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CONTRADICTORINESS

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Abstract

This article provides a new perspective on the phenomenon of contradictoriness, i.e. the observable fact that certain sentences *feel* contradictory. In the first part (§1-2), I present a series of empirical arguments aimed at showing that standard notions of contradiction—falsehood in every possible world, falsehood under all possible uniform substitutions of non-logical words—are of little use when it comes to characterising contradictoriness. In the second part (§3-5), I offer an account of this phenomenon; this account is stated in the form of a generalisation and has at its core a new theoretical notion—the notion of *predicate connection*.*

1 Introduction

Consider (1) below: (1)a *feels* contradictory or exhibits contradictoriness (presence of contradictoriness is signalled with a '^{c}'). (1)b, by contrast, doesn't: it is just perceived as false (absence of contradictoriness is signalled with a '$^{¢}$').

(1) a. ^{c} John was killed and he wasn't killed.
b. $^{¢}$ Donald Trump didn't serve as US President.

Why is this so? There are two well-known responses to this question:

(2) <u>The formalist's response:</u> (1)a, but not (1)b, exhibits contradictoriness because (1)a, unlike (1)b, is a *formal contradiction* (i.e. it is false under all possible uniform substitutions of non-logical words).

(3) <u>The romantic's response:</u> (1)a, but not (1)b, exhibits contradictoriness because (1)a, unlike (1)b, is a *necessary falsehood* (i.e. it is false in every possible world).

(2) is known to be too restrictive: (4)a, just like (1)a and unlike (1)b/(4)b, exhibits contradictoriness; however, (4)a isn't a formal contradiction.

(4) a. ^{c} John was killed and he didn't die.
b. $^{¢}$ Donald Trump didn't serve as US President.

* Acknowledgments to be added.

(3), on the other hand, can be invoked to explain both the contrast in (1) and that in (4): (4)a and (1)a, unlike (4)b/(1)b, are necessary falsehoods. Because of this clear advantage with its rival, (3) has gained, at least in semantics circles, the status of being 'the correct response'.

(3), as will be discussed, is incorrect: there are contingent falsehoods that exhibit contradictoriness as well as necessary falsehoods that don't. But, if (3) is incorrect, then, why is it that (1)a and (4)a *feel* contradictory? This article seeks to shed light on this question. Its structure is as follows: In §2, I show that the romantic's account of contradictoriness is wrong. In §3, I put forward a generalisation that makes sense of a great deal of linguistic data and offers a fresh theoretical perspective on the phenomenon of interest. In §4, I make two refinements to this generalisation, refinements that further extend its empirical reach. In §5, I discuss four problems that the proposed generalisation faces and hint at possible solutions.

2 The romantics' answer isn't right

The romantic believes (5) to be true:

(5) A sentence exhibits contradictoriness if and only if it is false in every possible world.

(5) isn't true, however. Consider the sentence in (6), for example. (6) is not a necessary falsehood—(6) is true in worlds in which every member of Linguae likes John, Benjamin isn't a member of Linguae, and Benjamin hates John.

(6) [CONTEXT I: it is common ground that Benjamin is a member of Linguae.]
[CONTEXT II: it is common ground that no member of Linguae likes John and, furthermore, that Benjamin is a member of Parlare (not of Linguae).]

[C-I: c | C-II: ¢] Every member of Linguae likes John, but Benjamin hates him.

Under the generalisation in (5), (6) is not expected to exhibit contradictoriness, neither in CONTEXT I nor in CONTEXT II (as established, (6) is not a necessary falsehood). (6), however, does exhibit contradictoriness in CONTEXT I (in CONTEXT II it is false but doesn't exhibit contradictoriness). (5) cannot be right then: as (6) discloses, it is possible for a contingent falsehood to exhibit contradictoriness.

Here's another argument against (5).

(7) [CONTEXT: it is common ground that the city of Tajiff is in Cuba.]

a. [c] Benjamin is in Tajiff and he isn't in Cuba.
b. [¢] Benjamin is in Tajiff and Tajiff isn't in Cuba.

(7)a and (7)b are true in exactly the same worlds—namely, in worlds in which Benjamin lives in Tajiff and Tajiff isn't in Cuba.[1] Despite having the same truth-conditions, (7)a and (7)b elicit different judgments in the stipulated context: (7)a exhibits contradictoriness; (7)a, by contrast, doesn't. This observation falsifies (5): according to (5), neither (7)a nor (7)b should exhibit contradictoriness (because neither (7)a nor (7)b are necessary falsehoods).

(5) can be refuted from the opposite direction: that is, it can be shown that not all necessary falsehoods feel contradictory. Consider (8), for example.

(8) a. [¢] Bachelors have wives.
b. [c] Bachelors have wives and aren't married.

Both (8)a and (8)b are necessary falsehoods: however, only (8)b exhibits contradictoriness—(8)a is simply perceived as false. (8), therefore, should persuade one that (5) is false; unless, of course, one wasn't persuaded that (8)a is a necessary falsehood, but, oddly enough, was persuaded that (8)b is one. To those in such a state, I offer this argument: if (8)a wasn't a necessary falsehood but a contingent one, it should be possible to imagine what it would take for it to be true. This doesn't seem to be possible, however, as the test below reveals.

(9) <u>The 'imagine if' test</u>

a. Bachelors have wives.
a'. ? Imagine if bachelors had wives.

b. Dogs don't bark.
b'. ✓ Imagine if dogs didn't bark.

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[1] I could have made the same point with (i):

(i) a. [¢] Benjamin is in Paris, and he isn't in France.
b. [c] Benjamin is in Paris, and Paris isn't in France.

The reason why I am using (7) and not (i) is purely methodological: (7) is like (6) in that the contextual information given is not, as a matter of fact, part of the common ground (I'm just asking the reader to assume that it is). (i) is different in this respect: the relevant contextual information (namely, that Paris is in France) is part of the common ground. As the parallelism between (7) and (i) discloses, this difference is orthogonal to the judgments that I am eliciting.

c. Elephants have red stripes.
c'. ✓ Imagine if elephants had red stripes.

Indeed, (9)a, when embedded under 'imagine if', reads as a non-sensical command: a bachelor *is* an unmarried man… so how could an unmarried man have a wife? (9)b and (9)c behave differently: these are generic sentences that, like (9)a, are known to be false but, unlike (9)a, can be felicitously embedded under 'imagine if'. To account for this contrast, one is compelled to recognise that (8)a is not a contingent but a necessary falsehood. (5), therefore, can't be right.

An additional argument can be given against (5): (5) entails that only false sentences can exhibit contradictoriness; this, however, isn't true. First, contradictoriness can be found in sentences whose truth value is unknown (e.g. (10)a) as well as in tautologies, either logical (e.g. (10)b) or contextual (e.g. (10)c).

(10) a. ^{c} Either John is an artist, or he isn't an artist and he is both single and married.
b. ^{c} Either John lives in Montmartre but doesn't live in Paris, or it's false that he lives in Montmartre but doesn't live in Paris.
c. ^{c} It's false that John lives in Montmartre but doesn't live in Paris.

In addition, contradictoriness can be found in sentences that aren't truth-bearers, such as questions, as shown below.

(11) a. ^{c} Is it true that Paul is single and married?
b. ^{c} Is it true that John lives in Toulouse but doesn't live in France?

Once again, (5) can't be right.

3 The nature of contradictoriness

It is clear that we lack an empirically adequate account of contradictoriness; in particular, we lack an account able to predict the following three contrasts:

Table 1

	Contradictoriness? Yes.	Contradictoriness? No.
(i)	(6) in CONTEXT I	(6) in CONTEXT II
(ii)	(7)a	(7)b
(iii)	(8)b	(8)a

In this section, I put forward a generalisation that does predict these contrasts and, unlike the romantic's account, is compatible with the observation that a non-false sentence may give rise to contradictoriness.

3.1 Towards a generalisation

3.1.1 Joint emptiness and predicate connection

To begin with, let's take a close look at the sentences in (12), all of which exhibit contradictoriness.

(12) [CONTEXT: it is common ground that Benjamin is a member of Linguae.]
a. ^{c} Every member of Linguae likes John, but Benjamin hates him.

[CONTEXT: it is common ground that Benjamin is a member of Linguae.]
b. ^{c} No member of Linguae likes John, but Benjamin loves him.

[CONTEXT: it is common ground that the city of Tajiff is in Cuba.]
c. ^{c} Benjamin is in Tajiff and he isn't in Cuba.

d. ^{c} None of the suspects are in France, and one of them isn't outside Paris.

e. ^{c} Paul didn't move and didn't stay still.

f. ^{c} Benjamin (both) smokes and doesn't smoke.

g. ^{c} Every bachelor is married.

h. ^{c} Some married bachelor came to the party.

i. ^{c} John is neither married nor unmarried.

In each of the examples above, I have underlined two one-place predicates (call them α and β). The first thing that I would like to note is this: either (α and β) or (α and not-β) or (not-α and β) or (not-α and not-β) are *jointly empty* throughout the context set (see definition below).

