Disjunction in a predictive theory of anaphora

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Abstract. In this paper I develop a dynamic semantics for a first-order fragment, which incorporates insights from work on anaphora in logic, and the trivalent approach to presupposition projection. The resulting system — EDS — has interesting features which set it apart, both conceptually and empirically, from earlier iterations of dynamic semantics. Conceptually, the meanings of the logical connectives are derived by systematically generalizing the Strong Kleene connectives into a dynamic setting — the system is thereby *predictive*, drawing a tight connection between the logic of presupposition projection and patterns of anaphoric accessibility. On the empirical side, EDS diverges sharply from earlier proposals. In this paper, I focus mainly on disjunction, arguing that EDS provides a simple and elegant account of the dynamics of disjunction, including traditionally problematic cases such as Partee disjunctions and program disjunctions.

Keywords: disjunction, presupposition, anaphora

1 Introduction

Dynamic theories of natural language semantics traffic in anaphoric information. Pretty much everyone agrees that indefinites and pronouns are special — indefinites *introduce* anaphoric information — Karttunen's *discourse referents* [1] — and pronouns *retrieve* anaphoric information. A pertinent question arises: to what extent to one needs to make reference to anaphoric information in the semantics of other expressions, such as logical vocabulary (*and*, *or*, etc.)? Many proposals submit that logical expressions may arbitrarily encode a complex set of instructions for regulating the flow of anaphoric information [2, 3].

In this paper, I'll develop a different kind of dynamic semantics, taking anaphoric dependencies in disjunctive sentences as a case study. I'll maintain the idea that indefinites and pronouns are special, but I'll explore the possibility that we can make use of independently motivated machinery for explaining *presupposition projection* — concretely, the Strong Kleene logic of indeterminacy — in order to help us understand *why* different logical expressions regulate anaphoric information in just the way that they do.

In Sect. 2 I provide a brief précis of Dynamic Predicate Logic (DPL) [3]. This will serve two purposes:

1. The Strong Kleene dynamic logic which I develop in this paper will make use of notions first made precise in DPL.

2. DPL will serve as a good representative of (a certain family of) dynamic theories of natural language semantics.

In Sect. 3 I'll discuss empirical challenges for DPL, focusing on the internal and external dynamics of disjunction. This naturally leads into Sect. 4, where I develop a new logic of anaphora: EDS. This logic is based on the idea that it's possible to embed the logic of presupposition projection into a dynamic setting by computing three DPL-style meanings in tandem, corresponding to the three truth values of trivalent logic. In Sect. 5, I embed EDS in a concrete discourse pragmatics, by adopting Heim's notion of an information state [2], together with a concrete bridge principle. This will be essential in order to understand how the permissiveness of EDS might be reigned in. Finally, in Sect. 6, I briefly survey some recent, related approaches to anaphora before concluding.

2 Dynamic Predicate Logic

DPL [3] provides a dynamic interpretation for a simple first-order calculus. The interpretation of a sentence is a *relation between assignments* — I will assume some familiarity with DPL in this paper, so the presentation will remain rather terse. Before sketching out the details, note that I depart in a couple of notable ways from the presentation of Groenendijk and Stokhof [3] (henceforth G&S). Firstly, following, e.g., van den Berg's presentation, *discourse referent introduction* is cashed out as random assignment. Furthermore, DPL interpretations are stated relative to a world of evaluation.[1]

(1) **Dynamic Predicate Logic:**
   a. if ϕ is atomic, then $[\![\phi]\!]^w := \{\, (g, h) \mid g = h \wedge [\phi]^{w,g} \text{ is } \textbf{true} \,\}$
   b. $[\![\varepsilon_v]\!]^w := \{\, (g, h) \mid g[v]h \,\}$
   c. $[\![\phi \wedge \psi]\!]^w := [\![\phi]\!]^w \circ [\![\psi]\!]^w$
   d. $[\![\neg\phi]\!]^w := \{\, (g, h) \mid g = h \wedge \{\, i \mid (g, i) \in [\![\phi]\!]^w \,\} = \emptyset \,\}$

Atomic sentences (1a) are *tests*, i.e., they do not introduce anaphoric information, but merely assess (classical) truth with respect to an assignment g. Random assignment (1b) is responsible for introducing anaphoric information: ε_v is a privileged tautology, which in a dynamic setting means that for every assignment g, there is some assignment h, s.t., $(g, h) \in [\![\varepsilon_v]\!]^w$. Concretely, ε_v indeterministically assigns a value to v — $g[v]h$ holds just in case g and h differ at most in the value they assign to v. Random assignment is used to introduce discourse referents, which are threaded from left-to-right via dynamic conjunction (1c), which is just relational composition.[2] For example, the sentence "there is

[1] This will later prove useful when embedding EDS in a concrete discourse pragmatics.
[2] The definition of relational composition (which is totally standard) is given below:

$$R \circ S := \{\, (g, i) \mid \exists h[(g, h) \in R \wedge (h, i) \in S] \,\}$$

This operation plays a central role in both DPL and EDS.

a bathroom" is translated into DPL as $\varepsilon_v \wedge B(v)$. The open sentence $B(v)$ is interpreted relative to the discourse referent introduced by random assignment, thereby narrowing down assignments just to those that map v to a bathroom in w. This is illustrated below in (2).

(2) $[\![\varepsilon_v \wedge B(x)]\!]^w$
 a. $= [\![\varepsilon_v]\!]^w \circ [\![B(x)]\!]^w$
 b. $= \{\, (g,h) \mid g[v]h \,\} \circ \{\, (g,h) \mid g = h \wedge g(v) \in I_w(B) \,\}$
 c. $= \{\, (g,h) \mid g[v]h \wedge h(v) \in I_w(B) \,\}$

More generally conjunction in DPL is associative, thanks to the associativity of relational composition:

(3) **Associativity of dynamic conjunction (DPL):**
$\phi \wedge (\psi \wedge \sigma) \iff (\phi \wedge \psi) \wedge \sigma$

Associativity, together with random assignment, underlies *Egli's theorem*, which ecapsulates the DPL account of discourse anaphora: "there is a^v bathroom, and it$_v$'s upstairs" is semantically equivalent to "there is a^v bathroom upstairs" in DPL.

