



Negation, modality, events, and truthmaker semantics

Lucas Champollion, Timothée Bernard, and Justin Bledin

New York University, Université Paris Cité, and Johns Hopkins University
champollion@nyu.edu, timothee.bernard@u-paris.fr, jbledin1@jhu.edu

Galway, Ireland – August 8-12, 2022 – ESLLI course



Overview of the course

- Day 1: Davidsonian event semantics, problems with negation.
- Day 2: Situation semantics, negation as a modality.
- Day 3: Negative events in compositional semantics.
- Day 4: Event semantics as exact truthmaker semantics.
- Day 5: Propositions as sets of events, and negative individuals.



Day 3



Recap from Day 2

- Events are *exact* verifiers of sentences by which they are described while situations are *inexact* verifiers.
- Situations are like partial worlds; p and $\neg p$ can fail to hold at a situation.
- Compatibility negation: $\neg p$ is true in s_1 iff $s_1 \perp s_2$ whenever p is true at s_2 .
- Relevant entailment is truth preservation at situations; this requires the premise to “have bearing” on the conclusion.
- Constraints on \perp or its complement C differentiate between multiple notions of negation.



Today's contents

- Negative events as events introduced by a negative sentence.
- A formalization without possible worlds.
- A compositional fragment.
- Applications to negative perception reports and negative causation reports.
- Negative events and possible world semantics.
- Treatment of modals and of attitude verbs.



Our goal: negative events as first-class citizens

- We first show how to constrain standard models of event semantics without possible worlds so as to make negative events available.
- “Negative event”: event introduced by a negative sentence.
- If sentence s describes a set of events P (s 's *radical*), we assume that the negation of s also describes some set of events, that we call anti- P events.
- Our first goal is to formalize this set.



Bedrock assumption

We are after the following property:

The negation of a sentence is true iff this sentence is false.

- We will not try to model the pragmatic effects of negation (Tian & Breheny 2019).

Examples related to expectations:

- (1) a. I saw the car not stop.
b. This is not a hotel.



The core of the semantics of negation: *Neg*

- $Neg : \langle \langle v, t \rangle, \langle v, t \rangle \rangle$ sends any set of events P to the set of anti- P events.
- Ex:
 - A raining event x satisfies $rain(x)$;
 - An anti-raining event x satisfies $Neg(rain)(x)$.
We often write " $x \in Neg(rain)$ " instead.
- Neg is not just any function of type $\langle \langle v, t \rangle, \langle v, t \rangle \rangle$; we require that Neg respects the Principle of Negation given later.
- Before, let's discuss a problem that arises from not using possible worlds.



What if all events occur?

- Do all events exist (i.e., occur in reality)?
- Let's assume so. Then, also assume that both of the following sentences are true:
 - (2) a. Mary did not stay.
b. John did not laugh.
- Let P_1 be the set of stayings by Mary and P_2 the set of laughings by John:
 - $P_1 = \emptyset$
 - $P_2 = \emptyset$
- What can we say about $Neg(P_1)$ (anti-Mary-staying events) and $Neg(P_2)$ (anti-John-laughing events)?



If all events occur, we have a problem

- If $P_1 = P_2$, then $Neg(P_1) = Neg(P_2)$.
- The two previous sentences in (2) are predicted to describe the same events!
- Further consequence: Seeing an anti-Mary-staying is predicted to be equivalent to seeing an anti-John-laughing. . .



The root of the problem, and possible directions

- Standard model-theoretical semantics:
 - $\llbracket AB \rrbracket_M = \llbracket A \rrbracket_M(\llbracket B \rrbracket_M)$
 - So $\llbracket Neg(P_i) \rrbracket_M = \llbracket Neg \rrbracket_M(\llbracket P_i \rrbracket_M)$
- One of our hypotheses must be changed.
 - ① $Neg : \langle \langle v, t \rangle, \langle v, t \rangle \rangle$ not used for negative event predicates?
 - ② $\llbracket AB \rrbracket_M \neq \llbracket A \rrbracket_M(\llbracket B \rrbracket_M)$? (see hyperintensional systems such as the one of Muskens 2007)
 - ③ $\llbracket P \rrbracket_M$ **is not the set of occurring P events?**



