# Negation, modality, events, and truthmaker semantics

Lucas Champollion, Timothée Bernard, and Justin Bledin

New York University, Université Paris Cité, and Johns Hopkins University
champollion@nyu.edu, timothee.bernard@u-paris.fr, jbledin1@jhu.edu

Galway, Ireland – August 8-12, 2022 – ESSLLI course

# Overview of the course

- Day 1: Davidsonian event semantics, problems with negation.
- Day 2: Situation semantics, negation as a modality.
- Day 3: Negative events in compositional semantics.
- Day 4: Event semantics as exact truthmaker semantics.
- Day 5: Propositions as sets of events, and negative individuals.

# Day 2

# Recap from Day 1

- In traditional event semantics, sentences existentially quantify over events.
- This has been generalized from action sentences to other natural language constructions.
- *John saw Mary leave*:
  $\exists e_1 \exists e_2.\ see(John, e_1, e_2) \wedge leave(e_2, Mary)$
- According to the standard analysis, the event quantifier takes scope below classical negation.
- This leaves little room for negated event descriptions such as *John saw Mary not leave*.
- Krifka (1989) introduced a nonstandard negation to deal with some temporal modifiers, but this does not really help with perception reports.

# Today's contents

- Exact vs. inexact verification.
- Situation semantics (Barwise & Perry 1983, Kratzer 1989, 2020, Muskens 1995).
- Relevant entailment.
- Perception reports in situation semantics.
- Situation-based nonstandard logics of negation.
- Possibility and possible worlds in situation semantics.
- Negation as a modality (Goldblatt 1974, Dunn 1993, 1996, 1999, Berto 2015).

# Events and exact verification

- Let's think about the relationship between a simple true action sentence and an event it describes.

**Events are exact verifiers**

It's raining.
$\exists e.rain(e) \approx$ There is a raining event.

- There might be more than one raining event. Any of them *verifies* the sentence (or: is *described* by the sentence).
- A verifying event for *It's raining* is an event that consists wholly of raining. It's an *exact* verifier.

# Situations and inexact verification

- Situation semantics is based on a different idea. A situation is like a visual scene or a partial world.

## Situations can be inexact verifiers

It's raining.
$\exists s.rain(s) \approx$ There is a situation that contains rain.

- There might be other things in the situation besides rain.
- A verifying situation for *It's raining* is a situation that contains raining – and perhaps other happenings and entities. That's why it's called an *inexact* verifier.

Selected references: Barwise & Perry (1983), Kratzer (1989, 2021), Muskens (1995)

# The intuition behind situation semantics

- Worlds are made up of situations.
- True sentences are true not only in the actual world as a whole but also in the situations comprising it.
- Unlike worlds, situations can be partial/incomplete.

**Example: a situation**

$s_1$ is a situation in which John sleeps, it's raining, and the radio is on.

- "It's raining" is true in $s_1$
- "John sleeps" is true in $s_1$
- "The radio is off" is false in $s_1$
- "Mary sleeps" is false (or at least not true) in $s_1$

# The algebraic view on situation semantics

- Situations form a set that is partially ordered by $\leq$. It is common to assume that they form a complete lattice.
- This is a familiar structure from mereology, but with the bottom element present (the *null situation*).
- Smaller situations contain less information and/or fewer participants than larger situations.
- One can think of situations as partial valuations of propositional formulas, though this imposes more structure on them than is needed. (The same is true of events, so this is true of both inexact and exact verification.)

# Inexact verification and persistence

- What distinguishes exact vs. inexact semantics isn't the partiality. It's the verification relation between verifiers and the statements they verify.
- In situation semantics, propositional letters are assumed to be *persistent*: if $p$ is true at $s_1$ and $s_1 \leq s_2$, then $p$ is true at $s_2$. This is often taken to be a desirable feature.
- We will later discuss failures of persistence in event semantics.

# Situation semantics clauses: Conjunction and disjunction

- The rules for conjunction and disjunction are standard.

**Conjunction**

$p \wedge q$ is true in $s$ iff $p$ and $q$ are both true in $s$.

**Disjunction**

$p \vee q$ is true in $s$ iff $p$ or $q$ is true in $s$.

- These clauses transfer persistence upwards:
  - Suppose that $p$ and $q$ are both persistent and true at $s$. Then they are both true at every situation that contains $s$, hence so is $p \wedge q$.
  - Similarly for $p \vee q$.

# Negation in situation semantics: It's complicated . . .

- Negation in situation semantics is a surprisingly complicated topic.
- There are several approaches and we will highlight one of them later.
- What they have in common is that since situations are partial, $p$ and $\neg p$ may both be false (or at least not true) at a given situation.
- Intuitively, this is the case when a situation lacks information about whether $p$.
- Some approaches also allow impossible situations where $p$ and $\neg p$ may both be true.

