Nasal (by a unification operation, according to the model in Reiss (2021)).

To reiterate, (29) illustrates the distinctive aspects of our model listed in (30):

(30) Lessons from Bantu nasalization

- There is no need for iterative application—in fact, Leduc (2021) argues that Karajá provides evidence that global (simultaneous) application of rules is not just possible, but required.
- Apparent locality (including adjacency—see the next section) arises as a result of specification on TRM.
- There is no need to characterize what segments can intervene between targets and triggers—this falls out of the specification of TRM

These three topics—iterativity, locality and the nature of interveners in 'non-local' processes—have been the focus of a tremendous amount of the phonology literature for decades. Our model offers a new perspective on all of these issues and their interaction.

For concreteness, consider that Jensen (1974, p.676) discusses the possibility of a phonological rule like (31):

(31) $\mathrm{C} \rightarrow \mathrm{C}^y$ / ___ X *i*

The intended effect of the rule is to palatalize any consonant that is followed at any distance (in the word) by the segment *i*. The symbol X is intended as a variable whose value can be any string of segments. So, the rule means 'palatalize a consonant if there is an *i* to its right, no matter how far away, and no matter what intervenes'. Jensen says "It is clear that we do not want to be able to have rules like [31], since such processes are never found in natural languages." According to Jensen "One thing wrong with [31] is that it allows the intervening material X to contain possible inputs [targets] to the rule." In other words, the rule as stated could map an input *katalani* to output k^y*atalani*, where X is interpreted maximally as *atalan*, the maximal substring between a C and an *i*, and none of the consonants in X are affected by the rule.

In contrast to Jensen, we want rules to have exactly this property of applying maximally. However, we want the rule to apply not only maximally, but to all relevant intervening material. This happens, not because the material is intervening, but because the intervening INRs happen to have found the same TRM as the further INR. Jensen, and most of the literature has the segment *i* 'looking for' targets to the left; in contrast, we have the INRs 'looking for' a TRM to the right. This parallels what we see in Lamba and Bemba where two laterals might 'see' the same nasal (to the left), even though that nasal is only superficially local to one of the two. The −Lateral specification of TRM is crucial to the 'transparency' of the leftmost *l*. The Change only applies if the CONDITION is satisfied—TRM must be +Nasal.

In the Kikongo/Tshiluba system, both laterals can find the same nasal, even at a distance, because the TRM is specified to be +Nasal and there is no CONDITION. For us, unlike Jensen, application of the Change to all relevant interveners in these languages is “a feature, not a bug” of the model.

11 Adjacency conditions

What if we want to formulate a rule to express the following: “Turn a ■ into a ○ if there is a ▲ to its immediate right”? This rule demands that the “trigger” segment, the black triangle, be to the *immediate* right of the “target” black square. Compare this to (14) which required merely that the trigger be anywhere to the right. Given our system, adjacency requires more specification than long-distance environments. However, we do not need to make our Search model more complex—we can capture adjacency conditions with the machinery we already have. Consider this formulation:

(32) Rule 5

- From a ■ Search for a SHAPE to the right and turn the ■ into a ○ if TRM is ▲
- Rule 5 Parameters

  Search: INR: ■, TRM: SHAPE, DIR: R

  Change: INPUT: INR, OUTPUT: ○, $\boxed{\text{CONDITION: TRM: }\blacktriangle}$

In (32) specification of TRM is the maximally general category SHAPE—this means that Search will terminate at the first symbol to the right of INR, no matter what it is. The condition on Change then demands that the TRM be a black triangle in order for the Change to apply. In other words, only a black triangle to the immediate right of the INR black square will trigger the change of INR to a white circle.

In our model we don’t need any additional mechanism to account for local rules, changes that are triggered by segments adjacent to the target. We can see that adjacency is accounted for by the same mechanisms that we used to describe opaqueness, above. Consider the effects of (32) with the following mapping:

(33) Mapping from (32)
■$_1$ ●$_2$ ▲$_3$ ■$_4$ ▲$_5$ $\rightsquigarrow$ ■$_1$ ●$_2$ ▲$_3$ ○$_4$ ▲$_5$

Obviously the black square in position 4 satisfies the INR specification. This INR finds the TRM in position 5, which is a black triangle, so position 4 is rewritten as a white circle.

