A Language of Thought Based Theory of English Grammar

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Abstract. This paper on the foundations of linguistics presents a theory of English syntax as a mapping between sentences and their semantic interpretation. In particular, and the main claim of this paper, English sentences map one to one with particular formula and structures of a language of thought, built up from more basic ones through syntactic and expressive expansion. The theory gives a specification of verb, preposition, and determiner ontology, addresses noun ontology, and defines the distinction between individual names, mass nouns, and count nouns. From these atomic cases, the structural basis of adjectival phrases and the semantic interpretation of the verb forms, word order of the verb and its arguments, the composition of verbs and prepositions, and the dative construction can all be given. All of this is possible based on the semantic theory the grammar is mapped to. The ontology is defined within a space-time structure which is paired with sets, (some of) whose elements are formula of the predicate calculus. By this means, we can give a formal definition of states, in a way that distinguishes them from rules, capacities, and other cases. Our intuitive notion of change is a syntactic property of the predicate calculus, and thus we can represent both static and dynamic states. In addition, a mechanical conception of knowledge allows us to integrate the dynamics of knowledge transfer underlying many speech acts. From these observations, a framework is laid out for the semantic interpretation of a large subset of English sentences.

Contents

1 Semantic Preliminaries

A primary feature of language is the semantic similarity of the same words in different orderings and phrasings. For instance, in (1a), “the monument” strikes us as being related to “the tourists loved to visit” in the same way it is in (1b). Sometimes called movement, such a property is considered “non-negotiable” for any syntactic theory of linguistics (Chomsky et al. (2019)). English’s use of such variation can be for specific reasons. For example, compare “the car is fast” with the noun phrase “the fast car.” The relation shared in both constructions between “fast” and “car” has an easy semantic gloss: the formula speed(car) $\geq \alpha$ applies, for some constant α. The difference can be seen by trying to use them as an argument in another structure as in (2).

(1) a. The monument the tourists loved to visit put the town on the map.
    b. The tourists loved to visit the monument.

(2) a. The fast car is in the driveway.
    b. The car is fast is in the driveway.

The noun phrase outputs an object (car), though it encodes the relation (speed(car) $\geq \alpha$), while the initial sentence outputs the relation itself, and not the object. This is the general pattern we find with English: “movement” occurs when there is a change in the ontology of the element outputted. In (1a), “the monument” is the object outputted. “The tourists loved to vist” is embedded information. And, as is common with language, one can make a list of such embedded elements, as in “The monument the tourists loved to visit, the historians hated to write about, or which stood behind the politicians in photos put the town on the map.” From these facts, we require structures for objects which embed information and structures for multiplicities.

Our plan for the presentation is as follows. In §2 we introduce the language of thought (LOT) with a brief overview, working towards generating a structure which handles sentences, and using the formula for postulating a few definitions. Along the way, we briefly address some details regarding noun reference (§2.4) and adjectival phrases (§2.5). In §3, trying to keep the presentation as non-technical as possible, we pick back up on the themes just above, namely word order, and give a more detailed account of verbs, prepositions, and their arguments and composition. §4 gives a completely formal presentation of a core English grammar, followed by example derivations of sentences and a brief discussion of interpretation. Lastly, §5 presents an extension of the core grammar, dealing with determiners, interrogative pronouns, tense, and the problem of a sentence’s core being the input for the creation of further structures of the LOT. We begin with laying out some preliminary semantic work, discussing change and then building up a partial, space-time based psychological theory.

1.1 Taxis

Our minds are dependent upon tracking change, from the positional change of flying birds, or increasing and decreasing values of a graph, to the change of possession when checking out at the grocery store, or a change in the family tree that occurs with a birth or marriage. In the case of a continuous substrate, we are particularly sensitive to critical points—local extrema and discontinuities. Breaking down the taxis—a biological unit of behavior—into a mathematical form, produces atoms which, along with constants, can compress all functions and in terms of the critical points we're sensitive to.

A taxis is movement at a fixed angle towards or away from a stimulus (Allaby (2009), Britannica (1999)).[1] Movement towards or away from the stimulus will occur so long as it's not at a right angle. So, a taxis is either moving towards or away from a zero distance, $\text{dist}(x, y) = 0$. If we generalize — so that spatial distance is only a special case, introducing color distance, temperature distance, sound pressure level distance, and so on, with the particular function irrelevant, so long as a metric is definable on the function's output (it's thus more accurate to write $\text{dist}(\phi(x), \alpha) = 0$ or $\text{dist}(\phi(x), \phi(y)) = 0$, where $\phi()$ is the relevant function) — then we may use this as a basis for our semantics.[2] (3a) and (3b) demonstrate the difference between a taxis being encoded or not in language. Further contrast with (3c) demonstrates the difference between an active and completed taxis.

(3) a. The book is long.
b. The book is getting/growing long.
c. The book got/grew long.

Although we would like the metric condition to handle all cases, empirical evidence suggests things are more complicated. The mind generally takes locomotion to be a change of the individual's location. When we say "Amber went to the store," we generally understand Amber changing locations, and this is in contrast to "the price of the stock rose this morning." Phenomenologically, in the formula $\phi(x)\text{R}\alpha$, x is changing in the former case, while α is changing in the latter case.[3] Locations are more akin to sets, with hierarchical containment relations (think of locations on a map, with cities inside states inside countries), and have no obvious metric defined over them that match with intuition. The same thing that's so for location is true of possession: possession is mutually exclusive but for containment. (If so-and-so is in community A, and so-and-so possesses x, then, relative to communities B and C, community A possesses x). It turns out that there is a formal difference between these two types of change which we'll specify in §2.1.

1.2 Knowledge Taxis

A taxis—in the generalized sense we are using it here—is applicable to mechanical movement and radiation such as light, heat, and sound, and thus can be used to capture speaking *to*

[1]Others use taxis for an orienting response which may co-occur with movement (Gallistel (1980), Lorenz (1939), Lorenz (1982)).

[2]Definitions of metrics are readily available, e.g. `https://proofwiki.org/wiki/Definition:Metric_Space` or `https://mathworld.wolfram.com/Metric.html` for references. (Accessed August 2, 2022.)

[3]It seems, by intuition, that this is correct: that the person is being mapped to locations rather than locations to the person, while the stock is being mapped to some number rather than the number to the stock; and that the locations stay invariant while the person moves around, while price—which specifies which number the stock gets mapped to—is undergoing change while the stock stays invariant. Such intuitions are discussed under the heading of thematic roles (see e.g. Harley (2010), Dowty (1991)).

someone. But there is a particular type of behavior that it cannot capture directly, and that is a read operation. A read operation has 3 steps, one of which is dependent on external behavior, making the computation open. The first step is a mechanical step, the second step the external input, and the third step the processing of the input into a form which is carried forward in time. For example, computers perform read operations, by initiating a mechanical arm which then is effected by changes in the external magnetic field, and then carries information about the external changes forward in time, to be read out later by a similar process. Or in the case of biology, we see a read process in our perceptual vocabulary. For instance, the muscular movement of the eyes towards a region of space is captured by LOOK, the changing light reflected from external elements onto our eyes is captured by another sense of LOOK, and then the carrying forward in time, and the effect of induced knowledge is captured with SEE. The 3 steps have distinct auditory perception vocabulary (LISTEN, SOUND, HEAR), but for all other sensory systems, some vocabulary term encodes more than one case.

The openness—the dependence on some external element—means that the end can't be known from the start, a key part of eschatological reasoning, or reasoning from a temporal end point.[4] Nonetheless, we may still describe this process in terms of a taxis, by developing a knowledge taxis based on the following intuitions.

> Suppose one goes down to the basement to search for a tool. One comes to the bottom of the stares, and turns to look at the various places. Over to the left is nothing but wall and floor, and one's mind decides it can't be there, and so one looks there no more. Over to the right on a table are various things in a pile, and above are cupboards, and one's mind decides that it could be there and so one goes to look there more. One gets closer and the process begins again. One looks and one's mind decides that this spot can't have the tool, or at this other spot is so much clutter and possible occlusion and so one looks further here or moves things around. At each step, one's mind determines whether this space may or may not have the tool, and one proceeds. Then, alas, one finds the tool, and one searches where one found it no more, and one searches in the basement no more.

Increasing or ceasing a behavior is evidence of a metric property and, in combination with the end from the search, for a taxis. Let's suppose that e.g. a given space is unresolved if one doesn't know whether one should look there more or not, and resolved when one knows that either one has obtained the desired result or when one knows the desired result is not to be found there. Then these values of resolved and unresolved are equivalent to a complete and incomplete taxis, respectively. Again, generalizing to other functions so that (a search of) space is only a special case, so long as one has a mechanism of coming closer to a resolved value, one has a viable taxis.[5] This gives us a means of retaining eschatological reasoning,

[4] For example, if one wants, or needs, at time t_1 to have have a higher velocity, then at time t_0—so eschatological reasoning tells us—add a force in that direction and prevent any other new forces. Fodor (2008) discusses "practical reasoning," and this is a variation of that.

[5] For instance, one might have that if the contrast with the background is greater than some amount, if the size is greater than some amount, and so on as a criteria of success, and getting closer, making the light brighter, squinting and so on as mechanisms. We can generate a metric from the success criteria, as follows.

though it now occurs at a level of abstraction above the usual taxes (location, temperature, brightness, and so on).

Resolution is the function which takes a formula as input, and outputs the current data's distance from the formula's data conditions, and thus is the basis for resolved and unresolved values. Any attempt to bring this function to a given value is a knowledge taxis.

1.3 A Space-Time Universe

In English, talking about adding and removing things from a set is natural. In mathematics, a set's extensional definition rules out any such dynamics. The problem we face is making these two frameworks align in some way. One approach is to use a sequence of sets, where removing or adding elements refers to the difference between the values (sets) of neighboring indexes of the sequence. We also want to talk about what holds true in a space-time region, but space-time defines positions and locations, not states. So, to get states attached to a space-time region, we amend a set to it, and insert the appropriate formula in the set. As it turns out, the relevant properties for a state is a property of how these amended space-time regions relate to one another. For instance, if "the cat is on the couch" holds true here in this house from 5 to 8, and here in this house is in Delaware, then "the cat is on the couch" holds true both here in Delaware, or on the east coast, or on Earth, or any other place that contains here, and from 6 to 7, or 7:30 to 7:31, or any time that's between 5 and 8. But, it doesn't necessarily hold in the kitchen nor at 8:05. Once we spell this out, it becomes apparent that more than one set is possible, and useful, so we'll attach multiple sets to the space-time regions.

We thus present the following. A *space* is an ordered pair $(W(t), \mathfrak{X})$, where $W(t)$ is a space-time region and $\mathfrak{X}$ is a collection of sets, with further details and constraints provided in appendix A. With this, we can define a space universe.

Def.: Let $(\mathcal{U}, \mathcal{H})$ be an ordered pair, where $\mathcal{U}$ is a set of spaces and $\mathcal{H}$ a set of functions, such that for spaces $(W_0(t_0), \mathfrak{X}_0)$ and $(W_1(t_1), \mathfrak{X}_1)$ in $\mathcal{U}$, $\exists!$ function $h \in \mathcal{H}$ from $\mathfrak{X}_0$ to $\mathfrak{X}_1$, and $\exists \Phi \in \mathfrak{X}_0$ that satisfy the conditions:

(a) Space-Time Uniqueness: $W_0 = W_1$ and $t_0 = t_1 \rightarrow \mathfrak{X}_0 = \mathfrak{X}_1$

(b) $h : \mathfrak{X}_0 \rightarrow \mathfrak{X}_1$ is one-to-one and onto

(c) Closure of Composition: For any $(W_2(t_2), \mathfrak{X}_2)$ with $h' \in \mathcal{H}$ unique from $\mathfrak{X}_1$ to $\mathfrak{X}_2$ and $h'' \in \mathcal{H}$ unique from $\mathfrak{X}_0$ to $\mathfrak{X}_2$, $h'(h(X)) = h''(X)$, for any $X \in \mathfrak{X}_0$.

(d) $t_0 = t_1$ and $W_0 \subseteq W_1 \rightarrow \Phi \subseteq h(\Phi)$

(e) $W_0 = W_1$ and $t_0 \subseteq t_1 \rightarrow h(\Phi) \subseteq \Phi$

Suppose that the criteria for knowledge is that n conditions are met, regardless of whether order matters. Let the points of the metric domain be a_0, the point where we have no information, the points $a_1, a_2, \ldots, a_{n-1}$ are the points where we have, respectively, $1, 2, \ldots,$ or $n-1$ pieces of positive information and no negative information, and a_n, is the point where we have n pieces of positive information or at least 1 piece of negative information. We define the distance between a_i and a_j as $d(a_i, a_j) = \frac{|j-i|}{n}$. This satisfies the metric conditions, and so a knowledge taxis has the goal of e.g. $\mathrm{d}(\mathrm{data}(x), a_n) = 0$.

Then $(\mathcal{U}, \mathcal{H})$ is a *space universe*. If a space $(W(t), \mathfrak{X})$ is in a space universe $((W(t), \mathfrak{X}) \in \mathcal{U})$, we say $\mathfrak{X}$ is a *set universe*.

An initial set and the functions in $\mathcal{H}$ can generate a collection of sets, which we may consider a class, and for which there is one set per space in the class. By considering selected spaces, where the spatial part is fixed and the temporal parts are sequential and non-intersecting, it's possible to characterize the elements of a class as evolving through time, and then the idea of adding and removing things to and from a set becomes sensible, as was desired. For the ease of communication, when working with just one space, we will always denote these sets of the same class with the same symbol.

Any static state may be put in Φ, and other states are applicable as well. For instance, a moving object has some velocity, and if this velocity stays within a set of values, then a velocity formula, e.g. $\text{vel}(x) \in A$, can also be placed in Φ, and so movement with little speed variation is a state. Similarly, consider the case of rates. So long as one doesn't have any variables, then the spatial part is satisfied. For the temporal part, we need an interpretation for temporal scales smaller than a, when the rate is $1/a$ or greater. The rate reading, for a given temporal point r, holds if given the rate in the form $1/a$, for all line segments of length a that contain r, there is at least one occurrence on the line segment, and more than one occurrence implies the temporal distance between the occurrences is a. That is, there can be an interval which has an occurrence at each end point, but otherwise the line segment will have just one occurrence. To satisfy this, there is, for real time processing, a "grace period" when a change of rate occurs. For instance, at the end of a song, it's not clear if the rate will change or not. Thus, the end is the time of the last note, though the time of knowing it's the last note occurs afterwards.

1.4 Set Universes

Every set universe has a set in the Φ class, and here we introduce sets that form more classes. In general, we wish to posit as few sets as possible, while also allowing enough distinctions to capture our psychology. Below we give intuitions behind, and justifications for, each set. We also define mechanism, which is key to the handling of dynamics. Later in the paper, a few other sets will be added as needed to introduce new classes.

Inputs (Λ). When we perceive a state, and consider its change, we must deal with two different ideas of what's in Φ: any new elements and all the old elements. Recall that a read operation involves an input, and so let us posit a set Λ that takes this input. But, there's no reason to believe that only Φ will be capable of obtaining new formula and removing old formula. Thus, we need this input to not only specify the formula, but also the set the formula's in, and so the elements of Λ will be proposals as defined in §2.1.

Capacities (H). Suppose we have some state holding and thus there's a formula in Φ. To make it concrete, suppose Frances is standing $5'6''$ tall. Contrast one's reaction to this as opposed to Aaron, who stands $12'$ tall. If that were to happen, one would probably feel disbelief. So, in order to capture this, we posit our second set H, which gives us a range of values that are expected and values outside the range correspond to our surprise, though these are not the only kinds of expectations and surprises we can have. Thus, if $[\phi(x) = \alpha] \in \Phi$ is comprehendible, then $[\phi(x) = A] \in H$ and $\alpha \in A$. But, this is only one case and we'll see that H handles more expectations than for just Φ.

Rules (Ψ). Suppose one is watching a tennis ball, and consider the following cases

(4) a. If the ball lies still—and nothing is pressing on it nor acting on it—then one finds this to be comprehensible.

b. If it rolls on the floor, and slowly decreases speed until it comes to a complete halt. If the start is understandable, then this entire scenario is understandable.

c. It rolls on the floor, and one can see ahead on its path is a ramp. If one is watching the tennis ball rolling down the ramp and, apropos of nothing, the ball suddenly stops, this leaves us confused, or looking for some unseen occluding element.

Given that (4a) is understandable and sudden movement from the (4a) situation would be incomprehensible or surprising, the basis for (4b) being understandable is a rule. This rule is approximately: no forces on the object implies the change in speed is negative. But, from (4c), we see that this needs to be modified, so a better approximation is: no forces on the object implies the change in speed is negative and bound by some amount. Thus, the movement is comprehensible by a rule telling us how much change in a value is reasonable. More generally, changes in states are taken to be incomprehensible without a corresponding rule.

If one thinks of rules and laws, one realizes that rules, unlike states, are assumed to continue to hold, and even well after we're told it's a rule. Thus, if someone says "no pushing," one understands that this has nothing to do with what currently holds true, or what current states there are. Without the explicit statement of removal, it'll be assumed that the rules still apply. And rules, also unlike states, are limited to being within a spatial region. The "universal" laws of science aren't very good examples, but the laws of countries applying only within the country's borders or the rules of sports applying only to the sports area are better examples of the spatial limitation. Rules are in the form of implications and implicitly tie two nearby temporal regions together. They can be used either as an expectation or explanation, as a validity condition or a computational instruction.

Key to these thought experiments are holding forces fixed. Besides background forces like gravity or contact events, there are what appear to be spontaneous but comprehensible forces, added and removed from a space by e.g. agents, which also needs to be addressed.

Statuses (D). Let us consider possessions, kinship, and memories. Such things don't seem to be states nor rules. A possession can be near or far, used often or nearly forgotten. Possession is justified recursively, with buying, inheriting, and forfeiting being rules for valid successors. Kinship relations continue holding, as if they're like abstract numbers, and yet can be changed with marriage, divorce, and birth. Likewise, memories have this carrying forward in time. All hold in light of some past event and continue to hold but for modification by similar events. This is functionally distinct, and so we give a distinct set D for the formula which encode these ideas.

Suppositions (Π). Consider putting a pot of water on the stove top and turning on the burner. Immediately after turning the burners on, one is sure that the water is not boiling. But, as time passes, if one doesn't have an eye on the water, one's certainty of whether the water is boiling or not decreases. Or another case: if one buys produce at the store, then the first day, one knows the produce is good, but after a few days, one is uncertain whether the produce has gone bad or not. If one gives somebody a command or task to do, at that point in time, one knows they didn't do the task, but later in the day one will be uncertain if they did. This functionality—of the resolution[6] of a state (or similar) starting at some value, and decreasing below that value at some point in time—is the characteristic of the set Π.

Mechanisms and Thermics. Let us return to the problem of comprehensible spontaneous forces. Consider Frances standing $5'6''$ tall again. Suppose, before one's very eyes, Frances begins to elongate, and grows to $5'8''$, $5'10''$, and keeps going. First note that, if she's growing at a good rate, then stops suddenly, it'll have the same effect as the ball stopping on the ramp. That is, the element of Ψ, which is the basis of comprehending how much change will occur, is not the same thing as comprehending the introduction of the formula to Ψ. To understand this better, we need to take a detour into physics.

The parts that lead us to be puzzled can all be characterized by introducing or removing a force, above some magnitude, into the space. Note the dynamics of introduce: this does not mean every force in the space, but only the forces in the space that were not there before. Once a force is introduced, a rule takes over. Similar remarks are possible for remove. In cases like contact events, one rule implies the introduction of another. But, with cases like agency, the forces may appear spontaneous to us and not governed by a rule. To handle these cases, we delve into another branch of physics.