# Classical entailment

- Classical entailment is truth preservation at possible worlds:

**Classical entailment**

$p$ classically entails $q$ iff in every world where $p$ is true, $q$ is true.

- Example: $p$ classically entails $q \vee \neg q$ because in every world where $p$ is true, $q \vee \neg q$ is true.
- *Stocks are down today. So either my train is late or it isn't.*
- This feels odd because the premise is not relevant to the conclusion.
- Such "paradoxes of implication" motivated work in *relevance logic* by Anderson & Belnap Jr. 1975, Anderson, Belnap Jr. & Dunn 1992, and others.

# Entailment in situation semantics

- Within situation semantics, we can define a notion of *relevant entailment*:

**Relevant entailment**

$p$ relevantly entails $q$ iff in every situation where $p$ is true, $q$ is true.

- This requires the premise to "have bearing" on the conclusion.
- Example: $p$ does not relevantly entail $q \vee \neg q$ because there are situations in which $p$ is true but $q \vee \neg q$ is not true.
- Can you think of such situations?

# Early application to perception reports

- Relevant entailment seems prima facie more appropriate for perception reports.
- Note: $p$ classically but not relevantly entails $p \wedge (q \vee \neg q)$.
- Now consider:

(1) a. John saw [Mary walk]$_p$.

b. John saw [Mary walk]$_p$ and [Bill walk]$_q$ or [not walk]$_{\neg q}$.

- Suppose John's field of view includes Mary but not Bill.
- Then (1a) can be true even if (1b) is false.

# Situation semantics and (negative) perception reports

**Barwise's (1981) analysis of (1a) and (1b)**

- $\exists s$. John sees $s \wedge s \in [\![\text{Mary walk}]\!]$
- $\exists s$. John sees $s \wedge s \in [\![\text{Mary walk}]\!] \wedge$
  $[s \in [\![\text{Bill walk}]\!] \vee s \in [\![\text{Bill not walk}]\!]]$

- For this analysis to work, we need to know when $s \in [\![\text{Bill not walk}]\!]$ holds.

# Compatibility negation

- For negation, one popular approach is to use *incompatibility* relations (e.g., Beall & Restall 2005, Berto 2015).
- Incompatibility relations between situations are usually associated with Goldblatt (1974) and Dunn (1993).
- We assume that some situations occur and others do not.
- $s_1 C s_2$ ("conforms") means that $s_1$ is compatible with $s_2$: i.e., neither rules out the occurrence of the other.
- When $s_1 C s_2$ does not hold, we write $s_1 \perp s_2$ ("conflicts").

**Compatibility negation**

$\neg p$ is true in $s_1$ iff $p$ is not true in any $s_2$ such that $s_1 C s_2$.

# The intuition behind compatibility negation

**Compatibility negation**

$\neg p$ is true in $s_1$ iff $p$ is not true in any $s_2$ such that $s_1 C s_2$.
*Equivalently:* ...iff $s_1 \perp s_2$ for any $s_2$ where $p$ is true.

Here is how Beall & Restall (2005) put it:

- If $\neg p$ is true in $s_1$ and $p$ is true in $s_2$, this must be because $s_1$ conflicts with $s_2$.
- If $p$ is not true in any $s_2$ that conforms with $s_1$, then $s_1$ has "ruled out" $p$; that is, $\neg p$ is true in $s_1$.
- This is similar to Kripke's semantics for negation in intuitionistic logic.

# Negation is similar to impossibility

- Compatibility negation for situations is similar to impossibility (necessity of negation) in modal logic:

**Compatibility negation in situation semantics**

$\neg p$ is true in $s_1$ iff for every $s_2$ such that $s_1 C s_2$, $p$ is not true in $s_2$.

**Impossibility in classical modal logic**

$\Box\neg p$ is true in $w_1$ iff for every $w_2$ accessible from $w_1$, $p$ is not true in $w_2$.

# Example: Law of excluded middle

- $p \vee \neg p$ is true in $s_1$
  - ...iff $p$ or $\neg p$ is true in $s_1$
  - ...iff $p$ is true in $s_1$ or whenever $p$ is true in $s_2$ then $s_1 \perp s_2$
- This can fail if $s_1$ is not a $p$-situation but is compatible with a $p$-situation.
- Can you think of a situation where an instance of the Law of excluded middle fails?

# Example: Classical contradictions

- $p \wedge \neg p$ is true in $s_1$
  - ...iff $p$ and $\neg p$ are both true in $s_1$
  - ...iff $p$ is true in $s_1$ and whenever $p$ is true in $s_2$ then $s_1 \perp s_2$
- If we assume that all situations are consistent ($s \not\perp s$), this can never be true.
- Alternatively, we can countenance impossible situations where $p$ and $\neg p$ may both be true.
- Such impossible situations will never be part of any possible world (more on this in a moment).