(4) **Egli's theorem (DPL):**
$\varepsilon_v \wedge (\phi \wedge \psi) \iff (\varepsilon_v \wedge \phi) \wedge \psi$

Dynamic negation (1d) will have an important role to play in the following discussion, so it's worth dwelling on. The definition of negation in DPL is tailored to ensure that negative sentences are *anaphorically inert*. Concretely, negative sentences are *tests*, which succeed if the scope ϕ is false with respect to an assignment. This means that placing, e.g., (2) in the scope of negation results in a sentence that is (a) anaphorically inert, and (b) has the truth-conditions of a negative existential statement.

(5) $[\![\neg(\varepsilon_v \wedge B(v))]\!]^w$
 a. $= \{\, (g,h) \mid g = h \wedge \{\, i \mid (g,i) \in [\![\varepsilon_v \wedge B(v)]\!]^w \,\} = \emptyset \,\}$
 b. $= \{\, (g,h) \mid g = h \wedge I_w(B) = \emptyset \,\}$

Existential quantification is defined syncategorematically in terms of random assignment and dynamic conjunction (6a). Disjunction (6b) and implication (6c) are defined syncategorematically in terms of classically equivalent sentences. A hallmark of DPL is that it matters exactly *which* classical equivalences one uses to define these connectives — the choise is *crucial* for constraining the flow of anaphoric information in complex sentences.

(6) **DPL definitions**
 a. $\exists_v \phi := \varepsilon_v \wedge \phi$
 b. $\phi \vee \psi := \neg(\neg\phi \wedge \neg\psi)$
 c. $\phi \to \psi := \neg(\phi \wedge \neg\psi)$

Since disjunction is the primary focus of this paper, I'll focus on (6b). Since a disjunctive sentence is a negative sentence it is anaphorically inert — a discourse referent introduced in a disjunction cannot be retrieved by a subsequent open sentence. To use DPL terminology, disjunction is *externally static* (by way of contrast, conjunction is externally *dynamic*). This is taken to be desirable, since as noted by G&S, anaphora from out of a disjunctive sentence is seemingly impossible in natural language:[3]

(7) Either this house is derelict, or there's a^v bathroom. #It_v's upstairs.

Furthermore, since each disjunct is itself a negative sentence, each disjunct is anaphorically inert, and therefore anaphora between disjuncts is impossible. In DPL terminology, disjunction is *internally static* (again, conjunction in contrast is internally dynamic). Similarly, G&S suggest that this is desirable on the basis of natural language — the following example is from [4, p. 245].

(8) #Either Jones owns a^v bicycle, or it_v's broken.

Implication will not be the main focus of this paper, but briefly — DPL implication is tailored to derive *universal* readings for the famous case of Donkey Sentences. I.e., (9) is taken to be equivalent to *every bathroom is upstairs.* More generally, *Egli's corollory* holds in DPL.[4] With regards to anaphora, implication is internally dynamic but externally static.

(9) If there's a^v bathroom, it_v's upstairs.

(10) **Egli's corollary**
$\exists_x \phi \to \psi \iff \forall_x (\phi \to \psi)$

DPL provides a simple and elegant logic of anaphoric information, but the way in which disjunction and implication are defined is rather *ad hoc* — as noted previously, it really matters which classical equivalences one uses to define disjunction and implication, and which operations one takes to be basic. For example, it wouldn't do to take dynamic disjunction to be a basic operation, and thereby define conjunction as $\neg(\neg\phi \vee \neg\psi)$ via de Morgan's equivalence. This would predict that anaphora should be impossible between conjuncts, despite the fact that the truth-conditional import of such a conjunction would be classical. There is therefore no obvious way of *determining* the DPL semantics of a logical operator given its truth-conditional import.

Furthermore, there are a multitude of possible connectives that are definable in DPL which would manipulate anaphoric information in a way which wouldn't correspond to their language counterparts. I take that it would be desirable to have a general recipe for determining how logical connectives manipulate anaphoric information, on the basis of their truth-conditional contribution.

[3] G&S importantly assume that a multi-sentence discourse is translated into DPL as a conjunctive sentence.
[4] N.b. the universal quantifier is defined as the dual of the existential.

This is related to the explanatory problem for dynamic semantics, discussed most frequently with respect to Heim's satisfaction theory presupposition projection [5] (see, e.g., [6, 7, 8, 9]). The conceptual problem is equally acute in the case of anaphora, and unlike the case of presupposition projection, fewer alternatives have been explored.[5]

3 Empirical challenges

3.1 Double negation

It has been continuously pointed out, including by G&S themselves and in much subsequent work, that the empirical predictions made by DPL often do not match up with our intuitions about natural language. The most straightforward challenge is that, in DPL, Double Negation Elimination (DNE) isn't valid. This is because, since any negative sentence is a test, a doubly-negated sentence is always a test. However, as pointed out by Krahmer and Muskens [15] and Gotham [16] among others, doubly-negated sentences often license anaphora. The following example is from [15].

(11) It's not true that John didn't bring an^v umbrella.
It$_v$ was purple and stood in the hallway.

One desideratum of the account developed in Sect. 4 is to have a dynamic logic in which DNE *is* valid. There are some potential objections which are worth immediately addressing. Gotham [16] suggests that the facts are more nuanced, in that doubly-negated sentences may carry inferences that their positive counterparts lack. His example is given in (12). His point is that this discourse sounds odd because it implies that John owns a single shirt; the conclusion is that $\neg\neg\exists_v\phi$ carries a uniqueness inference that $\exists_v\phi$ lacks.

(12) It's not true that John doesn't own a^v shirt. ?It$_v$'s in the wardrobe.

I however agree with Mandelkern (cited as p.c. in [16]) that it's possible to demonstrate that doubly-negated sentences do not systematically entail uniqueness. This can be shown by using a generalization of Heim's famous sage plant example.

(13) It's not the case that Sue didn't buy a^v sage plant.
In fact, she bought eight others along with it$_v$!

[5] As emphasized by Mandelkern and Rothschild [10] the kind of situation-based e-type approach to anaphora developed in [11] and refined in [12, 13] does *not* (in its current state, at least) consitute a viable alternative. E-type theories have not addressed in detail how to capture notions of anaphoric accessibility in complex sentences, beyond donkey sentences, and as shown in [14], were they to do so, they would require entries for the logical connectives which manipulate minimal situations in an apparently arbitrary fashion.

Moreover, the minimal positive counterpart of (12) to my ear also strongly implies that John only owns a single shirt. It's certainly an interesting question to ask how exactly such uniqueness inferences arise, but for the purposes of this paper, I'll be setting them to one side.

(14) John does own a^{v} shirt. ?It_{v}'s in the wardrobe.