Our solution to the problem of the spontaneous force is that they must be attributed to opaque insides of thermodynamic systems, which have a shared border with the relevant thermodynamic system we're working in (a space-time region). More often than not, this is a spatially embedded thermodynamic system. We use thermodynamic systems since they have the properties of being defined as a volume of space, which may be separated by the surroundings by walls, and have variables of state (Coopersmith (2015)). And since we may be interested in states of the system which may not be thermodynamic, we error on the side of caution and call such sources of spontaneous forces *thermics* (with some details available in appendix A). That is the first part of the solution. The second part of the solution stems from the classical definition of machines. A machine is defined as a function to and from sets of forces (Britannica (2019), Britannica (2021)). A *mechanism* is a machine which produces a taxis. That is, it takes input forces and maps them to some output forces which produce a taxis. An active mechanism will be interpreted as a rate of energy output being within a certain range, and so active mechanisms are in the set of states, Φ. Mechanisms, for this theory at least, are prime elements of comprehension: any mechanism that is in H may have its active form introduced or removed with comprehension.

Now let's return to Frances' growth. If the growth from $5'6''$ to $5'8''$ had occurred over a month, then we probably wouldn't be puzzled. This is because mechanisms have limitations which are stored in H. The growth rates in H are rather slow, and so given H, growth at such a fast rate leaves the situation incomprehensible and surprising. But, given such a puzzling mechanism is in place and the basis for some rule, then we still have a reaction to its sudden stopping.

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We now have an intuitive basis for the formal development that follows. Taxes will show up

[6] We mean the formal function defined in §1.2.

when discussing verb and preposition meaning, since many of them describe eschatological dynamics, and are definable by a rule translation of taxes. Knowledge taxes played a role in defining Π, and will play a role, along with mechanisms, in specifying a condition of word order variation. The sets of a set universe will be introduced into the LOT by appearing in particular formula, and these formula will bring the sets, and thus spaces, into definitions. In the next section, we walk through how this applies to the basic parts of language in nouns, verbs, prepositions, and adjectives. We start with a general observation about the basis of polysemy.

2 LOT and Basic Vocabulary

Upon looking for an underlying LOT, one discovers that language often doesn't pronounce functions and relations. When functions are pronounced (e.g. "speed," "price"), it is to refer to their outputs rather than the function itself. Let us demonstrate the phenomenon. (5a) might be glossed as owner(prize) = violinist.[7] And we might then try to gloss (5b) as owner(headache) = violinist. However, this seems like a strange gloss. Headaches are not the kind of things people own; it's the kind of thing we experience. Thus, a better gloss is experiencer(headache) = violinist. This, for obvious reasons, makes sentence meaning—and especially verb meaning—puzzling. The view presented here is that any function (and relation) which satisfies certain constraints of the verb, will suffice. These constraints are specified by means of the definitions of verbs and prepositions; attempts at some definitions are presented in §2.2. For the time being, when we present a formula such as $\phi(x)\mathrm{R}\alpha$, understand that most often, only x and α get pronounced.

(5) a. The violinist got a prize.
    b. The violinist got a headache.

2.1 Formula of the LOT

1. *Basic Formula.* A basic formula is either of the form $\phi(x)\mathrm{R}\alpha$ or $\alpha\mathrm{R}\phi(x)$. Although the logical difference of these forms is trivial, it matches the distinction between BE and HAVE's arguments, and so we treat them as distinct.

2. *Formula.* While basic formulas play an important role in syntax, more robust formula play an important role in definitions. These formula use the general rules of the predicate calculus, so that any relation combined with terms forms an atomic formula, combinations of atomic formula by logical connectives ($\wedge, \vee, \rightarrow, \neg$) are formula, and quantifier expressions ($\forall c, \exists c$) over a formula is a formula.[8] In fact, the formula we are concerned with are more robust than this, as relations may take terms *and formula* as input, which is necessary for defining proposals. Basic formula will be denoted φ and σ while general formula will be denoted ρ and γ.

[7] Such a gloss ignores taxis complications for the sake of simplicity.

[8] Presentations of the predicate calculus are readily available in texts on mathematical logic, for instance in Church (1956) or Kleene (1952).

3. *Proposal.* A proposal is defined recursively as any formula or term in a set, or any proposal in a set. To give examples, $(\phi(x)\mathrm{R}\alpha) \in \Phi$ is a proposal and so is $[(\phi(x)\mathrm{R}\alpha) \in \Phi] \in \Lambda$. The sets of the proposal are the sets of a set universe, and so proposals are defined in some space $S = (W(t), \mathfrak{X})$. Thus, proposals are the point at which the space universe enters the LOT. It's useful to speak of a Φ proposal when we're talking about a basic formula in Φ, and more generally an X proposal when we're talking about any term or formula in X. Proposals we denote with u and μ.

Change. If at two different times, the speed of a car is different, e.g. speed(car) $= \alpha$ at one time and speed(car) $= \omega$ at another time, then we know that we have change. Contrast this to speed(car) $= \alpha$ at one time and temperature(car) $= \alpha$ at another time, or speed(car) $= \alpha$ at the initial time and speed(cheetah) $= \alpha$ at a second time, and it's clear that there is a simple syntactic condition for when we consider there to be change, namely given $\phi(x) = \alpha$ and $\psi(y) = \omega$, there is change when $\phi = \psi$, $x = y$, and $\alpha \neq \omega$. This reduction of the property of change to local syntactic conditions is significant, as computations can be defined over the syntax, and thus this property (Fodor (2001)). Two formula which meet these conditions, minus the non-equivalence, we call *compatible*. By the syntax alone, we can develop a method of changing elements in a set/class while also checking for consistency. That is, if at t_0, $(\phi(x) = \alpha) \in \Phi$ and at time t_1, we have an instruction for $(\phi(x) = \omega) \in h(\Phi)$, we can check for compatible formula, and remove them, and thus keep things consistent, while recursively constructing the current set based on the previous set.

When the relation is not equality, but containment, we also have a syntactic condition for change. Given $a\mathrm{R}b$ and $c\mathrm{R}'d$, where $\mathrm{R}, \mathrm{R}' \in \{\in, \notin\}$, we have change whenever $a = b$, $c = d$, and $\mathrm{R} \neq \mathrm{R}'$.

4. *Statement.* A statement is a formula which contains a proposal, all of the quantifiers are out front (prenex normal form), and each atomic formula (a formula with no logical connectives) is connected to every other atomic formula by some shared term. Thus, all atomic formula, by transitivity, can be traced back to the proposal, and plays a role in describing the proposal. Statements we denote with A and U.

5. *Momentary Conditions.* The conjunction of statements we call momentary conditions. Each proposal is understood to be defined in the same space. Momentary conditions are leading us to moments and validity values. A subset of moments correspond to the skeletons of English sentences, and validity values are the basis of verbs and prepositions. We will write these as $\bigwedge_{i=0}^{n} A_i$ or just $\bigwedge U_i$ at times.

6. *Partial Formula.* If one leaves out a single term from a well-formed formula, all of its syntactic properties are recoverable. Further, given the removed term, a unique reconstruction is possible. This we prove in Appendix B. Such formula we call partial formula. If one thinks back to the knowledge taxes, they will need to be defined in terms of partial formula. For basic formula, formula more generally, proposals, statements, and momentary conditions, each will have a corresponding partial form, which can be reconstructed into their entire forms by being combined with a single term. Partial formula are embedded into what we'll call files, and will handle our treatment of adjectival phrases. We denote any partials with a grave mark, e.g. $\grave{x}, \grave{\varphi}, \grave{\bigwedge} A_i$.

Let us give some more theoretical context. We say that a partial formula is a *property* when it contains a partial term, or when a term is missing from a function. We say a partial formula is a *query* when a term is missing from a relation. Besides these two cases, its also possible to have a partial formula by removing a relation rather than a term, and still retain syntactic information. This we call a *possibility.* Possibilities are the basis of yes/no questions, and would be the top action of searching for the tool in the basement, followed by property based knowledge taxes. Thus we have "Is it in the basement?" → "What's in the basement?" → "Is it in the corner?" → "What's in the corner?" and so on, with sublocations the basis for recursive implementation.

2.2 On Verb and Preposition Definitions

For a perceived state to be understandable, the state must have a capacity condition met, and if it's a static state, then no rule applies to it. But if it's not a static state, then some rule must apply and imply the change of state. These conditions make the state understandable, which we can interpret as valid. But we may have multiple possible constraints, and this is the idea behind validity values. Even besides the multiple perceptual comprehension cases, we have possession, ethics, and reasoning comprehension. (Comprehension is to be distinguished from agreement or approval.) So it makes sense to talk of it being valid in many different ways, letting a value, which we call a validity value, correspond to each kind of validity. The application of a verb, it can be noticed, is similar to these comprehension constraints. Thus we posit that for each verb, there is a corresponding validity value and the momentary condition which specifies that validity value we take as the verb's definition.

Before we give our first two examples, we need a definition. We want to distinguish a static state from a dynamic state, and so we don't want any rules to apply to the state. To help us speak precisely, let us give the definition, followed by our first two validity values.

(6) *free*(φ): For some $\psi(), x, \mathrm{R}, \alpha, \omega$, if $(\forall\gamma, \rho, ([\varphi \in \Phi \to \gamma] \notin \Psi, [(\psi(x)\mathrm{R}\omega) \notin \Phi \to \rho] \notin \Psi,$ $\varphi = \psi(x)\mathrm{R}\alpha$, and $\psi(x) \sqsubseteq \rho))$, then φ is *free.*[9]

(D1) BE(φ): If $(\varphi \in \Phi) \in \Lambda$, $\varphi \in \Phi$, and φ is free, then φ is BE-valid.

(D2) HAVE(φ): If $\varphi \in D$, then φ is HAVE-valid

If one looks at the various uses of BE and HAVE, the definitions above allow for the entire range. Both lack dynamics, with BE capturing states that hold, and HAVE capturing the various statuses, such as kinship, things that had happened, and possession.

The general problem of providing definitions is, at this point, a matter of problem solving and with no fixed procedure. In general, one wants to look at a given verb's meaning across multiple domains, and see what's common to all cases, which is easier said than done. Domains include (a) locations and location transfer, (b) objects and object transfer, (c) knowledge and knowledge transfer, (d) actions and action transfer, and (e) time and passing time. For instance, consider the verbs GET and TAKE:

(7) a. Get out!

b. The cat got some catnip

[9]The notation $a \sqsubseteq b$ means a is a substring of b. See the definition of TO for a specification of what the second condition is meant to rule out.

c. Did you get the message the landlord left?

d. Can you get your younger sibling to soccer after school?

e. The day gets closer.

(8) a. We're taking off soon.

b. Ellen took a pamphlet and passed the stack.

c. It takes a few tries to get used to it.

d. Allen took the GRE test.

e. The days took alot out of us.

Let's focus on the object transfer case. What's the difference between Ellen taking and getting a pamphlet? To tease this out, consider the dynamic properties of possession. In an environment of trust (e.g. one is willing to leave one's stuff out, presuming it won't be stolen if one walks away for at least short periods of time), let's suppose there is some unpossessed, but possessable object. For instance, let it be a free periodical. The rules of behavior under this condition are different from the rules of behavior after the object is possessed. Consider two people going to obtain the last available periodical at the same time. In such a case, what occurs to the periodical must be based on consensus of the two individuals; if, on the other hand, one had gotten their shortly before hand — but long enough that the dynamics imply it's one's possession — then consensus no longer needs to be formed, and any insistence or aggression by the other person would be a force on decreased trust. There is still the capacity for others to ask for the object, but not to take the object as they please.

If we step back from this, we can see that when there is a change in possession, the possessor has increased capacity, in terms of decreased preconditions for any use of the object. On the other hand, for anybody else, there is a decrease in capacity with regards to the use of the object, in terms of increased preconditions to use the object (and not reduce trust: a decrease in trust itself would imply decreased capacities). And we find that this is so for many other cases. For instance, instead of the unpossessed element lying around, it may be part of a process that assigns it a possessor, such as when prizes or merchandise are given away as part of a contest. Further, even in cases of the possessor of the object changing, e.g. by gift giving, we still have this same change in capacity.

By imagination, the imagery of GET is a positive taxis, either of the object to Ellen, or Ellen to the object, while the imagery of TAKE is a negative taxis of the object from its previous location, with the perspective being invariant. (That is, one doesn't see the objects path, but the initial location, with the object and then without the object.) The former is consistent with imagery for the person that obtains an increased capacity (possession), while the latter is consistent with imagery for decreased capacity (object is no longer freely available). This interpretation—of increased and decreased capacities, respectively—can work over other domains as well, but there is some degree of freedom with these capacity claims since there's lots of variables, and it's easy to make just so stories about each case. Nonetheless, the emotional tone matches with intuitions, including capturing the emotional difference of "The days got/took a lot out of us." So, let us first give a way of denoting increased and decreased capacities, then provide tentative definitions:

(9) $\nearrow(\sigma, \varphi \in X) := \exists \psi, x, \mathrm{R}, \alpha, \alpha', \varphi \in X, \sigma = (\psi(x)\mathrm{R}\alpha), \varphi = (\psi(x)\mathrm{R}\alpha')$, and $|\alpha| > |\alpha'|$.[10]

(10) $\searrow(\sigma, \varphi \in X) := \exists \psi, x, \mathrm{R}, \alpha, \alpha', \varphi \in X, \sigma = (\psi(x)\mathrm{R}\alpha), \varphi = (\psi(x)\mathrm{R}\alpha')$, and $|\alpha| < |\alpha'|$.

(D3) GET(φ): For some σ, θ, if $(\varphi \in \Phi) \in \Lambda$, $\varphi \notin \Phi$, $[\varphi \in \Phi \rightarrow \sigma \in H] \in \Psi$ and $\nearrow(\sigma, \theta \in H)$, then φ is GET-valid.

(D4) TAKE: For some σ, θ, if $(\varphi \in \Phi) \in \Lambda$, $\varphi \notin \Phi$, $[\varphi \in \Phi \rightarrow \sigma \in H] \in \Psi$ and $\searrow(\sigma, \theta \in H)$, then φ is TAKE-valid.

A similar phenomenology for both location transfer and possession transfer suggest that PASS has these dynamics sequentially combined, describing an initial coming closer, and then movement away. Thus, the following definition.

(D5) PASS(φ): For some σ, σ', θ, if $(\varphi \in \Phi) \in \Lambda$, $\varphi \notin \Phi$, $(\varphi \notin \Phi \rightarrow \sigma' \in H) \in \Psi$, $\nearrow(\sigma', \theta \in H)$, $(\varphi \in \Phi \rightarrow \sigma \in H) \in \Psi$, and $\searrow(\sigma, \theta \in H)$, then φ is PASS-valid.

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The preposition TO clearly captures a positive taxis. The only trouble with defining a positive taxis is that the rules of the taxis no longer apply at completion, so they must be removed.[11] So a taxis may be captured by two rules as follows:

(D6) TO($[x_0, x_1]$): For some α, if $\rho \in \Psi \wedge \rho = [\phi(x_0)\mathrm{R}x_1] \in \Phi \rightarrow \sigma$, where

$$\sigma = (\rho \notin \Psi \text{ and } [(\phi(x_0)\mathrm{R}x_1) \notin \Phi \rightarrow (\frac{\mathrm{d}(\mathrm{dist}(\phi(x_0), x_1))}{\mathrm{d}t} \geq \alpha) \in \Phi] \notin \Psi),$$

and $[(\phi(x_0)\mathrm{R}x_1) \notin \Phi \rightarrow (\frac{\mathrm{d}(\mathrm{dist}(\phi(x_0), x_1))}{\mathrm{d}t} \geq \alpha) \in \Phi] \in \Psi$, then $[x_0, x_1]$ is TO-valid.

For verbs, our input type was a basic formula, and we see that for TO, and prepositions more generally, our input is a vector.

IN we take to be a dynamic condition. If one has a grocery cart, and moves it around, one assumes that everything in the cart also moves to the new location. We can capture this with the following definition:

(D7) IN($[x_0, x_1]$): If $[\forall \alpha, (\phi(x_1)\mathrm{R}\alpha) \in \Phi \rightarrow (\psi(x_0)\mathrm{R}\alpha) \in \Phi] \in \Psi$, then $[x_0, x_1]$ is IN-valid.

The quantifier out front of the rule formula blocks a computational process that would otherwise make the change itself. (Any change instead occurs by perception, though the rule sets an expectation.)

ON is similar in some sense to IN. If one moves a table with an object on it, while one expects an effect, it is not the same effect as for the table, but instead being knocked over or falling off the table. This distinction can thus to be stated with the following:

[10] If α and α' are finite, then one may take $|X|$ as cardinality; if they are some infinite domain (such as a finite interval of the real numbers), then $|X|$ must be interpreted as a measure. For references, see `https://mathworld.wolfram.com/Measure.html` or `https://proofwiki.org/wiki/Definition:Measure_(Measure_Theory)`. Accessed on August 4, 2022.

[11] But see note in appendix A on rules.

(D8) ON($[x_0, x_1]$): If $\rho \in \Psi$ and $\rho = [\exists \alpha, \omega, (\phi(x_1)\mathrm{R}\alpha) \notin \Phi \rightarrow ((\psi(x_0)\mathrm{R}'\omega) \notin \Phi$ and $\rho \notin \Psi)]$, then $[x_0, x_1]$ is ON-valid.

Note that these work for various other domains besides physical objects at a location. This format also works for a few other cases. We introduce a new set in the set universe, Θ, which contains all the thermics in the space.

(D9) OF($[x_0, x_1]$): If $[x_0 \in \Theta \rightarrow x_1 \in \Theta] \in \Psi$, then $[x_0, x_1]$ is OF-valid.

(D10) WITH($[x_0, x_1]$): For some u and mechanism $m()$, if $[(m(x_1) = u) \in \Phi \rightarrow (m(x_0) = u) \in \Phi] \in \Psi$, then $[x_0, x_1]$ is WITH-valid.

Then a few opposites follow immediately from what has already been presented:

(D11) OUT($[x_0, x_1]$): If $[\forall \alpha, (\phi(x_1)\mathrm{R}\alpha) \in \Phi \rightarrow (\psi(x_0)\mathrm{R}\alpha) \in \Phi] \notin \Psi$, then $[x_0, x_1]$ is OUT-valid.

(D12) OFF($[x_0, x_1]$): If $r \notin \Psi$ and
$r = [\exists \alpha, \omega, (\phi(x_1)\mathrm{R}\alpha) \notin \Phi \rightarrow ((\psi(x_0)\mathrm{R}'\omega) \notin \Phi$ and $r \notin \Psi)]$, then $[x_0, x_1]$ is OFF-valid.

Once we have IN and OUT, we have OPEN and CLOSE, which indicate the change in capacity of changing whether something's in or out of an object:

(D13) OPEN(φ): For x_0 such that $\varphi = [\psi(x_0) = x_1]$, for some $\psi'()$, $\psi^*()$, R', R^*, x_2, x_3, and for all $\mathrm{R} \in \{\in, \notin\}$, if $(\varphi \in \Phi) \in \Lambda$, $\varphi \notin \Phi$, $[\varphi \in \Phi \rightarrow [m(x_3) = u] \in H] \in \Psi$, and $[m(x_3) = u] \notin H$, where $u = ([\forall \alpha, \psi'(x_0)\mathrm{R}'\alpha \rightarrow \psi^*(x_2)\mathrm{R}^*\alpha]\mathrm{R}\Psi)$, then φ is OPEN-valid.

(D14) CLOSE(φ): For x_0 such that $\varphi = [\psi(x_0) = x_1]$, for some $\psi'()$, $\psi^*()$, R', R^*, x_2, x_3, and for all $\mathrm{R} \in \{\in, \notin\}$, if $(\varphi \in \Phi) \in \Lambda$, $\varphi \notin \Phi$, $[\varphi \in \Phi \rightarrow [m(x_3) = u] \notin H] \in \Psi$, and $[m(x_3) = u] \in H$, where $u = ([\forall \alpha, \psi'(x_0)\mathrm{R}'\alpha \rightarrow \psi^*(x_2)\mathrm{R}^*\alpha]\mathrm{R}\Psi)$, then φ is CLOSE-valid.