Now consider the black square INR in position 1. This INR initiates a Search that finds a TRM in position 2 (since any SHAPE will do) , but since position 2 does not contain a black triangle, the black square in position 1 is unchanged in the output. The INR in position 1 cannot keep searching beyond position 2 and ‘see’ the black triangle in position 3. We could say that the black circle in position 2 is 'opaque', but this term adds nothing to our understanding—the rules are fully characterized by understanding the specifications of the various parameters.

12 'Icy targets' and Sanskrit *nati*

With this reconceptualization of locality and even adjacency as derived from opaque long-distance effects, we can get simple accounts of a wide range of phenomena. Consider the following rule that turns any square into a triangle of the same color if the first black shape to its right is a circle.

(34) Rule 6

- From a ▨ SEARCH for B to the right and turn the ▨ into a ◭ if TRM is ◍
- Rule 6

  SEARCH: INR: ▨, TRM:B, DIR: R

  CHANGE: INPUT: INR, OUTPUT: ◭, CONDITION: TRM: ◍

This rule uses only mechanisms we have seen before. Now consider some mappings of strings by this rule.

(35) Mapping from (34)

a. $\blacksquare_1\ \bigcirc_2\ \blacksquare_3\ \bigcirc_4\ \blacktriangle_5 \rightsquigarrow \blacksquare_1\ \bigcirc_2\ \blacksquare_3\ \bigcirc_4\ \blacktriangle_5$

b. $\blacksquare_1\ \bigcirc_2\ \square_3\ \bigcirc_4\ \blacktriangle_5 \rightsquigarrow \blacksquare_1\ \bigcirc_2\ \square_3\ \bigcirc_4\ \blacktriangle_5$

c. $\blacksquare_1\ \bigcirc_2\ \blacksquare_3\ \bigcirc_4\ \bullet_5 \rightsquigarrow \blacksquare_1\ \bigcirc_2\ \blacktriangle_3\ \bigcirc_4\ \bullet_5$ "Icy" ■

d. $\blacksquare_1\ \bigcirc_2\ \square_3\ \bigcirc_4\ \bullet_5 \rightsquigarrow \blacktriangle_1\ \bigcirc_2\ \triangle_3\ \bigcirc_4\ \bullet_5$ "Transparent" □

e. $\square_1\ \bigcirc_2\ \blacksquare_3\ \bigcirc_4\ \blacktriangle_5 \rightsquigarrow \square_1\ \bigcirc_2\ \blacksquare_3\ \bigcirc_4\ \blacktriangle_5$

f. $\square_1\ \bigcirc_2\ \square_3\ \bigcirc_4\ \blacktriangle_5 \rightsquigarrow \square_1\ \bigcirc_2\ \square_3\ \bigcirc_4\ \blacktriangle_5$

g. $\square_1\ \bigcirc_2\ \blacksquare_3\ \bigcirc_4\ \bullet_5 \rightsquigarrow \square_1\ \bigcirc_2\ \blacktriangle_3\ \bigcirc_4\ \bullet_5$ "Icy" ■

h. $\square_1\ \bigcirc_2\ \square_3\ \bigcirc_4\ \bullet_5 \rightsquigarrow \triangle_1\ \bigcirc_2\ \triangle_3\ \bigcirc_4\ \bullet_5$ "Transparent" □

There can be no change from input to output in examples (a,b,e,f) because the CHANGE can only be triggered by a black circle and these input strings have no black circles. In (c), the black square INR in position 1 finds as its TRM the black square in position 3. The black square in position 1 does not change in the output because its TRM square in position 3 is not a circle!

The black square in position 3 finds as its TRM the black shape in position 5, which also satisfies the condition of being a circle, so position 3 maps to a triangle (leaving its color intact) in the output.