Besides, developing a dynamic logic in which DNE is valid will have positive ramifications elsewhere, for example in the treatment of disjunction. I take it that developing a dynamic logic in which DNE is valid is a reasonable starting point; doubly-negated sentences undoubtedly differ from their positive counterparts in certain respects, but this is somewhat unsurprising, especially from a Gricean perspective.[6] Having outlined the problems associated with negation in DPL, I now turn to the main focus of this paper: disjunction.

3.2 Partee disjunctions

As noted, DPL disjunction is internally static. A famous example originally due to Barbara Partee (henceforth: *Partee disjunctions*) suggests that this isn't quite right for natural language. (15) is in a sense doubly surprising in the context of DPL, since as well as seemingly involving an anaphoric dependency between disjuncts, it also seemingly involves anaphoric information introduced by a negative sentence (the first disjunct).

(15) Either there's no^{v} bathroom, or it_{v}'s upstairs.

At this stage, it's worth establishing some desiderata for the eventual treatment of Partee disjunctions, since there is some disagreement in the literature on their truth-conditions. For example, [15] suggests that (15) has universal truth conditions, by analogy with the DPL treatment of donkey sentences. Their analysis predicts that (15) implies that *every bathroom is upstairs*. Related to the discussion of double negation, Gotham claims that (15) carries a conditional uniqueness inference, i.e., *if there is a bathroom, then there is exactly one*. Even if universal/uniqueness readings exist, I argue here that both are at least sometimes too strong. Much like donkey sentences,[7] Partee disjunctions can have existential readings. (16) is true just in case (a) Gabe has no credit card, (b) Gabe has at least one credit card and paid with one of his credit cards. Crucially for the present point, (16) is true if Gabe has a credit card he paid with, and one that he didn't.

[6] If ϕ and $\neg\neg\phi$ are equivalent, then choosing to use a sentence of the form $\neg\neg\phi$ is naturally expected to trigger a *Manner* implicature. I leave the interesting question of the pragmatics of doubly-negated sentences to future work.

[7] (16) is in fact modelled after the following well-known example used to motivate existential readings of donkey sentences (attributed by [17, p. 63] to Robin Cooper).

(1) Yesterday, every person who had a credit card paid his Bill with it.

(16) Either Gabe doesn't have a credit card, or he paid with it.

(16) is already incompatible with uniqueness given the provided context, but just to drive home the point, I provide a disjunctive variant of Heim's sage plant sentence (following [10]).

(17) Either Sue didn't buy a^v sage plant,
or she bought eight others along with it$_v$.

It's important to mention at this point that the possibility of anaphora in Partee disjunctions parallels facts concerning presupposition projection. Despite the fact that the a definite description typically presupposes uniqueness, (18) lacks a corresponding uniqueness inference.

(18) Either there isn't a bathroom, or the bathroom is upstairs.

The account of Partee disjunctions which I develop in Sect. 4 leans on this parallel, ultimately unifying (15) and (18) by generalizing the Strong Kleene logic of indeterminacy to a dynamic setting.

3.3 Program disjunctions

G&S themselves observe that there are cases in which an externally static disjunction makes the wrong predictions. They give the example in (19) — more generally, anaphora from out of a disjunctive sentence is possible when each disjunct contains a parallel indefinite.[8]

(19) A^v professor or an^v assistant professor will attend the meeting of the university board. He$_v$ will report to the faculty.

They use this data to motivate a completely distinct disjunction operator, which they dub *program disjunction*, which is internally static but externally dynamic, and thus captures the data in (19) (although Partee disjunctions are still out of reach). The details won't be important for our purposes, but note that the fact that an alternative, externally static semantics for disjunction is possible in DPL conjures up the same conceptual worry that I've already raised — namely, it's not clear *why* logical expressions manipulate anaphoric information in just the way that they do.

Moreover, once disjunction can be translated into an externally static operator, it's not clear why it only occurs in the kind of instructions instantiated by (19). If disjunction can be externally dynamic, why should anaphora be *impossible* out of a disjunctive sentence elsewhere? Ideally, one would settle on whether the treatment of disjunction is externally static or dynamic. The semantics I'll ultimately end up with will be closer in spirit to G&S's program disjunction. In fact, it turns out that there is a problem with the data motivating G&S's externally static disjunction, which I turn to now.

[8] This observation is often attributed to the later [18]. The intuition behind the analysis is already implied by the fact that the two indefinites are annotates with the same variable.

3.4 Anaphora and contextual entailment

G&S's general project involves capturing surface generalizations about anaphora in complex sentences by picking just the right semantics for logical expressions. Rothschild [19] made an observation that shows that this simple picture overlooks the important role of the discourse context. Consider: ordinarily, anaphora out of a disjunctive sentence is impossible, as illustrated by (20).

(20) Either it's a weekday, or a^v critic is watching our play.
#They_v look unhappy.

Rothschild points out that when a witness to the indefinite is subsequently (locally, in this case) contextually entailed, anaphora is possible.

(21) Either it's a weekday, or a^v critic is watching out play.
If it's Saturday today, I want them_v to give us a good review.

Elliott [20] shows that this a very general problem for DPL — other operators, which were thought to be externally static, such as implication, allow for anaphora in similar circumstances; the ultimate suggestion is that a logic which gives connectives an externally dynamic semantics by default is desirable. Later, in Sect. 5, I'll have more to say about how to account for restrictions on anaphora out of disjunction.

Having surveyed some of the most pressing conceptual and empirical issues for DPL,[9] in the next section I begin to develop a new logic for anaphora, building on the Strong Kleene logic of indeterminacy.

4 EDS

EDS stands for *Existential Dynamic Semantics*, or alternatively *Externally-Dynamic Dynamic Semantics*, and it has some signature logical properties which distinguish it from DPL and related theories. I'll explore these properties in more detail later, but briefly:

- Double Negation Elimination is valid in EDS.
- Egli's theorem doesn't hold, but rather a weaker equivalence.
- De Morgan's equivalences hold.
- The logical connectives are a generalization of the Strong Kleene trivalent connectives into a dynamic setting.

[9] For an excellent recent overview of DPL, which expands on many issues which I don't have the space to discuss here, see [21].

4.1 The basics

At the core of EDS is the idea that pronouns are variables which semantically *presuppose* the existence of an assigned value at a given evaluation point (see especially [22]). This is implemented in the logic formally by emulating partial assignments using an privileged value in the domain of individuals $\#_e$, which corresponds intuitively to the 'unknown' individual. Concretely, assignments are *total* functions from a stock of variables to $D \cup \{ \#_e \}$.