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Consider DO such as "the student did the assignment" and "the committee did the decorating." In both cases, there's a plan in place for a future event (e.g. grading or party), a precondition for that future and DO indicates the precondition being satisfied. A difficulty though, is how to treat the precondition in the LOT. The claim, which we can't back up in a strong way, is that "the assignment" and "the decorating" are in a formula in Φ. For instance, when one looks over at the paper, one remembers one has to do the assignment. When one goes to the gymnasium or comes across another reminder about the party, one knows the decorating needs to occur. Such things may be in Φ by some symbolic encoding, similar to how letters invoke lexical items upon sighting, though in this case, it's a temporary symbol. If we combine this with the general idea of completion, both in these cases and with the DO plus infinitive cases, the result is the following definition:

(D15) DO(φ): If $\varphi \notin \Phi \in \Lambda$ and $\varphi \in \Phi$, then φ is DO-valid.

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There are various ways it's imaginable that these definition attempts will be found to be inadequate, especially with regards to under specification, but the general idea of what a definition of a verb or preposition will look like—a specification of conditions, definable in a single space, centering around proposals and independent of particular domains—is the key claim of this theory of verb and preposition definitions.

2.3 Structures of the LOT

We now want to diverge from predicate formula, in order to build up towards sentences and develop structures that have the capacity to embed information and take arguments.

7. *Decomposition.* Since parts of formula are ignored while other parts get translated, we postulate structures which allow us to get the translated parts on their own. We start with e.g. a basic formula and replace each object or value with a variable. An ordered pair combines this formula with a vector of the initial objects and values, and the terms are now external. Thus, $\phi(x)\mathrm{R}\alpha$ becomes $(\phi(c_0)\mathrm{R}c_1, [x_0, x_1])$, using c to denote variables. This we call a *decomposition*. There are other cases besides basic formula, and worth mentioning is a mechanical case, which allows for "causal" argument alternations.

8. *Object Files.* For each thermic and value, there is a corresponding object file. An object file has two mandatory parts and one optional part, with the one part being optional since embedded information need not occur. The mandatory parts are the object itself, and a *head*, which is a partial formula which gives a description of the object. In terms of notation, we borrow the form of set builder notation, and write the object equal to its object file, so that e.g. $z = \langle z |\ \grave{\rho} \rangle$ or $z = \langle z |\ \grave{\rho}, \grave{\bigwedge} A_i \rangle$, where z is the object, $\grave{\rho}$ is the head, and $\grave{\bigwedge} A_i$ are any modifiers—adjectives or adverbs. The latter object file we call an *extension* of the former object file. For thermics, we typically have $\langle x |\ \mathrm{type}(\ ) \leq n \rangle$.

9. *Moments.* For verbs, we want the arguments to be external, thus implying a vector like with decomposition, but we also have adverbs which implies an object file structure. So, the object that corresponds to a verb does not have an object file, but is rather an ordered pair of an object file and a vector. This object is a *moment*, and the notation will be $y = (\langle v |\ \mathrm{valid}(\varphi, S) = \ \rangle, [x_0, \ldots, x_n])$, where v is a validity value. The function valid() has two inputs. φ is an element such that $(\varphi, [x_0, \ldots, x_n])$ is a decomposition, and S is a space. The function is to be understood such that, if $(\varphi, [x_0, \ldots, x_n])$ is a decomposition of σ, then σ is v-valid in S implies $\mathrm{valid}(\sigma, S) = v$.

10. *Partial Structures.* Just as there were partial formula, so we have partial structures, so that partial decompositions and partial moments will be like decompositions and moments, but missing a single term.

2.4 Noun Reference

Nunberg (1979) gives the basic inspiration for our approach to noun reference. To take an example of the problem, the noun "cat" can refer to a particular feline ("the cat rubbed against the owner when hungry"), a sequence of phonemes ("Cat has a K sound"), a sequence of graphemes ("Cat has 3 letters"), a kind ("A cat is a mammal"), and a substance ("cat splattered everywhere from the horrible accident"). Further, for any noun, there are creative extensions that can occur, such as Nunberg's example of a wait staff using a customer's order to refer to the customer, so that "ham sandwich" can refer to a person.

The theory we propose is a specification of an idea for a recursive formula found in Nunberg (1979). For any ω, ω is referable from ω, and if ω' is referable from ω and $[\phi(\omega^*)\mathrm{R}\omega'] \in \Phi$, then ω^* is referable from ω.

This theory gives a lot of paths, and not all of them will be ones usable for the sake of reference. Thus, this gives necessary, but not sufficient conditions for reference. There is likely a set of fixed paths, defined in terms of only the functions that make it up, and that all reference must be one of these paths or but a small extension of such a path, as with the "ham sandwich" case.

To give a tentative database structure that would match with the "cat" references above, we might have, using unexplained schema and short hand to be brief,

(A) phone(CAT) $\leq$ /kɑt/,	(D) type(x') $\leq$ /kɑt/,	(G) type(z') $\leq$ $\mathfrak{tissue}$,
(B) word($\mathfrak{cat}$) $\leq$ CAT	(E) type(x) $\leq$ $\mathfrak{cat}$,	(H) whole(z) $\leq x$,
(C) word(c-a-t) $\leq$ CAT,	(F) type(z) $\leq$ $\mathfrak{bone}$,	(I) whole(z') $\leq x$

each in Φ. Then the sequence of phonemes may be referred to by the basic referable condition, or by (D). The sequence of graphemes may be referred to by (A) and (C), the type may be referred to by (A) and (B). A cat may be referred to by (A), (B) and (E). For the substance, we have (A), (B), (E), and (H) and/or (I). For each path, besides the basic referable condition, (A) may be replaced by graph(CAT) $\leq$ c-a-t in the case of writing.

2.5 Adjectival Phrases

In this grammar, adjectives and adverbs don't get as much treatment as they merit. Adverbial phrases will be included loosely in the core grammar on the same basic model as adjectival phrases. Given the structures and formula we have covered so far, we can say that an adjectival phrase is any partial formula or structure in an object file, with the exception of the head of the file. Adjectival phrases may be adjectives, prepositions, sentential phrases absent a single noun (partial moments), and their conjunctions.

The most common style of English usage is for adjectives to go before the noun, while prepositions and sentential phrases go after the noun. However, all three modifiers are capable of going before or after the noun, as demonstrated in (11) and (12). There is a slight but distinct phenomenological difference in the cases. The phenomenology is consistent with being more immediate and certain for the after position. This suggests a set distinction, but adjectives presumably correspond to Φ proposals, while prepositions correspond to Ψ proposals. Thus, if there is a set distinction, the set must be able to take proposals as input. We thus introduce another set, which handles the left cases, and will be significant for §5.1.

(11) The the-basket-is-hiding youths, happy, are playing hide and seek.

(12) A purple, in-the-room elephant was talked about without a thought.

Beliefs (E). We've discussed surprise and confusion to motivate sets, and in that same vein, consider a case of being upset. Each year, Jens looks forward to the town's annual St. Olaf's Day festival. But budget cuts, various logistics, and red tape have caused this years festival to be canceled. Having believed that the festival was to take place and having already begun to look forward to and prepare for the festival, this has been quite a disappointment for Jens. Returning to our theoretical concerns, there is not puzzlement: no rule change is uncounted for and it is not outside the realm of anyone or anything's capacity. Further, there is no disbelief at an inexplicable change of state. Thus, we have some other set. This is our set of beliefs, which we denote E. We have a commitment to our beliefs less there are sufficient forces against, such as perception or frequent opposition and so on.

If one thinks about our beliefs: the belief that the annual festival will be some particular day, or one's belief about it being a full moon tomorrow night, it's clear one needs not only a formula but also to have temporal information. If one thinks back to suppositions (or Π proposals), those too will require there to be temporal information. Thus, the full psychological theory should actually have these sets take ordered pairs as input, but we will only represent these sets as taking a formula or proposal without regard to the temporal information.

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Predicate formula and proposals allow us to build up momentary conditions, which are expressive enough to present verb definitions. They are also identifiable by their syntax alone, and so we can, and will, define computations over them. With object files, we have a mechanism of embedding information, and thus a means of representing adjectival and adverbial phrases. With decomposition and moments, we have a means of externalizing the arguments of verbs and prepositions. Combined with the rules for reference of nouns, this leaves us with the tools for interpreting basic sentences. But often English has composition of verbs and prepositions; or the verb is in a form, such as a participle, which gives it a different meaning than the definition we gave; or an argument is not being pronounced and it's not clear how this maps to a basic formula; or the word order can vary in ways not yet covered, and with accompanying semantic effects. These are the topics of the next section, in which we develop tools for interpreting more complex sentences.

3 English Analysis

3.1 Absent Arguments

English has various forms of argument alternations (Levin (1993)), and at least some of these involve a missing argument. Under what conditions this may occur is puzzling, but we might start with some observations. Certain cases of space-time can be deleted, as with "are you going?" But consider some prepositional cases. Unlike with (15), Both cases of (13) and (14) retain the same meaning.

(13) a. Nastasya turned the light on
b. Nastasya turned on the light

(14) a. Hank took his shoes off
b. Hank took off his shoes

(15) a. Siobhan got her coat off
b. Siobhan got off her coat

Upon some reflection, the interpretation we give of this, for all but (15b), is that the right object of the preposition is absent. In fact, (15b) has such a reading too, when the preposition receives the same articulation markers as the verb, but those markers are beyond the scope of this paper. These right argument deletions are also found with certain intransitive constructions:

(16) Ali ate.

(17) The car starts.

What conditions distinguish when these right argument absences can and can't occur? Considering what the light is on, and considering what Ali eats, it's clear that there could be a plurality, and perhaps this plurality is a basis for the unpronounced argument. Another possibility is direct orientation, though there are puzzles here, as to why "the child draws" seems incomplete, though one is oriented towards what one draws. Such is the difficulty. Orientation-towards by the speaker allows for the absence of singular subjects, and sometimes an "auxiliary" verb, too. Thus, one can orient towards a car and say "gets 25 to the gallon" or "getting fixed tomorrow."

These facts about orientation we mention as it will be used later in the grammar, but for our purposes now, we merely need an interpretation within the LOT. And to that end, we treat these as quantifier expressions. Just as functions are not (usually) expressed within English, so we take these quantified variables to also be unexpressed. Thus, we take (16) and (17) to get the interpretation of (18) and (19), and likewise (13) and (14) to get the interpretation of (20) and (21).

(18) $\exists x$, Ali ate x.

(19) $\exists x$, The car starts x.

(20) $\exists x$, Nastasya turned the light on x.

(21) $\exists x$, Hank took his shoes off x.

There remains the question as to why (13) and (14) have 2 forms. Phenomenologically, their distinction is that the (a) cases are more immediate and the (b) cases are more past and settled. We thus take the difference to be whether the consequent of the rule is just about to be in place or is already in place, respectively.

3.2 The Dative

Consider the arguments to the right of the verb in (22) and (23). We have already seen from our posited definitions that prepositions are restricted to Ψ proposals. In (22) and (23), the (c) sentences' two right arguments can be placed into a HAVE sentence and preserve the functions and relations, suggesting they represent information in D. For (22a), we see that the two right arguments can be placed in a BE sentence. In terms of accommodating such diversity, this leads to various possibilities, including the definitions of the verbs being ambiguous with regards to the relevant sets, but we posit a different solution. We claim instead that the (b) and (c) cases are constructions external to the verb definition which may be made under certain constraints. So that it pertains to the verb definition, we propose the constraint to be that the zeroth proposal of the verb definition implies the Ψ-proposal(s) or D-proposal. Under such a condition, these constructions may be used.

(22) a. The teacher made the quiz challenging.

b. The teacher made a quiz for the students.
c. The teacher made the students a quiz.

(23) a. The professor taught the material.
b. The professor taught the material to the class.
c. The professor taught the class the material.

The way that the dative will be most prominent in this study is when one of the arguments is a moment. That is, when it has the form $x\mathrm{R}\psi(y) \in D$, where y is a moment. This will be the main way that verb composition occurs. But there is an exception, namely, the passive construction. The passive construction works by having the past participle compose with the verb in the same way prepositions do. As we mentioned in the last section, the consequent of the preposition's Ψ proposals may hold or not, giving us two separate constructions. And as mentioned in this section, the zeroth proposal of the verb implies the Ψ proposal. The case without the consequent holding includes both the preposition composition and the passive. And since it seems too robust for the zeroth proposal to imply the whole past participle definition, the composition will be assumed to imply only the zeroth proposal. Note that with most prepositions, the zeroth proposal and its whole definition are equivalent.

3.3 Order of Verb's Arguments

Consider some categories of word order variation of verbs and their arguments:

(24) Commands without a subject
Ex.: "Print."

(25) Questions with do-support
Ex.: "Did the function print anything?," "What did the function print?"

(26) Auxiliary verb questions without do-support
Ex.: "Is the computer a function?," "Can the computer run anything?"

(27) The existence of 'SO V N' constructions.
Ex.: "The computer ran all day, and so did the function."

(28) The existence of 'N HAVE V-en N' constructions.
Ex.: "The computer had printed something."

All of these cases are either cases of the verb preceding the subject ((25)-(28)), or no subject ((24)), relative to the finite declarative form. In contrast to these cases, we also have similar cases with a distinct order:

(29) Non-finite verb with a subject
Ex: "The function had the computer run all day" and "The function got the computer printing Fibonacci numbers."

(30) Interrogative pronoun as subject of non-auxiliary verbs
Ex.: "What ran the function?"

(31) The existence of 'N HAVE N V-en' constructions.
Ex.: "The computer had something printed."

Let us introduce three principles which, along with the dative construction, allow us to handle all of these cases.

Principle 1. The first argument of the finite form of a verb is absent in the non-finite form.[12] Formally, we will handle this with the existential quantifier as we did in §3.1 with the right argument. The reason for this hypothesis is (1) that one can get the criteria above to work out and (2) each form has a demonstration of this property. For the infinitive, it's commands. For the present participle, it's nominal phrases like the subject of (32a) and adverbial constructions as in (32b). For the past participle, we have the passive construction, and the 'N HAVE N V-en' construction. Principle 1 creates a problem for constructions like (29) as there is a non-finite verb, a subject, and no other verb next to it. This we claim is handled by the dative transformation, so that the moment and the missing argument are the two dative-arguments.

(32) a. playing in the sand is fun when you're a kid.
b. playing in the kitchen, the kids were keeping out of everyone's way.

Principle 2. A verb has all its arguments to its right only when it expresses an unresolved moment. This is applicable for questions in an obvious way, but for commands and other cases, it's less obvious. Commands such as "be quiet" and statements such as "you are being quiet" and "you should be quiet" have a quality to them that is lacking in "you are quiet." That Π applies — that they are not yet states and that its unresolved whether they will be states in the future, or if they do hold, then the current state still leaves the future of the state unresolved — can account for this distinction. With dynamic verbs, the completion—expressed by the finite forms—is expressed to be in the future by the infinitive and present participle. Thus, this is consistent with the end being in Π and being unresolved.

Principle 3. If an active mechanism can output the zeroth proposal of the verb, then the all right argument construction, or principle 2, isn't available. The zeroth proposal, so called because it's the proposal of A_0 in the momentary condition $\bigwedge_{i=0}^{n} A_i$, is normally some basic formula in Φ, if one ignores Λ. (The definitions of §2.2 were presented so that the zeroth proposal occurs first.) When a mechanism can output the zeroth proposal, u, then we have $m(x) = u$, which if active, is in Φ. When this is possible, we say the verb is *mechanizable.* This constraint must be understood right to left: what is to the right of a verb can change how this rule applies to it, since a verb to its right can force a particular reading, and in particular it can remove the possibility of a mechanical construction. Most of the time, a mechanisms availability can be determined by the presence of 2 argument structures, one understood as adding the cause/source of the other, e.g. "the town got a prize" and "the mayor got the town a prize." However, certain cases, like "give" with the possession domain, don't have the non-mechanism argument option. This principle gives otherwise indistinct cases separate syntactic structures, so let's look at what is happening, going through the list one by one, using all three principles.

[12] The past participle of GO may be an exception, though having both "Sam is gone" and "Sam has gone" suggest something irregular relative to comparable cases.

By principle 1, the infinitive in (24) has one less argument than the finite form. The remaining arguments are to its right by principle 2, since the infinitive definition implies it's unresolved (discussion above and §3.4), and in general, non-finite verbs are not mechanizable. The details of why this is interpreted as a command rather than a question we leave vague, but we note that Searle's (1985) observation that questions are commands is consistent with this approach. That addresses (24), so let us address questions. Suppose we have a mechanizable verb. Since principle 3 applies, then principle 2 can't apply, so the expression of unresolved can't occur and a semantic precondition of a question can't apply. Thus, only non-mechanizable verbs occur as the first verb. These verbs are BE, the modal verbs, DO constrained by a composing infinitive, and HAVE constrained by a (unresolved) past participle, and this accounts for (25) and (26). The contrasting case of (30) we handle with a technicality. Principle 3 blocks the all right *construction*. It won't be handled until the extension, but these interrogative pronouns have the structure of a noun plus a partial moment. When the first argument of the moment is the one missing, then it becomes ambiguous as to whether the source was resolved or not, and so assuming unintelligent, modular processing, the partial moment can satisfy the unresolved condition (but need not).

For (28) and (31), by definition of HAVE, we don't need the dative transformation, though the same technique of having the moment be an argument applies. The distinction between the two cases is whether the past participle is resolved or not, leading to the different argument orders. A question that arises is why the past participle occurs in this construction and not the infinitive or present participle. We cannot say for certain, but (29) is handled by the argument structures we've been claiming exist in D (one case by definition of HAVE, the other by the dative construction), plus a mechanism, and it may be that, just as with GIVE and the possessive domain, the mechanism is mandatory for whatever reason.

There is one case that remains. Lacking a definition of SO, it's hard to say what's happening with (27), but we may note that principle 3 applies and restricts this construction.

3.4 Verb Forms

The infinitive, present participle, past participle, finite present, and finite past are the five possible forms of English verbs. The finite forms will be addressed later in §5.2. For now we are focusing on the non-finite forms. One thing we want to understand is how these forms interact with the verbs' definitions to produce their particular outputs. So let us note a few distinctions that exist.

For the present participle, it has been noted that verbs whose finite forms imply completion lack such completion with the present participle (Comrie (1976)). Thus "the dog is getting its bone" is incomplete, though "the dog gets its bone" implies completion. We might take this absent final state as defining this form, except that for state verbs, the present participle implies the same state as the finite form, and so it isn't absent. This can be seen with the similarity of (33a) and (33b). Another option is to take the state—the static one or the one for completion—to be uncertain, and so in Π. This matches anticipation which can occur with the dynamic case, and for the static case, we may note that this uncertainty is so presumed in certain instances, that its lack allows us to make inferences. For instance, if a dog is lying around, one finds nothing out of the ordinary about this. But if it lies around for a long time, making the uncertainty of their state unnecessary, then the owner can infer illness or malady from this. We see that (33b) has more of a suspense feeling and a less settled atmosphere, especially if appearing in a narrative. Thus, we assume the zeroth proposal of the verb is in Π for the present participle. Otherwise there is a matching with the verbs usual meaning.

(33) a. The dog lies on the floor.

b. The dog is lying on the floor.

For the various uses of the infinitive, we have that it is not actually the case, nor has it begun to be the case, that whatever typically occurs for the verb, is occurring. If one gives a command, the person is not currently working towards that task; the various modals with the infinitive often imply it's not currently happening, though CAN is sometimes an exception (e.g. "I can see you"). Thus, we want the meaning to capture the lacking of current status. This can be done by putting every proposal of its definition in Π. In general, one isn't guaranteed that arriving at the tea cup will give one an increased capacity by possession, and so that too must be uncertain when the action "get the tea cup" is not active.