# Example: *Ex falso quodlibet*

- *Ex falso quodlibet* (EFQ), the principle of explosion, says that anything follows from a logical falsehood. A single inconsistency leads to everything whatsoever being true.
- $p \wedge \neg p$ classically entails $q$.
    - It is vacuously the case that every possible $[p \wedge \neg p]$-world is a $q$-world, because there are no such worlds.
- *Paraconsistent logics* are logics in which EFQ does not hold (e.g., Priest's 1979 "Logic of Paradox").
- Relevant entailment over possible and impossible situations is paraconsistent.
    - Some $[p \wedge \neg p]$-situations are not $q$-situations.
    - But these situations are all impossible, in the sense that they self-conflict.

# Possibility and possible worlds in situation semantics

- We call a situation $s$ *possible* iff $sCs$, otherwise *impossible*.
- We can say that $w$ is a *world* iff $wCw$ and for any $s$, either $w \perp s$ or $s$ is part of $w$.
- Equivalently, we can say that $w$ is a *world* iff $wCw$ and for any $w^+$ that properly contains $w$, $w^+$ is impossible.
- So a possible world is a maximal possible situation.
- This is a common style of definition: e.g., Pollock (1967), Plantinga (1978), Kratzer (1989).

# Negation as a modality: A brief history

- The semantic analysis of negation as a modal operator in terms of an incompatibility relation can be traced back to Goldblatt's (1974) *orthologic*.
- Goldblatt was trying to provide an intensional model theory for quantum logic, inspired by Kripke's work in modal logic.
- Dunn (1993, 1996, 1999) has investigated many variations of incompatibility negation.
- Logicians and philosophers continue to explore different non-classical negation operators defined in terms of compatibility and incompatibility (Berto 2015).

# Orthologic

- Goldblatt (1974) is the locus classicus for orthologic, in which $\perp$ (rather than $C$) is the starting point and there is no $\leq$.
- Goldblatt imposes the following constraints on $\perp$:
  - $\perp$ is irreflexive (so no impossible situations)
  - $\perp$ is symmetric (a very intuitive and common constraint).
- A set of situations $X$ is said to be $\perp$-*closed* iff any situation $s_1$ is outside $X$ only if $s_1$ is compatible with some situation that conflicts with every situation in $X$.
- In other words, $X$ includes every situation that conflicts with every situation that conflicts with all the situations in $X$.
- Models assign a $\perp$-closed interpretation to each propositional letter. This extends to arbitrary formulas.
- There are only two truth values.
- Entailment is truth preservation at all situations.

# The clauses of orthologic in Goldblatt (1974)

## Conjunction

$p \wedge q$ is true in $s_1$ iff $p$ and $q$ are both true in $s_1$.

## Negation

$\neg p$ is true in $s_1$ iff for every $s_2$ where $p$ is true, $s_1 \perp s_2$.

This is a compatibility negation.

# Negation as a modality: Dunn

- Dunn (1993, 1996, 1999) has investigated many variations of orthologic.
- The clause for negation remains as in Goldblatt (1974).
- The flexibility comes from variation in the conditions placed on $\perp$, as in Kripke-style modal logics where different constraints are imposed on the accessibility relation between possible worlds.
- A partial order $\leq$ is also introduced, as in situation semantics.

# Correspondence theory for the modality of negation

## Compatibility negation

$\neg p$ is true in $s_1$ iff for every $s_2$ such that $s_1 C s_2$, $p$ is false in $s_2$.
Equivalently: ...iff for every $s_2$ where $p$ is true, $s_1 \perp s_2$.

- $A \vdash \neg\neg A$ corresponds to $\forall s_1 \forall s_2.\ [s_1 C s_2 \rightarrow s_2 C s_1]$ (symmetry)
- $A \vdash B, A \vdash \neg B / A \vdash \neg C$ corresponds to $\forall s_1 \forall s_2.\ [s_1 C s_2 \rightarrow s_1 C s_1]$ (shift-reflexivity)
- $A \vdash B, A \vdash \neg B / A \vdash C$ corresponds to $\forall s_1.\ [s_1 C s_1], \forall s_1 \forall s_2.\ [s_1 C s_2 \rightarrow s_2 C s_1]$
- $\neg\neg A \vdash A$ corresponds to $\forall s_1 \exists s_2.\ [s_1 C s_2 \wedge \forall s_3.\ [s_2 C s_3 \rightarrow s_3 = s_1]]$
- See Horn & Wansing (2020) (SEP) for more discussion.