In order to simplify the presentation, I'll consider a language with variables and no constants. Since I emulate partiality via the unknown individual, an atomic sentence ϕ receives the obvious (static) trivalent interpretation, where the truth of the atomic sentence at g is unknown just in case the value of any of the variables in the sentence is unknown at g. This is formalized below in (22), where the third truth-value is **unknown**.[10][11]

(22) **Static semantics for atomic sentences**

$$[P(v_1, \ldots, v_n)]^{w,g} = \begin{cases} \textbf{unknown} & g(v_1) = \#_e \ldots \vee \ldots g(v_n) = \#_e \\ \textbf{true} & [P(v_1, \ldots, v_n)]^{w,g} \text{ is not } \textbf{unknown} \\ & \text{and } \langle g(v_1), \ldots, g(v_n) \rangle \in I_w(P) \\ \textbf{false} & [P(v_1, \ldots, v_n)]^{w,g} \text{ is not } \textbf{unknown} \\ & \text{and } \langle g(v_1), \ldots, g(v_n) \rangle \notin I_w(P) \end{cases}$$

There are a number of possibilities for making a DPL-style relational semantics *partial* (see especially [24] for discussion). In EDS, the main innovation is that each of the three truth-values in a trivalent logic corresponds to a DPL-style relational meaning in a dynamic setting, i.e., it keeps track of anaphoric information associated with verification, falsification, and the 'unknown' case in tandem.[12] EDS is therefore a *trivalent* logic; in order to formalize this idea, I recursively define $[\![.]\!]^w_+$, $[\![.]\!]^w_-$, $[\![.]\!]^w_?$, (corresponding to the true, false, and unknown respectively).

(23) **Atomic sentences in EDS**

a. $[\![P(v_1, \ldots, v_n)]\!]^w_+ := \{ (g, h) \mid g = h \wedge |P(v_1, \ldots, v_n)|^{w,h} \text{ is } \textbf{true} \}$

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[10] Since I'm exclusively concerned with *anaphoric* presuppositions here, I make the simplifying assumption that all predicates are bivalent, i.e., if all of the values of the variables are *known* then an atomic sentence is always either **true** or **false**. One way of extending the logical language in order to model (non-anaphoric) presuppositions while maintaining bivalent predicates would be to incorporate Beaver's unary presupposition operator [23].

[11] Note that there are different ways in which to interpret the third truth value in a trivalent setting, e.g., as standing in for *undefinedness*. Here, it is explicitly referred to as "unknown", since this framing is a natural fit for the Strong Kleene logic of indeterminacy, which is exploited extensively later in the paper. Undefinedness typically goes together with Weak Kleene logic. I'm grateful to an anonymous reviewer for pressing me to clarify this point.

[12] This builds on the dynamic system developed in [25, 26], in which outputs are paired with bivalent truth-values.

b. $[\![P(v_1,\ldots,v_n)]\!]^w_- := \{\,(g,h) \mid g = h \land |P(v_1,\ldots,v_n)|^{w,h}$ is **false** $\}$
c. $[\![P(v_1,\ldots,v_n)]\!]^w_? := \{\,(g,h) \mid g = h \land |P(v_1,\ldots,v_n)|^{w,h}$ is **unknown** $\}$

Since the logic is trivalent, I give explicit truth and falsity conditions. N.b. that according to (24), a sentence with a free variable v will be **unknown** at g if $g(v) = \#_e$.

(24) **Truth and falsity in EDS**
a. $[\phi]^{w,g}$ is **true** if $\{\,h \mid (g,h) \in [\![\phi]\!]^w_+\,\} \neq \emptyset$
b. $[\phi]^{w,g}$ is **false** if $[\phi]^{w,g}$ is not **true** and $\{\,h \mid (g,h) \in [\![\phi]\!]^w_-\,\} \neq \emptyset$
c. $[\phi]^{w,g}$ is **unknown** otherwise

4.2 Negation

Negation in EDS is a flip-flop operator, defined as in (25) (see also [15]). N.b. that presuppositions project. This is a generalization of Strong Kleene negation, in the sense that each cell in the Strong Kleene truth table is interpreted as a DPL-style relational meaning, as opposed to a truth value.

(25) **Negation in EDS**
a. $[\![\neg\phi]\!]^w_+ := [\![\phi]\!]^w_-$
b. $[\![\neg\phi]\!]^w_- := [\![\phi]\!]^w_+$
c. $[\![\neg\phi]\!]^w_? := [\![\phi]\!]^w_?$

It follows straightforwardly from the flip-flop definition that Double-Negation Elimination is valid:

(26) **Double Negation in EDS:** $\phi \iff \neg\neg\phi$

As I've already discussed, it seems desirable to have a dynamic logic in which (26) holds. The statement of (25) is of course extremely straightforward. In the following I'll show that flip-flop negation makes good predictions in tandem with the other logical operators, once defined.

4.3 Connectives and embedding Strong Kleene

Now for the logical connectives. I've gestured several times towards the idea that the semantics of the logical connectives is a generalization of Strong Kleene trivalent logic to a dynamic setting. It's now time to make this idea precise. What kind of information does a truth table encode for a binary connective $*$? Well, given the truth values of two sentences ϕ, ψ it tells us how to compute the truth-value of the complex sentence $\phi * \psi$. Each cell in a truth table therefore expresses the result of apply some function from pairs of truth values, to truth values. In a dynamic setting, the values of sentences ϕ, ψ are not truth-values but rather relations. It's therefore natural to interpret each cell in a truth table as specifying a *relational composition*. The classical truth value tells us which polarity the resulting relation belongs to, on the basis of the polarities of the input relations. Exactly how this works will become more readily apparent once I go through some concrete examples, so let's start with the simplest case: conjunction.