This leaves only the past participle. For the past participle, we note that it implies not only completion, but the temporal space after the completion. These means that, if $u \in \Lambda$, then u will now hold, and if $(\rho \to \mu) \in \Psi$ and ρ, then μ holds. The former case will be produced by the thus-function and the latter case by the then-function. The discussion of change (§2.1) will play a crucial role in being able to define these functions. Further, we see that the past participle has both resolved and unresolved argument orders: both left and right and all right arguments. We will posit that its zeroth proposal is in Π like with the present participle. Cases of it being resolved we aren't being precise enough to capture, but is chalked up to the Π time being in the future, and so it's resolved at the time of reference, even if it won't be afterwards.

Putting this all together gives us definitions for these forms of the verb. In §4.1, we give more precise definitions, but here is the general idea of the definitions for the non-finite forms:

(34) *Infinitive*: If $\bigwedge A_i$ implies φ is v-valid, then $\bigwedge A'_i$, the momentary condition gotten by replacing all proposals u in $\bigwedge A_i$ with $u \in \Pi$, implies φ is $\textsc{inf}(v)$-valid.

(35) *Present Participle*: If $\bigwedge A_i$ implies φ is v-valid, then $\bigwedge A'_i$, the momentary condition gotten by replacing the proposal u in A_0 with $u \in \Pi$, implies φ is $\textsc{ing}(v)$-valid.

(36) *Past Participle*: If $\bigwedge A_i$ implies φ is v-valid, and $\bigwedge A'_i = \textsc{then}(\textsc{thus}(\bigwedge A_i))$, then $\bigwedge U_i$, the momentary condition gotten by replacing the proposal u in A'_0 with $u \in \Pi$, implies φ is $\textsc{en}(v)$-valid.

3.5 Preposition Composition

In §3.2, we saw transformations of a verb's arguments and composition by having one verb as an argument of another. With prepositions, the means of composition is another distinct process. To see this, let us look at the prepositions "into" and "onto," and for simplicity we'll stick with "into," though "onto" is also applicable. Attending to its meaning, we see that the conditions of "in" occurs at the end, which is on condition of the taxis completion. One way of understanding this is that the conditional of TO gets transformed so that the resulting consequent (of the conditional) is a conjunction of the initial consequent and IN. (Symbolically, if IN:= $a \to b$ and TO:= $c \to d$, then INTO:= $c \to (d \wedge (a \to b))$.) The concern from here is whether this works in other cases.

There are not many examples in English, but of the few "out of" also is consistent with this construction. Let us consider the construction "x without z." The problem here is that, by the semantics, we have OUT and the negation of WITH. That is, if one is without a paddle, this is inconsistent with doing something with a paddle. But by the formula above and OUT implying the formula is not in Ψ, we would have that either OUT or the negation of WITH, but not both, is perfectly acceptable. And *or* implies the negation of WITH is optional, contrary to its semantics. On the other hand, consider "x from out (of) z." If we gloss FROM as a negative taxis that is not complete until a certain distance occurs, so that the condition of FROM, namely being a certain distance away, isn't satisfied, then we have OUT and the negation of FROM consistent with the meaning of "from out (of)," where the semanttics is interpreted as consistent with "just came out of." This suggests that WITHOUT is not an outlier, but instead part of the general pattern.

We can denote the general formula for the composition of $(a \to b)\mathrm{R}\Psi$ and $(c \to d)R'\Psi$ as $(c \to (d\ f(\mathrm{R}')\ (a \to b)\mathrm{R}\Psi))\mathrm{R}'\Psi$, where $f(\in) = \wedge$ and $f(\notin) = \vee$.

∼ ∼ ∼

While we have tried to present the nuances that underlie the grammar in as simple a manner as possible in this section, they contribute a great deal of complexity to the rules posited below. The dative transformation and verb-preposition composition, as well as mechanization and argument absences will complicate the presentation of decomposition and, more generally, the translation from formula to structures and then lexical items. Preposition composition and the forms of the verb require the definition of functions from validity values to validity values, with the past participle's definition having the most elaborate prerequisites to present. Resolved and unresolved moments, and restrictions from mechanization, force distinct rules for mapping from structures to linear order, as does preposition composition when there is an absent right argument. Nevertheless, once these phenomena are made precise, they help form the core grammar of English.

4 A Core English Grammar

The grammar we present may be considered a production grammar as it maps from the LOT to externalization. Another possible grammar is a parsing grammar, which maps externalizations to the LOT. Such a grammar involves articulation cues to distinguish structure types, and for the sake of being practical, had to be foregone. The first part of the grammar, §4.1, specifies the structures of the LOT. The second part §4.2, which may be considered the grammar proper, specifies the translation of the LOT, up to particular lexical elements, which we just claim exist. The grammar itself is followed up by examples so as to make concrete the otherwise abstract presentation. In §4.3, we provide derivations at the level of

LOT structure, and in §4.4 we provide interpretations for very basic sentences at the level of LOT formula.

4.1 The Language of Thought

We begin, in L1 and L2, developing the rules of the LOT formula (§2.1). L3 develops the verb forms (§3.4), with most of the space dedicated to the past participle. L4 transitions from the formula to structures by establishing preposition composition (§3.2 and §3.5), decomposition, and the validity function (§2.3). L5 introduces object files (§2.3) and their embedded modifiers (§2.5) to round out the LOT.

Formula

(L1.1) *Objects.* Thermics, values, and variables are objects. (We don't use the usual definition of terms, so as to avoid functions.)

(L1.2) *Basic Formula.* If $\phi()$ is a function, R any relation, and x_0 and x_1 objects, then $\phi(x_0)\mathrm{R}x_1$ and $x_0\mathrm{R}\phi(x_1)$ are *basic formula.*

(L1.3) *Basic Partial Formula.* If $\phi()$ is a function, R any relation, and x an object, then $\phi(x)\mathrm{R}$, $\phi(\ )\mathrm{R}x$, $x\mathrm{R}\phi(\ )$, an $\mathrm{R}\phi(x)$ are basic partial formula.

(L1.4) *Formula.* A formula corresponds to the formula of the predicate calculus extended so that relations may also take formula. So any object is a term, and any function made up of terms is a term. Then any relation made up of terms is a formula, any relation made up of terms and formula is a formula, if ρ and γ are formula, then $\neg\rho$, $\rho \wedge \gamma$, $\rho \vee \gamma$, and $\rho \rightarrow \gamma$ are formula, and if c is a variable, then $\exists c, \rho$ and $\forall c, \rho$ are formula.

(L1.5) *Partial Formula.* Any function made up of terms and one missing input is a partial term, and any function made up of terms and one partial term is a partial term. Any relation made up of terms, relations, and one missing input is a partial formula, any relation made up of terms, formula, and one partial term is a partial formula, and any relation made up of terms, formula, and one partial formula is a partial formula. If ρ is a partial formula, γ a formula or partial formula, and c a variable, then $\neg\rho$, $\rho \wedge \gamma$, $\gamma \wedge \rho$, $\rho \vee \gamma$, $\gamma \vee \rho$, $\rho \rightarrow \gamma$, $\gamma \rightarrow \rho$, $\forall c, \rho$ and $\exists c, \rho$ are partial formula.

(L1.6) *Well-Formed Space.* If $W_1(t_1)$ is any space-time region and $\mathfrak{X}_1$ is any collection of sets such that for some $(W_0(t_0), \mathfrak{X}_0) \in \mathcal{U}$, there is a one-to-one correspondents h from $\mathfrak{X}_1$ to $\mathfrak{X}_0$, then $S = (W_1(t_1), \mathfrak{X}_1)$ is a *well-formed space.*

(L1.7) *Proposal.* u is a *proposal* if, for any object x, for any formula ρ, $\mathrm{R} \in \{\in, \notin\}$, and X a set in a set universe of a well-formed space,

i) $u = x\mathrm{R}X$,

ii) $u = \rho\mathrm{R}X$, or

iii) There is a proposal μ such that $u = \mu\mathrm{R}X$

(L1.8) *Partial Proposal.* $\grave{u}$ is a *partial proposal* if, for any partial formula $\grave{\sigma}$, partial term $\grave{x}$, $R \in \{\in, \notin\}$, and X a set in a set universe of a well-formed space,

i) $\grave{u} = \mathrm{R}X$,

ii) $\grave{u} = \grave{x}\mathrm{R}X$,

iii) $\grave{u} = \grave{\sigma}\mathrm{R}X$, or

iv) There is a partial proposal $\grave{\mu}$ such that $\grave{u} = \grave{\mu}\mathrm{R}X$

(L1.9) *String.* Any formula may be interpreted as a string. If $\rho = a_0 a_1 \ldots a_n$ is any formula, we distinguish with the string function λ, and we write $\lambda(\rho) = \lambda(a_0)^\frown\lambda(a_1)^\frown \ldots ^\frown \lambda(a_n) = a_0^\frown a_1^\frown \ldots ^\frown a_n$ for a string.

(L1.10) *Null String.* There is a null string $\emptyset$ such that for any string s, $s^\frown\emptyset = \emptyset^\frown s = s$

(L1.11) *Substring.* Let s be a string such that $s = s_0^\frown \ldots ^\frown s_n$. Then for any i and j, $0 \leq i \leq j \leq n$, $s' = s_i^\frown s_{i+1}^\frown \ldots ^\frown s_j$ is a *substring* of s, and we denote this $s' \sqsubseteq s$. Furthermore, if $\lambda(\rho) \sqsubseteq \lambda(\gamma)$, then $\rho \sqsubseteq \gamma$.

(L1.12) *Partial Formula Completion.* If $\grave{\varphi}$ is any partial formula, and x an object such that $x \not\sqsubseteq \grave{\varphi}$, then $\grave{\varphi} \oplus x$ is defined as the unique basic formula made up of the symbols of $\grave{\varphi}$, x, and no other symbols. The existence and uniqueness is shown in appendix B.

Likewise, if $\grave{u}$ is a partial proposal and $x \not\sqsubseteq \grave{u}$, then $\grave{u} \oplus x$ is the unique well-formed proposal gotten by the insertion of x into $\grave{u}$.

Momentary Conditions

(L2.1) *Substitution Function.* There is a substitution function, $\textsc{sub}()$, from triplets of strings to strings, such that $\textsc{sub}(c^a_b) = c_0^\frown a^\frown c_1^\frown a^\frown \ldots ^\frown a^\frown c_n$, where $c = c_0^\frown b^\frown c_1^\frown b^\frown \ldots ^\frown b^\frown c_n$ and for each i, $0 \leq i \leq n$, $b \not\sqsubseteq c_i$. We also use the notation

$$\textsc{sub}(c^{a_0,a_1,\ldots,a_k}_{b_0,b_1,\ldots,b_k}) := \textsc{sub}(\ldots(\textsc{sub}(\textsc{sub}(c^{a_0}_{b_0})^{a_1}_{b_1})\ldots)^{a_k}_{b_k})$$

(L2.2) *Atomic and Predicate Formula.* A formula is *atomic* when it contains no logical symbols $\wedge$, $\vee$, $\rightarrow$, $\neg$, $\forall$, nor $\exists$. A formula is a *predicate formula* when relations contain only terms and no formula.

(L2.3) *See Relation.* Let γ be any formula and suppose x_0 and x_1 are objects such that $x_0, x_1 \sqsubseteq \gamma$. x_0 sees x_1 in γ if

i) there is an atomic predicate formula ρ such that $x_0, x_1 \sqsubseteq \rho \sqsubseteq \gamma$, or

ii) there is a z such that x_0 sees z in γ, and z sees x_1 in γ

(L2.4) *Subproposal.* Let s be any string. u is a subproposal of s if u is a proposal, $u \sqsubseteq s$, and for any u' such that $u' \sqsubseteq s$, u' is a proposal implies $u' \sqsubseteq u$.

(L2.5) *Quantified Term.* L is a quantified term if $L = [\forall c]$ or $L = [\exists c]$ where c is a variable.

(L2.6) *Statement.* A formula A is a *statement* if there is a subproposal u of A and

i) $A = u$,

ii) $A = u \wedge \gamma$, or

iii) there is an A', A' is a statement and $\lambda(A) = L^\frown A'$

for some formula γ and sequence of quantified terms L, such that for each term x in A, there is a term z in u such that x sees u. We say that u is the proposal of A, and denote this $\text{prop}(A) = u$.

(L2.7) *Partial Statement.* $\grave{A}$ is a *partial statement* if $\grave{u}$ is a partial proposal, $\grave{u} \sqsubseteq \grave{A}$, and for any term $x \not\sqsubseteq \grave{A}$, $\grave{A} \oplus x$ is a statement.

(L2.8) $\bigwedge_{i=a}^{a} X_i := X_a$, and $\bigwedge_{i=a}^{k} X_i := \bigwedge_{i=a}^{k-1} X_i \wedge X_k$, for $k > a$.

(L2.9) *Momentary Condition.* If $A_0, A_1, \ldots,$ and A_n are statements, then $\bigwedge_{i=0}^{n} A_i$ is a *momentary condition* whenever for every i, $1 \leq i \leq n$, there is a term x in $\text{prop}(A_i)$ and a term z in $\text{prop}(A_0)$ such that x sees z in $\bigwedge_{i=0}^{n} A_i$.

(L2.10) *Partial Momentary Condition.* $\grave{\bigwedge} A_i$ is a *partial momentary condition* whenever each A_i is a statement or a partial statement, and there is at least one partial statement. Furthermore, for $x \not\sqsubseteq A_i$, for all i, $(\grave{\bigwedge} A_i) \oplus x = \bigwedge f(A_i, x)$ where $f(A_i, x) = A_i \oplus x$ if A_i is a partial statement and $f(A_i, x) = A_i$ otherwise.

Verb Forms

(L3.1) *Infinitive.* If $\bigwedge_{i=0}^{n} U_i$ implies φ is v-valid, $\bigwedge_{i=0}^{n} A_i$, and $\forall i, 0 \leq i \leq n$, $A_i = \text{SUB}({U_i}_{\text{prop}(U_i)}^{\text{prop}(U_i) \in \Pi})$, then φ is $\text{INF}(v)$-valid.

(L3.2) *Present Participle.* If $\bigwedge_{i=0}^{n} U_i$ implies φ is v-valid, $\bigwedge_{i=0}^{n} A_i$, $\forall i, 1 \leq i \leq n$, $U_i = A_i$, and $A_0 = \text{SUB}({U_0}_{\text{prop}(U_0)}^{\text{prop}(U_0) \in \Pi})$, then φ is $\text{ING}(v)$-valid.

(L3.3) *Compatible.* Suppose u and μ are proposals such that $u = [\phi(x) = \alpha \in X_0 \in \cdots \in X_n]$ and $\mu = [\psi(z) = \omega \in Z_0 \in \cdots \in Z_m]$. u and μ are *compatible* if $\phi() = \psi()$, $x = z$, $m = n$, and for $i, 0 \leq i \leq n$, $X_i = Z_i$. Otherwise, u and μ are *incompatible.*

(L3.4) *But Function.* We can now define change with preservation. For proposals $u = (\rho \mathrm{R} X)$ and $\mu = \gamma \mathrm{R}' Y$, where $\mathrm{R}, \mathrm{R}' \in \{\in, \notin\}$ and X and Y are sets, let

$$f(u, \mu) := \begin{cases} 1 & \text{if } (X = Y \text{ and } \rho = \gamma) \text{ or } u \text{ and } \mu \text{ are compatible} \\ 0 & \text{otherwise} \end{cases}$$

Then for two sets of proposals A and B,

$$\text{BUT}(A, B) := \{u | \ u \in B \text{ or } (u \in A \text{ and } \forall \mu \in B, f(u, \mu) = 0)\}.$$

(L3.5) *Conjunctive Subtraction.*

$$(\bigwedge_{i=a}^{n} A) \setminus A_j := \begin{cases} \bigwedge_{i=a+1}^{n} A & \text{if } j = a \\ \bigwedge_{i=a}^{n-1} A & \text{if } j = n \\ \bigwedge_{i=a}^{j-1} A \wedge \bigwedge_{i=j+1}^{n} A & \text{if } a < j < n \end{cases}$$

Further, $(\bigwedge_{i=a}^{n} A) \setminus \bigwedge_{j\in X} A$ is defined as $(\ldots((\bigwedge_{i=a}^{n} A) \setminus A_{j_0}) \setminus \ldots) \setminus A_{j_k}$, where $X = \{j_0, \ldots, j_k\}$, and for each $j \in X$, $a \le j \le n$.

(L3.6) *Thus Function.* The thus function removes all the Λ-proposal statements from an initial momentary condition ($\bigwedge A_i$) and adds them as non-Λ proposal statements to a new momentary condition ($\text{THUS}(\bigwedge A_i)$). We'll first separate out all the Λ-proposal based statements ($\bigwedge_{j\in X} A_j$), now processed outside of Λ ($\mathcal{L}$), and all the non-Λ-proposal based statements ($\bigwedge U_i$). Then we'll replace compatible elements ($\bigwedge B_i$) and join them with the remaining Λ-based proposals ($\bigwedge_{M\in\mathcal{M}} \text{prop}(M)$).

Let $\bigwedge A_i$ be a momentary condition. Define
$X := \{i|\ \lambda(\text{prop}(A_i)) = \rho^\frown \text{R}^\frown \Lambda\}$, $\bigwedge_{i=0}^{m} U_i := (\bigwedge_{i=0}^{n} A_i) \setminus \bigwedge_{j\in X} A_j$,
$\mathcal{L} : \{L|\ j \in X \text{ and } \lambda(L) = \text{SUB}(A_{j\,\rho^\frown \text{R}^\frown \Lambda}^{\ \rho})\}$, and for any set $\mathcal{F}$ of statements and any statement A, define $h()$ as

$$h(A, \mathcal{F}) := \begin{cases} \text{prop}(F) & \text{if } \exists F \in \mathcal{F}, \text{BUT}(\{\text{prop}(A)\}, \{\text{prop}(F)\}) = \{\text{prop}(F)\} \\ A & \text{otherwise} \end{cases}$$

Finally, let $\bigwedge_{i=0}^{k} B_i = \bigwedge_{i=0}^{m} h(U_i, \mathcal{L})$ and $\mathcal{M} = \mathcal{L} \setminus \{B_0, \ldots, B_k\}$. The thus function is $\text{THUS}(\bigwedge_{i=0}^{n} A_i) = \bigwedge B_i \wedge \bigwedge_{M\in\mathcal{M}} \text{prop}(M)$

(L3.7) *Then Function.* The then function takes all the conditional rules, and outputs the consequent of satisfied antecedents. We first separate out the cases with satisfied antecedents ($\mathcal{U}$), and then separate out their consequences ($\mathcal{L}$). We then replace the initial momentary condition with the compatible consequences ($\bigwedge B_i$), and finally combine these with the remaining consequences ($\mathcal{M}$).

Let $\bigwedge_{i=0}^{n} A_i$ be a momentary condition. Define
$\mathcal{U} := \{U|\ \exists j, U = A_j \text{ and } \exists i, \rho, \text{prop}(A_j) = [(\text{prop}(A_i) \to \rho) \in \Psi]\}$. For any formula γ define $f()$ as

$$f(\gamma) := \begin{cases} \{\gamma\} & \text{if } \gamma \text{ is a statement} \\ f(\rho) \cup f(\rho') & \text{if } \gamma = \rho \wedge \rho' \text{ and } \rho \text{ is a statement or } \rho' \text{ is a statement} \\ \emptyset & \text{otherwise.} \end{cases}$$

Further, for any proposal u, define

$$g(u) := \begin{cases} f(\gamma), & \text{if } \exists \rho, \text{prop}(u) = [(\rho \to \gamma) \in \Psi] \\ \emptyset & \text{otherwise} \end{cases}$$

Again, define for a collection $\mathcal{F}$ of statements and a statement A,

$$h(A, \mathcal{F}) := \begin{cases} \text{prop}(F) & \text{if } \exists F \in \mathcal{F}, \text{BUT}(\{\text{prop}(A)\}, \{\text{prop}(F)\}) = \{\text{prop}(F)\} \\ A & \text{otherwise} \end{cases}$$

Let $\mathcal{L} = \bigcup_{U \in \mathcal{U}} g(U)$, $\bigwedge_{i=0}^{n} B_i = \bigwedge_{i=0}^{n} h(A_i, \mathcal{L})$, and $\mathcal{M} = \mathcal{L} \backslash \{B_0, \ldots, B_n\}$. The then function is $\text{THEN}(\bigwedge_{i=o}^{n} A_i) = \bigwedge_{i=0}^{n} B_i \wedge \bigwedge_{M \in \mathcal{M}} \text{prop}(M)$.