(13) **Joint emptiness**

Let α and β be two one-place predicates, W a set of worlds, and $\mathcal{D}$ the set of all possible individuals. Then,

α and β are jointly empty throughout W iff, for any $w \in W$, $\{x \in \mathcal{D}: [\![\alpha]\!]^{w}(x) = 1\} \cap \{x \in \mathcal{D}: [\![\beta]\!]^{w}(x) = 1\} = \emptyset$.

For example, consider the two underlined predicates in (12)d, namely 'in Tajiff' and 'isn't in Cuba': these predicates are jointly empty throughout the context set—i.e given what is known, it is no possible for an individual to be both in the extension of 'in Tajiff' and in the extension of 'isn't in Cuba'. Consider now the two underlined predicates in (12)c, namely ‘likes John’ and ‘loves (John)’: ‘**doesn’t** like~~s~~ John’ (the negation of ‘likes John’) and ‘loves (John)’ are jointly empty throughout the context set. Finally, consider the two underlined predicates in (12)e, namely ‘in Paris’ and ‘outside of Paris’: ‘**not** in Paris’ (the negation of ‘in Paris’) and ‘**not** outside Paris’ (the negation of ‘outside Paris’) are jointly empty throughout the context set.

I would like to make a second observation: these predicates (the underlined predicates in each example) are ‘connected via some entity’. But what does this mean? To a first approximation, it can be said that two predicates are connected iff their meanings are entangled with one another in such a way that, even if these predicates didn’t mean what they in fact mean but something else, this entanglement would nonetheless persist.

This sounds rather cryptic, though the notion I am after is rather intuitive; a thought experiment, I think, can help here. Let’s imagine that Steve and Mary have a car accident and end up in hospital. The doctors, after examining them, conclude that their respective lexicons have suffered alterations: Steve now believes that ‘like’ means the same as ‘respect’, ‘hate’ the same as ‘adore’, ‘love’ the same as ‘despise’, ‘move’ the same as ‘sing’, and ‘stay still’ the same as ‘play piano’; Mary, in turn, believes that ‘like’ means the same as ‘supervise’, ‘hate’ the same as ‘promote’, ‘love’ the same as ‘help’, ‘move’ the same as ‘dance’, and ‘stay still’ the same as ‘play guitar’. No other alteration is detected in the lexicons of Steve and Mary. Let’s now take a look at (12)a, (12)b, and (12)e, repeated below as (14)a-c.

(14) [CONTEXT: it is common ground that Benjamin is a member of Linguae.]
a. ^{c} Every member of Linguae likes John, but Benjamin hates him.

[CONTEXT: it is common ground that Benjamin is a member of Linguae.]
b. ^{c} No member of Linguae likes John, but Benjamin loves him.

c. ^{c} Paul didn’t move and didn’t stay still.

The observation is this: if faced with the task of interpreting (14)a-c (and if assured that (14)a-c are true), Steve and Mary, despite the fact that they will map the underlined predicates to different meanings, will converge on something that pertains to the meaning of these predicates: from (14)a, they will both conclude that Benjamin (who is known to be a member of Linguae) is in the extension of ‘likes John’ as well as in the extension of ‘hates (John)’; from (14)b, they will both conclude that Benjamin (who is known to be a member of Linguae) is in the anti-extension of ‘likes John’ as well as in the extension of ‘loves (John)’; from (14)c, finally, they will both conclude that Paul is in the anti-extension of ‘move’ as well as in the anti-extension of 'stay still'. I will call these facts connection facts: in (14)a, 'likes John' and 'hates (John)' can be said to be positively connected (or pos-connected) via an entity (namely, Benjamin); in (14)b, 'likes John' and 'loves (John)' can be said to be cross-connected via an entity (also Benjamin); in (14)c, 'move' and 'stay still' can be said to be negatively connected via an entity (namely, Paul).

The basic intuition being introduced, let's now proceed to give a formal definition of predicate connection.

(15) **Predicate connection v.1**

Let α and β be two one-place predicates, μ a clause, C the context set, and $\mathcal{D}$ the set of all possible individuals. P and Q are two one-place predicate variables and f a variable over assignment functions from $\{P, Q\}$ to $\mathcal{D}_{\langle s,\langle e,t\rangle\rangle}$.

Definition. Then α and β are connected via an element of $\mathcal{D}$ in μ relative to C iff…

(i) α and β are both constituents of μ,
(ii) α isn't dominated by β nor is β dominated by α, and
(iii) μ′—a clause just like μ except that α has been replaced by P and β by Q—satisfies (a) and at least one of the other three conditions:[2]

(a) $\exists f \exists w \in C([\![\mu']\!]^{w,f} = 1)$

(b) $\forall f \forall w \in C([\![\mu']\!]^{w,f} = 1 \rightarrow \exists x \in \mathcal{D} \text{ s.t. } [\![P]\!]^{w,f}(x) = 1 \wedge [\![Q]\!]^{w,f}(x) = 1)$
In such a case, we say that α and β are **pos-connected** (via some entity, in μ/C).

(c) $\forall f \forall w \in C([\![\mu']\!]^{w,f} = 1 \rightarrow \exists x \in \mathcal{D} \text{ s.t. } [\![P]\!]^{w,f}(x) = 0 \wedge [\![Q]\!]^{w,f}(x) = 0)$
In such a case, we say that α and β are **neg-connected** (via some entity, in μ/C).

(d) $\exists v_1, v_2 \in \{0,1\} \text{ s.t. } v_1 \neq v_2 \wedge$
$\forall f \forall w \in C\ ([\![\mu']\!]^{w,f} = 1 \rightarrow \exists x \in \mathcal{D} \text{ s.t. } [\![P]\!]^{w,f}(x) = v_1 \wedge [\![Q]\!]^{w,f}(x) = v_2)$[3]
In such a case, we say that α and β are **cross-connected** (via some entity, in μ/C).

(15) is straightforward; if turned into a recipe, it would go more or less like this: first, identify a clause that has two constituents of predicative type (call one of these constituents α and the other β); second, replace α by P and β by Q (by so doing, one makes sure that the calculation of connection facts doesn't depend on the meaning of α and β); third, check whether the impoverished clause (the clause in which α has been replaced by P and β by Q) satisfies (iii)b, (iii)c, or (iii)d—for example, an impoverished clause of the form

[2] I'm assuming the following (non-standard) intepretation rule: if γ is an element of $\{P, Q\}$, then, for any w and for any f, $[\![\gamma]\!]^{w,f} = f(\gamma)(w)$; if γ is not an element of $\{P, Q\}$, then, for any w and for any f, $[\![\gamma]\!]^{w,f} = [\![\gamma]\!]^{w}$. To avoid clutter, I am omitting g, the assignment function that deals with the 'real' (as opposed to the artificially introduced) variables. This omission is harmless.
[3] It's worth noting that, by deleting '$v_1 \neq v_2\ \wedge$', a condition that covers (iii)b, (iii)c, and (iii)d can be generated; if μ′ satisfied this condition and (iii)a, it would mean that α and β are connected *simpliciter* (either pos-, neg-, or cross-connected).

'John is *P* and *Q*' satisfies (iii)b, which means that α and β are pos-connected; by contrast, an impoverished clause of the form 'John is *P* and not *Q*' satisfies (iii)c, which means that α and β are cross-connected.

There are two elements of (15) that require an explanation—namely, (15)ii and (15)iii.a. Let's start with (15)ii. This condition makes sure that step (iii) is defined: if α were to be dominated by β (or vice versa), it wouldn't be possible to replace both α by *P* and β by *Q* and, hence, it would not be possible to generate μ′. Let's now move to (15)iii.a; what is this condition for? The answer is straightforward: (15)iii.a's function is to prevent (15)iii from being satisfied trivially: indeed, if (15)iii.a wasn't there, (15)iii could be satisfied in cases in which (15)iii.b, (15)iii.c, and (15)iii.d are vacuously true.

It is worth noting that, according to (15), predicates, when connected, are connected <u>in a clause</u>, which may or may not be the matrix clause. Take, for example, (16).

(16) Either John is French, or he <u>is English</u> and <u>very wealthy</u>.

The predicates 'is English' and 'very wealthy' are not connected in the matrix clause: indeed, 'Either John is French or he is *P* and *Q*' satisfies neither (iii)b, nor (iii)c, nor (iii)d. However, these two predicates are pos-connected in the second disjunct: as can easily be checked, '(John) *P* and *Q*' satisfies (iii)b.