The change affects position 3 but is not 'transmitted' to position 1. The black squares are an example of what has been called an 'icy target' in the literature (Jurgec, 2011), a segment that undergoes a change (as a target) but does not transmit it further (as a trigger). We can see that this behavior is nothing special in our system—it just follows from the nature of our rule.

In (d) the black square in position 1 can 'see' all the way to the first black segment in position 5. Since that TRM is not only black, but happens to be a circle, the CONDITION for Change is met. So, the shape in position 1 changes to a triangle of the same color. The white circles and the white square are transparent because they do not contain the property that defines a TRM (being black). However, the white square in position 3 changes to a white triangle, because it also finds as its TRM the black circle in position 5.

Examples (g,h) are consistent with what we have seen. In (g) the black square in position 3 can itself undergo the change, but it 'blocks access' by the white square in position 1 to the black circle in position 5. In (h) both white squares in positions 1 and 3 find their TRM in position 5, and they both change, since that TRM is a black circle.

These examples are sufficient to understand so-called 'icy-targets' in very simple terms. Let's think of S and the characterizations of INR, TRM and the CONDITION-on-TRM as sets of features.[15] Given a rule R, with initiator specification INR_R and terminator specification TRM_R, a segment S will be an icy target with respect to R if it fulfills these conditions:

(36) Set theoretic conditions on 'icy targets' of a rule R

i. S must contain all the features that characterize INR_R (since S has to be able to undergo Change): $S \supseteq \text{INR}_R$. In other words, S must be a member of the class of segments that can be an initiator for R.

ii. S must contain all the features that characterize TRM_R (since S has to be able to terminate Search by other segments and stop them from 'looking beyond S; this is what makes S 'opaque'): $S \supseteq \text{TRM}_R$. In other words, S must be a member of the class of segments that can be a terminator for R.

iii. S must NOT fulfill the $\text{CONDITION-on-TRM}_R$ that triggers Change (since S does not itself trigger the Change). $S \not\supseteq$ CONDITION-on-TRM_R. In other words, S must not have all the properties needed to trigger Change.

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[15] The characterizations of INR and TRM yield natural classes, which are sets of segments, and thus sets of sets of features. The characterization of S is just a set of features. See Bale and Reiss (2018) for notational suggestions on differentiating these types. In brief, they use features in square brackets for natural classes and features in normal set (curly) brackets for segments. Here, just consider the set of features used to define *intensionally* a natural class of TRM or INR segments, and compare that to the set of features that define a given segment S.

Note that we analyze icy targets, not as failing to *transmit* a change, but as failing to trigger the same change that it undergoes. Our model lacks 'transmission' since there is no iterativity.

Now that we have derived the notion of icy target from our model, we can show how simply we can derive a famously challenging pattern, the so-called *nati* retroflexion of Sanskrit, which Jurgec (2011) treats as exemplifying icy targets. Our simple assumption is that the rule targets just one segment, [n], and it happens to be 'icy'.[16]

In Sanskrit, the dental (+Coronal, +Anterior, +Distributed) nasal becomes retroflexed if preceded by a retroflex continuant (Schein and Steriade, 1986). The retroflexion does not occur if any coronal intervenes between the target and trigger. For clarity, we will use schematic forms. Note, that in our model, *nati* involves a *leftward* search from an INR /n/ {+Cor, +Nas}. The blocking effect of intervening coronals like /t/ tells us that TRM is {+Cor}. So far, so good: /n/ is a superset of the INR description {+Cor, +Nas}, satisfying condition (36i); and /n/ is a superset of the TRM description {+Cor}, satisfying condition (36ii). Now, we know that only retroflex continuants like /ɻ/ and /ʂ/ actually trigger *nati* retroflexion. So, although all coronals are TRMs, there is a further CONDITION for Change to apply: TRM must also contain the features +Continuant and −Anterior: that is, for the Change to apply, TRM must be a superset of {+Continuant, −Anterior, +Coronal}. The segment /n/ is *not* a superset of this set, so condition (36iii) is met. So, /n/ is an icy target for this rule. There happen to be no non-icy targets.