(L3.8) *Past Participle.* If $\bigwedge_{i=0}^{n} U_i'$ implies φ is v-valid, $\bigwedge_{j=0}^{m} U_j = \text{THEN}(\text{THUS}(\bigwedge_{i=0}^{n} U_i'),$ $\forall i, 1 \leq i \leq m$, $U_i = A_i$, $A_0 = \text{SUB}(U_{0\,\text{prop}(U_0)}^{\text{prop}(U_0) \in \Pi})$, and $\bigwedge A_i$, then φ is $\text{EN}(v)$-valid.

Validity Formula

(L4.1) *Decomposition.* An ordered pair $(\varphi', [x_1, \ldots, x_n])$ is a *decomposition* of φ if, for variables $c_0, c_1, \ldots, c_n$

i) *Prepositional Case.* $\varphi = [x_1, \ldots, x_n]$ and $\varphi' = [c_1, \ldots, c_n]$,

ii) *Basic Case.* $n = 2$, φ is a basic formula, and $\varphi' = \text{SUB}(\varphi_{x_1, x_2}^{c_1, c_2})$,

iii) *Mechanical Case.* there is a σ and decomposition of σ, $(\sigma', [x_2, \ldots, x_n])$, $\varphi = [(m(x_1), \sigma)]$, and $\varphi' = [(m(c_1), \sigma')]$, where m is a mechanism,

iv) *Left Quantification.* There is a decomposition of σ, $(\sigma', [x_0, \ldots, x_n])$, $\varphi = \exists c_0, \text{SUB}(\sigma_{x_0}^{c_0})$, and $\varphi' = [\exists c_0, \sigma']$. In this case, we say that φ and φ' are *left-quantified*, or

v) *Right Quantification.* There is a decomposition of σ, $(\sigma', [x_1, \ldots, x_{n+1}])$, $\varphi = \exists c_0, \text{SUB}(\sigma_{x_0}^{c_0})$, and $\varphi' = [\exists c_{n+1}, \sigma']$. In this case, we say that φ and φ' are *right-quantified.*

(L4.2) *Partial Formula Decomposition.* If $(\varphi, [x_0, \ldots, x_n]$ is a decomposition, then $(\dot{\varphi}, [x_0, \ldots, x_{i-1}, x_{i+1}, \ldots, x_n])$, where $\dot{\varphi} = \text{SUB}(\varphi_{c_i}^{\emptyset})$, is a partial decomposition. Further, we extend $\oplus$ so that $(\dot{\varphi}, [x_0, \ldots, x_{i-1}, x_{i+1}, \ldots, x_n]) \oplus x_i = (\varphi, [x_0, \ldots, x_{i-1}, x_i, x_{i+1}, \ldots, x_n])$.

(L4.3) *Validity Function.* There is a validity function valid() such that for any formula φ, validity value v, and well-formed space S, if φ is v-valid in S, then $\text{valid}(\varphi, S) = v$.

(L4.4) *Prep.-Verb Internal Composition.* If $\bigwedge U_j$ implies σ is v-valid, and $\bigwedge U_j'$ implies φ is v'-valid, $\bigwedge A_i$ and $\bigwedge A_i = [\text{THUS}(\text{prop}(U_0)) \to \text{prop}(U_0')] \in \Psi \wedge \bigwedge U_j \wedge \bigwedge_{k \geq 1} U_k'$, then φ is $\chi(v, v')$-valid.[13]

(L4.5) *Prep.-Verb External Composition.* If $\bigwedge U_j$ implies σ is v-valid, and $\bigwedge U_j'$ implies φ is v'-valid, $\bigwedge A_i$ and $\bigwedge A_i = \bigwedge U_j' \wedge [\text{THUS}(\text{prop}(U_0)) \to \text{prop}(U_0')] \in \Psi \wedge \bigwedge U_j \wedge \bigwedge U_k$, then φ is $\pi(v, v')$-valid.

(L4.6) *Prep.-Preposition Composition.* Let v and v' be validity values such that if $\bigwedge A_i$, then $[x_0, x_1]$ is v-valid, and if $\bigwedge U_i$, then $[x_0, x_1]$ is v'-valid, where $U_0 = [(\rho \to \gamma)\text{R}\Psi]$. Then there is a validity value $\xi(v, v')$ such that if $\bigwedge U_i'$, $U_0' = [\rho \to (\gamma\ f(\text{R})\ \bigwedge A_i)\text{R}\Psi]$, where $f(\in) = \wedge$ and $f(\notin) = \vee$, and for $i, 1 \leq i \leq n, U_i' = U_i$, then $[x_0, x_1]$ is $\xi(v, v')$-valid.

(L4.7) *Function Composition.* For any two functions $f()$ and $g()$ and validity values v and v', $f(g(v, v')) = g(f(v), v')$.

[13]Note φ may be a vector when v' corresponds to a preposition.

Objects

(L5.1) *Object File.* For each value α, there is an object file $\langle\alpha|\ \grave{\rho}\rangle$, where $\grave{\rho}$ is a partial formula, and we write $\alpha = \langle\alpha|\ \grave{\rho}\rangle$ to show the relation between the objects and their files.

(L5.2) *Thermic.* If x is a thermic then x is an object with object file $x = \langle x|\ \text{type}(\ ) \leq n\rangle$, for some n.

(L5.3) *Moment.* A moment is an ordered pair of an object file and vector, such that $(\langle v|\ valid(\varphi', S) = \rangle, [x_0, \ldots, x_n])$ is a moment whenever there is a φ such that $(\varphi', [x_0, \ldots, x_n])$ is a decomposition of φ.

(L5.4) *Partial Moment.*

i) If $(\grave{\varphi}', [x_1, \ldots, x_n])$ is a partial decomposition, then $(\langle v|\ \text{valid}(\grave{\varphi}', S) = \rangle, [x_1, \ldots, x_n])$ is a partial moment.

ii) If $\grave{y}$ is a partial moment and $(\varphi', [x_0, \ldots, x_{i-1}, x_i, x_{i+1}, \ldots, x_n])$ is a decomposition, then $(\langle v|\ \text{valid}(\varphi, S) = \rangle, [x_1, \ldots, x_{i-1}, \grave{y}, x_{i+1}, \ldots, x_n])$ is a partial moment.

For any $x \not\sqsubseteq \grave{y}$, $\grave{y} \oplus x$ is the unique moment obtained from x and $\grave{y}$.

(L5.5) *Value.* Each value has an object file, where its head identifies it, e.g.
$0 = \langle 0|\quad \in \mathbb{N} \wedge \forall n, s(n) \neq \rangle$.

(L5.6) *Left Arguments.* Let $\bigwedge A_i$ be a partial momentary condition. $\bigwedge A_i$ is *left* if $\forall i$, $\text{prop}(A_i)$ is of the form $s^\frown E$, or $s^\frown E^\frown R^\frown \Lambda$, where $R \in \{\in, \notin\}$.

(L5.7) For any object file $\langle z|\ \grave{\rho}\rangle$, we may extend the object file to $\langle z|\ \grave{\rho}, \grave{\bigwedge} A_i\rangle$ where A is a partial momentary condition such that $\grave{\bigwedge} A_i \oplus z$ is a momentary condition.

4.2 Translation

(A0.0) *Translation Function.* We denote our translation function as $\tau()$, which takes elements of the LOT as input and outputs a string of pseudo-lexical elements. By "pseudo-lexical elements," we mean a symbol which stands in a place, such that, in a more robust theory, it would be replaced by a lexical element or phoneme sequence. $\tau()$ may be defined recursively from atomic cases, but the presentation here begins with the sentence and works towards the atomic cases. A1-A3 lay out the structures for verbs and prepositions (§3.1-3.3). A4 addresses adjectival and (some) adverbial phrases, and A5 gives the translations for nouns (§2.1, §2.3-2.5). Wrapping up, A6 gives a first approximation of the LOT structures that match English sentences. We write $\tau(a) \Rightarrow b$ to denote the optional nature of the translations.

Finite Verbs

(A1.1) *Atomic.* A validity value v is atomic if for every v', v^*, and function f defined from pairs of validity values to validity values, $f(v', v^*) \neq v$.

(A1.2) *Verbal.* If φ is v-valid and v is atomic implies φ is a basic formula, then v is *verbal.*

(A1.3) *Finite.* A validity value v is *finite* if

i) v is atomic and verbal and $\forall v'$, $v \neq \text{INF}(v')$ and $v \neq \text{ING}(v')$ and $v \neq \text{EN}(v')$, or

ii) $\exists v', v^*, f()$ such that v' is finite and $v = f(v', v^*)$.

If v is verbal and not finite, then v is *non-finite.*

(A1.4) *Mechanism Translation.* If $\bigwedge A_i$ implies σ is v-valid and in S, $\bigwedge A_i$, $[m(x_0) = \text{THUS}(\text{prop}(A_0))] \in \Phi$, and $\tau(\bigwedge A_i) \Rightarrow \tau(\ (\langle v|\ \text{valid}(\varphi', S) = \rangle, [x_1, \ldots, x_n]))$, then $\tau(\bigwedge A_i) \Rightarrow \tau((\langle v|\ \text{valid}((m(c_0), \varphi'), S) = \rangle, [x_0, x_1, \ldots, x_n]))$.

(A1.5) *Dative Translation.* Let v be verbal and finite. If $\bigwedge A_i$ implies σ is v-valid, $u = \text{prop}(A_0)$, and $[u \to \varphi \in D] \in \Psi$ in S, then $\tau(\bigwedge A_i) \Rightarrow \tau((\langle v|\ \text{valid}(\varphi', S) = \rangle, [x_0, x_1]))$, where $(\varphi', [x_0, x_1])$ decomposes φ.

(A1.6) *Basic Translation.* Let v be verbal and finite. If $\bigwedge A_i$ implies φ is v-valid in S, then $\tau(\bigwedge A_i) \Rightarrow \tau((\langle v|\ \text{valid}(\varphi', S) = \rangle, [x_0, x_1]))$ and $\tau(\bigwedge A_i) \Rightarrow \tau((\langle v|\ \text{valid}(\exists c_1 \varphi', S) = \rangle, [x_0]))$

(A1.7) *Mechanizable.* If, for a verb and its argument types (thermic, moment, value), there is a mechanical construction, then the moment is *mechanizable*, whatever the criteria for there being a mechanical construction may be.

(A1.8) *Atomic Validity Value.* If $y = (\langle v|\ \text{valid}(\varphi, S) = \rangle, [x_0, \ldots, x_n])$ is a moment, $\tau(\bigwedge A_i) \Rightarrow \tau(y)$, v is atomic, and prop(A_0) is resolved, then $\tau(y) \Rightarrow \tau(x_0)^\frown\tau(\langle v|\ \text{valid}(\varphi, S) = \rangle)^\frown\tau(x_1)^\frown \ldots ^\frown \tau(x_n)$

(A1.9) *Mechanism Free.* If $y = (\langle v|\ \text{valid}(\varphi, S) = \rangle, [x_0, \ldots, x_n])$ is a moment, $\tau(\bigwedge A_i) \Rightarrow \tau(y)$, and v is non-mechanizable and atomic, then

(1) if prop(A_0) is unresolved, $\tau(y) \Rightarrow \tau(\langle v|\ \text{valid}(\varphi, S) = \rangle)^\frown\tau(x_0)^\frown \ldots ^\frown \tau(x_n)$

(2) $\tau(y) \Rightarrow \tau(x_0)^\frown\tau(\langle v|\ \text{valid}(\varphi, S) = \rangle)$.

Prepositions

(A2.1) *Prepositional.* If φ is v-valid and v is atomic implies φ is a vector, then v is *prepositional.*

(A2.2) *Prep. Comp. Translation.* If $\bigwedge U_j$ implies φ is v'-valid, $\bigwedge A_i$ holds, and $\tau(\bigwedge U_j) \Rightarrow \tau((\langle v'|\text{valid}(\varphi', S) = \rangle, [x_0, \ldots, x_n]))$, then

i) if $\bigwedge A_i$ implies φ is $\chi(v, v')$-valid, $\tau(\bigwedge A_i) \Rightarrow \tau((\langle \chi(v, v')|\text{valid}(\varphi', S) = \rangle, [x_0, \ldots, x_n]))$

ii) if $\bigwedge A_i$ implies φ is $\pi(v,v')$-valid, $\tau(\bigwedge A_i) \Rightarrow \tau((\langle \pi(v,v')|\text{valid}(\varphi', S) = \rangle, [x_0, \ldots, x_n]))$

(A2.3) *Right Quantified Free.* If $y = (\langle \chi(v,v')|\ \text{valid}(\varphi, S) = \rangle, [x_0, \ldots, x_n])$,
$y' = (\langle \pi(v,v')|\ \text{valid}(\varphi, S) = \rangle, [x_0, \ldots, x_n])$, and φ is not right quantified, then

i) $\tau(y) \Rightarrow \text{SUB}(\tau((\langle v|\ \text{valid}(\varphi, S) = \rangle, [x_0, \ldots, x_n]))^{\tau(v')\frown\tau(x_n)}_{\tau(x_n)})$

ii) $\tau(y') \Rightarrow \text{SUB}(\tau((\langle v|\ \text{valid}(\varphi, S) = \rangle, [x_0, \ldots, x_n]))^{\tau(v')\frown\tau(x_n)}_{\tau(x_n)})$

(A2.4) *Right Quantified.* If $y = (\langle \chi(v,v')|\ \text{valid}(\varphi, S) = \rangle, [x_0, \ldots, x_n])$,
$y' = (\langle \pi(v,v')|\ \text{valid}(\varphi, S) = \rangle, [x_0, \ldots, x_n])$, and φ is right quantified, then

i) $\tau(y) \Rightarrow \text{SUB}(\tau((\langle v|\ \text{valid}(\varphi, S) = \rangle, [x_0, \ldots, x_n]))^{\tau(v)\frown\tau(v')}_{\tau(v)})$

ii) $\tau(y') \Rightarrow \text{SUB}(\tau((\langle v|\ \text{valid}(\varphi, S) = \rangle, [x_0, \ldots, x_n]))^{\tau(x_n)\frown\tau(v')}_{\tau(x_n)})$

(A2.5) *Preposition Translation.* Let v be prepositional. If $\bigwedge A_i$ implies φ is v-valid in S, then

i) $\tau(\bigwedge A_i) \Rightarrow \tau((\langle v|\ \text{valid}(\varphi', S) = \rangle, [x_o, x_1]))$, and

ii) $\tau(\bigwedge A_i) \Rightarrow \tau((\langle v|\ \text{valid}(\exists c_1 \varphi', S) = \rangle, [x_o]))$.

(A2.6) *Prep. Composition.* $\tau(\xi(v,v')) = \tau(v)\frown\tau(v')$.

Non-Finite Verbs

(A3.1) *Non-Finite Translation.* Let v be verbal and finite. If $\bigwedge A_i$ implies σ is $f(v)$-valid, where $f()$ is $\text{INF}()$, $\text{ING}()$, or $\text{EN}()$, $\bigwedge U_i$ implies σ is v-valid, and
$\tau(\bigwedge U_i) \Rightarrow \tau(\ (\langle v|\ \text{valid}(\varphi, S) = \rangle, [x_0, \ldots, x_n]))$, then
$\tau(\bigwedge A_i) \Rightarrow \tau(\ (\langle f(v)|\ \text{valid}(\exists c_0, \varphi, S) = \rangle, [x_1, \ldots, x_n]))$.

(A3.2) *Validity Value.* For each validity value v and function $f()$ from validity values to validity values, $\exists V$, $\tau(f(v)) \Rightarrow V$.

Adjectival Phrases

(A4.1) *File Extension.* If $\langle z|\ \grave{\rho}, \bigwedge U_i\rangle$ is an extension of $\langle z|\ \grave{\rho}\rangle$, then
$\tau(\langle z|\ \grave{\rho}\rangle) \Rightarrow \tau(\langle z|\ \grave{\rho}, \bigwedge U_i\rangle)$

(A4.2) *Left Adjectives and (Some) Adverbs.* Let $\grave{\bigwedge}_{i=0}^{n} U_i$ be a partial momentary condition that is left. Then

$$\tau(\langle z|\ \grave{\rho}, \grave{\bigwedge} U_i\rangle) \Rightarrow \tau(\grave{\bigwedge} U_i)\frown\tau(\langle z|\ \grave{\rho}\rangle)$$

(A4.3) *Right Adjectives and (Some) Adverbs.* Let $\grave{\bigwedge}_{i=0}^{n} U_i$ be a partial momentary condition that is not left. Then

$$\tau(\langle z|\ \grave{\rho}, \grave{\bigwedge} U_i\rangle) \Rightarrow \tau(\langle z|\ \grave{\rho}\rangle)\frown\tau(\grave{\bigwedge} U_i)$$

(A4.4) *Momentary Condition.* If $\bigwedge_{i=a}^{n} A_i$ is a momentary condition such that $\bigwedge_{i=0}^{n} A_i = \bigwedge_{i=0}^{m_0} A_i \wedge \bigwedge_{i=m_0}^{m_1} A_i \wedge \cdots \wedge \bigwedge_{i=m_k}^{n} A_i$, then

$$\tau(\bigwedge_{i=a}^{n} A_i) \Rightarrow \tau(\bigwedge_{i=0}^{m_0} A_i)^\frown \tau(\bigwedge_{i=m_0}^{m_1} A_i)^\frown \ldots {}^\frown \tau(\bigwedge_{i=m_k}^{n}).$$

(A4.5) *Partial Translations.* For any $\grave{z}$, x, and z, if $\grave{z} \oplus x = z$, then $\tau(\grave{z}) = \textsc{sub}(\tau(z)^{\emptyset}_{\tau(x)})$

Nouns

(A5.1) *Object.* If J is an object file of x, then $\tau(x) \Rightarrow \tau(J)$.

(A5.2) *Object File.* If $\langle z|\ \grave{\rho}\rangle$ is an object file, then $\tau(\langle z|\ \grave{\rho}\rangle) \Rightarrow \tau(\grave{\rho})$ and $\tau(\langle z|\ \grave{\rho}\rangle) \Rightarrow \tau(z)$

(A5.3) *Adjectives.* There are partial formula $\grave{\rho}$ such that $\exists A, \tau(\grave{\rho}) \Rightarrow A$.

(A5.4) *Functions.* There are function ψ such that for any x and R, $\exists N$, $\tau(\psi(x)\mathrm{R}\ ) \Rightarrow \tau(\Psi()) \Rightarrow N$ and $\tau(\ \mathrm{R}\psi(x)) \Rightarrow \tau(\Psi()) \Rightarrow N$.

(A5.5) *Values.* There are values α such that there is an N, $\tau(\alpha) \Rightarrow N$.

(A5.6) *Type.* For any type n, $\exists N$, $\tau(\text{type}(\ ) \leq n) \Rightarrow \tau(n) \Rightarrow N$.

Sentences

(A6.1) *Primary Moment.* If $\bigwedge A_i$ implies φ is v-valid, v is finite, and $\tau(\bigwedge A_i) \Rightarrow \tau(y)$, then y is a primary moment.

(37) *Basic Sentence.* If y is a primary moment, T is a finite string of pseudo-lexical elements, and $\tau(y) \Rightarrow T$, then T is a well-formed sentence.

We don't present (37) as part of the grammar, as we'll be giving a modified version in the extension, but it's a helpful approximation at this stage in the presentation.