Having made these remarks, let me illustrate in a bit more detail how (15) works: let's take (14)c for example and ask whether 'move' and 'stay still' are connected in the matrix clause. Since the matrix clause is 'Paul didn't move and didn't stay still', and since we want to find out whether 'move' and 'stay still' are connected in it, the relevant μ′ is 'Paul didn't *P* and didn't *Q*' (if 'move' is identified with α and 'stay still' with β). 'Paul didn't *P* and didn't *Q*' satisfies both (15)iii.a and (15)iii.c: 'Paul didn't *P* and didn't *Q*' satisfies (15)iii.a as there is an intepretation of *P* and *Q* such that 'Paul didn't *P* and didn't *Q*' is true in some world in C (say, 'Paul didn't <u>read *Crime and Punishment*</u> and didn't <u>read *War and Peace*</u>'); and 'Paul didn't *P* and didn't *Q*' satisfies (15)iii.c as the following statement is the case:

> for any f and for any world w in C, if $[\![\text{Paul didn't } P \text{ and didn't } Q]\!]^{w,f} = 1$, then $[\![P]\!]^{w,f}(\text{Paul}'_w) = 0$ and $[\![Q]\!]^{w,f}(\text{Paul}'_w) = 0$ (i.e. there is an $x \in \mathcal{D}$ such that $[\![P]\!]^{w,f}(x) = 0$ and $[\![Q]\!]^{w,f}(x) = 0$).

Thus, according to (15), 'move' and 'stay still' are *neg-connected* (in the matrix clause of (14)c).

It should be noted that, given how predicate connection is defined, if any two predicates α and β are connected in some clause, the application of predicate negation to α (or β, or to both α and β) won't have the effect of disconnecting them: predicate negation is only expected to change the type of connection that holds between α and β (on the assumption, of course, that predicates have a bivalent semantics). Take, for example, the matrix clause of (14)c: in this clause, 'move' and 'stay still', as discussed, are *neg-connected* ('Paul didn't *P* and didn't *Q*' satisfies both (15)iii.a and (15)iii.c), 'didn't move' and 'stay still', by contrast, are cross-connected (i.e. 'Paul didn't *P* and didn't *Q*' satisfies both (15)iii.a and (15)iii.d), whereas 'didn't move' and 'didn't stay still' are pos-connected (i.e. 'Paul *P* and *Q*' satisfies both (15)iii.a and (15)iii.b).

To sum up, the sentences in (12) all exhibit contradictoriness: I have pointed out that, in each of these sentences, there are two predicates α and β of which two things are true: (i) (α and β) or (α and not-β) or (not-α and β) or (not-α and not-β) are jointly empty throughout the context set, and (ii) α and β are connected (in the sense made explicit in (15)).[4]

3.1.2 Contradictoriness

With (13) and (15) on board, I am now in a position to put forward the generalisation in (17):

(17) **Contradictoriness v.1**

Let α and β be two one-place predicates, μ a clause, C the context set, and $\mathcal{D}$ the set of all possible individuals. *P* and *Q* are two one-place predicate variables and *f* a variable over assignment functions from {*P*, *Q*} to $\mathcal{D}_{\langle s,\langle e,t\rangle\rangle}$.

Generalisation. S exhibits contradictoriness in C iff…

(i) α and β are connected in μ, and
(ii) one of the following statements is the case:

(a) α and β are pos-connected in μ and jointly empty throughout the context set.
(b) α and β are neg-connected in μ and not-α and not-β are jointly empty throughout the context set.
(c) α and β are cross-connected in μ, μ′—a clause just like μ except that α has been replaced by *P* and β by *Q*—is such that $\forall f \forall w \in C([\![\mu']\!]^{w,f} = 1 \rightarrow \exists x \in \mathcal{D}([\![P]\!]^{w,f}(x) = 1 \wedge [\![Q]\!]^{w,f}(x) = 0))$, and α and not-β are jointly empty throughout the context set; or α and β are cross-connected in μ, μ′ is such that $\forall f \forall w \in C([\![\mu']\!]^{w,f} = 1 \rightarrow \exists x \in \mathcal{D}([\![P]\!]^{w,f}(x) = 0 \wedge [\![Q]\!]^{w,f}(x) = 1))$, and not-α and β are jointly empty throughout the context set.

As explicitly stated, I take (17) to be a generalisation, a description of the facts, and not a theory, which should explain the facts and not just describe them. That said, I think it is possible to glean from (17), if not

[4] This is true of (12)h on the assumption that 'every' carries an existence presupposition that its restrictor isn't empty.

a theory, the scaffolding of one. Indeed, it is natural to conceptualise (17) along the following lines: connection facts impose certain constraints on what predicates can mean; when those constraints are violated, contradictoriness follows. As research in this domain expands and matures, this conceptual framing of (17) may reveal itself as inadequate—for presentation purposes, however, it is helpful, and I will thus make use of it.

Let's first consider pos-connection; if two predicates α and β are pos-connected, then α and β are constrained as follows: it has to be the case that α and β aren't incompatible given what is known. Thus, if α and β are pos-connected and jointly empty throughout the context set, then there is a constraint that has been violated (contradictoriness thus follows). Let's now move to neg-connection; if two predicates α and β are neg-connected in clause μ, then α and β are constrained as follows: it has to be the case that not-α and not-β aren't aren't incompatible given what is known. Thus, if α and β are neg-connected and not-α and not-β are jointly empty throughout the context set, then there is a constraint that has been violated (contradictoriness thus follows). Finally, let's consider cross-connection, which has two possible realisations: (i) If two predicates α and β are cross-connected in clause μ and μ′—a clause just like μ except that α has been replaced by P and β by Q—is such that $\forall f \forall w \in C([\![\mu']\!]^{w,f} = 1 \rightarrow \exists x \in \mathcal{D}([\![P]\!]^{w,f}(x) = 1 \wedge [\![Q]\!]^{w,f}(x) = 0)$, then then α and β are constrained as follows: it has to be the case that α and not-β aren't incompatible given what is known. Thus, if α and β are cross-connected in μ, μ′ is such that $\forall f \forall w \in C([\![\mu']\!]^{w,f} = 1 \rightarrow \exists x \in \mathcal{D}([\![P]\!]^{w,f}(x) = 1 \wedge [\![Q]\!]^{w,f}(x) = 0)$, and α and not-β are jointly empty throughout the context set, then there is a constraint that has been violated (contradictoriness thus follows). (ii) If two predicates α and β are cross-connected in clause μ and μ′—a clause just like μ except that α has been replaced by P and β by Q—is such that $\forall f \forall w \in C([\![\mu']\!]^{w,f} = 0 \rightarrow \exists x \in \mathcal{D}([\![P]\!]^{w,f}(x) = 0 \wedge [\![Q]\!]^{w,f}(x) = 1)$, then then α and β are constrained as follows: it has to be the case that not-α and β aren't incompatible given what is known. Thus, if α and β are cross-connected in μ, μ′ is such that $\forall f \forall w \in C([\![\mu']\!]^{w,f} = 0 \rightarrow \exists x \in \mathcal{D}([\![P]\!]^{w,f}(x) = 0 \wedge [\![Q]\!]^{w,f}(x) = 1)$, and not-α and β are jointly empty throughout the context set, then there is a constraint that has been violated (contradictoriness thus follows).

(17) (correctly) predicts all the sentences in (12) to exhibit contradictoriness; let's take once again (12)a, (12)b, and (12)e (repeated below as (18)a, (18)b, and (18)c).

(18) [CONTEXT: it is common ground that Benjamin is a member of Linguae.]
a. ^{c} Every member of Linguae likes John and Benjamin hates him.

[CONTEXT: it is common ground that Benjamin is a member of Linguae.]
b. ^{c} No member of Linguae likes John and Benjamin loves him.

c. ^{c} Paul didn't move and didn't stay still.

Let's begin with (18)a; (17)i is satisfied: (18)a has two predicates (namely, 'likes John' and 'hates (John)') that are connected in the matrix clause. (17)ii is also satisfied (because (17)ii.a is the case): 'likes John' and 'hates (John)' are pos-connected in the matrix clause and jointly empty throughout the context set. Let's now move to (18)b; (17)i is satisfied: (18)b has two predicates—namely, 'likes John' and 'loves (John)'—that are connected in the matrix clause. (17)ii is also satisfied (because (17)ii.c is the case): 'likes John' and 'loves (John)' are cross-connected in the matrix clause, 'No member of Linguae *P* and Benjamin *Q*' (if 'likes John' is identified with α and 'loves John' with β[5]) is such that $\forall f \forall w \in C([\![\text{No member of Linguae } P \text{ and Benjamin } Q]\!]^{w,f} = 1 \rightarrow \exists x \in \mathcal{D}([\![P]\!]^{w,f}(x) = 0 \wedge [\![Q]\!]^{w,f}(x) = 1))$, and '**doesn't** like~~s~~ John' and 'loves John' are jointly empty throughout the context set. Finally, let's consider (18)c; (17)i is satisfied: (18)a has two predicates that are connected in the matrix clause (it has more than two in fact: 'move'/'stay still', 'didn't move'/'stay still', 'move'/'didn't stay still', and 'didn't move'/'didn't stay still' are all pairs of connected predicates). No matter which of these pairs one picks, one of the three statements in (17)ii comes out true—for example, 'move' and 'stay still' are neg-connected and '**doesn't** move' and '**doesn't** stay still' are jointly empty throughout the context set; 'didn't move' and 'didn't stay still' are pos-connected and 'didn't move' and 'didn't stay still' are jointly empty throughout the context set; etc.