In Fig. 1, I give the Strong Kleene 'truth-table' for conjunction in EDS. Just as in Strong Kleene semantics, a conjunctive sentence is only verified if both conjuncts are verified, but here *verification* is interpreted in a dynamic sense — in order to compute the positive extension of the conjunctive sentence, compute the relational composition of the positive extensions of the conjuncts. Falsification is a weaker requirement — there are many different ways in which conjunctive sentences can be falsified in Strong Kleene logic, and in some cases one of the conjuncts is unknown. The negative extension of the conjunctive sentence is the union of all of the dynamic falsifications. The unknown extension is also computed by taking the union of all of the unknown cases, computed dynamically.[13]

$\phi \wedge \psi$	$[\![\psi]\!]^w_+$	$[\![\psi]\!]^w_-$	$[\![\psi]\!]^w_?$
$[\![\phi]\!]^w_+$	$\circ, +$	$\circ, -$	$\circ, ?$
$[\![\phi]\!]^w_-$	$\circ, -$	$\circ, -$	$\circ, -$
$[\![\phi]\!]^w_?$	$\circ, ?$	$\circ, -$	$\circ, ?$

Fig. 1. Strong Kleene conjunction in EDS

EDS has a left-to-right bias directly encoded in the recipe it uses for lifting Strong Kleene semantics into a dynamic setting, since relational composition is non-commutative. Below, I write out the information encoded informally in Fig. 1 as the semantics of conjunction in EDS.[14]

(27) **Conjunction in EDS**

a. $[\![\phi \wedge \psi]\!]^w_+ := [\![\phi]\!]^w_+ \circ [\![\psi]\!]^w_+$

b. $[\![\phi \wedge \psi]\!]^w_- := [\![\phi]\!]^w_- \circ [\![\psi]\!]^w_{+,-,?} \cup [\![\phi]\!]^w_{+,?} \circ [\![\psi]\!]^w_-$

c. $[\![\phi \wedge \psi]\!]^w_? := [\![\phi]\!]^w_+ \circ [\![\psi]\!]^w_? \cup [\![\phi]\!]^w_? \circ [\![\psi]\!]^w_{+,?}$

Before I discuss a concrete application, i.e., modelling discourse anaphora, a remark is in order on the generality of this picture. As an anonymous reviewer points out, the question of what kind of information a truth-table encodes of course generalizes beyond just binary connectives. The picture outlined here can

[13] The generalization of Strong Kleene trivalent semantics to a dynamic setting will out of necessity remain rather impressionistic in this paper. The procedure of lifting truth-functional operators into a dynamic setting has however been made precise in important work by Charlow [26]. Simon Charlow (p.c.) points out that the recipe for lifting the Strong Kleene semantics used here can be formalized as a lifting of the Strong Kleene connectives into the `State.Set` applicative, following [26]. See [20] for more details.

[14] In order to keep the definitions relatively terse, I take advantage of the convention that $[\![\phi]\!]^w_{+,-,?}$ is understood as $[\![\phi]\!]^w_+ \cup [\![\phi]\!]^w_- \cup [\![\phi]\!]^w_?$.

be generalized as follows: one can think of a classical truth-table as encoding a function f from a sequence of n truth-values to a truth-value (where $n > 0$). The polarized dynamic interpretations that EDS deals in can in turn be encoded as a pair consisting of a truth-value and a relation. For each 'cell' in a derived dynamic truth table, I can state a general recipe: it takes as its input f, and a sequence of polarized relation pairs, and gives back a polarized relation; this is formalized below in (28). See [20] for more details.

(28) $$f((t_1, R_1), \ldots, (t_n, R_n)) := \begin{cases} (f(t_1), R_1) & n = 1 \\ (f(t_1, \ldots, t_n), R_1 \circ \ldots \circ R_n) & n > 1 \end{cases}$$

Now let's turn to a concrete appliation of the EDS semantics for conjunction: discourse anaphora. First let's define discourse referent introduction in EDS. Since negation is a flip-flop operator in EDS, it's important to ensure that a negated existential statement doesn't introduce anaphoric information, while preserving DNE. This is accomplished by syncategorematically defining existential quantification in terms of (a) conjunction (27), (b) DPL-style random assignment (29), and (c) a 'positive closure' operator. Positive closure simply ensures that its negative extension is always a test (i.e., anaphorically inert).

(29) **Random assigment in EDS**

a. $[\![\varepsilon_v]\!]^w_+ := \{\, (g, h) \mid g[v]h \,\}$

b. $[\![\varepsilon_v]\!]^w_- := \emptyset$

c. $[\![\varepsilon_v]\!]^w_? := \emptyset$

(30) **Positive closure in EDS**

a. $[\![\dagger\phi]\!]^w_+ := [\![\phi]\!]^w_+$

b. $[\![\dagger\phi]\!]^w_- := \{\, (g, h) \mid g = h \wedge [\phi]^{w,g}$ is **false** $\}$

c. $[\![\dagger\phi]\!]^w_? := [\![\phi]\!]^w_?$

Existential quantification is defined syncategorematically, just as in DPL but with the addition of †.

(31) **Existential quantification in EDS**

$\exists_v \phi := \dagger(\varepsilon_v \wedge \phi)$

I'll now establish some useful facts relating to the treatment of discourse anaphora in EDS. Note that $[\![\phi \wedge \psi]\!]^w_+$ is a simple relational composition. Consequently, concentrating just on the positive extension, associativity holds (32), and therefore the account of discourse anaphora from DPL is maintained. This is easy to see, since the positive extension of random assignment is the same as DPL random assignment, and positive closure is vacuous with respect to positive extensions.

(32) **Positive associativity of conjunction in EDS:**

$[\![(\phi \wedge (\psi \wedge \sigma))]\!]^w_+ = [\![(\phi \wedge \psi) \wedge \sigma]\!]^w_+$

The interaction between negation and discourse referent introduction is one respect in which EDS substantially departs from DPL. In DPL, negative existential statements are anaphorically inert by dint of the special properties of negation. In EDS, conversely, negation is more classical — DNE is valid — and negated existential statements are anaphorically inert by dint of the special properties of *positive closure*, which ensures that an existential statement is a *negative test* (i.e., its negative extension is a test). This is illustrated in (33).

(33) **Negative existential statements are negative tests:**
$[\![\exists_v P(v)]\!]^w_-$
a. $= [\![\dagger(\varepsilon_v \wedge P(v))]\!]^w_-$
b. $= \{\, (g,h) \mid g = h \wedge [\varepsilon_v \wedge P(v)]^{w,g}$ is **false** $\}$
c. $= \{\, (g,h) \mid g = h \wedge I_w(P) = \emptyset \,\}$

There's more to be said about the negative extension of conjunctive sentences, where (as I'll show), one observes failures of associativity. First though, I'll discuss the semantics of disjunction on EDS, illustrating how it resolves the vexing problem of Partee disjunctions.

4.4 Disjunction

Just as with conjunction, the semantics of disjunction in EDS is a lifting of the Strong Kleene trivalent semantics into a dynamic setting. This is illustrated in Fig. 2. With conjunction, there was essentially one way of dynamically verifying the sentence, but many ways of dynamically falsifying. With disjunction, the situation is the reverse: there are many ways of dynamically verifying, but only one way of dynamically falsifying.