Consider these schematic input-output mappings:[17]

(37) Schematic Sanskrit *nati* data
i. /ɻakana/ ⇝ [ɻakaɳa] *k* is transparent—not TRM
ii. /ɻatana/ ⇝ [ɻatana] *t* is opaque—TRM, but not retroflex
iii. /ɻakanana/ ⇝ [ɻakaɳana] *n* is 'icy'—INR and TRM, but not retroflex

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[16] We do not pretend to offer a full account of all the details of *nati*, including potential morphological factors and additional phonological factors discussed recently by Ryan (2017). On the one hand, we are using this simplified version of the Sanskrit facts to illustrate components of our model, and the factors presented by Ryan will perhaps be handled by future elaborations of our model. On the other hand, there are some obvious alternatives to Ryan's 'gang effect' solutions that have not been fully explored. For example, it may be possible to avoid reference to morphological conditioning by considering the possibility that Sanskrit has both an underlying +Ant non-retroflexed /n/ (which occurs after tautomorphemic adjacent stops) and another underlying coronal nasal that is unspecified for Anterior. This distinction can potentially yield a phonological account for the different behavior of certain nasals. If *nati* is a feature-filling process, then only the underspecified nasal will be affected. This kind of account has been exploited elsewhere to explain away apparent exceptional phonological behavior, for example, by Inkelas and Orgun (1995); Reiss (2021). This paper is not the place to explore such matters.

[17] Actual forms corresponding to these cases are brahmaṇi 'brahman-LOC sg' with only non-coronals consonants between the retroflex /r/ and the nasal; mṛd-nā- 'be gracious-PRESENT' with the coronal /d/ blocking retroflexion of the /n/; and prā́ṇanam 'breathing', with retroflexion of the leftmost nasal, but not the one to its right. The symbol [ṇ] corresponds to the IPA [ɳ].

The segment /ɻ/ is a retroflex coronal continuant, so it is a TRM and it satisfies the CONDITION on TRM that triggers the Change. What's the status of the segments /k,t,n/? In (i), INR /n/ finds TRM /ɻ/ and the Change applies, yielding a retroflex [ɳ]. The /k/ is not a TRM, so it is transparent. In (ii), the INR /n/ finds TRM /t/, but the condition on TRM that it be a retroflex continuant is not met by /t/. Thus, the /n/ does not change, because the /t/ is opaque. In (iii), both /n/'s are INRs. The leftmost one finds TRM /ɻ/ and changes to [ɳ]; but the rightmost /n/ finds as its TRM the /n/ to its left, and the Change does not apply. The segment /n/ is a so-called 'icy' target.

This account of *nati* and other 'icy targets' requires no special mechanisms beyond the streamlined model we have independently proposed. So-called icy target behavior can be described in simple set-theoretic terms. Our account of Sanskrit retroflexion involves INRs characterized by features on two tiers, Cor and Nasal, and TRMs characterized by features on just one tier (the +Cor that leads to termination of Search) or three tiers (if we include the +Cont and −Ant that constitute the CONDITION), so this is an example of the insufficiency of stating relations among the segments involved in rules on a single tier, as discussed in section 2.

13 Scope and rule parameters

We have treated unbounded Search, unbounded in the sense that it continues as far as necessary to find a TRM, as the basic operation that underlies even local or adjacent phonological processes. Principle 1 is important to getting to this point—it is necessary to divorce the study of what is happening, the Change, from where it is happening. We have proposed that the environment, the *where*, is best understood as satisfaction of the Search specifications along with additional CONDITIONS on Change, if any.

To use a concept familiar to linguists, we can think of our rule in terms of the scope of feature specifications. In (14) the specifications 'black' and 'triangle' both have scope over the definition of TRM in the Search. In (22), the specification 'triangle' has scope over the definition of TRM in the Search and the specification 'black' has scope over the definition of CONDITION in the Change. In (32), the specifications 'black' and 'triangle' both have scope over the definition of CONDITION in the Change. Note that in none of these cases do we refer to the properties of INR or to the INPUT and OUTPUT of the rule—these sometimes share features with other parts of rules, as when a rule happens to be an assimilation rule, but that is not an essential feature of the model of environments we are developing.