# Logical pluralism about negation: Berto (2015)

*Not everything called negation in the logical literature deserves that name, but more than one item does . . . Such a view is pursued by grounding the meaning of negation in a single (albeit twofold) core notion: the concept of compatibility, together with its polar opposite, incompatibility. The features of (in)compatibility set precise constraints on what counts as a negation.*

# Day 2: Summary

- Events are *exact* verifiers of sentences by which they are described while situations are *inexact* verifiers.
- Situations are like partial worlds; $p$ and $\neg p$ can fail to hold at a situation.
- Compatibility negation: $\neg p$ is true in $s_1$ iff $s_1 \perp s_2$ whenever $p$ is true at $s_2$.
- Relevant entailment is truth preservation at situations; this requires the premise to "have bearing" on the conclusion.
- Constraints on $\perp$ or its complement $C$ differentiate between multiple notions of negation.

- Anderson, Alan Ross & Nuel Belnap Jr. 1975. *Entailment, Volume I*. Princeton: Princeton University Press.
- Anderson, Alan Ross, Nuel Belnap Jr. & J. Michael Dunn. 1992. *Entailment, Volume II*. Princeton: Princeton University Press.
- Barwise, Jon. 1981. Scenes and other situations. *The Journal of Philosophy* 78(7). 369–397. https://doi.org/10.2307/2026481.
- Barwise, Jon & John Perry. 1983. *Situations and attitudes*. MIT Press.
- Beall, JC & Greg Restall. 2005. *Logical pluralism*. Oxford, UK: Oxford University Press. https://doi.org/10.1093/acprof:oso/9780199288403.001.0001.
- Berto, Francesco. 2015. A modality called 'negation'. *Mind* 124(495). 761–793. https://doi.org/10.1093/mind/fzv026.
- Dunn, J. Michael. 1993. Star and perp: two treatments of negation. *Philosophical Perspectives* 7. 331.

https://doi.org/10.2307/2214128.
https://doi.org/10.2307%2F2214128.

- Dunn, J. Michael. 1996. Generalized ortho negation. In Heinrich Wansing (ed.), *Negation*, vol. 7 (Perspektiven der Analytischen Philosophie / Perspectives in Analytical Philosophy), 3–26. de Gruyter. https://doi.org/10.1515/9783110876802.3.
- Dunn, J. Michael. 1999. A comparative study of various model-theoretic treatments of negation: a history of formal negation. In Dov M. Gabbay & Heinrich Wansing (eds.), *What is negation?*, vol. 13 (Applied Logic), 23–51. Springer. https://doi.org/10.1007/978-94-015-9309-0_2.
- Goldblatt, Robert I. 1974. Semantic analysis of orthologic. *Journal of Philosophical Logic* 3(1-2). 19–35. https://doi.org/10.1007/bf00652069. https://doi.org/10.1007/bf00652069.
- Horn, Laurence R. & Heinrich Wansing. 2020. Negation. In Edward N. Zalta (ed.), *The Stanford Encyclopedia of*

*Philosophy*, Spring 2020. Stanford, CA: Metaphysics Research Lab, Stanford University. https://plato.stanford.edu/archives/spr2020/entries/negation/.

- Kratzer, Angelika. 1989. An investigation of the lumps of thought. *Linguistics and Philosophy* 12(5). 607–653. https://doi.org/10.1007/bf00627775.
- Kratzer, Angelika. 2020. Situations in natural language semantics. In Edward N. Zalta (ed.), *The Stanford Encyclopedia of Philosophy*, Fall 2020. Stanford, CA: Metaphysics Research Lab, Stanford University. https://plato.stanford.edu/archives/fall2020/entries/situations-semantics/.
- Kratzer, Angelika. 2021. Situations in natural language semantics. In Edward N. Zalta (ed.), *The Stanford Encyclopedia of Philosophy*, Winter 2021. Stanford, CA: Metaphysics Research Lab, Stanford University. https://plato.stanford.edu/archives/win2021/entries/situations-semantics/.
- Krifka, Manfred. 1989. Nominal reference, temporal constitution and quantification in event semantics. In Renate Bartsch,

Johan van Benthem & Peter van Emde Boas (eds.), *Semantics and contextual expression*, 75–115. Dordrecht, Netherlands: Foris. http://amor.cms.hu-berlin.de/~h2816i3x/Publications/Krifka%201989%20Nominal%20Reference.pdf.

- Muskens, Reinhard. 1995. *Meaning and partiality*. Stanford, CA: CSLI Publications.
- Plantinga, Alvin. 1978. *The nature of necessity*. Oxford University Press. https://doi.org/10.1093/0198244142.001.0001.
- Pollock, John L. 1967. The logic of logical necessity. *Logique et Analyse* 10(39/40). 307–323. https://www.jstor.org/stable/44084498.
- Priest, Graham. 1979. The logic of paradox. *Journal of Philosophical Logic* 8. 219–241.