$\phi \vee \psi$	$[\![\psi]\!]^w_+$	$[\![\psi]\!]^w_-$	$[\![\psi]\!]^w_?$
$[\![\phi]\!]^w_+$	$\circ, +$	$\circ, +$	$\circ, +$
$[\![\phi]\!]^w_-$	$\circ, +$	$\circ, -$	$\circ, ?$
$[\![\phi]\!]^w_?$	$\circ, +$	$\circ, ?$	$\circ, ?$

Fig. 2. Strong Kleene disjunction in EDS

The Strong Kleene truth-table in (2), where each cell is interpreted as a relational composition, corresponds to the EDS semantics of disjunction laid out below.

(34) **Disjunction in EDS**
a. $[\![\phi \vee \psi]\!]^w_+ := [\![\phi]\!]^w_+ \circ [\![\psi]\!]^w_{+,-,?} \cup [\![\phi]\!]^w_{-,?} \circ [\![\psi]\!]^w_+$
b. $[\![\phi \vee \psi]\!]^w_- := [\![\phi]\!]^w_- \circ [\![\psi]\!]^w_-$
c. $[\![\phi \vee \psi]\!]^w_? := [\![\phi]\!]^w_- \circ [\![\psi]\!]^w_? \cup [\![\phi]\!]^w_? \circ [\![\psi]\!]^w_{-,?}$

The crucial insight which will underlie the account of Partee disjunctions in EDS is that, one way of dynamically verifying a disjunctive sentence is by composing the negative extension of the first disjunct with the positive extension of the second. In EDS, since DNE is valid, a negative extension can introduce a discourse referent. In order to go through how this works, *I*l work through a simple example (35).

(35) Either there's no^v bathroom, or it's upstairs.
$\neg\exists_v B(v) \vee U(v)$

First, let's spell out the negative and positive extensions of the first disjunct; the positive extension tests whether there are no bathrooms (thanks to positive closure), and the negative extension introduces a bathroom discourse referent.

(36) $[\![\neg\exists_v B(v)]\!]^w_+ = [\![\exists_v B(v)]\!]^w_- = \{\, (g,h) \mid g = h \wedge I_w(B) = \emptyset \,\}$

(37) $[\![\neg\exists_v B(v)]\!]^w_- = [\![\exists_v B(v)]\!]^w_+ = \{\, (g,h) \mid g[v]h \wedge h(v) \in I_w(B) \,\}$

The second disjunct is an open sentence, so it has a standard trivalent test semantics. In order to compute the positive extension of the disjunctive sentence, I consider all ways of dynamically verifying the disjunction.

- One salient possibility is that one verifies the disjunction by falsifying the first disjunct, and verifying the second disjunct. Falsifying the first disjunct introduces a bathroom discourse referent which is dynamically retrieved when verifying the second disjunct (38).
- Another way of verifying the disjunction is by verifying the first disjunct, in which case the second disjunct is irrelevant — this is captured in Strong Kleene semantics, by taking the relational composition with the positive/negative/unknown extension of the second disjunct. Since the second disjunct is a test, this is equivalent to the positive extension of the first disjunct (39).
- Finally, I union everything together in (40).

(38) $[\![\neg\exists_v B(v)]\!]^w_- \circ [\![U(v)]\!]^w_+$
$= [\![\exists_v B(v)]\!]^w_+ \circ [\![U(v)]\!]^w_+$
$= \{\, (g,h) \mid g[v]h \wedge h_v \in I_w(B) \wedge h_v \in I_w(U) \,\}$

(39) $[\![\neg\exists_v B(v)]\!]^w_+ \circ [\![U(v)]\!]^w_{+,-,?} = \{\, (g,h) \mid g = h \wedge I_w(B) = \emptyset \,\}$

(40) $[\![\neg\exists_v B(v) \vee U(v)]\!]^w_+ = \{\, (g,h) \mid g[v]h \wedge h_v \in I_w(B) \wedge h_v \in I_w(U) \,\}$
$\cup \{\, (g,h) \mid g = h \wedge I_w(B) = \emptyset \,\}$

This captures the attested *existential* truth-conditions of Partee disjunctions, which I argued for in Sect. 3.2; the positive extension of the Partee disjunction will be non-empty if, either: (a) there is a bathroom upstair (in which case, introduce a bathroom upstairs discourse referent), or (b) (there is no bathroom).

This is (arguably) a desirable result! There is however a pressing issue that arises under the EDS semantics for disjunction, which underlies an apparent issue for the semantics more generally. Namely, Partee disjunctions *conditionally* introduce discourse referents. More generally, our semantics for disjunction is *externally dynamic* in the sense of [3]. This seems, on the face of it, incompatible with the evidence that disjunction is externally static, as discussed way back in Sect. 2.

Manifestations of this problem can be seen elsewhere. For example, although I won't discuss this in detail, de Morgan's equivalences are valid in EDS.[15] One consequence of this is that $\neg\exists_v B(x) \vee U(x)$ is equivalent to $\neg(\exists_v B(v) \wedge \neg U(v))$ via de Morgan's and DNE. This means that negated conjunctions can conditionally introduce discourse referents too. Another equivalent sentence in EDS[16] is $\exists_v B(x) \to U(x)$ — similarly, G&S argue that material implication is externally static, but in EDS the implicational sentence conditionally introduces a discourse referent.

In the next section, I'll show that, far from being a fatal problem, making external dynamicity the ordinary case is a desirable feature for a dynamic logic. I've already provided some empirical evidence for this in the form of program disjunctions, discussed in Sect. 3.3, and Rothschild's observation, discussed in Sect. 3.4. EDS will capture both of these datapoints, while maintaining a certain degree of restrictiveness, once integrated into a theory of discourse pragmatics.

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[15] I'll simply note here that the validity of de Morgan's in the presence of anaphoric dependencies seems independently desirable given our intuitions about natural language. The following sentences are all arguably truth-conditionally equivalent.

(1) a. Either there's no bathroom, or it's upstairs.

b. It's not the case that there's a bathroom and it's not upstairs.

c. If there's a bathroom, then it's upstairs.

[16] Assuming a Strong Kleene semantics for material implication. Something interesting to note here is that EDS predicts existential truth conditions for donkey sentences, unlike, e.g., [3]. Egli's corrolary therefore doesn't hold.

This is by no means a bad prediction — it has been widely reported that such existential readings are attested for donkey sentences, as alluded to in fn. 7 (see also [17, 27]). The following example, for example, is clearly true if Gabor owns two credit cards but only pays with one of them.

(1) If Gabor has a^x credit card, he'll pay with it$_x$.