14 Nested Rules

Obviously, there are real phonological processes that are more complex than what can be handled with our simple examples. In this section, we explore some simple ways in which our rule syntax can be expanded so as to have greater empirical coverage, but we leave for future work the demonstration that such mechanism are actually needed.

Let's first address the question of how to express two-sided rules like this: $a \rightarrow b$ / c ___d, equivalently $cad \rightarrow cbd$. One approach is to perform simultaneous rightward and leftward Search out from the INR segment a. Another option, which we follow here, is to perform nested Search: first use INR c to Search for TRM a; then start a new Search with INR a for TRM d and apply Change to the INR of the second, embedded Search.

One might be tempted to express such a rule as follows

(38) Rule 7

- One kind of nesting

  Search$_i$: INR$_i$:c; TRM$_i$:a; DIR:R

    Search$_j$: $\boxed{\text{INR}_j}$:TRM$_i$; TRM$_j$:d; DIR:R

    Change: INPUT:$\boxed{\text{INR}_j}$; OUTPUT:b
- Is this right? $a \rightarrow b$ / c ___d
- No—see (39)!

There is a problem with the formulation in (38). Recall that long-distance application is the default in our system, so as written, the Search parts of the rule actually mean "starting from a c Search rightward until an a is found; then Search from the a until a d is found". This means that the distance from c to a and from a to d is not bounded by the rule. We could informally denote this rule thus:

(39) $a \rightarrow b$ / $c \ldots$___$\ldots d$

Since the TRMs are specific symbols (a and d), Search will not in general stop at the segment to the immediate right of each INR.

In order to ensure that each Search finds a right-adjacent TRM, we need the specification of TRM to be more general. We'll use the symbol ς as a variable to mean 'any segment', analogous to SHAPE above.[18]

(40) Rule 8

- $a \rightarrow b$ / c ___d

  Search$_i$: INR$_i$:c; TRM$_i$:ς; DIR:R

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[18] We can also call ς a partial description defining the set of all segments. See Bale et al. (2020) for a way to avoid a feature ±Segment, and to instead use the notation '[ ]' to mean 'any segment'.

$$\textsc{Search}_j\text{: }\boxed{\text{INR}_j}\text{:TRM}_i\text{; TRM}_j\text{:}\varsigma\text{; DIR:R}$$

$$\textsc{Change}\text{: INPUT:}\boxed{\text{INR}_j}\text{; OUTPUT:}b\text{;}$$

$$\text{CONDITION: TRM}_i\text{:}a\text{, TRM}_j\text{:}d$$

Indices function to share information between parts of the rule. Information flows between the first SEARCH and the second, since the TRM of the former (TMR_i) is the value assigned to INR_j, the initiator of the second SEARCH. CHANGE is subject to two conditions, one on the TRM of the first SEARCH and one on the TRM of the second SEARCH. There is also information flow between the CONDITION and the two TRMs. If the first SEARCH is successful, the second is triggered, and if the second is successful, the CHANGE is triggered, and applied if the CONDITION is satisfied.

So, we need something like (40) to express a rule with a two-sided adjacency condition. Recall that this result is not suprising in light of our analysis of the distinction between long-distance nasalization in Kikongo and Tshiluba vs. local nasalization in Bemba and Lamba.

If two nested instances of SEARCH are possible, it is natural to ask if the pattern can be repeated indefinitely. Does phonology allow, in principle, an unbounded number of nested instances of SEARCH yielding the possibility of rules like this for an an arbitrary integer n:

(41) Unbounded nesting with INPUT:$\boxed{\text{INR}_n}$

$$\textsc{Search}_1\text{: INR}_1\text{; TRM}_1\text{; DIR:R}$$

$$\textsc{Search}_2\text{: INR}_2\text{:TRM}_1\text{; TRM}_2\text{; DIR:R}$$

$$\textsc{Search}_3\text{: INR}_3\text{:TRM}_2\text{; TRM}_3\text{; DIR:R}$$

$$\vdots$$

$$\textsc{Search}_n\text{: }\boxed{\text{INR}_n}\text{:TRM}_{n-1}\text{; TRM}_n\text{; DIR:R}$$

$$\textsc{Change}\text{: INPUT:}\boxed{\text{INR}_n}\text{; OUTPUT}$$

Each SEARCH-instance has the TRM of the previous one as the value of its own INR, and the CHANGE takes the INR of the last SEARCH as its INPUT. This would just be a generalization of the rule in (38) to allow unbounded nesting. Of course, CONDITIONS like those in (40) can be combined with unbounded nesting, potentially.

An obvious question is whether a rule like the following is possible: $a \rightarrow b$ / ___cd, that is $acd \rightarrow bcd$. Such a rule can be stated if we allow the CHANGE to refer back to the INR of the higher SEARCH. As above, we need to carefully distinguish the following rules:

(42) i. $a \rightarrow b$ / ___cd

Change a to b when it occurs immediately to the left of a c that is to the immediate left of a d

ii. $a \rightarrow b \,/\, \_\_\ldots c \ldots d$

Change a to b when it occurs anywhere to the left of a c that occurs anywhere to the left of a d

It is not clear that rules like (42i) are needed by the phonology, and (42ii) seems even more unlikely as a candidate rule of an attested language. However, we need to appreciate that in our system, (42i) is a special case of (42ii) that we get by adding CONDITIONs on TRMs, so if it can generate the former, it can generate the latter as well. In order to not get sidetracked by the use of CONDITIONs, we use (42ii) to illustrate the structure of interest:

(43) Rule 9

- $a \rightarrow b \,/\, \_\_\ldots c \ldots d$

Search$_i$: $\boxed{\text{INR}_i}$:a; TRM$_i$:c; DIR:R
Search$_j$: INR$_j$:TRM$_i$ TRM$_j$:d; DIR:R
Change : INPUT:$\boxed{\text{INR}_i}$; OUTPUT:b

In (43) the INPUT parameter of Change is the highest level INR, INR$_i$. Here's the general case:

(44) Unbounded nesting with INPUT:$\boxed{\text{INR}_1}$

Search$_1$:$\boxed{\text{INR}_1}$; TRM$_1$; DIR:R
Search$_2$: INR$_2$:TRM$_1$; TRM$_2$; DIR:R
Search$_3$: INR$_3$:TRM$_2$; TRM$_3$; DIR:R
$\vdots$
Search$_n$: INR$_n$:TRM$_{n-1}$; TRM$_n$; DIR:R
Change: INPUT:$\boxed{\text{INR}_1}$; OUTPUT

It is an empirical question whether human phonology needs such power—can the Change in a rule look back to the highest level Search and have scope over the INR of that Search? In (41) the INPUT to Change is INR$_n$ of the last level Search, whereas in (44), the INPUT to Change is INR$_1$ of the first level Search. Can the INPUT to Change potentially be any one of the intermediate INRs? Many questions remain, but they are empirical questions that can be asked in the context of a research program based on the explicit model we propose.

15 Prospects and Conclusions

Note that both (38,39) and (43), and their generalized versions in (41) and (44), respectively, illustrate that our model reduces rules, which are functions mapping strings to strings, to combinations of binary relations between segments. For example, the rule $a \rightarrow b$ / $c \ldots \_\_ \ldots d$ is deconstructed into a SEARCH relation between c and a, a SEARCH relation between a and d, and a CHANGE relation between a and b. We can better explore the nature of human phonology by building and understanding the combinatoric space defined by the simple pieces we posit, a feature inventory and SEARCH and CHANGE relations.

To be fully explicit we need to provide a semantics for the rules we propose. We can start this task along the lines of the rule interpretations given in Bale and Reiss (2018). Here's a sketch of the interpretation for (39):

(45) Interpretation for (39): Informally $a \rightarrow b$ / $c \ldots \_\_ \ldots d$
This rule is a function that maps a string $x_1 \ldots x_n$ to a string $y_1 \ldots y_n$ such that

- IF x_i is a, and there exists $j < i$ s.t. x_j is c, and there exists $k > i$ s.t. x_k is d, then y_i is b
- ELSE $y_i = x_i$

In other words, in the mapping from input string to output string, just change the segments that are a's between a c and a d, at any distance.