4.3 Derivations

The translation function produces a mapping between the LOT and lexical items, at least if enriched with a suitable lexical domain. Our task now will be to show that this mapping does produce sentences of English. Below, we will consider the formulas of the LOT and noun interpretations. Here, we begin with showing that certain sentences can be derived from structures, especially moments, of the LOT. We use a shorthand of ordered pairs of a verb and/or preposition, and a vector of arguments to give the general idea of what the underlying moment structure would be like. A few cases, namely questions, refer to rules of the extension.

(38) The teacher had played the piano.

Derivation: By the basic construction (A1.6), (play, [the teacher, piano]). By the non-finite construction (A3.1), y_1: (play, [piano]). By the basic construction (A1.6), y_0: (have, [the teacher, y_1]). By unresolved (A1.9), "piano" to the right, and by resolved (A1.8), "the teacher" to the left.

(39) *The teacher gets played the piano.

Derivation: By the basic construction (A1.6), (play, [the teacher, piano]). By the non-finite construction (A3.1), y_1: (play, [piano]). By the basic construction (A1.6), (get, [x_0, x_1]). By the dative construction (A1.5), *(get, [the teacher, y_1]). Presumed semantic violation: no state to imply the past participle is satisfied *and* an increased capacity.

(40) The town got the lights to flash words.

Derivation: By the basic construction (A1.6), (flash, [the lights, words]). By the non-finite construction (A3.1), y_1: (flash, [words]). By the prep. construction (A2.5), (to, $[z_0, z_1]$). By the dative construction (A1.5), (to, [the lights, y_1]). By the basic construction (A1.6), (get, $[x_0, x_1]$). By prep. composition (A2.2), (π(get, to), [the lights, y_1]). By the mechanism construction (A1.4), (π(get, to), [the town, the lights, y_1]).

(41) The dog is liking hating the cat.

Derivation: By the basic construction (A1.6), (hate, [the cat]). By the mechanism construction (A1.4), (hate, [the dog, the cat]). By the non-finite construction (A3.1), y_2: (hate, [the cat]). By the basic construction (A1.6), (like, [y_2]). By the mechanism construction (A1.4), (like, [the dog, y_2]). By non-finite again, y_1: (like, [y_2]). By the dative construction (A1.5), y_0: (be, [the dog, y_1]). By unresolved, "the cat" and "liking" to the right, by resolved, "the dog" to the left.

(42) The dog is liking the cat hating the chipmunk.

Derivation: By the basic construction (A1.6), (hate, [the chipmunk]). By the mechanism construction (A1.4), (hate, [the cat, the chipmunk]). By the non-finite construction (A3.1), y_2: (hate, [the chipmunk]). By the dative construction (A1.5), (like, [the cat, y_2]). By the mechanism construction (A1.4), (like, [the dog, the cat, y_2]). By non-finite again, y_1: (like, [the cat, y_2]). By dative again, y_0: (be, [the dog, y_1]). By unresolved, "the chipmunk" and "the cat" to the right, by resolved, "the dog" to the left.

(43) Mary was given a trip around the world.

Derivation: By the prep. construction (A2.5), y_2: (around, [a trip, the world]). By the basic construction (A1.6), (give, $[z_0, z_1]$). By the dative construction (A1.5), (give, [Mary, a trip]). By the mechanism construction (A1.4), (give, [x_0, Mary, a trip]). By the non-finite construction (A3.1), y_1: (give, [Mary, a trip]). By the basic construction (A1.6), (be, $[x_1, x_2]$). By prep. composition (A2.2), (χ(is, give), [Mary, a trip]). Suppose $\tau(\grave{\bigwedge} A_i \oplus$ a trip$) \Rightarrow \tau(y_2)$ and $\langle$a trip$|\ \grave{\rho}, \bigwedge A_i\rangle$. By right adjective (A4.3) and partial translation (A4.5), τ(a trip) $\Rightarrow$ a trip around the world.

(44) The book is to be released in August.

Derivation: By the basic construction (A1.6), (release, [x_2, the book]). By the non-finite construction (A3.1), (release, [the book]). By the basic construction (A1.6), (be, [x_0, x_1]). By prep. composition (A2.2): (χ(be, release), [the book]). By the non-finite construction (A3.1), y_2: (χ(be, release), []).

By the prep. construction (A2.5), y_1: (in, [y_2, August]). Suppose $\tau(\grave{\bigwedge} U_i \oplus y_2) \Rightarrow \tau(y_1)$ and ($\langle$ χ(be, release)$|$ $\grave{\rho}, \grave{\bigwedge} U_i\rangle$, []). By right adjective (A4.3) and partial translation (A4.5), $\tau(y_2) \Rightarrow y_2$ in August

By the prep. construction (A2.5), (to, [z_2, z_3]). By the dative construction (A1.5), (to, [the book, y_2]). By prep. composition (A2.2), (χ(be, to), [the book, y_2].

(45) Does he understand what's going on?

Derivation: By the prep. construction (A2.5), (on, [what, x_4]), By the basic construction (A1.6), (go, [x_2, x_3]). By prep. composition (A2.2), (χ(go, on), [x_1]). By the non-finite construction (A3.1), y_3: (χ(go, on), []). By the dative construction (A1.5), (be, [what, y_3]). By the non-finite construction (A3.1)y_2: (is, [y_3]).

By the basic construction (A1.6), (understand, [he, what]). By the non-finite construction (A3.1), y_1: (understand, [what]). By the interrogative complement (A7.3), τ(what) $\Rightarrow$ what$^\frown y_2$. By the dative construction (A1.5), (do, [he, y_1]).

(46) *Understand(s) he what's going on?

Derivation: y_1: (understand, [he, what]) of the last construction, by the last construction. UNDERSTAND, if finite, is mechanizable, and so can't have arguments to the right; if non-finite, then not a primary moment.

(47) What did you tell her?

Derivation: By the basic construction (A1.6) and the mechanism construction (A1.4), (tell, [you, her, x]). By the non-finite construction (A3.1), (tell, [her, x]). By partial moment $\grave{y}_1$: (tell, [her]). By the dative construction (A1.5), y_0: (do, [you, $\grave{y}_1$]). By the interrogative complement (A7.3), τ(what) $\Rightarrow$ what$^\frown y_0$. By unresolved, "you" and "her" to the right.

(48) Who played the trombone?

Derivation: By the basic construction (A1.6), (play, [x, the trombone]). By partial moment, y_1: (play, [the trombone]). By the interrogative complement (A7.3), τ(who) $\Rightarrow$ who$^\frown y_1$.

(49) Was the dog happy?

Derivation: By the basic construction (A1.6), (be, [the dog, happy]). By unresolved (A1.9), "the dog" to the right.

(50) Have you played the game?

Derivation: By the basic construction (A1.6), (play, [you, the game]). By the non-finite construction (A3.1), y_1: (play, [the game]). By the basic construction (A1.6) and def. of HAVE, (have, [you, y_1]). By unresolved (A1.9), "you" to the left.

(51) *Had you any tea?

Derivation: By the basic construction (A1.6), (have, [you, some tea]). By the mechanism construction (A1.4), (have, [you, yourself, some tea]). Therefore, (have, [you, any tea]) is mechanizable, and so arguments can't be to its right.

4.4 Basic Interpretations

Consider interpreting the following sentences:

(52) The lighter is on the window sill.

(53) The sun is killing our eyes.

(54) The computer does run the function.

For (52)), we have preposition composition, so we need a basic formula in Φ that implies the Ψproposal. It's not clear what it would be, nor why it seems mandatory to have location information translated with a Ψ proposal or a demonstrative such as "here" or "there." Nonetheless, letting $\varphi =$ [location(lighter) $\subseteq$ window sill] or $\varphi =$ [position(lighter) $= \alpha$], for some α, we may have $\varphi \in \Phi$ and $(\varphi \in \Phi \to \gamma \in \Psi) \in \Psi$, where $\gamma \in \Psi$ corresponds to "the lighter on the window sill."

With (53) and (54), we demonstrate how the mechanism associated with one verb may be the state of another verb. If sun $\in$ satisfies$(y) \in D$, where $y \approx (\exists c, c$ killing our eyes), then a state that would imply this would be an active mechanism. In particular, the mechanism that outputs the zeroth proposal of "the sun kills our eyes." This is the formula we presume implicit in (53). Similarly, upon the completion of an act, it's reasonable to expect its active mechanism removed from Φ, and this is compatible with DO's definition, and so the interpretation for (54). However, we may note that completion of the act is incompatible with our characterization of the infinitive meaning we gave in §3.4. This may be taken as evidence against the current theory, but a simple solution consistent with this framework is to introduce a new function, so that one gets e.g. computer $\in$ resolves(y) in D, where $y \approx (\exists c, c$ run the function). So then, in total, this formula in D is implied by the removal of the mechanism from Φ.

(55) The student had passed the stack of papers.

(56) The student is passed the stack of papers.

Consider the past participle with HAVE (55) and in the passive construction (56). For (55), by the definition of HAVE, no inference need be in place, and what would normally be a dative transformation simply occurs as the basic argument. For the passive construction (56), note that the dative transformation isn't applicable. In the form $(x \in$ satisfies$(y)) \in D$, x is not present with the passive. Instead we have composition, just as with preposition composition.

Thus, in this case, there is a formula in the space which is EN(PASS)-valid, and we need an implicit state which implies this. When giving someone a possession, a common scenario is to be oriented at the neck towards them, and for there to be a "grace period" which occurs until one orients away from the person, during which one can change one's mind about giving the object. This orientation is one possibility for the state that holds. A second is the stack being on the student's desk, or in the student's hands. Each of these are the introduction of a state which would then imply the EN(PASS) zeroth proposal—that the completion state of PASS is in Π. Since the state holds as per the definition of BE, the zeroth proposal of the past participle is not yet unresolved, and so the left argument occurs.

(57) The car is fast.

Having looked at interpretations at the level of the formula, let us look at interpreting the terms within the formula. Earlier we gave a gloss of (57) as speed(car) $\geq \alpha$. There are various reasons why this is wrong. Let us first focus on the fact that the current speed can be zero, and more generally its current behavior need not be demonstrating that the car is fast. By definition of BE, we have a state—a formula in Φ, and thus this presents a conflict, as our intuition is more indicative of a rule, capacity, or other abstraction whose semantics don't stem from here and now. There are certain states which are similar in nature. For instance, consider mass within a Newtonian framework. While mass is considered a property based on empirical observation, and so is a state, it also effects the rule for how forces effect an object, with higher masses producing less acceleration per force. That is, the state acts as an indicator of how a rule will apply, when it is applicable, though it need not be applicable now.

Thus, suppose FAST is encoding such a property, and let us consider what it might be. We have an intuition of "fast" relating to high speeds and we also have cases of "fast" meaning firmly holds still (Merriam-Webster (nd)) (though limited to right constructions). We know that two people or two cars of the same weight may be capable of different speeds, and so we can rule out mass itself. Since toy cars can also be fast, we can rule out self-generating forces. One option that is left is that objects vary in their capacity to retain their speeds. If two objects are given forces, but one is prone to have a decay of speed greater than the other, or a higher "leaking" of energy to its surrounding, then the total effect of the forces across time will lead to the difference in capacity to reach high speeds. But "car" does some of the work for our semantic intuitions: compare "fast stove" and it's clear that "car" contributes the movement intuition. Thus, "fast" need not apply to speed, and so we have fast computers, fast games, and so on. The LOT is set up so that framing this in terms of forces (some object dependent constant times a second order derivative of distance) allows for the general pattern of function abstraction, discussed at the beginning of §2. If we consider a force as a conversion of the form of energy, then FAST is roughly that there is a force that converts energy into some particular form, and the object has a high rate of retention of that energy form (from that force). Here, "force" must be understood as a type, or in terms of its sense, rather than picking out a particular instance.

The capacity to retain a force's energy is also applicable to the holding still meaning, and it does so by contingency of current arrangements, and not a general property which one would presume across time and so hold as a belief. In terms of an LOT formula and structure, we may have energy$_{\mathrm{F}}$-retention(car) $= \alpha$, where energy$_{\mathrm{F}}$-retention() indicates a property which is conditional, just as mass corresponds to a property of a condition which need not hold. In total, FAST itself can be interpreted as

$$\langle \alpha | \; \exists x, F, (\text{energy}_{\mathrm{F}}\text{-retention}(x) = \;\;) \wedge \;\; \geq \alpha' \rangle,$$

where α' indicates some significant point for a capacity or rule, and the partial formula is the basis for translating α as "fast."

(58) An example of self reference is (58)

(59) An example is a particular instance of a general phenomenon.

(60) a. The presenter is his sister.

b. The presenter has a brother.

To demonstrate the nature of reference, let us look at (58) and (59) and the issue of multiple reference. While $\langle x | \; \text{type}(\;) \leq \mathfrak{example} \rangle$ handles "example" in (58), it fails to do so for (59). Instead, in (59), "example" refers to the type itself. From there, the problem of the interpretation is difficult to say. The function may be e.g. meaning$(n) \leq n'$, with the second noun phrase also denoting a type, but it's not clear at this point. In (58), "(58)" acts like "ham sandwich" in a language game played by academics, to borrow Wittgenstein's (2009) term, and is an example of the creative extension of reference. Its reference is a sentence which one can locate by that number, whatever a sentence's ontology turns out to be.

Consider (60), which is meant to be over the same relation, namely p and q are two individuals, q is the presenter p's brother, thus p is q's sister. (60a) can be interpreted as type$(p) \leq q \; \mathfrak{sister} \in \Phi$. We can translate p as "presenter," given type$(p) \leq \mathfrak{presenter}$ as the head of its file. The second argument is a type, derived from an individual and type (see the possesive in §5.1), and the type can just be translated directly. For (60b), we have $p \in \text{sister}(q) \in D$. p again gets the same translation as before, and for q, we have type$(q) \leq \mathfrak{brother}$, parallel to the structure for p. Thus, this produces the two interpretations, based on the rules presented, however there is a problem worth mentioning. In (60b), there is nothing that prevents us from using e.g. type$(q) \leq \mathfrak{father}$, type$(q) \leq \mathfrak{child}$, and so on, which gives us an incorrect meaning. We may note that this speech act introduces an individual or type, otherwise the (60a) form is used. So, this is a problem we leave open, but see the related marks in §5.1.

Let us briefly touch on the relation of A5 to §2.4. One might expect that (A5.6) should be the recursive rule of §2.4. But, we may note that there is a distinct possible behavior with regards to adjectives. In particular, referring to the sound of /kɑt/ as "cat," while simultaneously using adjectives sounds off. Thus, "nasally cat out of his mouth had a K," has the effect of problem solving, rather than smooth processing, which can occur without adjectives. Contrast "the nasally articulation of cat out of his mouth had a K." It isn't impossible for such meanings and structures to develop, perhaps within a language game, but they're not the default and so we take the distinction and dual approach to be merited.

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One other problem of interpretation is the case of plurals and groups. If we consider a sentence like (61) or (62), we may note that it may be true even if not every player plays. That is, so long as some subset of the team/players is playing, it's an accurate statement. Restrictions on the size of this subset varies case by case. Sometimes, a single element may suffice. Thus, "the team scored" can be followed by the question, "which player scored?" since typically, a single player need score for the team to score. Other times, the whole set is required. If "the team won" is followed up with "which player won?," incomprehension of the game will generally be taken, as every player wins if the team wins. Which case is applicable is not dependent upon the verb, but on the domains and ontology that underlie the event. Implicit in the case of winning is that the team is a team for the particular competition they won; but if the team wins in a different competition, the use is different. Thus, in the case of a hockey team, one can say "the team won the quiz bowl," and unlike with a hockey game, this can refer to a subset of the team rather than the whole team.

(61) The team is playing hockey.

(62) The players are playing hockey.

The main question for the LOT is the conditions under which such things can be translated as this or that. In particular, given the currently proposed framework, it seems that two reasonable hypotheses are that the basic formula may be filled by single elements, and multiple such formula meet the requirements of v-validity in a space, in which case a descriptor like "team" and "players" allows for a compression of multiple moments under a single description; or the alternative is that the group or set is forming one of the terms of the basic formula. And these two options are not mutually exclusive, so that there can be cases of one and cases of the other. Thus, although we have dedicated a great deal of this space to interpreting sentences in a LOT, there are still basic issues and details to be explored further.

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In the core grammar, object files, moments, and momentary conditions translated into moments are given a pseudo-lexical interpretation. The valid translations are restricted to finite translations of primary moments. The moment is translated into a collection of object files and objects (arguments), with the objects in turn being translated as object files and possibly further arguments. The object files are then translated into pseudo-lexical items, and may also produce further translations of objects with adjectival and adverbial phrases. We now wish to start from a different point: from the moments discussed already as objects in short term memory which then generate further structure and translations. Here, we have second order translations akin to e.g. the preposition composition translations. This topic is followed up by a treatment of tense, newly needed translation rules, and a more complete account of English sentences. We will motivate the use of second order translations over objects in short term memory with our introduction to determiners. But we begin with stipulating two more sets of the set universe.

5 Extending the Grammar

Reports (Σ). If one is asked to take the reading from a meter, with the implicit instruction of reporting this back, then the result is something one needs to keep in short term memory, and is quite different then perception or belief. Phenomenologically, this is consistent with concentration and focus. Functionally, this set will be characterized by resolution greater than that of being resolved. This is something we haven't really gone over enough to make precise. Basic-level categories have been proposed (Hajibayova (2013)), and in those terms, Σ contains information more specific than basic categories. It is also functionally characterized by requiring work to be brought forth in time for use, e.g. for recall or for implementing in an action. This requirement of work makes it distinction from beliefs and statuses.

Inquiries (Q). If the pot is on the stove, and one plans to put the noodles in after the water begins boiling, one needs to know that the water is boiling. That is, one must have a mechanism of resolving this. Likewise, when going to get the reading from the meter or figuring out some computation, one needs a mechanism to do this. Whenever we have some resolution below some threshold—res(φ)$< \alpha$, and we expect to do work and increase it's value, then we have the characteristic of Q. Further, like reports, this is functionally characterized by requiring work to keep elements in the set, and thus is distinct from Π.

5.1 Determiners and Related Vocabulary

A distinction made in narrative studies and linguistics, though with varying terminology, is the distinction between discourse time—the time of transmitting information—and content time—the time of what's being talked about (Chatman (1974), Enç (1987), Genette (1983), Tonhauser (2020)). The use of determiners follow dynamics which are applicable at discourse time. For instance, if one is talking to a neighbor, one might ask "Did you hear about what happened at the park?" But, if writing for a national newspaper, descriptions of events at the park would have to start with "a park," not "the park," though later one could use the definite article. A plausible explanation for this is that the indefinite article introduces the reference, while the definite article refers to an already introduced reference. Neighbors being familiar with the park don't need it introduced. But note that "introduced" applies to the discourse time, and the reference need not be introduced in the content time. Thus, discourse time determines this dynamic.

On the other hand, the particular abstract type assigned for reference is applicable at content time, and need not be applicable at discourse time. Thus, telling of a court case that happened some time ago, one can refer to an individual, who was the defendant in that court case, as "the defendant" even if they are not, at discourse time, the defendant of any court case. This presents a conflict between the noun phrase having properties at discourse time and at content time. Recall that Π and E will require their elements to be ordered pairs, with part of the ordered pair specifying temporal information. Then, the idea we propose is that determiners are moments, whose space-time region corresponds to discourse time, and which centers around the proposal $(\text{role}(n) = x, S) \in E$. We will simplify this to just $\text{role}(n) = x \in E$ below. Also, note that the space-time region is often larger than the content time of the sentence and corresponds to the space-time region of e.g. a scene.