The empirical picture is however much more complicated, and donkey sentences do often have stronger, universal readings. Relatedly, [15] reports that Partee disjunctions have universal readings — unlike donkey sentences, to my knowledge very little work has been done examining the distribution of existential and universal readings of Partee disjunctions. A detailed discussion of universal readings will have to wait for another occasion, but see [28] for one recent approach.

5 Discourse pragmatics

5.1 Update

In order to give an account of the dynamics of disjunction, it's important to understand how discourse referents are introduces in context. In certain dynamic theories, such as Heim's *File Change Semantics* [2], the relationship between the semantic value of ϕ and what it means to assert ϕ is almost trivial, since on such theories sentences themselves denote updates on information states (see also [29]). Since EDS is a relational theory, much like DPL, as well as encoding partiality, I need to state a concrete bridge principle in order to integrate EDS with a Heimian notion of information states. Update in EDS is defined as in (41).

(41) **Update in EDS:**

$$c[\phi] = \begin{cases} \bigcup_{(w,g)\in c} \{\, (w,h) \mid (g,h) \in [\![\phi]\!]^{w}_{+} \,\} & \forall (w,g) \in c \begin{bmatrix} [\phi]^{w,g} \text{ is } \textbf{true} \\ \text{or } [\phi]^{w,g} \text{ is } \textbf{false} \end{bmatrix} \\ \text{undefined} & \text{otherwise} \end{cases}$$

Here, I take information states to be sets of world-assignment pairs [2]. 'Initial' states (i.e., those where no discourse referents have been introduced) are those paired with the unique assignment which maps every variable to $\#_e$ (I'll write the initial assignment as []). Updating an information state c with a sentence ϕ is *defined* just in case ϕ is *contextually bivalent*.[17] If defined, the updated information state is computed by gathering up, at each evaluation point $i \in c$, the positive extension of ϕ at i. EDS sentences therefore update information states by (i) eliminating worldly possibilities, and (ii) introducing discourse referents, i.e., expanding anaphoric possibilities.

An immediate consequence of the notion of update in (41) is that an open sentence $P(v)$ presupposes at c that v is 'defined' at every evaluation point $i \in c$. This is exactly the notion of *familiarity* introduced by Heim [2, 31], but here derived from a partial DPL-like dynamic semantics plus a generalization of Stalnaker's bridge. To be precise: *definedness* is a condition placed on individual evaluation points, whereas *familiarity* is a (derivative) universal condition placed on the entire input context.

5.2 Disjunction and contingency

Disjunctive assertions in natural language are subject to a contingency requirement (43). I state this formally as a felicity condition on assertion in (43), making use of a notion of *worldly content* defined in (42) — the idea here is just that it's possible to retrieve the 'classical' Stalnakerian content of a Heimian information

[17] This is what von Fintel calls 'Stalnaker's bridge' [30], in the context of a dynamic setting.

state. This captures the intuition that a disjunctive sentence cannot be felicitously asserted if one of the disjuncts is contextually trivial.[18]

(42) **Worldly content:** $W(c) := \{ w \mid \exists g[(w, g) \in c] \}$

(43) **Contingency requirement:**
Assertion of a sentence of the form $\phi \vee \psi$ is felicitous in c iff $\mathbf{W}(c[\phi])$ and $\mathbf{W}(c[\psi])$ are non-empty proper subsets of $\mathbf{W}(c)$.

The update rule in (41), together with the contingency requirement in (43) accounts for G&S's observations concerning the apparent external staticity of disjunction, as well as Rothschild's observation, discussed in Sect. 3.4. To see why, consider the simple example in (44). The first disjunct $\exists_v P(x)$ is contextually trivial at c, unless some worlds in c are worlds s.t., $I_w(P) = \emptyset$. This guarantees that, so long as (43) is satisfied, updating an information state with (44) will result in an updated information state containing at least some non-P worlds, where discourse referents aren't introduced. This means that a subsequent open sentence such as $Q(v)$ cannot be felicitously asserted, since (44) can't make v *familiar*.

(44) $\exists_v P(v) \vee Q(a)$

Crucially, if the non-P worlds are subsequently eliminated, v might become familiar later in the discourse, for example if an assertion is made that contextually entails the first disjunct. In this case, anaphora will be possible since familiarity will be satisifed.

This general explanatory strategy can be extended to other apparently cases of external staticity, once the contingency requirement in (43) is generalized to other complex sentences. For example, assertion of sentences of the form $\neg(\phi \wedge \psi)$ typically requires that $\neg\phi$ and $\neg\psi$ are not contextually trivial.

One interesting thing to note is that the requirement as stated in (43) doesn't quite work as stated for Partee disjunctions, since it doesn't take into account the possibility of an anaphoric dependency between disjuncts. I address this issue in detail in [32].

5.3 Program disjunctions

I'm now in a position to explain why program disjunctions are an apparent exception to the more general properties of disjunctive assertions in discourse. Following Groenendijk and Stokhof's DPL account [3], I assume that what makes program disjunctions special is that each disjunct is an existential statement introducing a discourse referent at the same variable. A schematic case is provided in (45). *Unlike* DPL, EDS accounts for the behavior of program disjunctions

[18] Various pragmatic justifications can be given for the formal contingency requirement stated in (43). What is important for my purposes is that if a disjunctive sentence is asserted by a speaker s in a context which trivializes one of the disjuncts, the assertion is judged to be 'odd'.

without having to assume that disjunction is ambiguous between externally static and externally dynamic variants.

(45) $\exists_v P(v) \vee \exists_v Q(v)$

The contingency requirement insists that there be some P worlds, and some non-P worlds in c for (45) to be assertable, as well as some Q-worlds, and some non-Q worlds. Once (45) is asserted however, all non-P, non-Q worlds will be eliminated. This leaves only P-worlds and Q-worlds, each of which is associated with a discourse referent at v. (45) therefore makes v familiar, and subsequent anaphora is (accurately) predicted to be possible by EDS.[19]

5.4 Internal staticity

There is a loose end from Sect. 3 that I have yet to address in the more permissive setting of EDS — namely, why is disjunction internally static? The problematic data is given below.

(46) #Either there's a^v bathroom, or it$_v$'s upstairs.

