

Negation, modality, events, and truthmaker semantics

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Overview of the course

- Day 1: Davidsonian event semantics, problems with negation.
- Day 2: Situation semantics, negation as a modality.
- Day 3: Negative events in compositional semantics.
- Day 4: Event semantics as exact truthmaker semantics.
- Day 5: Propositions as sets of events, and negative individuals.



Day 2

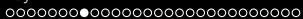


Recap from Day 1

- In traditional event semantics, sentences existentially quantify over events.
- This has been generalized from action sentences to other natural language constructions.
- *John saw Mary leave*:
 $\exists e_1 \exists e_2. \text{see}(\text{John}, e_1, e_2) \wedge \text{leave}(e_2, \text{Mary})$
- According to the standard analysis, the event quantifier takes scope below classical negation.
- This leaves little room for negated event descriptions such as *John saw Mary not leave*.
- Krifka (1989) introduced a nonstandard negation to deal with some temporal modifiers, but this does not really help with perception reports.

The algebraic view on situation semantics

- Situations form a set that is partially ordered by \leq . It is common to assume that they form a complete lattice.
- This is a familiar structure from mereology, but with the bottom element present (the *null situation*).
- Smaller situations contain less information and/or fewer participants than larger situations.
- One can think of situations as partial valuations of propositional formulas, though this imposes more structure on them than is needed. (The same is true of events, so this is true of both inexact and exact verification.)



Inexact verification and persistence

- What distinguishes exact vs. inexact semantics isn't the partiality. It's the verification relation between verifiers and the statements they verify.
- In situation semantics, propositional letters are assumed to be *persistent*: if p is true at s_1 and $s_1 \leq s_2$, then p is true at s_2 . This is often taken to be a desirable feature.
- We will later discuss failures of persistence in event semantics.

Situation semantics clauses: Conjunction and disjunction

- The rules for conjunction and disjunction are standard.

Conjunction

$p \wedge q$ is true in s iff p and q are both true in s .

Disjunction

$p \vee q$ is true in s iff p or q is true in s .

- These clauses transfer persistence upwards:
 - Suppose that p and q are both persistent and true at s . Then they are both true at every situation that contains s , hence so is $p \wedge q$.
 - Similarly for $p \vee q$.

Negation in situation semantics: It's complicated ...

- Negation in situation semantics is a surprisingly complicated topic.
- There are several approaches and we will highlight one of them later.
- What they have in common is that since situations are partial, p and $\neg p$ may both be false (or at least not true) at a given situation.
- Intuitively, this is the case when a situation lacks information about whether p .
- Some approaches also allow impossible situations where p and $\neg p$ may both be true.

Classical entailment

- Classical entailment is truth preservation at possible worlds:

Classical entailment

p classically entails q iff in every world where p is true, q is true.

- Example: p classically entails $q \vee \neg q$ because in every world where p is true, $q \vee \neg q$ is true.
- *Stocks are down today. So either my train is late or it isn't.*
- This feels odd because the premise is not relevant to the conclusion.
- Such “paradoxes of implication” motivated work in *relevance logic* by Anderson & Belnap Jr. 1975, Anderson, Belnap Jr. & Dunn 1992, and others.



Entailment in situation semantics

- Within situation semantics, we can define a notion of *relevant entailment*:

Relevant entailment

p relevantly entails q iff in every situation where p is true, q is true.

- This requires the premise to “have bearing” on the conclusion.
- Example: p does not relevantly entail $q \vee \neg q$ because there are situations in which p is true but $q \vee \neg q$ is not true.
- Can you think of such situations?



Early application to perception reports

- Relevant entailment seems prima facie more appropriate for perception reports.
- Note: p classically but not relevantly entails $p \wedge (q \vee \neg q)$.
- Now consider:

- (1)
- a. John saw $[Mary\ walk]_p$.
 - b. John saw $[Mary\ walk]_p$ and $[Bill\ walk]_q$ or $[not\ walk]_{\neg q}$.

- Suppose John's field of view includes Mary but not Bill.
- Then (1a) can be true even if (1b) is false.