There are a few reasons why we wish to have this role() function, and not just rely on type(). First, we have that one can refer to “the car” in a parking lot full of cars.[14] Why type() couldn’t refer to any of the cars (that one won’t be driving in), even though it applies, is a puzzle that role() solves. Second, Tonhauser (2020), citing Enç, gives examples—contrary to the characterization above—that the type can be applied even when it is not applicable at the content time. Thus, “fugitive” may be used after the individual is jailed and “president” may be used to discuss the current president as a child, before their presidency (Tonhauser (2020)). Through the role() function, we can have a transformation from a count noun to a name. The car example demonstrates the spatial property, and the Tonhuaser and Enç cases demonstrate the temporal property for names (see below). Thus, once a role is established in one content time, one can use this to refer to the same individual at other content times.

But, there is another issue with role() to be addressed. We see with the “fugitive” example a case of roles changing, and as Propp (1968) pointed out, the relation between individuals and roles can vary greatly, from being one-to-one to being many-to-many. For instance, at a grocery store, one and the same person may be the shopper, customer, and consumer, and many such individuals may satisfy this. Or, these roles may be split between many individuals. For instance: someone promises to help someone and buy their groceries, and then sends their assistance to go shop for the groceries, and so the shopper, customer, and consumer roles are split among many individuals. And, in the middle of shopping, it’s possible for someone to take over and so on. But the problematic case comes from when one individual has many roles. Thus, consider the following introduction to a folk tale.

> Once on a time a poor couple lived far, far away in a great wood. The wife was brought to bed, and had a pretty girl, but they were so poor they did not know how to get the babe christened, for they had no money to pay the parson’s fees. So one day the father went out...
>
> Asbjørnsen and Moe (2010)

Our minds know the same person to be “a pretty girl” and “the babe,” the same person to be “the wife” and a member of “a poor couple,” and the same person to be “the father” and a member of “a poor couple,” though it’s not explicitly stated. The conflict with the role() hypothesis is that the definite article is used with a new type rather than the previous type the individual was introduced with. If one looks at the formal structures behind these terms, we see that there are possible solutions. Consider the following schema for “wife,” “father,” and “couple,” where $\rightleftharpoons$ denotes spouses and $\downarrow$ denotes parent(s) and child(ren).

(63) COUPLE: $\langle (a_0, a_1) |\ a_0 \rightleftharpoons a_1 \rangle$

(64) WIFE: $\langle a_0 |\ \exists c_1, (\ \rightleftharpoons c_1) \wedge (\text{gender}(\ ) = f) \rangle$

(65) FATHER: $\langle a_0 |\ \exists c_1, c_2, (\ \rightleftharpoons c_1) \downarrow c_2) \wedge (\text{gender}(\ ) = m) \rangle$

It seems clearer now how “wife” and “father” are easily inferred from “couple,” as they have overlapping structures. We know that people are capable of solving something like an

[14] I do not recall, and have failed to find, the original source of this example, but it is not my own observation.

algebraic equation with kinship terms (Read (2001)), so such skills are not foreign to what we know about our cognition. Further, "customer," "consumer," and "shopper" can be specified by a conjunction of kinship type relations with the store, and a distinct taxis. (Items to a container, items and money to a new possessor, and items to mouth or other agent-internal space, respectively.) "A girl" and "the babe" on the other hand, don't fit such a schema. So there's still plenty to work out, but we have at least addressed many issues which motivate this approach and seeming problems with it.

We take determiners to be the definite and indefinite articles, and any words or constructions which are found to be in mutually exclusive use with the articles and one another. Thus, "every," but not "all" is a determiner since "every" is always used without "a(n)" or "the" while "all" can accompany "the." We write $h(X)$ to indicate a set of content time. We begin with cases that have all their semantic background introduced. After these definitions, we address the name, mass, and count distinction, and its relation to these determiners, and then expand to more cases below.

(D16) THE(x): For some n, if $\text{role}(n) = x \in E$, then x is THE-valid

(D17) POSSESSIVE(x): For some n, if $\text{role}(x'\ n) = x \in E$ and x is not a type, then x is GENITIVE(x')-valid.[15]

(D18) AN(x): For some n and for all z, if $[(\text{type}(x) \leq n) \in h(\Phi)] \in E$, $[(\text{role}(n) = x) \in E] \in \Pi$ and $(\text{role}(n) = z) \notin E$, then x is AN-valid.

(D19) ANOTHER(x): For some n, x', if $\text{role}(n) = x' \in E$, $x \neq x'$, and $[(\text{type}(x) \leq n) \in h(\Phi)] \in E$, then x is ANOTHER-valid.

(D20) ANY(n): For some n', If $[\exists z, (\text{type}(z) \leq n) \in h(\Phi)] \in \Pi$, and $\forall$x$([(u \wedge \mu) \rightarrow (\text{role}(n') = x \in E) \in \Pi] \in \Psi)$, where $u = [(\text{type}(x) \leq n \in h(\Phi)) \in E]$ and $\mu = [\forall z(\text{role}(n') = z \notin E)]$, then n is ANY-valid.

(D21) SOME(n): For some x, if $[\text{type}(x) \leq n \in h(\Phi)] \in E$, $\text{role}(n) = x \notin E$, and $\text{role}(n) = x \in E \notin \Pi$, then n is SOME-valid.

Note that some of these take an object, and some take a type as argument, but either may be in place for a sentence.

As we know, the use of determiners varies based on whether we have a name, count noun, or mass noun. Given space universes as well as thermics, it's possible to give formal definitions of these 3 distinctions, where names correspond to an identity criteria. These definitions require two parts, matching with Frege's (1993) reference and sense distinction. For, as observed by Frege (1960) himself, number (and thus a count noun) depends on the predicate just as much as what's being predicated. The same set of people may be one group but five individuals. The 3 distinctions can be formally given as follows:

[15] If one contrasts the possessive with compound nouns, then the reasonable conclusion is that the possessive constructs a subtype from a non-type and a type, while compound nouns construct a subtype from (at least) two types. Thus, we take this to be the difference between "a lawn's chair" and "a lawn chair."

Def.: Let x be a thermic and n be a type, such that $\langle x|\ \text{type}() = n\rangle$ and let $S_0 = (W_0(t_0), \mathfrak{X}_0)$, $S_1 = (W_1(t_1), \mathfrak{X}_1)$, $S_2 = (W_2(t_2, \mathfrak{X}_2)$, and $S_3 = (W_3(t_3), \mathfrak{X}_3)$ be spaces in some $(\mathcal{U}, \mathcal{H})$ such that $W_0 \cup W_1 \subseteq W_2$, $t_0 \cap t_1 = t_2$, $W_0 \cap W_1 = W_3$, and $t_0 \cup t_1 \subseteq t_3$. Let us refer to $X \in \mathfrak{X}_i$ as X_i. Then

I) n is a *name type* of x if $\forall x'$ such that type of x' is n, $x \in \Theta_0$ and $x' \in \Theta_1$ implies $x, x' \in \Theta_3$.

II) n is a *count type* of x if $\forall x'$ such that type of x' is n, $x \in \Theta_0, x' \in \Theta_1$, and $W_0 \cap W_1 = \emptyset$ implies $x, x' \in \Theta_2$

III) n is a *mass type* of x if $\forall x'$ such that type of x' is n, $x \in \Theta_0, x' \in \Theta_1$, and $W_0 \cap W_1 = \emptyset$ implies $\exists z, x \cup x' \subseteq z$, type of z is n, and $z \in \Theta_2$. (For thermics x and z, $x \subseteq z$ means the space-time region of x is a subset of the space-time region of z.)

When n is an identity type for x, we write $\langle x|\ ID(\ ) = n\rangle$.

Even after doing this, though, there are still issues which persist with regard to the reference of nouns. In particular we need further axioms which specify the thermic property of extension, or mutual exclusivity of their space-time regions, while simultaneously dealing with subthermics. But, it should be clear how, having a unique one in the space, mass nouns don't have a thermic introduced at discourse time. We can stipulate that when type is sufficient for a unique thermic, then no determiner is needed. The use of "the" with mass nouns appears to indicate either a subthermic identified by a container or the retention of the previous thermic, which is not guaranteed by the mass type. (For instance, water may come and go.)

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We now want to address "every" and "no," and while the numerical part is understood, the part of this theory that is less understood is that this numerical property describes the size of the intersection of two sets. Thus, we introduce a way of talking about this.

A lattice is a mathematical structure, with a set and a partial order defined over the set. A partial order is such that any two elements need not be comparable. For a lattice, for any two elements x and y,

(66) $\exists u, (x \leq u, y \leq u, \text{ and } \forall u', ((x \leq u' \text{ and } y \leq u') \rightarrow u \leq u'))$.

(67) $\exists v, (v \leq x, v \leq y, \text{ and } \forall v', ((v' \leq x \text{ and } v' \leq y) \rightarrow v' \leq v))$.

Such u and v are the least upper bound and greatest lower bound, respectively. These are equivalent to products, so we write $x \vee y = u$ (read: "x join y equals u") and $x \wedge y = v$ (read: "x meet y equals v"). For a more thorough presentation of, or introduction to, lattices, see e.g. Nation (nd).

For any given object z, there is a set Z whose elements are the elements of a lattice, and $\zeta(z)$ is the top element of Z. That is, $\forall z' \in Z, z' \vee \zeta(z) = \zeta(z)$ and $z' \wedge \zeta(z) = z'$. What the set Z is we cannot fully specify, so this is like functions and relations in not being pronounced. However, the general form for thermics and moments are as follows.

For any set, using the power set with set inclusion as the partial order, one can form a lattice. For a set of thermics, extending this partial order to elements, we have

(68) $x \subseteq y \to x \leq y$

(69) $x \leq \{x\}$

(70) $x \leq x$

Thus, $\zeta(z)$ would correspond to the initial set.

For validity values, we have that actions can be described in terms of a sequence of moments, and this collection of moments has a partial order defined over it. For instance, if one is grocery shopping, the order in which one picks the items from the shelves does not matter, and if one has help, they can even be picked out simultaneously. But they must be picked from the shelves before one checks out, and they must be checked out before one leaves the store. The necessity of being before is then the basis for one moment being "less than" another. Thus, for validity values, $\zeta(v)$ is the last moment, which all other moments, including the one the validity value is the object file of, are structurally less than.

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Another function we want is $\eta(x, y)$, which is the set of all y, possibly restricted by a space-time constraint, which satisfy all but one argument or validity value. Thus, we have:

(71) For some $a, 0 \leq a \leq n_m$,

$$\begin{aligned}\eta(z, y_0) := \{x_a |\ \exists S_0, S_1, \ldots, S_m, y_0 &= (\langle v_0 | \text{valid}(\varphi, S_0) = \rangle, [x_0, \ldots, x_{n_0}]), \ldots, \\ y_i &= (\langle v_i | \text{valid}(\varphi, S_i) = \rangle, [x_{n_i+1}, \ldots, x_a, \ldots, x_{n_i}]), \ldots, \\ y_m &= (\langle v_m | \text{valid}(\varphi, S_1) = \rangle, [x_{n_{m-1}+1}, \ldots, x_{n_m}]), \\ &\text{and } \forall j, 1 \leq j \leq m, \exists k \leq n_j, y_j = x_k\}.\end{aligned}$$

Given that this is a set, the lattice we defined earlier for objects is applicable. This leads us to our next definitions, which are in mutually exclusive constructions with determiners.

(D22) EVERY(n): If $\{x|\ \text{type}(x) \leq n\} \wedge \eta(x, y) = \{x|\ \text{type}(x) \leq n\} \in E$, then n is EVERY-valid.

(D23) NO(n): If $\{x|\ \text{type}(x) \leq n\} \wedge \eta(x, y) = \emptyset] \in E$, then n is NO-valid.

With the zeta and eta functions, the semantics of some other terms, which are presumably adverbs, becomes more apparent. We give the following definitions, and discuss the intuitions for "just" as an exemplar case below.

(D24) NOT(x): For some X, if $x \wedge X = \emptyset \in E$, then x is NOT-valid

(D25) ONLY(x): For some X, if $x \vee X = x \in E$, then x is ONLY valid.

(D26) JUST(x): For some $f(), z$'s, and α, if $f(z_0, \ldots, z_{i-1}, x, z_{i+1}, \ldots, z_n) < \alpha \in E \in \Lambda$ and $f(z_0, \ldots, z_{i-1}, x, z_{i+1}, \ldots, z_n) \geq \alpha \in E$, then x is JUST-valid.

(D27) EVEN(x): For some $f(), z$'s, and α, if $f(z_0, \ldots, z_{i-1}, x, z_{i+1}, \ldots, z_n) > \alpha \in E \in \Lambda$ and $f(z_0, \ldots, z_{i-1}, x, z_{i+1}, \ldots, z_n) \leq \alpha \in E$, then x is EVEN-valid.

The set X for NOT and ONLY is usually $\eta(z, y)$, but it's possible for nouns to have other possibilities, e.g. "the only child." Below, (72) indicates that among the many possible things she could be doing right now, and especially the many issues which she could be addressing, it's only the case of grabbing a coat, with a concern for the cold. Thus we might take $f()$ to be trouble($\zeta(v)$). (73) gives us the same interpretation as only, thus f() can be taken as $|\eta(x, y)|$. Finally, (74) may be interpreted as dist(time(y), NOW). The statement can be made as a protest or excuse, but all of that appears to be prosodic, with "just" merely referring to temporal distance.

(72) She's just grabbing a coat in case it's cold.

(73) Just the large companies are exempt from the regulations.

(74) We just got here.

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Similar to the non-object file reference discussed above and to pronouns, interrogative pronouns do not take left modifiers. They instead can be found either by themselves or with a modifying primary moment or partial primary moment. Thus, we present the following definitions, and give them their own special translation rules below.

(D28) WHO(x): For some y, if $\langle x|\ \exists c, \text{ID}(\ ) \leq c\rangle \in Q$ and $\exists \grave{y}, ([\exists c, (\langle x|\ \text{ID}(\ ) \leq c\rangle \in \Sigma \rightarrow y \in \Sigma)] \in \Psi$ and $y = \grave{y} \oplus x)$, then x is WHO-valid.

(D29) WHAT(x): For some y, if $\langle x|\ \exists c, \text{type}(\ ) \leq c\rangle \in Q$, and $\exists \grave{y}, ([\exists c, (\langle x|\ \text{type}(\ ) \leq c\rangle \in \Sigma \rightarrow y \in \Sigma)] \in \Psi$ and $y = \grave{y} \oplus x)$, then x is WHAT-valid.

(D30) WHOSE(x): For some y, x', if $\langle x|\ \exists c, \text{ID}(\ ) \leq c\rangle \in Q$, x' is GENITIVE(x)-valid, and $\exists \grave{y}, ([\exists c, (\langle x|\ \text{ID}(\ ) \leq c\rangle \in \Sigma \rightarrow y \in \Sigma)] \in \Psi$ and $y = \grave{y} \oplus x'$, then x is WHOSE-valid.

(D31) WHICH(x): For some y, n, if $\langle x|\ \exists c, \text{type}(\ ) \leq c\rangle \in Q$, type($x$) $\leq n \in h(\Phi) \in E$, and $\exists \grave{y}, \exists c, [(\langle x|\ \text{type}(\ ) \leq c\rangle \in \Sigma \rightarrow y \in \Sigma) \in \Psi$, $c < n$ and $y = \grave{y} \oplus x]$, then x is WHICH-valid.

(D32) WHERE(z): For some y, if $(\langle z|\ \exists c, \text{type}(\ ) \leq c\rangle \in Q)$, $z \in \text{source}(y) \in D$, and $[\exists c, (\langle z|\ \text{type}(\ ) \leq c\rangle \in \Sigma \rightarrow y \in \Sigma)] \in \Psi$, then y is WHERE-valid.

(D33) WHEN(y): For some $y', \varphi, \varphi', v, v', \vec{x}, \vec{z}$, if $y \in Q$, $\exists c, [y = (\langle v|\ \text{valid}(\varphi, c) = \rangle, \vec{x})$ and $y' = (\langle v'|\ \text{valid}(\varphi', c) = \rangle, \vec{z})]$, and $(y \in \Sigma \rightarrow y' \in \Sigma) \in \Psi$, then y is WHEN-valid.

(D34) HOW(y): For some y', if $y \in Q$, $[y \in \Sigma \rightarrow y' \in \Sigma] \in \Psi$, and $y' = \zeta(y)$, then y is HOW-valid.

Missing is WHY, and while it appears to be related to forces, it's definition is still elusive. It will also be useful to have the following definition for translation.

(L6.1) *Wh- Validity Value.* Suppose $\bigwedge A_i$ implies z is v-valid. v is Wh- if prop(A_0) is a Q proposal and z is an object or term.

5.2 Tense

In §3.4, we gave the definition of the non-finite verb forms as a function of the verb definition. Now, we wish to do a similar thing with the finite forms, which are differentiated by tense. In short, tense is the intersection (present), or lack there of (past), of the space-time region of the verb's moment with the space-time region of the discourse time—normally the speech act. There are two complications of this picture, to describe tense accurately. There is an embedded, recursive tense complication and an aspectual variation which involves multiple moments.

Let us note the fixed nature of time for the finite form as opposed to a non-finite form. (75a) is ambiguous as to whether the phrase to the right of the subject occurs at discourse time or content time, but for (75b) the phrase occurs at discourse time. Using the observations and analysis of Enç (1987), although we haven't discussed these constructions, we can go further and describe the conditions for embedded tenses. Here we mean constructions like "You know I laughed about that." We analyze this as corresponding to an LOT structure of the form $y = (\langle \text{KNOW} |\ \text{valid}(\varphi, S_i) = \rangle, [x_0, y'])$. In these constructions, we have that, for present tense, the moment's space must intersect *all* relevant spaces: the space of discourse time, the space of any moment it's an argument of, and any moment that moment is an argument of, and so on.

(75) a. The clock, ticking in the corner, was covered in mud.

b. The clock, which ticks in the corner, was covered in mud.

Another detail, this time coming from aspectual concerns, also effects the tense function. "The clock ticks" has an iterative reading, and "they play golf on Tuesday(s)" has a habitual reading, as Comrie (1976) uses these terms. These cases suggest that we not only have present tense when the moment is in the discourse time space, but also when the rate of similar moments—with differences stemming only from the spaces of the moments—is greater than 0, or at least is in a "grace period." The way we will approach this is to have a function which allows not only a single moment, but also a set of moments which all have the same validity, but with different space-times.

To begin with the rate case, let lub(t) and glb(t) be defined, so that $\forall r \in t, r \leq \text{lub}(t)$, and $\forall r' \in t$, if $\forall r, r \leq r'$, then $\text{lub}(t) \leq r'$. Similarly for $\text{glb}(t)$. To define rate, so as to define the set overlapping the current time, we have the following definitions.

(L6.2) When y is a moment such that $y = (\langle v |\ \text{valid}(\varphi, S) = \rangle, [x_0, \ldots, x_n])$, then y is a moment of S.

(L6.3) *Moment Set Time.* Let Y be a set of moments. We may assign Y a temporal part t_Y such that $t_Y := \{r |\ \exists y, y' \in Y\ y$ is a moment of $S = (W(t), \mathfrak{X})$, y' is a moment of $S' = (W'(t'), \mathfrak{X}')$, and $\text{glb}(t) \leq r \leq \text{lub}(t')$.

(L6.4) *Right Before Relation.* Let y and y' be moments, with y a moment of $S = (W(t), \mathfrak{X})$ and y' a moment of $S' = (W'(t'), \mathfrak{X}')$. We say $y \lhd y'$ ("y is right before y'") in Y whenever $\text{lub}(t) \leq \text{lub}(t')$, and $y'' \in Y$ is a moment of $S'' = (W''(t''), \mathfrak{X}'')$ implies $\text{lub}(t'') \leq \text{lub}(t)$ or $\text{lub}(t') \leq \text{lub}(t'')$.

(L6.5) *Rate of Occurence.* Let D be the set defined as $D := \{d | \exists y, y', y \triangleleft y' \text{ in } Y \text{ and } d = \text{dist}(lub(y), lub(y'))\}$. The rate of Y, denoted rate(Y), is defined as the interval (a, b) such that $a = \text{glb}(D)$ and $b = \text{lub}(D)$.