In fact, in order to capture Partee disjunctions, it seems essential to allow for anaphoric information to pass between disjuncts, so (46) seems to constitute something of a mystery. In fact, the infelicity of (46) in a context where v isn't familiar is expected on the basis of the contingency requirement. Consider the LF of (46):

(47) $\exists_v B(v) \vee U(v)$

If the first disjunct is true, the second is contextually bivalent, but if the first disjunct is false, the truth of the second disjunct is partial, and dependent on the input assignment. Eliding the full computation, the positive and negative extensions of (47) is given below:

(48) $$[\![\cdot]\!]^w_+ = \{\, (g,h) \mid g[v]h \wedge g(v) \in I_w(B) \,\} \cup \{\, (g,h) \mid g = h \wedge I_w(B) = \emptyset \wedge h(v) \in I_w(U) \,\}$$

(49) $$[\![\cdot]\!]^w_- = \{\, (g,h) \mid g = h \wedge I_w(B) = \emptyset \wedge h(v) \notin I_w(U) \,\}$$

[19] There seem to be information-structural constraints on program disjunctions in natural language which are beyond the remit of EDS. For example, it seems that some degree of parallelism is required to hold between the disjuncts. Singular anaphora, by my reckoning, is extremely difficult in the following example:

(1) Either a^v linguist sneezed, or the meeting was interrupted by a^v philosopher. ?She was very rude.

I speculate that this is related to constraints on co-indexing. I leave this interesting issue to future work on program disjunctions.

For the disjunctive sentence to be assertable, every $(w, g) \in c$ should be such that either w is a B-world, or w is a non-B world and $g(v)$ is defined. Given the contingency requirement then (suitably generalized to allow for anaphoric dependencies), there should be non-B parts of c, in which case (47) requires a familiar discourse referent v in order to be assertable.

In fact, the empirical picture is potentially even more nuanced than this. Filipe Hisao Kobayashi (p.c.) observes that anaphora seems to be possible in (50).

(50) Either there's a^v bathroom upstairs, or it$_v$'s downstairs.

Here I'll tentatively suggest that the contrast between examples like (46) and (50) is due to the different Logical Forms available to existential statements. Concretely, an existential statement in natural language can be translated either as discourse anaphora (51a) or as a existentially-quantified formula (51b). In a theory such as DPL, (51a) and (51b) are equivalent (Egli's theorem). In EDS on the other hand, (51a) and (51b) are positively equivalent but negatively distinct, due to the fact that conjunction isn't associative.[20]

(51) a. $\exists_v B(v) \wedge U(v)$
b. $\exists_v (B(v) \wedge U(v))$

Concretely, the negative extension of (51b) is always a test, due to positive closure taking widest scope. The negative extension of (51a) on the other hand conditionally introduces a discourse referent. In (50), I conjecture, the first disjunct is translated as in (51a). I leave a more detailed assessment of examples such as (50) to future research.

6 Comparison to alternatives

Although it will be impossible to provide a detailed comparison between EDS and related proposals, some parallels and correspondences are worth mentioning.

The semantics of existential quantification in EDS — decomposed into positive closure, conjunction, and random assignment, is closely related to the system developed in Mandelkern's work [33]. Mandelkern develops a logic of anaphora which is bivalent and classical, but supplemented with an extra dimension of meaning — *witness bounds*. The witness bounds of an existential statement ensure that a discourse referent is conditionally introduced if there is a witness to the existential statement, and thereby maintains external staticity of a negated existential statement while validating DNE. The workings of witness bounds are highly reminiscent of my positive closure operator, and there are other compelling logical correspondences between my theory of Mandelkern's which deserve further

[20] This kind of suggestion raises the issue of how exactly natural language sentences can be mapped compositionally to EDS Logical Forms. After all, one doesn't want to allow for too much flexibility, otherwise the resulting grammar won't be sufficiently constrained.

exploration. One respect however in which the theories diverge is that, in EDS, the familiarity requirement associated with a pronoun/free variable is just an ordinary presupposition. In Mandelkern's theory, the correspondence between presupposition projection and anaphoric accessibility is not straightforwardly captured.

Hofmann [34, 35] tackles many of the same problems discussed here within the context of a much more expressive system based on CDRT [36] and intensionalized discourse referents [37]. An appealing property of Hofmann's system is that it can handle modality and modal subordination. This is important in accounting for certain cases of anaphora from out of a negative sentence, such as (52).

(52) Colin doesn't own a^v car, but it$_v$ would be a Subaru.

It remains to be seen to what extent Hofmann's insights can be incorporated into EDS, in order to expand its empirical remit.

Finally, it would be remiss of me not to mention the connection between EDS and earlier work by Rothschild [19], which also attempts to account for patterns of anaphoric accessibility using the trivalent account presupposition projection, and which thereby consistitutes an important precursor to EDS. There are a couple of important differences between EDS and Rothschild's proposal — here, a *left-to-right* bias arises due to the way in which the Strong Kleene connectives are lifted into a dynamic setting (i.e., using relational composition). On Rothschild's account, the trivalent logic itself must be given a left-to-right bias [38, 39, 40] in order to account for linear asymmetries in anaphora. Furthermore, in order to account for, e.g., Partee disjunctions, Rothschild stipulates that classically transparent material may be freely inserted into Logical Forms. This mechanism is somewhat ad-hoc and leads to concerns of over-generation. EDS consistutes a clear improvement, in the sense that Partee disjunctions are follow from standard dynamic mechanisms for capturing cross-sentential anaphora.

7 Conclusion and Outlook

In this paper, I've sketched a new kind of dynamic logic: EDS. EDS incorporates the insights of Groenendijk & Stokhof's *Dynamic Predicate Logic*, and trivalent approaches to presupposition projection. A core tenet of EDS is that the dynamics of the logical connectives should not be stipulated, but rather arise as a generalization of the Strong Kleene connectives into a dynamic setting. This approach is conceptually appealing, as it maintains a certain degree of predictiveness while establishing a tight connection between patterns of anaphoric accessibility and presupposition projection, following, e.g., [19].

EDS is *more classical* than orthodox logics of anaphora such as DPL in important respects — for example, DNE is valid. This is an important result, as empirical evidence suggests that classical equivalences such as DNE and de Morgan's don't break down in the presence of anaphoric dependencies. There are also striking respects in which EDS differs from DPL. To recap, neither Egli's theorem nor Egli's corrolary hold in EDS. This is surprising, since Egli's theorem is often framed as *the* central logical property of dynamic theories. Instead, a weaker variant of Egli's theorem holds, just with respect to *positive* extensions. Another major departure is that EDS predicts existential readings across the board, including for donkey sentences.

Much work remains to be done in investigating the inferential properties of EDS, and extending the central ideas outlined here to a broader empirical domain, encompassing quantification, plurality, and modality.

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