Situation semantics and (negative) perception reports

Barwise's (1981) analysis of (1a) and (1b)

- $\exists s. \text{John sees } s \wedge s \in \llbracket \text{Mary walk} \rrbracket$
- $\exists s. \text{John sees } s \wedge s \in \llbracket \text{Mary walk} \rrbracket \wedge$
 $[s \in \llbracket \text{Bill walk} \rrbracket \vee s \in \llbracket \text{Bill not walk} \rrbracket]$
- For this analysis to work, we need to know when $s \in \llbracket \text{Bill not walk} \rrbracket$ holds.



Compatibility negation

- For negation, one popular approach is to use *incompatibility* relations (e.g., Beall & Restall 2005, Berto 2015).
- Incompatibility relations between situations are usually associated with Goldblatt (1974) and Dunn (1993).
- We assume that some situations occur and others do not.
- $s_1 C s_2$ (“conforms”) means that s_1 is compatible with s_2 : i.e., neither rules out the occurrence of the other.
- When $s_1 C s_2$ does not hold, we write $s_1 \perp s_2$ (“conflicts”).

Compatibility negation

$\neg p$ is true in s_1 iff p is not true in any s_2 such that $s_1 C s_2$.

The intuition behind compatibility negation

Compatibility negation

$\neg p$ is true in s_1 iff p is not true in any s_2 such that $s_1 C s_2$.

Equivalently: ... iff $s_1 \perp s_2$ for any s_2 where p is true.

Here is how Beall & Restall (2005) put it:

- If $\neg p$ is true in s_1 and p is true in s_2 , this must be because s_1 conflicts with s_2 .
- If p is not true in any s_2 that conforms with s_1 , then s_1 has “ruled out” p ; that is, $\neg p$ is true in s_1 .
- This is similar to Kripke's semantics for negation in intuitionistic logic.

Negation is similar to impossibility

- Compatibility negation for situations is similar to impossibility (necessity of negation) in modal logic:

Compatibility negation in situation semantics

$\neg p$ is true in s_1 iff for every s_2 such that $s_1 Cs_2$, p is not true in s_2 .

Impossibility in classical modal logic

$\Box \neg p$ is true in w_1 iff for every w_2 accessible from w_1 , p is not true in w_2 .



Example: Law of excluded middle

- $p \vee \neg p$ is true in s_1
 - ... iff p or $\neg p$ is true in s_1
 - ... iff p is true in s_1 or whenever p is true in s_2 then $s_1 \perp s_2$
- This can fail if s_1 is not a p -situation but is compatible with a p -situation.
- Can you think of a situation where an instance of the Law of excluded middle fails?

Example: Classical contradictions

- $p \wedge \neg p$ is true in s_1
 - ... iff p and $\neg p$ are both true in s_1
 - ... iff p is true in s_1 and whenever p is true in s_2 then $s_1 \perp s_2$
- If we assume that all situations are consistent ($s \not\perp s$), this can never be true.
- Alternatively, we can countenance impossible situations where p and $\neg p$ may both be true.
- Such impossible situations will never be part of any possible world (more on this in a moment).

Example: *Ex falso quodlibet*

- *Ex falso quodlibet* (EFQ), the principle of explosion, says that anything follows from a logical falsehood. A single inconsistency leads to everything whatsoever being true.
- $p \wedge \neg p$ classically entails q .
 - It is vacuously the case that every possible $[p \wedge \neg p]$ -world is a q -world, because there are no such worlds.
- *Paraconsistent logics* are logics in which EFQ does not hold (e.g., Priest's 1979 "Logic of Paradox").
- Relevant entailment over possible and impossible situations is paraconsistent.
 - Some $[p \wedge \neg p]$ -situations are not q -situations.
 - But these situations are all impossible, in the sense that they self-conflict.

Possibility and possible worlds in situation semantics

- We call a situation s *possible* iff sCs , otherwise *impossible*.
- We can say that w is a *world* iff wCw and for any s , either $w \perp s$ or s is part of w .
- Equivalently, we can say that w is a *world* iff wCw and for any w^+ that properly contains w , w^+ is impossible.
- So a possible world is a maximal possible situation.
- This is a common style of definition: e.g., Pollock (1967), Plantinga (1978), Kratzer (1989).