A solution: noneism

- Let's assume that events may be either actual or nonactual (but not both).
- Consequence:
 - *Mary did not stay* $\nrightarrow (P_1 = \emptyset)$;
no staying event by Mary occurred, still, some might not occur.
 - Similarly with *John did not laugh*.
 - So, one can assume that $P_1 \neq P_2$.
- We assume quantifiers (\exists , \forall) are *possibilist*: they range over both actual and nonactual entities.
- Is this an exotic position?



With possibilist \exists , we need an explicit actuality predicate

- The set of actual events is denoted by the predicate *actual* : $\langle v, t \rangle$.
- *Instantiated* $\stackrel{\text{def}}{=} \lambda P. [\exists e. \text{actual}(e) \wedge P e]$
- Caution:
 - (3) a. It is raining.
 - b. $*_{(\text{too weak})} \exists e. \text{rain}(e)$
 - c. $\exists e. \text{actual}(e) \wedge \text{rain}(e)$



A constraint on *Neg*: the Principle of Negation

Principle of Negation

$$(4) \quad \forall P. [\exists e \in P. \text{actual}(e)] \leftrightarrow \neg[\exists e \in \text{Neg}(P). \text{actual}(e)]$$

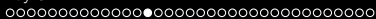
$$(\forall P. \text{Instantiated}(P) \leftrightarrow \neg \text{Instantiated}(\text{Neg}(P)))$$

- (\rightarrow ; *No Gluts*) If an anti- P event occurs, then no P event does.
- (\leftarrow ; *No Gaps*) If no P event occurs, then an anti- P event does.
- We now *assume* the Principle of Negation; but later we embed our theory into Truthmaker semantics and *derive* it from first principles.



Basic linguistic assumption

- Linguistic negation (*not*) lexicalizes *Neg*.
 - (5) a. It is not raining.
 - b. $\exists e. \text{actual}(e) \wedge e \in \text{Neg}(\text{rain})$
 - c. $\neg \exists e. \text{actual}(e) \wedge \text{rain}(e)$ (by Principle of Neg.)



Direct consequences of the Principle of Negation

Excluded middle

$Instantiated(Neg(P)) \vee Instantiated(P)$

Ex: $(6a) \vee (6b)$

Noncontradiction

$\neg(Instantiated(Neg(P)) \wedge Instantiated(P))$

Ex: $\neg((6a) \wedge (6b))$

- (6)
- a. Mary did not eat.
 - b. Mary ate.



Some other properties of *Neg*

Downward-entailingness

Instantiated(Neg(P)) \rightarrow *Instantiated(Neg(P \cap Q))*

Ex: (7a) \rightarrow (7c)

Double negation cancellation

Instantiated(Neg(Neg(P))) \rightarrow *Instantiated(P)*

Ex: (7d) \rightarrow (7b)

- (7)
- a. Mary did not eat.
 - b. Mary ate.
 - c. Mary did not eat an egg.
 - d. Mary did not not eat.



The “negativity” of negative events

It is to be observed, that the nature of the act, whether positive or negative, is not to be determined immediately by the form of the discourse made use of to express it. An act which is positive in its nature may be characterized by a negative expression: thus, not to be at rest, is as much as to say to move. So also an act, which is negative in its nature, may be characterized by a positive expression: thus, to forbear or omit to bring food to a person in certain circumstances, is signified by the single and positive term to starve. Bentham (1789: ch. 7, §10)

- If there is an ontological difference between positive and negative events (see also e.g., Mossel 2009 and references therein),
 - it is not reflected by the use of negation;
 - we have not been trying to capture it.



Can an event be both positive and negative?

- Recall that in our sense: A negative event is any event described by a negated sentence.
- One may assume that some events are both positive and negative.
- Good candidates found there:
 - (8) a. Mary did not leave.
 - b. Mary stayed.