3.1.2.1 Promise fulfilled

At the outset of this section, I made a promise: to put forward a generalisation able to account for the three contrasts that the romantic's account cannot handle—see Table 1, contrasts (i), (ii), and (iii). (17) is that generalisation.

Let's start with contrast (i); as discussed, (6), repeated below as (19), exhibit contradictoriness in CONTEXT I but not in CONTEXT II.

(19) [CONTEXT I: it is common ground that Benjamin is a member of Linguae.]
[CONTEXT II: it is common ground that no member of Linguae likes John and, furthermore, that Benjamin is a member of Parlare (not of Linguae).]

[C-I: ^{c} | C-II: $^{¢}$] Every member of Linguae likes John, but Benjamin hates him.

How does (17) account for this contrast? The first thing to note is that CONTEXT I, unlike in CONTEXT II, entails that Benjamin is a member of Linguae; this difference alters the connection facts and, ultimately, the contradictoriness facts. Let's see this:

[5] Note that, if 'likes John' is identified with β and 'likes (John)' with α instead, (17)ii.c is also satisfied.

<u>Sentence</u>: ‘Every member of Linguae <u>likes John</u> but Benjamin <u>hates him</u>.’

CONTEXT I | C ⊨ Benjamin is a member of Linguae
For any f and for any world $w \in$ C, if $[\![$Every member of Linguae P and Benjamin $Q]\!]^{w,f} = 1$, then $[\![P]\!]^{w,f}(\text{Benjamin}'_w) = 1$ and $[\![Q]\!]^{w,f}(\text{Benjamin}'_w) = 1$.

‘likes John’ and ‘hates (John)’ are thus pos-connected in the matrix clause of (19); contradictoriness is expected because ‘likes John’ and ‘hates (John)’ are jointly empty throughout the context set.

CONTEXT II | C ⊨ Benjamin is not a member of Linguae
For any f and for any world $w \in$ C, if $[\![$Every member of Linguae P and Benjamin $Q]\!]^{w,f} = 1$, it’s not the case that $[\![P]\!]^{w,f}(\text{Benjamin}'_w) = 1$ and $[\![Q]\!]^{w,f}(\text{Benjamin}'_w) = 1$, it’s not the case that $[\![P]\!]^{w,f}(\text{Benjamin}'_w) = 0$ and $[\![Q]\!]^{w,f}(\text{Benjamin}'_w) = 0$, it’s not the case that $[\![P]\!]^{w,f}(\text{Benjamin}'_w) = 1$ and $[\![Q]\!]^{w,f}(\text{Benjamin}'_w) = 0$, and it’s not the case that $[\![P]\!]^{w,f}(\text{Benjamin}'_w) = 0$ and $[\![Q]\!]^{w,f}(\text{Benjamin}'_w) = 1$.

‘likes John’ and ‘hates (John)’, as shown above, are not connected via Benjamin (nor are they connected via any other entity, as can easily be checked); hence, these predicates are not expected to induce contradictoriness.

Let’s now move to contrast (ii), repeated in (20) below.

(20) [CONTEXT: it is common ground that the city of Tajiff is in Cuba.]

a. $^{\text{c}}$ Benjamin is in Tajiff and he isn’t in Cuba.
b. $^{\not\text{c}}$ Benjamin is in Tajiff and Tajiff isn’t in Cuba.

Through (20)a and (20)b are truth-conditionally equivalent, they are not equivalent when it comes to connection facts: indeed, ‘is in Tajiff’ and ‘isn’t is Cuba’ are connected (pos-connected to be precise) in the matrix clause of (20)a but not in the matrix clause of (20)b. To see this, let’s generate the relevant (impoverished) LFs.

(21) [CONTEXT: it is common ground that the city of Tajiff is in Cuba.]

a. Benjamin P and he Q. | ‘is in Tajiff’ has been replaced by P and ‘isn’t in Cuba’ by Q.
b. Benjamin P and Tajiff Q. | ‘is in Tajiff’ has been replaced by P and ‘isn’t in Cuba’ by Q.

If μ' in (15)iii is instantiated as (21)a, then (15)iii-a and (15)iii-b are both true, which means that ‘is in Tajiff’ and ‘isn’t is Cuba’ are pos-connected in the matrix clause of (20)a. Conversely, if μ' in (15)iii is instantiated as (21)b, then neither (15)iii-b, nor (15)iii-c, nor (15)iii-d are true, which means that ‘is in Tajiff’ and ‘isn’t is Cuba’ aren’t connected in the matrix clause of (20)b. The generalisation in (17) thus accounts for the contrast between (20)a and (20)b: in (20)a, ‘is in Tajiff’ and ‘isn’t in Cuba’ are pos-connected and jointly empty throughout the context set—contradictoriness is thus expected under (17); in

(20)b, 'is in Tajiff' and 'isn't in Cuba' are not connected—and, according to (17), connection is a pre-requisite for contradictoriness.

Finally, let's consider contrast (iii), repeated in (22).

(22) a. ¢ Bachelors have wives.
b. c Bachelors have wives and aren't married.

The observation is that (22)b exhibits contradictoriness while (22)a doesn't (it just feels false). Provided that 'bachelors' is treated as a referential expression—a kind-denoting expression, as in Carlson (1977)[6]—(17) can be invoked to make sense of this contrast. Indeed, on such an analysis, there are just two constituents of predicative type in (22)a—namely, 'have wives' and 'wives'—, and these two constituents aren't connected ('have wives' dominates 'wives'); thus, (22)a is not expected to exhibit contradictoriness under the generalisation in (17). In (22)b, on the other hand, there are two constituents of predicative type—namely, 'have wives' and 'aren't married'—that are pos-connected (via the kind individual denoted by 'bachelors') and jointly empty throughout the context set; thus, under the generalisation in (17), (22)b is expected to exhibit contradictoriness.[7]

For the same reason that it predicts contradictoriness in (22)b, (17) also predicts contradictoriness in (23)b.

(23) a. ¢ Bachelors have wives.
b. c Most of the bachelors in this room have a wife.

Indeed, in (23)b, just like in (22)b, there are two predicates—namely, 'bachelors in this room' and 'has a wife'—that are pos-connected (in the matrix clause) and jointly empty throughout the context set.[8]

Another good feature of (17) is that it has no issues dealing with the sentences in (10) and (11)—collected in (24) below.

(24) QUESTIONS
a. c Is it true that <u>Paul is *single* and *married*</u>?
b. c Is it true that <u>John *lives in Toulouse* and *doesn't live in France*</u>?

[6] Remember that, on Carlson's (1977) influential account, individuals consist of two basic sorts: 'object' individuals and 'kind' individuals.
[7] This (good) prediction relies of course on a theoretical assumption—namely, that 'bachelors' in (22)a-b denotes, or can denote, a kind. For recent discussions on this issue, see Liebesman (2011) and Leslie (2015).
[8] For any f and for any w, if $[\![\text{most of the P Q}]\!]^{f,w} = 1$, then there's x such that $[\![P]\!]^{f,w}(x) = 1$ and $[\![Q]\!]^{f,w}(x) = 1$. Hence, 'bachelors in this room' and 'has a wife' are pos-connected.

TAUTOLOGIES
c. ᶜ Either John *lives in Montmartre* but *doesn't live in Paris*, or it's false that he *lives in Montmartre* but *doesn't live in Paris*.
d. ᶜ It's false that John *lives in Montmartre* but *doesn't live in Paris*.

UNKNOWN TRUTH VALUE
e. ᶜ Either John is an artist, or he isn't an artist and he is both *single* and *married.*

None of the sentences in (24) are false—(24)a-b are not even truth-bearers!—; nonetheless, these sentences are expected to exhibit contradictoriness under (17). Indeed, the underlined clause in each of these sentences has two pos-connected predicates—highlighted in italics—that are jointly empty throughout the context set.

The promise has been fulfilled.

3.1.3 Gradience in contradictoriness judgments

Consider the pair of sentences in (25).

(25) a. ᶜ John lives in Paris but doesn't live in France.
b. ᶜ John was killed but didn't die.

These sentences differ from each other in an important respect: (25)a is not a necessary falsehood (there are possible worlds in which it is true), whereas (25)b is a necessary falsehood. This difference, though real, is orthogonal to (17): according to (17), what matters is whether the sentence has a clause that contains two predicates that are connected; and, if it does, whether one of the statements in (17)ii is the case. (As can be easily checked, (25)a and (25)b are both expected to exhibit contradictoriness under (17): the underlined predicates in each of these sentences are pos-connected and jointly empty throughout the context set.)

This is *prima facie* a good result: (25)a and (25)b do as a matter of fact exhibit contradictoriness. It could be argued, however, that (17), precisely because it isn't sensitive to whether the target sentence is (or isn't) a necessary falsehood, cannot make sense of the following observation (which I take to be uncontroversial): (25)a's contradictoriness is less pungent than that of (25)b.