The interpretation of the adjacency-based version in (40) is just something along these lines:

(46) Interpretation for (40): Informally $a \rightarrow b$ / $c \_\_ d$
This rule is a function that maps a string $x_1 \ldots x_n$ to a $y_1 \ldots y_n$ such that

- IF x_i is a, x_{i-1} is c, and x_{i+1} is d, then y_i is b
- ELSE $y_i = x_i$

Of course, x_{i-1}, x_i and x_{i+1} in (46) are just a special case of x_j, x_i and x_k, respectively, in (45). These interpretations of SEARCH-based rule syntax are explicit and can be generalized to all our examples.

The closely related notions of adjacency, locality, opaqueness and transparency have played a major role in the phonological literature over decades and across theoretical frameworks. In addition to the strictly local phonology work mentioned in the introduction, some other notable work includes Howard (1972); Jensen (1974); Odden (1980, 1994); Bakovic (2000) and Kimper (2011), which all tackle difficult problems and offer creative solutions. These works all attest to the challenge of defining phonological environments in a way that can make all, or almost all phenomena, even those that appear to be non-local, be understood in terms of fundamentally local relations.

We have proposed a contrarian bottom-up model of phonological processes that embraces non-locality. The approach is bottom-up in the sense that we develop a system based on toy examples, and hope to show that applying this model to attested patterns can lead to insight when applied to 'real' data. In our formalism the simplest phonological relation constituting a rule environment is one that is unbounded with respect to distance in segment strings. Locality and adjacency are not derived in the perhaps intuitively obvious manner of, say, defining immediate precedence as a special case of precedence in strings.[19] Instead, we express adjacency (and other forms of apparent locality) with the same mechanism that yields the kinds of behavior described in the literature as segment "opaqueness". We have attempted to relate our defetishization of locality to a concomitant defetishization of COPY by reconceptualizing the notions of target and trigger—INR and TRM are the closest parallel to those notions in our model, but they reflect an improvement because they allow us to maintain Principle 1.

Here are the primary claims of the paper:

- Rule environments are best conceived as (potentially nested) SEARCH procedures.
- If the SEARCH terminates successfully and the CONDITION is satisfied, a rule's CHANGE applies. Otherwise, the output form is identical to the input.
- Notions like copy, agreement, harmony and identity are not essential to phonological rules (aside from some uses of α-notation).
- Opaqueness, transparency and 'icy-ness' of segments have a merely descriptive status derivable from our rule parameters.
- Common phonological processes do not necessarily best reflect the fundamental components of phonological computation.
- There should be no constraints or primitives of any kind that are specific to phonological 'constructions' like vowel harmony or place assimilation

We hope we have offered a new perspective on old questions concerning adjacency, locality, opaqueness and transparency.

We have clarified the independence of the changes effected by rules and the environments that rules define. We have proposed that the difficult question of specifying possible intervening material between a rule target and trigger can be avoided by reconceptualizing rules in terms of satisfaction of SEARCH procedures. Many questions remain open, but we hope to have moved the discussion in the direction of discovery of general representational and computational properties of the phonological

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[19] For work focussing on precedence and immediate precedence, see especially Raimy (2000); Shen (2016) and Papillon (2020).

system, and away from 'taxonomic artifacts' such as 'vowel harmony', 'final devoicing', and 'place assimilation'. Recall that we reject McCarthy's view that "the goal of phonology is the construction of a theory in which cross-linguistically common and well-established processes emerge from very simple combinations of the descriptive parameters of the model" (McCarthy, 1988, 84). Instead, our goal is to "abstract from the welter of descriptive complexity certain general principles governing computation that would allow the rules of a particular language to be given in very simple forms" (Chomsky, 2000a, 122).

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