Negation as a modality: A brief history

- The semantic analysis of negation as a modal operator in terms of an incompatibility relation can be traced back to Goldblatt's (1974) *orthologic*.
- Goldblatt was trying to provide an intensional model theory for quantum logic, inspired by Kripke's work in modal logic.
- Dunn (1993, 1996, 1999) has investigated many variations of incompatibility negation.
- Logicians and philosophers continue to explore different non-classical negation operators defined in terms of compatibility and incompatibility (Berto 2015).

Orthologic

- Goldblatt (1974) is the locus classicus for orthologic, in which \perp (rather than C) is the starting point and there is no \leq .
- Goldblatt imposes the following constraints on \perp :
 - \perp is irreflexive (so no impossible situations)
 - \perp is symmetric (a very intuitive and common constraint).
- A set of situations X is said to be \perp -closed iff any situation s_1 is outside X only if s_1 is compatible with some situation that conflicts with every situation in X .
- In other words, X includes every situation that conflicts with every situation that conflicts with all the situations in X .
- Models assign a \perp -closed interpretation to each propositional letter. This extends to arbitrary formulas.
- There are only two truth values.
- Entailment is truth preservation at all situations.

The clauses of orthologic in Goldblatt (1974)

Conjunction

$p \wedge q$ is true in s_1 iff p and q are both true in s_1 .

Negation

$\neg p$ is true in s_1 iff for every s_2 where p is true, $s_1 \perp s_2$.

This is a compatibility negation.

Negation as a modality: Dunn

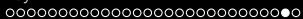
- Dunn (1993, 1996, 1999) has investigated many variations of orthologic.
- The clause for negation remains as in Goldblatt (1974).
- The flexibility comes from variation in the conditions placed on \perp , as in Kripke-style modal logics where different constraints are imposed on the accessibility relation between possible worlds.
- A partial order \leq is also introduced, as in situation semantics.

Correspondence theory for the modality of negation

Compatibility negation

$\neg p$ is true in s_1 iff for every s_2 such that $s_1 C s_2$, p is false in s_2 .
Equivalently: ... iff for every s_2 where p is true, $s_1 \perp s_2$.

- $A \vdash \neg\neg A$ corresponds to $\forall s_1 \forall s_2. [s_1 C s_2 \rightarrow s_2 C s_1]$ (symmetry)
- $A \vdash B, A \vdash \neg B / A \vdash \neg C$ corresponds to $\forall s_1 \forall s_2. [s_1 C s_2 \rightarrow s_1 C s_1]$ (shift-reflexivity)
- $A \vdash B, A \vdash \neg B / A \vdash C$ corresponds to $\forall s_1. [s_1 C s_1], \forall s_1 \forall s_2. [s_1 C s_2 \rightarrow s_2 C s_1]$
- $\neg\neg A \vdash A$ corresponds to $\forall s_1 \exists s_2. [s_1 C s_2 \wedge \forall s_3. [s_2 C s_3 \rightarrow s_3 = s_1]]$
- See Horn & Wansing (2020) (SEP) for more discussion.



Logical pluralism about negation: Berto (2015)

Not everything called negation in the logical literature deserves that name, but more than one item does . . . Such a view is pursued by grounding the meaning of negation in a single (albeit twofold) core notion: the concept of compatibility, together with its polar opposite, incompatibility. The features of (in)compatibility set precise constraints on what counts as a negation.



Day 2: Summary

- Events are *exact* verifiers of sentences by which they are described while situations are *inexact* verifiers.
- Situations are like partial worlds; p and $\neg p$ can fail to hold at a situation.
- Compatibility negation: $\neg p$ is true in s_1 iff $s_1 \perp s_2$ whenever p is true at s_2 .
- Relevant entailment is truth preservation at situations; this requires the premise to “have bearing” on the conclusion.
- Constraints on \perp or its complement C differentiate between multiple notions of negation.



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