Though it is true this contrast cannot be derived from (17), it can nonetheless be made sense of within the proposed framework. The predicates 'lives in Paris' and 'doesn't live in France' are jointly empty throughout the context set but not throughout Logical Space; as a result, it is possible to 'fix' (25)a by revising the context set—namely, by mentally recruiting a set of possibilities throughout which 'living in Paris' and 'not living in France' aren't jointly empty (e.g. a context set according to which Paris is an independent republic and no longer belongs to France). Such a strategy isn't available in (25)b: here the predicates 'was killed' and 'didn't die' are not just jointly empty throughout the context set but also throughout Logical Space; as result, it is not possible to 'fix' (25)b-c via context set revision: 'was killed' and 'didn't die' are jointly empty throughout any set of worlds.

4 Two refinements

4.1 Adding in local contexts

A (generally) nice consequence of (17) is that it predicts automatic projection: a sentence exhibits contradictoriness in C iff it contains a clause μ that contains two predicates that are connected in μ and meet some further conditions. Because of this, (17) predicts contradictoriness in all the sentences in (24). Sometimes, however, contradictoriness is not automatically inherited by the matrix clause. Consider, for example, (26).

(26) [CONTEXT: it is common ground between Alex and Sally that Paris is in France and also that Jo, their 5-year-old son, believes that Paris is in Italy.]

a. [Alex tells Sally:] ^{c} Benjamin lives in Paris but not in France
b. [Alex tells Sally:] $^{\not c}$ Jo believes that Benjamin lives in Paris but not in France.

(17) makes a bad prediction here: it predicts (26)b to exhibit contradictoriness (and, as far as I can tell, it doesn't). This problem arises because, in (17), the notion of context that is in operation is global (i.e. C is the context set) while (26)b, at least intuitively, calls for a local notion of context: intuitively, what matters is not whether the proposition $\{w : \text{Paris is in France in } w\}$ is in the common ground but rather whether Jo believes that proposition to be true.

Here I will not discuss local contexts in any great detail: suffice it to say that the local context of an expression *E* is typically identified with the information contributed by the preceding syntactic environment of *E* and the common ground. Local contexts are primarily invoked in theories of presupposition projection; consider, for example, the contrast in assertability between (27)a and (27)b:

(27) [CONTEXT: it is common ground between Alex and Sally that there is no king of France and also that Jo, their 5-year-old son, believes that France has a king.]

a. [Alex tells Sally:] # The king of France is rich.
b. [Alex tells Sally:] Jo believes that the king of France is rich.

The (arguably) most influential account of this contrast, which originates in Stalnaker (1974, 1978) and Karttunen (1974), goes as follows: (27)a is not assertable because the presupposition of the clause 'the king of France is rich' (i.e. there is a unique king of France) contradicts—and hence isn't satisfied—in its local context (the local context of 'The king of France is rich' is C, the context set, as there is no linguistic material that precedes the clause); (27)b, by contrast, is assertable because the presupposition of the clause 'the king of France is rich' is satisfied in its local context (in (27)b, the local context of 'the king of France is rich' is not C but the set of worlds compatible with those beliefs that are attributed to Jo in C[9]).

Let's now return to (26)b; as discussed, according to (17), (26)b should exhibit contradictoriness: this is because 'lives in Paris' and '(does) not live in France', which are pos-connected in the embedded (underlined) clause, are jointly empty throughout the context set.[10] Now, if what mattered was whether 'lives in Paris' and '(does) not live in France' are jointly empty (or not) throughout the local context of the clause in which they are pos-connected, the contrast between (26)a and (26)b would be accounted for: 'lives in Paris' and '(does) not live in France' are jointly empty throughout the local context of 'Benjamin lives in Paris but not in France' in (26)a (namely, the context set, which entails that Paris is in France) but not throughout the local context of 'Benjamin lives in Paris but not in France' in (26)b (namely, the set of worlds compatible with those beliefs that are attributed to Jo in C, which does not entail that Paris is in France).

Thus, to deal with cases such as (26)b, it seems reasonable to modify (17) as follows:

(28) **Contradictoriness v.2**

Let S be a sentence, μ a clause of S, α and β two one-place predicates, C the context set, **Cμ the local context of μ**, and $\mathcal{D}$ the set of all possible individuals. *P* and *Q* are two one-place predicate variables and *f* a variable over assignment functions from {*P*, *Q*} to $\mathcal{D}_{\langle s,\langle e,t\rangle\rangle}$.

Generalisation. S exhibits contradictoriness in C iff…

[9] This result (namely, that the local context of *p* in a sentence of the form 'X believes that *p*' is the set of worlds compatible with those beliefs that are attributed to X in C) is obtained both dynamic (e.g. Karttunen 1974, Heim 1992) and non-dynamic theories of presupposition projection (e.g. Schlenker 2009).

[10] Note that the embedded clause is the offending clause here: as it can be easily checked, the predicates 'lives in Paris' and '(does) not live in France' are not connected in the matrix clause of (26)b.

(i) α and β are connected in μ, and
(ii) one of the following statements is the case:

(a) α and β are pos-connected in μ and jointly empty throughout **Cμ**.
(b) α and β are neg-connected in μ and not-α and not-β are jointly empty throughout **Cμ**.
(c) α and β are cross-connected in μ, μ′—a clause just like μ except that α has been replaced by *P* and β by *Q*—is such that $\forall f \forall w \in \mathbf{C\mu}([\![\mu']\!]^{w,f} = 1 \rightarrow \exists x \in \mathcal{D}([\![P]\!]^{w,f}(x) = 1 \wedge [\![Q]\!]^{w,f}(x) = 0))$, and α and not-β are jointly empty throughout **Cμ**; or α and β are cross-connected in μ, μ′ is such that $\forall f \forall w \in \mathbf{C\mu}([\![\mu']\!]^{w,f} = 1 \rightarrow \exists x \in \mathcal{D}([\![P]\!]^{w,f}(x) = 0 \wedge [\![Q]\!]^{w,f}(x) = 1))$, not-α and β are jointly empty throughout **Cμ**.

The definition of predicate connection also needs to be adjusted along the same lines; consider, for example, (29)b.

(29) [CONTEXT: it is common ground between Alex and Sally that Ringo Starr was a member of The Beatles, and also that Jo (mistakenly) thinks that Ringo Starr wasn't a member of The Beatles but the singer of Black Sabbath]

a. [Alex tells Sally:] ᶜ Every member of The Beatles was a hippy and Ringo Starr wasn't a hippy.
b. [Alex tells Sally:] ¢ Jo believes that <u>every member of The Beatles was a hippy and Ringo Starr wasn't a hippy.</u>

Indeed, according to the definition in (15), the predicates 'was a hippy' and 'wasn't a hippy' are pos-connected (via Ringo Starr) in (29)b's underlined clause; since 'was a hippy' and 'wasn't a hippy' are jointly empty throughout the local context of this clause, (17) ends up (wrongly) predicting (29)b to exhibit contradictoriness. This problem is corrected in (30): according to this revised definition of predicate connection, the set of worlds that matters when calculating connection facts isn't the global context but the local context of the relevant clause.

(30) **Predicate connection v.2**

Let α and β be two one-place predicates, μ a clause, **Cμ the local context of μ**, and $\mathcal{D}$ the set of all possible individuals. *P* and *Q* are two one-place predicate variables and *f* a variable over assignment functions from {*P*, *Q*} to $\mathcal{D}_{\langle s,\langle e,t\rangle\rangle}$.

Definition. α and β are connected via an element of $\mathcal{D}$ in μ **relative to Cμ** iff…

(i) α and β are both constituents of μ,
(ii) α isn't dominated by β nor is β dominated by α, and
(iii) μ′—a clause just like μ except that α has been replaced by *P* and β by *Q*—satisfies (a) and least one of the other three conditions:

(a) $\exists f \exists w \in \mathbf{C\mu}([\![\mu']\!]^{w,f} = 1)$

(b) $\forall f \forall w \in \mathbf{C\mu}([\![\mu']\!]^{w,f} = 1 \rightarrow \exists x \in \mathcal{D} \text{ s.t. } [\![P]\!]^{w,f}(x) = 1 \wedge [\![Q]\!]^{w,f}(x) = 1)$
In such a case, we say that α and β are **pos-connected** (via some entity, in μ/Cμ).

(c) $\forall f \forall w \in \mathbf{C\mu}([\![\mu']\!]^{w,f} = 1 \rightarrow \exists x \in \mathcal{D} \text{ s.t. } [\![P]\!]^{w,f}(x) = 0 \wedge [\![Q]\!]^{w,f}(x) = 0)$
In such a case, we say that α and β are **neg-connected** (via some entity, in μ/Cμ).

(d) $\exists v_1, v_2 \in \{0,1\} \text{ s.t. } v_1 \neq v_2 \wedge$
$\forall f \forall w \in \mathbf{C\mu}([\![\mu']\!]^{w,f} = 1 \rightarrow \exists x \in \mathcal{D} \text{ s.t. } [\![P]\!]^{w,f}(x) = v_1 \wedge [\![Q]\!]^{w,f}(x) = v_2)$
In such a case, we say that α and β are **cross-connected** (via some entity, in μ/Cμ).

Under this revised definition, the predicates 'was a hippy' and 'wasn't a hippy', as shown below, are not pos-connected (via Ringo Starr) in (29)b's underlined clause.