(L6.6) *Active.* Y is *active* in $S = (W(t), \mathfrak{X})$ if $\text{lub}(t) \leq \text{lub}(t_Y) + f(\text{rate}(Y))$. $f()$ will have to be determined empirically, but it could be e.g. average or maximum.

Y will need to be translatable, and one of the well-formed structures of the language. In addition, we extend $\eta(x, Y) := \{x | \forall y \in Y, \eta(x, y)\}$.

With this, we can define tense:

(L6.7) *Moment Containment.* Let $\bigwedge A_i$ be a momentary condition in $S = (W(t), \mathfrak{X})$. A moment y is in S in $\bigwedge A_i$ if $y \in X$, $X \in \mathfrak{X}$, and y is in a moment y' in $\bigwedge A_i$ if y is an argument of y' in $\bigwedge A_i$.

(L6.8) *Spatial Tense Function.* For each space S, there is a function $\text{tense}_S()$ such that, given $S = (W(t), \mathfrak{X})$, $S' = (W'(t'), \mathfrak{X}')$, and $y = (\langle v | \text{ valid}(\varphi, S') = \rangle, [x_0, \ldots, x_n])$,

$$\text{tense}_S(y) := \begin{cases} 1 & \text{if glb}(t) \leq \text{lub}(t') \leq \text{lub}(t) \\ 0 & \text{otherwise} \end{cases} \text{ and } \text{tense}_S(Y) := \begin{cases} 1 & \text{if } Y \text{ is active} \\ 0 & \text{otherwise} \end{cases}$$

(L6.9) *Moment Tense Function.* Given $S^* = (W^*(t^*), \mathfrak{X}^*)$, for each moment $y^* = (\langle v^* | \text{ valid}(\varphi^*, S^*) = \rangle, [z_0, \ldots, z_n])$ such that y is in ω (a space or moment), there is a function

$$\text{tense}_{y^*}(y) = \begin{cases} 1 & \text{if tense}_\omega(y) = 1, \text{ and glb}(t^*) \leq \text{lub}(t') \leq \text{lub}(t^*) \\ 0 & \text{otherwise} \end{cases},$$

and

$$\text{tense}_{y^*}(Y) = \begin{cases} 1 & \text{if tense}_\omega(Y) = 1, \text{ and } Y \text{ is active in } S^* \\ 0 & \text{otherwise} \end{cases}$$

5.3 Extension Rules

We see that for both determiners and the related vocabulary, and for tense, we are giving conditions at discourse time, whereas the core grammar dealt with conditions defined within a content time space. Just as the content time translation stems from a momentary condition, the extension too, will begin with a momentary condition, but defined at discourse time. The content time information occurs mainly at A_0, either with a Q, Σ, or D proposal, as discussed in the next section. Determiners then give extra statements attached to this initial momentary condition, and the translation as a whole proceeds by (A4.4). We thus extend the translation function, presenting the information in the same recursive order as previously set.

(A7.1) *Discourse Time Modifiers.* For some a, b, $0 \leq a, b \leq n$, if there is a moment y such that $\tau(y) \subseteq \tau(\bigwedge_{i=0}^{a} A_i)$, and $\tau(z) \subseteq \tau(y)$, and $\bigwedge_{i=a+1}^{b} A_i$ implies z is d-valid, then $\tau(\bigwedge_{i=0}^{n} A_i) \Rightarrow \text{SUB}(\tau(\bigwedge_{i=0}^{n} A_i \setminus \bigwedge_{i=a}^{b} A_i)_{\tau(z)}^{\tau(d)^\frown\tau(z)})$

(A7.2) *Partial Moment Sets.* If $\grave{Y}$ is a set of partial moments and there is a T such that, $\forall \grave{y} \in \grave{Y}$, $\tau(\grave{y}) \Rightarrow T$, then $\tau(\grave{Y}) \Rightarrow T$

(A7.3) *Interrogative Complement.* If $\bigwedge A_i$ implies z is v-valid, v is Wh-, and

i) y is a primary moment such that $([z \in \Sigma \to y \in \Sigma] \in \Psi) \sqsubseteq (\bigwedge A_i)$ or

ii) y is a set of primary moments such that $([z \in \Sigma \to y \in \Sigma] \in \Psi) \sqsubseteq (\bigwedge A_i)$ or

iii) y is a partial primary moment such that there is a y', $y' = y \oplus z$ and $([z \in \Sigma \to y' \in \Sigma] \in \Psi) \sqsubseteq (\bigwedge A_i)$,

iv) y is a set of partial primary moment such that there is a Y, $Y = y \oplus z$ and $([z \in \Sigma \to Y \in \Sigma] \in \Psi) \sqsubseteq (\bigwedge A_i)$

then $\tau(z) \Rightarrow \text{SUB}(\tau(\bigwedge A_i)^{\tau(z)\frown\tau(y)}_{\tau(z)})$.

(A7.4) *Tense, Base Case.* Let $\bigwedge A_i$ be a moment in a space S. If $z = (\langle v|\ \text{valid}(\varphi, S') = \rangle, [x_0, \ldots, x_n])$ is a primary moment or $z = \{y|\ \exists c, y = (\langle v|\ \text{valid}(\varphi, c) = \rangle, [x_0, \ldots, x_n])\}$ is a set of primary moments, and $z \sqsubseteq \bigwedge A_i$ for some set X, and z is not an argument of any moment, then

i) $\text{tense}_S(z) = 1$ implies $\tau(v) \Rightarrow \tau(\text{present}(v))$

ii) $\text{tense}_S(z) = 0$ implies $\tau(v) \Rightarrow \tau(\text{past}(v))$

(A7.5) *Tense, Recursion.* Let $\bigwedge A_i$ be a momentary condition, and suppose $y \sqsubseteq \bigwedge A_i$, for some $y = y = (\langle v|\ \text{valid}(\varphi, S) = \rangle, [x_0, \ldots, x_n])$. If $z = (\langle v'|\ \text{valid}(\varphi', S') = \rangle, [x'_0, \ldots, x'_m])$ or $z = \{y'|\ \exists c, y' = (\langle v'|\ \text{valid}(\varphi', c) = \rangle, [x'_0, \ldots, x'_n])\}$, then

i) $\text{tense}_y(z) = 1$ implies $\tau(v') \Rightarrow \tau(\text{present}(v'))$ and

ii) $\text{tense}_y(z) = 0$ implies $\tau(v') \Rightarrow \tau(\text{past}(v'))$

(A7.6) *Low Resolution Noun.* If $\text{res}(\text{type}(x) \leq\ ) < \alpha$, for some α, then $\tau(\langle x|\ \exists c, \text{type}(\ ) = c\rangle) \Rightarrow \emptyset$

(A7.7) *Genitive.* If $\exists N, \tau(x') \Rightarrow N$, then $\exists N', \tau(\text{GENITIVE(X')}) \Rightarrow N'$.

(A7.8) *Moment Sets.* If Y is a set of moments and there is a T such that, $\forall y \in Y, \tau(y) \Rightarrow T$, then $\tau(Y) \Rightarrow T$.

We mentioned that one can have an absent argument when directly oriented at something (§3.1). We will use Ω as a relation, so that we have $x\ \Omega\ z$ as a condition. There are multiple ways this condition can be met, but not all of them are fully spelled out, and must be investigated empirically. But one way that $x\ \Omega\ z$ is satisfied is if x is oriented towards z, and given that people may be orient themselves in multiple ways (e.g. eyes, neck, shoulders), for a person, Ω is satisfied when x is oriented toward z at the hips. Various other conditions, such as a head nod in the direction of z, will also suffice.

(A7.9)

i) If p is a speaker, $p\ \Omega\ x$, and A a statement such that $\text{prop}(A) = [(x\text{R}\phi(y)) \in D]$, or $\text{prop}(A) = [(x\text{R}\phi(y)) \in D]$, then $\tau(A) \Rightarrow \tau(y)$.

ii) If p is a speaker, $p\ \Omega\ x$, and A a statement such that $\text{prop}(A) = [y \in \Sigma]$ or $\text{prop}(A) = [y \in Q]$, $y = \langle v|\ \text{valid}(\varphi, S) = \rangle, [x, z_1, \ldots, z_n]$, and $\grave{y} \oplus x = y$, then $\tau(A) \Rightarrow \tau(\grave{y})$.

iii) If p is a speaker, $p\ \Omega\ x$, and A a statement in S such that $\text{prop}(A) = [y \in \Sigma]$ or $\text{prop}(A) = [y \in Q]$, $y = \langle v|\ \text{valid}(\varphi, S) = \rangle, [p, x, z_2, \ldots, z_n]$, $\text{tense}_S(y) = 0$, and $\grave{y} \oplus p = y$, then $\tau(A) \Rightarrow \tau(\grave{y})$.

5.4 English Sentences

Chomsky (2002) gave us a formulation of how to approach natural language syntax, adopting soundness and completeness goals from the foundations of mathematics and logic to the linguistics setting. We may note that the current theory is lacking on both fronts. For, we have the problem of conjunctions and its variations, including gapping (see e.g. Johnson (2014)), and have not addressed comparatives and superlatives, nor pronouns, and thus fall short of completeness. And on the soundness side, we require constraints on when certain rules are applied, such as right argument quantification, or dative and prepositional constructions, or moments as an argument, as allowing these freely generates many noncoherent sentences. Each such constraint can be considered a second order validity type, and the sentences of English are the translations such that, for each second order validity type v, the translation is v-valid.

Here we introduce CORE valid to correspond to the main structure of English sentences, though they will technically require other validity types, too. We see from the interrogative pronouns that there are simple proposals we may use for introducing the content time: the relevant moments being in Q or Σ. But we also have the aspectual complication of there being sets of moments. Besides these cases, there is also the direct orientation case, only briefly mentioned in this paper. In total, the core structure of English sentences can be accommodated in the following way, but still must be enriched with e.g. DETERMINER valid, and whatever else. Thus, although we fall short of soundness and completeness, we have a general framework for which we can in principle handle the problems that still remain.

(A8.1) *Declaratives and Yes/No Questions.* Let T be a finite string such that $\tau(\bigwedge A_i) \Rightarrow T$. If $\bigwedge A_i$ is a momentary condition such that $\text{prop}(A_0) = [y \in \Sigma]$ or $\text{prop}(A_0) = [y \in Q]$, and y is a primary moment, then T is CORE valid.

(A8.2) *Questions and Answers.* Let T be a finite string such that $\tau(\bigwedge A_i) \Rightarrow T$. If $\bigwedge A_i$ is a momentary condition such that $\text{prop}(A_0) = [\langle z|\grave{\rho}\rangle \in Q]$, then T is CORE valid.

(A8.3) *Commands and Familiar Speech.* Let T be a finite string such that $\tau(\bigwedge A_i) \Rightarrow T$. If $\bigwedge A_i$ is a momentary condition such that $\text{prop}(A_0) = [x \in \phi(y) \in D]$ or $\text{prop}(A_0) = [(x \in \phi(y) \in D) \in \Pi]$, p is the speaker, and $p\ \Omega\ x$ holds, then T is CORE valid.

(A8.4) *Aspect Variation.* Let T be a finite string. If $\bigwedge A_i$ is a momentary condition such that $\text{prop}(A_0) = [\{y_0, y_1, \ldots, y_m\} \in \Sigma]$, $\text{prop}(A_0) = [x\text{R}\phi(\{y_0, y_1, \ldots, y_m\}) \in D]$, $\text{prop}(A_0) = [\{y_0, y_1, \ldots, y_m\} \in Q]$, or $\text{prop}(A_0) = [(x\text{R}\phi(\{y_0, y_1, \ldots, y_m\}) \in D) \in \Pi]$, and for each j, $0 \leq j \leq m$, $\tau(\textsc{sub}(\bigwedge A_{i_{\{y_0,y_1,\ldots,y_m\}}}^{y_j}) \Rightarrow T$ and for any finite string T', $\tau(\textsc{sub}(\bigwedge A_{i_{\{y_0,y_1,\ldots,y_m\}}}^{y_j}) \Rightarrow T'$ implies T' is CORE-valid, then T is CORE-valid.

A Clarifications on Space-Time

Def.: A *space-time* is a set X of 4-dimensional points taken as Cartesian coordinates (a, b, c, d) that satisfy 8 inequalities (76) defined over the 8 functions $f_x(y, z, t)$,$f_y(x, z, t)$, $f_z(x, y, t)$, $f_t(x, y, z)$, $g_x(y, z, t)$, $g_y(x, z, t)$, $g_z(x, y, t)$, $g_t(x, y, z)$

(76) a. $f_x(b, c, d) \leq a \leq g_x(b, c, d)$ b. $f_y(a, c, d) \leq b \leq g_y(a, c, d)$
c. $f_z(a, b, d) \leq c \leq g_z(a, b, d)$ d. $f_t((a, b, c) \leq d \leq g_t(a, b, c)$

We take the coordinates a, b, and c as the points' *spatial part* and d as its *temporal part.*

Def.: A *space-time region* is a space-time such that the spatial functions are independent of time, that is, $\forall d, d', f_x(y, z, d) = f_x(y, z, d')$, $f_y(x, z, d) = f_y(x, z, d')$, $f_z(x, y, d) = f_z(x, y, d')$, $g_x(y, z, d) = g_x(y, z, d')$, $g_y(x, z, d) = g_y(x, z, d')$, $g_z(x, y, d) = g_z(x, y, d')$, and f_t and g_t are constants.

Def.: A *thermic* is a set of points (a, b, c, d) defined by an ordered pair of a space-time X and a set of space-times D, such that for all $Y \in D$, $(a, b, c, d) \in X$ and $(a, b, c, d) \notin Y$, and f_t and g_t are constants. That is, thermics unlike space-times allow holes.

States are defined by the axioms

(77) $t_0 = t_1$ and $W_0 \subseteq W_1 \rightarrow \Phi \subseteq h(\Phi)$

(78) $W_0 = W_1$ and $t_0 \subseteq t_1 \rightarrow h(\Phi) \subseteq \Phi$

It'd be nice if other vocabulary besides, e.g. rules and capacities, satisfied analogous axioms. Unfortunately, a glimpse at the empirical record shows that people may add and remove rules and capacities (by for instance, passing and repealing laws), thus making (79) unlikely. But, it may be possible that such laws, even if they can be added and removed can satisfy (79) by making them dependent on a state that holds (e.g. the rule is on the books).

(79) $W_0 = W_1$ and $t_0 \subseteq t_1 \rightarrow X \subseteq h(X)$

(80) $W_0 = W_1$ and $t_0 \subseteq t_1 \rightarrow h(X) \subseteq X$

(81) $t_0 = t_1$ and $W_0 \subseteq W_1 \rightarrow X \subseteq h(X)$

(82) $t_0 = t_1$ and $W_0 \subseteq W_1 \rightarrow h(X) \subseteq X$

Regardless, the following modifiers are worthwhile to consider as properties of sets. A set X is *lawful* if (79) and (82) hold. A set X is *realistic* if (79) and (81) hold, and a set x is *regional* if (80) and (82) holds. Ψ, perhaps, if the dynamics can be salvaged by states, is lawful and capacities are likely realistic. We use the term "realistic" since arguments against something being real often take the form of it being disputed in another time or place. Although not sets, examples of "regional" are nouns like "party," "play" within certain games, "storms," and "songs." They are characterized by having rates of some phenomenon which are dissimilar to their neighboring spatial and temporal regions. They can typically "die," thus parties and storms can die down, songs can die out, and plays can be called dead by a referee.

B Partial Formula Well-Formedness

Lemma 1 $,\frown,$ $(\frown,$, $,\frown)$*, and* $(\frown)$ *are not substrings of any well-formed term.*

Proof. $,\frown,$ is not an object nor variable, and for a 1-ary function, we have by inspection that $,\frown,$ is not a substring of $f(x)$. Now suppose it's true of any term built from an (n+1)-ary function $g(x_0, x_1, \ldots, x_n)$. We take the (n+2)-ary function, and built a term $h(x_0, \ldots, x_n, x_{n+1})$. We see that $(\frown x_0\frown,\frown x_1\frown,\frown \ldots\frown,\frown x_n$ is a substring of $g(x_0, x_1, \ldots, x_n)$, and by hypothesis, $,\frown,$ is not a substring of this. The remainder of the string can be inspected, and thus we have the result. By the same methods, one can show it's so for the other strings, too. □

Lemma 2 $,\frown,$ $(\frown,$, $,\frown)$*, and* $(\frown)$ *are not substrings of any well-formed atomic formula.*

Proof. We prove this, and other relation cases, for the binary relations with the syntax of the main part of the paper. The more general case can use the syntax of functions and follow the format of the proof above.

Let φ be a well-formed atomic formula. Then $\varphi = x\mathrm{R}\alpha$, for some terms x and α. We know, by lemma 1 that e.g. $,\frown, \not\sqsubseteq x$ and $,\frown, \not\sqsubseteq \alpha$, and it's clear that $,\frown, \not\sqsubseteq \mathrm{R}$, and since $\lambda(\varphi) = x\frown\mathrm{R}\frown\alpha$, $,\frown, \not\sqsubseteq \varphi$. By a similar induction argument, the same holds when the relation takes a formula as input. □

Lemma 3 *For all atomic formula, a relation is not an end point.*

Proof. For the base case, we have $x\mathrm{R}z$, where x and z are terms, and R a relation. We clearly see that the first element nor the last element is the relation, and so it holds. And for induction, we have e.g. ρRz or $\rho R\gamma$. It can only be so if it's so for ρ or γ, and these cases the hypothesis rules out. □

Def.: a is a *combination* of b into c if $\exists c', c^*$, $c = c'\frown c^*$ and $a = c'\frown b\frown c^*$.

Proposition 1 *For any object z, and partial term $f(x_0, \ldots, x_i, \ , x_{i+1}, \ldots, x_n)$, there is one and only one combination α of z into $f(x_0, \ldots, x_i, \ , x_{i+1}, \ldots, x_n)$ such that α is a well-formed term.*

Proof. Let $b = \lambda(f(x_0, \ldots, x_i, \ , x_{i+1}, \ldots, x_n))$, and let $\alpha = b_0\frown \ldots\frown b_k\frown z\frown b_{k+1}\frown \ldots\frown b_m$. Since $\exists b', b^*$ such that $\alpha = b'\frown z\frown b^*$, then a is a combination of z into b. Since b is a partial term, we have $x_0, \ldots, x_n$ are well-formed terms, and z is a well-formed term by hypothesis, so $f(x_0, \ldots, x_i, z, x_{i+1}, \ldots, x_n)$ is a well-formed term by functional composition, thus α is a well-formed term. Let $\alpha' = c'\frown z\frown c^*$ be some combination of z into b, and let $c' = b_0\frown \ldots\frown b_l$. If $l > k$, then $b_k\frown b_{k+1} = ,\frown,$ or $b_k\frown b_{k+1} = (\frown,$ or $b_k\frown b_{k+1} = ,\frown)$ or $b_k\frown b_{k+1} = (\frown)$ is a substring of b' and thus a'. If $l < k$, then $b_k\frown b_{k+1}$ is a substring of b^* and thus α'. Therefore, $l \neq k$ implies α' is not well-formed, by lemma 1, and so α is unique. □

Proposition 2 *For any object z, and partial atomic formula $\dot{\varphi}$, there is one and only one combination σ of z into $\dot{\varphi}$ such that σ is a well-formed formula.*

The proof is a bit tedious, so we just give the general idea. We proceed by cases. The first case of a partial term just repeats the format of the proof above, but with lemma 2 rather than lemma 1. For the second case, we have a missing term, showing such a term exists proceeds as before. For uniqueness, the initial case will place the term as an end point of the string, so any other case will violate lemma 3. This handles all the predicate cases; if one wants to extend this to all formula, one needs to also show that relations preserve the uniqueness for partial formula just as we showed for partial terms. But there is no new technique needed, and so this concludes the outline of the proof. Besides this, it's proper to show that these properties also hold even after constructions with logical symbols.

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