> μ: 'Every member of The Beatles was a hippy and Ringo Starr wasn't a hippy.'
> Cμ ⊨ Ringo Starr wasn't a member of The Beatles
> For any f and for any world $w \in$ Cμ, if $[\![$Every member of The Beatles P and Ringo Starr $Q]\!]^{w,f} = 1$, then it's not the case that $[\![P]\!]^{w,f}(\text{Ringo-Starr}'_w) = 1$ and $[\![Q]\!]^{w,f}(\text{Ringo-Starr}'_w) = 1$, it's not the case that $[\![P]\!]^{w,f}(\text{Ringo-Starr}'_w) = 0$ and $[\![Q]\!]^{w,f}(\text{Ringo-Starr}'_w) = 0$, it's not the case that $[\![P]\!]^{w,f}(\text{Ringo-Starr}'_w) = 1$ and $[\![Q]\!]^{w,f}(\text{Ringo-Starr}'_w) = 0$, and it's not the case that $[\![P]\!]^{w,f}(\text{Ringo-Starr}'_w) = 0$ and $[\![Q]\!]^{w,f}(\text{Ringo-Starr}'_w) = 1$.

Though 'was a hippy' and 'wasn't a hippy' are jointly empty throughout the local context of (29)b's underlined clause, no contradictoriness is now expected: according to (30), 'was a hippy' and 'wasn't a happy' aren't pos-connected via Ringo Starr in (29)b's underlined clause (nor are they pos-connected via any other entity).

To conclude, it might be worth illustrating how (28)/(30) account for the following contrast:

(31) [CONTEXT: it is common ground between Alex and Sally that Ringo Starr was a member of The Beatles, and also that Jo (mistakenly) thinks that Ringo Starr wasn't a member of The Beatles but the singer of Black Sabbath]

a. ¢ Jo believes that <u>every member of The Beatles was a hippy and Ringo Starr wasn't a hippy.</u>
b. c Jo knows that <u>every member of The Beatles was a hippy and Ringo Starr wasn't a hippy.</u>

According to (30), 'was a hippy' and 'wasn't a hippy' aren't connected, neither in the underlined clause nor in the matrix clause of (31)a. No contradictoriness is thus expected. These predicates, however, are connected (pos-connected to be precise) in the matrix clause of (31)b: this is because a sentence of the form 'X knows *p*', under standard assumptions, is defined only if '*p*' is true in every world in C. Indeed, for any *f* and for any world *w* in the local context of (31)b's matrix clause (that is, in C), if $[\![$Jo knows that every member of The Beatles P and Ringo Starr $Q]\!]^{w,f} = 1$, then $[\![$every member of The Beatles P and Ringo Starr

Q ⟧^{w,f} = 1; and if ⟦every member of The Beatles P and Ringo Starr Q ⟧^{w,f} = 1, then ⟦P⟧^{w,f}(Ringo-Starr′$_w$) = 1 and ⟦Q⟧^{w,f}(Ringo-Starr′$_w$) = 1. (28) then makes the correct prediction: it predicts (31)b to exhibit contradictoriness (i.e. 'was a hippy' and 'wasn't a hippy' are pos-connected in the matrix clause of (31)b and jointly empty throughout C).

4.2 *n*-way contradictoriness

Consider (32), for example.

(32) ^{c} John married Jane, then married Paula, but never married twice.

This example exhibits contradictoriness; however, according to (17), it shouldn't: indeed, 'married Jane' and 'married Paula' are pos-connected, so are 'married Jane' and 'never married twice', and 'married Paula' and 'never married twice'; however, 'married Jane'/'married Paula' aren't jointly empty throughout the context set (nor are 'married Jane'/'never married twice' or 'married Paula'/'never married twice'). (32) appears to be a case of three-way contradictoriness, a case in which *three* connected predicated induce contradictoriness (it's not possible for an individual to be in the extension of 'married Jane' and 'married Paula' and, at the same time, be in the extension of 'never married twice').[11] To deal with such a case, our definitions need to be generalised as below, i.e. to any number of predicates greater than one.

(33) **Predicate connection v.3**

Let $I \coloneqq \{1, \dots, n\}$ be a subset of $\mathbb{N}$, A a set of one-place predicates of the same cardinality as I, X a set of one-place predicate variables of the same cardinality as I, h a surjective indexing function from I to A, k a surjective indexing function from I to X, μ a clause, Cμ the local context of μ, and $\mathcal{D}$ the set of all possible individuals. α_i is the element of X that h indexes with i, P_i the element of X that k indexes with i, and f a variable over assignment functions from X to $\mathcal{D}_{\langle s,\langle e,t\rangle\rangle}$.

Definition. The elements of A are connected via an element of $\mathcal{D}$ in μ relative to Cμ iff…

(i) the elements of A are all constituents of μ,
(ii) no two elements of A dominate each other in μ, and
(iii) μ′—a clause just like μ except that, for every $\alpha \in A$, α has been substituted by the unique $P \in X$ that bears the same index as α (namely, the unique $P \in X$ such that $k^{-1}(P) = h^{-1}(\alpha)$)— satisfies (a) and one of the other three conditions:[12]

[11] Thanks to Daniel Rothschild for drawing my attention to such cases.
[12] I'm assuming the following intepretation rule: if γ is an element of X, then, for any w, ⟦γ⟧^{w,f} = f(γ)(w); if γ is not an element of X, then, for any w, ⟦γ⟧^{w,f} = ⟦γ⟧^{w}. To avoid clutter, as in the previous definitions, I am omitting g, the assignment function that deals with the 'real' (as opposed to the artificially introduced) variables. (As in the previous definitions, the omission is harmless.)

(a) $\exists f \exists w \in C\mu([\![\mu']\!]^{w,f} = 1)$

(b) $\forall f \forall w \in C\mu([\![\mu']\!]^{w,f} = 1 \rightarrow \exists x \in \mathcal{D}$ s.t. $\bigwedge_{i=1}^{n} ([\![P_i]\!]^{w,f}(x) = 1))$
In such a case, we say that the elements of A are **pos-connected** (via some entity, in μ/C_μ).

(c) $\forall f \forall w \in C\mu([\![\mu']\!]^{w,f} = 1 \rightarrow \exists x \in \mathcal{D}$ s.t. $\bigwedge_{i=1}^{n} ([\![P_i]\!]^{w,f}(x) = 0))$
In such a case, we say that the elements of A are **neg-connected** (via some entity, in μ/C_μ).

(d) $\exists v_1, ..., v_n \in \{0,1\}$ s.t. $0 < \sum_{i=1}^{n} v_i < n \wedge \forall f \forall w \in C\mu([\![\mu']\!]^{w,f} = 1 \rightarrow \exists x \in \mathcal{D}$ s.t. $\bigwedge_{i=1}^{n} ([\![P_i]\!]^{w,f}(x) = v_i))$
In such a case, we say that the elements of A are **mix-connected** (via some entity, in μ/C_μ).

(34) **Contradictoriness v.3**

For any mix-connecting entity x (for any element of $\mathcal{D}$ via which the elements of A are mix-connected in $\mu/C\mu$), there are two associated non-empty and disjoint sets of indices—namely,

$I^{(+,x)} := \{i \in I : \forall f \forall w \in C\mu([\![\mu']\!]^{w,f} = 1 \rightarrow [\![P_i]\!]^{w,f}(x) = 1)\}$, and
$I^{(-,x)} := \{i \in I : \forall f \forall w \in C\mu([\![\mu']\!]^{w,f} = 1 \rightarrow [\![P_i]\!]^{w,f}(x) = 0)\}$.

These sets are put to use in (ii)c below.

Generalisation. S exhibits contradictoriness in C iff…

(i) the elements of A are connected in $\mu/C\mu$, and
(ii) one of the following statements is the case:

(a) The elements of A are pos-connected in $\mu/C\mu$ and, for any $w \in C\mu$, $\bigcap_{i \in I} \{y \in \mathcal{D} : [\![\alpha_i]\!]^{w}(y) = 1\} = \emptyset$.
(b) The elements of A are neg-connected in $\mu/C\mu$ and, for any $w \in C\mu$, $\bigcap_{i \in I} \{y \in \mathcal{D} : [\![\alpha_i]\!]^{w}(y) = 0\} = \emptyset$.
(c) The elements of A are mixed-connected in $\mu/C\mu$, and there is a mix-connecting entity x such that, for any $w \in C\mu$, $(\bigcap_{i \in I^{(+,x)}} \{y \in \mathcal{D} : [\![\alpha_i]\!]^{w}(y) = 1\}) \bigcap (\bigcap_{i \in I^{(-,x)}} \{y \in \mathcal{D} : [\![\alpha_i]\!]^{w}(y) = 0\}) = \emptyset$.

The definitions look more complex now but nothing of substance has changed: the difference is that our definitions can now deal with cases such as (32).

In the next section, I discuss some open problems; purely for the ease of exposition, I will just make reference to the 'binary' definitions of predicate connection and contradictoriness given in the previous section—i.e. **Predicate connection v.2** (in (30)) and **Contradictoriness v.2** (in (28)). The problems that will be discussed, however, are common to all the versions of predicate connection and contradictoriness so far considered.

5 Open problems

5.1 The constituency requirement

Contradictoriness v.2 has it that α and β, the predicates that will (or will not) induce contradictoriness, have to be constituents of a clause. But is this always the case? Consider (35), for example.

(35) a. ^{c} John killed George, but John didn't kill George.
b. ^{c} John killed George, but George didn't die.

(35)a exhibits contradictoriness; this is expected under **Contradictoriness v.2**: 'killed George' and 'didn't kill George' are pos-connected and jointly empty throughout the local context of the matrix clause (the context set). (35)b also exhibits contradictoriness; this, however, is unexpected: 'killed George' and 'didn't die' are not pos-connected (nor are they jointly empty throughout the context set). For **Contradictoriness v.2** to predict contradictoriness in (35)b, 'John killed' would need to be treated as a constituent; however, under standard assumptions, 'John killed' is not a constituent.

What does (35) tell us about **Predicate connection v.2**? Here's a possible response: it tells us that this definition needs to be paired with a grammar that implements a notion of constituency that is more flexible than the generative one—in particular, a grammar in which 'John killed' can act as a constituent. Grammars of this sort do exist: in a categorial grammar, for example, both parses '[[John killed] [George]]' and '[[John] [killed George]]' are in principle possible.[13] Thus, one idea would be to adjust **Predicate connection v.2** so that μ stands for a set—namely, the set of (truth-conditionally equivalent) parses that a clausal string can have in a categorial grammar; in (35)b, such a set would be {'[[[John Killed] [George]] but [[George] [didn't die]]]', '[[[John] [killed George]] but [[George] [didn't die]]]'}. **Predicate connection v.2** could then be reformulated as follows: 'two constituents α and β of predicative type are connected in μ via an element of $\mathcal{D}$ relative to Cμ iff there is a clause ψ in μ such that (...)'. On this syntactically more liberal formulation, 'John killed' and 'didn't die' would come out pos-connected (via George); contradictoriness would then be expected—'John killed' and 'didn't die' are jointly empty throughout the context set.

A different response should be considered though; indeed, (35) can also be interpreted as follows: the assumption on which **Predicate connection v.2** is built upon—namely, that only constituents can be

[13] The combinatory rules of a categorial grammar enable a transitive verb to combine with the subject first, an operation that yields a compound constituent that can then combine with the object. For an overview of categorical grammar, see Steedman (1993) and Steedman and Baldridge (2011). Categorial grammars were first proposed in Ajdukiewicz (1935) and Bar-Hillel (1953).

connected—is incorrect. Of course, the contradictoriness-inducing predicates need to be, in some sense, part of the contradictoriness-exhibiting sentence, but the relevant notion of parthood could be in principle silent as to whether these predicates are constituents (either in the generative-grammar sense or categorial-grammar sense). It is left for further research to determine whether the notion of connection can be defined without requiring the connected parts to be constituents.

To sum up, I don't think that contrasts such as (35) pose an existential threat to the proposed generalisation. However, it does suggest that its present formulation is not entirely adequate—in particular, it seems clear that there are one-place predicate meanings that, despite not being meanings of linguistic constituents (at least not in the generative-grammar sense), should nonetheless be considered when checking for contradictoriness.

5.2 Negated possibility modals

As Benjamin Spector noted to me, **Contradictoriness v.2** fails to make sense of the contrasts reported in (36) and (37).

(36) a. ^{c} It is not true that Benjamin lives in Toulouse but not in France.
b. $^{\not c}$ It can't be true that Benjamin lives in Toulouse but not in France.
c. $^{\not c}$ It's (just) not possible that Benjamin lives in Toulouse but not in France.

(37) a. ^{c} It is not true that Benjamin is married and doesn't have a wife.
b. $^{\not c}$ It can't be true that Benjamin is married and doesn't have a wife.
c. $^{\not c}$ It's (just) not possible that Benjamin is married and doesn't have a wife.

For some reason, contradictoriness vanishes in the scope of a negated possibility modal such that 'it can't be true that' or 'it is not possible that', but it is retained in the scope of sentential negation (e.g. 'it is not true that'). Why is this so, I do not know. The account presented here, has, as far as I can tell, no resources to make sense of such contrasts.[14]

Matthew Mandelkern (p.c.) has suggested to me that (36)b-c/(37)b-c, but not (36)a/(37)a, might have metalinguistic readings. This could explain why a sentence like (37)b 'escapes' contradictoriness: it escapes contradictoriness because it can be read as 'It can't be true that «Benjamin is married and doesn't have a wife»' (that is, it can be read as a statement about a sentence). For this to amount to an account of the

[14] Note that local contexts aren't of much help here: even if the local context of *p* 'it's not possible that *p*' was the set of all possible worlds, 'married' and 'doesn't have a wife' would still come out pos-connected in the embedded clause; and these two predicates are jointly empty throughout any set of worlds.

contrasts reported in (36) and (37), one would need an explanation for why (36)b-c/(37)b-c, but not (36)a/(37)a, have metalinguistic readings. In this absence of this explanation, (36) and (37) do constitute a challenge to the generalisation given.

5.3 Higher-order predicate connection

The sentence in (38) clearly exhibits contradictoriness; this, however, isn't expected under **Contradictoriness v.2**.

(38) [c] None or all of the students speak French, and exactly half of them speak French.

Indeed, for **Contradictoriness v.2** to predict contradictoriness in this case, 'speak French' and 'speak French' would need to be cross-connected ('speak French' and its negation are jointly empty throughout the context set). These predicates, however, aren't cross-connected, as shown below.[15]

> μ: 'None or all of the students speak French and exactly half of them speak French.'
> μ′: 'None or all of the students *P* and exactly half of them *Q*.'
> $C\mu = C$
>
> If 'speak French' and 'speak French' were to be cross-connected, then one of the two statements below would be true:
>
> (i) $\forall f \forall w \in C([\![\mu']\!]^{w,f} = 1 \rightarrow \exists x \in \mathcal{D}([\![P]\!]^{w,f}(x) = 1 \wedge [\![Q]\!]^{w,f}(x) = 0)$
> (ii) $\forall f \forall w \in C([\![\mu']\!]^{w,f} = 1 \rightarrow \exists x \in \mathcal{D}([\![P]\!]^{w,f}(x) = 0 \wedge [\![Q]\!]^{w,f}(x) = 1)$
>
> It is trivial to find counterexamples to both (i) and (ii), however. Let's suppose that the extension of 'students' is constant throughout C-worlds, e.g. for every w in C, $\{x : [\![\text{students}]\!]^{w}(x) = 1\}$ = {John, Paul, Mary, Jo}. Furthermore, let's suppose that f_1 and w_1 are such that w_1 is an element of C, $\{x : [\![P]\!]^{f_1,w_1}(x) = 1\}$ = {John, Paul, Mary, Jo}, and $\{x : [\![Q]\!]^{f_1,w_1}(x) = 1\}$ = {John, Paul}. This is a counterexample to (ii): $[\![\mu']\!]^{f_1,w_1} = 1$ yet there is no $x \in \mathcal{D}$ such that $[\![P]\!]^{f_1,w_1}(x) = 0$ and $[\![Q]\!]^{f_1,w_1}(x) = 1$. Let's now suppose that w_2 is also an element of C, $\{x : [\![P]\!]^{f_1,w_2}(x) = 1\} = \emptyset$, and $\{x : [\![Q]\!]^{f_1,w_2}(x) = 1\}$ = {John, Paul}. This is a counterexample to (i): $[\![\mu']\!]^{f_1,w_2} = 1$ yet there is no $x \in \mathcal{D}$ such that $[\![P]\!]^{f_1,w_2}(x) = 1$ and $[\![Q]\!]^{f_1,w_2}(x) = 0$. Thus, 'speak French' and 'speak French' aren't cross-connected in the matrix clause of (38).

However, as Benjamin Spector pointed out to me, a natural extension of the proposed generalisation could accommodate cases such as (38). Indeed, though according to **Predicate connection v.2** only first-order predicates can be connected, this definition can (easily) be extended to also generate connection facts between generalised quantifiers (namely, second-order predicates); and, under such an extension, (38)'s

[15] As a matter of fact, they aren't connected in any way.

generalised quantifiers—namely, 'none or all of the students' and 'exactly half of (the students)—do come out pos-connected in the matrix clause (via some element in $\mathcal{D}_{\langle e,t\rangle}$). Indeed, let P^2 and Q^2 be variables of type $\langle s,\langle\langle e,t\rangle,t\rangle\rangle$ and f a variable over assignment functions from $\{P, Q, P^2, Q^2\}$ to $\bigcup\{\mathcal{D}_{\langle s,\langle e,t\rangle\rangle}, \mathcal{D}_{\langle s,\langle\langle e,t\rangle,t\rangle\rangle}\}$; then, for any f and for any $w \in$ C, if $[\![P^2$ speak French and Q^2 speak French$]\!]^{w,f} = 1$, then $[\![P^2]\!]^{w,f}([\![$speak French$]\!]^{w,f}) = 1$ and $[\![Q^2]\!]^{w,f}([\![$speak French$]\!]^{w,f}) = 1$ (this just follows from the meaning of 'and'). (38)'s perceived contradictoriness would then be expected: 'none or all of the students' and 'exactly half of (the students)' are jointly empty throughout the context set, i.e. there is no world w in C and no one-place predicate denotation X such that $[\![$exactly half of (the students)$]\!]^{w}(X) = 1$ and $[\![$none or all of the students$]\!]^{w}(X) = 1$.

In sum, the proposed generalisation fails to account for examples such as (38). One possible solution, as discussed, would be to extend it to allow second-order predicates to also induce contradictoriness. It is left for future research to determine whether such a solution is appropriate.

5.4 Conditionals

The parallelism between (39) and (40) suggests that, when it comes to contradictoriness, indicative conditionals behave just like conjunctions.

(39) a. ^{c} If John was killed, then John didn't die.
b. ^{c} If John didn't die, then John was killed.

(40) a. ^{c} John was killed and John didn't die.
b. ^{c} John didn't die and John was killed.

As discussed, the generalisation proposed predicts (40)a-b to exhibit contradictoriness: the underlined predicates are pos-connected in the matrix clause and jointly empty throughout the context set. The generalisation proposed, however, fails to predict contradictoriness in (39)a-b, at least under standard assumptions concerning the meaning of conditional sentences. Indeed, under standard assumptions, the indicative conditional 'If John α, then he β' can be true in a world w despite John not being in the extension of α at w; in other words, under standard assumptions, the conditional 'If John α, then he β' doesn't entail that its antecedent is true. For this reason, the underlined predicates in (39)a-b are not expected to come out pos-connected (unlike the predicates in (40)a-b): indeed, given a material-conditional or strict-conditional analysis of the indicative conditional, it is not the case that, for any f and for any $w \in$ C, if $[\![$If John P, then he didn't $Q]\!]^{w,f} = 1$, then $[\![P]\!]^{w,f}(\text{John}'_w) = 1$ and $[\![Q]\!]^{w,f}(\text{John}'_w) = 1$.

This failure of the proposed generalisation suggests, at least *prima facie*, that **Predicate connection v.2** is stronger than it needs to be. Indeed, there is a weaker formulation of predicate connection, given in (41) below, that can help with cases such (39)a-b (at least under certain assumptions).

(41) **Predicate connection v.2′ (weakened version)**

Let α and β be two one-place predicates, μ a clause, Cμ the local context of μ, and $\mathcal{D}$ the set of all possible individuals. *P* and *Q* are two one-place predicate variables and *f* a variable over assignment functions from {*P*, *Q*} to $\mathcal{D}_{\langle s,\langle e,t\rangle\rangle}$.

Definition. α and β are connected via an element of $\mathcal{D}$ in μ relative to Cμ iff…

(i) α and β are both constituents of μ,
(ii) α isn't dominated by β nor is β dominated by α, and
(iii) μ′—a clause just like μ except that α has been replaced by *P* and β by *Q*—satisfies (a) and least one of the other three conditions:

(a) $\exists f \exists w([\![\mu']\!]^{w,f} = 1)$

(b) $\forall f \forall w \in C\mu([\![\mu']\!]^{w,f} = 1 \rightarrow \exists w' \in C\mu\ \exists x \in \mathcal{D} \text{ s.t. } [\![P]\!]^{w',f}(x) = 1 \wedge [\![Q]\!]^{w',f}(x) = 1)$
In such a case, we say that α and β are **pos-connected** (via some entity, in μ/Cμ).

(c) $\forall f \forall w \in C\mu([\![\mu']\!]^{w,f} = 1 \rightarrow \exists w' \in C\mu\ \exists x \in \mathcal{D} \text{ s.t. } [\![P]\!]^{w',f}(x) = 0 \wedge [\![Q]\!]^{w',f}(x) = 0)$
In such a case, we say that α and β are **neg-connected** (via some entity, in μ/Cμ).

(d) $\exists v_1, v_2 \in \{0,1\} \text{ s.t. } v_1 \neq v_2\ \wedge$
$\forall f \forall w \in C\mu([\![\mu']\!]^{w,f} = 1 \rightarrow \exists w' \in C\mu\ \exists x \in \mathcal{D} \text{ s.t. } [\![P]\!]^{w',f}(x) = v_1 \wedge [\![Q]\!]^{w',f}(x) = v_2)$
In such a case, we say that α and β are **cross-connected** (via some entity, in μ/Cμ).

Not much has changed—only the existential quantifier '$\exists w' \in C\mu$' has been appended next to '$\exists x \in \mathcal{D}$'. Let's take pos-connection, for example; according to (41), for two predicates α and β to be pos-connected in μ, it is no longer required that, for every *f* and for every $w \in C\mu$, if $[\![\mu']\!]^{w,f} = 1$, then $\exists x \in \mathcal{D}$ s.t. $[\![P]\!]^{w,f}(x) = 1 \wedge [\![Q]\!]^{w,f}(x) = 1$; all that is required is that, for every *f* and for every $w \in C\mu$, if $[\![\mu']\!]^{w,f} = 1$, then $\exists w' \in C\mu$ $\exists x \in \mathcal{D}$ s.t. $[\![P]\!]^{w',f}(x) = 1 \wedge [\![Q]\!]^{w',f}(x) = 1$.

Let's see how **Predicate connection v.2′** can help with (39)a-b. I'm going to assume that an (unembedded) indicative conditional is a modal statement that expresses the speaker's beliefs, and that the speaker's belief state in a C-world is a subset of C. Furthermore, I'll assume the following 'strict' semantics for the indicative conditional: $[\![\text{if } p, \text{ then } q]\!]^{w}$ is defined only if there is a world $w' \in B_w$ such that $[\![p]\!]^{w'} = 1$ (where B_w is the speaker's belief state in *w*); whenever defined, $[\![\text{if } p, \text{ then } q]\!]^{w} = 1$ iff in every world $w' \in B_w$, if $[\![p]\!]^{w'} = 1$, then $[\![q]\!]^{w'} = 1$.

With these assumptions on board, **Predicate connection v.2′** delivers the right results for (40)a-b, i.e. it predicts 'was killed' and 'didn't die' to be pos-connected (via John) both in the matrix clause of (40)a as well as in the matrix clause of (40)b. Indeed, for any f and for any $w \in$ C, if $[\![$if John P, then John $Q]\!]^{w,f} =$ 1, then it must be the case that there is a world $w' \in B_w$ (a subset of C) such that $[\![$John $P]\!]^{w',f} = 1$, and also that in every world $w' \in B_w$, if $[\![$John $P]\!]^{w',f} = 1$, then $[\![$John $Q]\!]^{w',f} = 1$. From this is follows that, for any f and for any $w \in$ C, if $[\![$if John P, then John $Q]\!]^{w,f} = 1$, then there is at least one world $w \in$ C such that $[\![P]\!]^{w,f}(\text{John}'_w) = [\![Q]\!]^{w,f}(\text{John}'_w) = 1$. Thus, if predicate connection is defined as in (41), 'was killed' and 'didn't die' would be expected to induce contradictoriness in (40)a-b ('was killed' and 'didn't die' are jointly empty throughout the context set).

Predicate connection v.2′, I believe, is an attractive modification of **Predicate connection v.2**: if two predicates are connected according to **Predicate connection v.2**, they will also be connected according to **Predicate connection v.2′**, as the latter is weaker than the former; unlike **Predicate connection v.2**, however, **Predicate connection v.2′** has the advantage that, at least under some assumptions, it delivers the connection facts that one needs to predict contradictoriness in indicative conditionals. Caution is needed, however. Though **Predicate connection v.2′** helps with (39)a-b, it does not help with (42)a-b, which also exhibit contradictoriness.

(42) a. ^{c} If John had been killed, then he wouldn't have died.
b. ^{c} If John wouldn't have died, then he had been killed.

Indeed, a counterfactual conditional does not presuppose that, as far as the speaker knows, the antecedent may be true (rather, it presupposes that, as far as the speaker knows, the antecedent is false); as a result, in (42)a-b, connection between the relevant predicates cannot be established via **Predicate connection v.2′**. A challenge for future research would be to determine whether it is possible to weaken **Predicate connection v.2′** even further so that it can also handle counterfactuals.

6 Conclusion

Contradiction and contradictoriness are different things. Contradiction is a theoretical notion—e.g. falsehood in every possible world, falsehood under all possible uniform substitutions of non-logical words. Contradictoriness, by contrast, is a phenomenon. In this paper, I have shown that standard notions of contradiction are of little use when it comes to describing the phenomenon of contradictoriness. I have also developed the notion of predicate connection—and put forward a generalisation that makes use of this notion—in an attempt to provide an adequate characterisation of this phenomenon. Whether I succeeded, it is not yet clear—it depends, to a large extent, on whether the problems noted in the previous section can be overcome within the conceptual boundaries of the account proposed.

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