The scope of *Neg*: a difficulty

- (9)
- a. Mary did not leave.
 - b. $*$ _(too strong)
 $\exists e. \text{actual}(e) \wedge \text{ag}(e) = \text{Mary} \wedge e \in \text{Neg}(\text{leave})$
 - c. $\exists e. \text{actual}(e) \wedge e \in \text{Neg}(\lambda e'. \text{ag}(e') = \text{Mary} \wedge \text{leave}(e'))$

- Can you spot unwanted consequences of (9b)?
- There is an apparent mismatch between the syntactic scope and the semantic scope of negation.
- This is a challenge for compositional semantics.
How can it be overcome?



Scope mismatch: a semantic solution

- Bernard & Champollion's (2018) solution:
 - directly compositional (Jacobson 2012);
 - based on continuation (Barker & Shan 2014).
- (You might remember continuations from day one, in the discussion of Champollion 2015.)
- Negation denotes a higher-order function that takes a verb phrase and a subject as its arguments, internally combines them, and applies the *Neg* function to the result of the combination:

$$\llbracket \text{not} \rrbracket = \lambda P \lambda f. \text{Neg}(\lambda e'. P e' \wedge f e')$$

- f is a *continuation*.



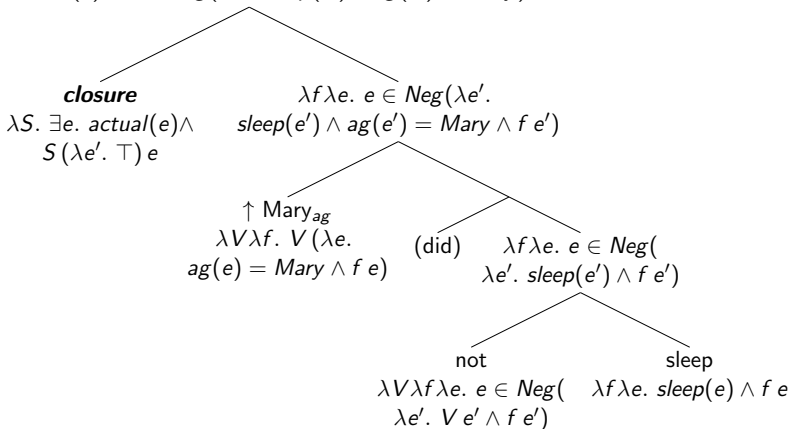
$$\exists e. \text{actual}(e) \wedge e \in \text{Neg}(\lambda e'. \text{sleep}(e') \wedge \text{ag}(e') = \text{Mary})$$


Figure: Bernard & Champollion's (2018) derivation of “Mary did not sleep”.



Scope mismatch: a syntactic solution

- VP-internal subject hypothesis (Koopman & Sportiche 1991): Subjects are base-generated within a verbal projection and then move out of it.
- The base position is lower than a possible negation.
- An item subject to movement leaves a *trace* behind: a free variable bound by an operator inserted at the end of the movement.
- With a nonquantified subject, this is equivalent to interpreting the subject below negation.

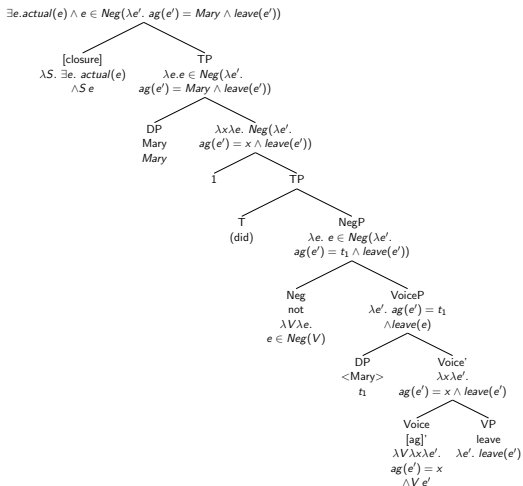


Figure: A derivation of *Mary did not leave* based on the VP-internal subject hypothesis.



Back to nonfinite perception reports

- Event-based analysis of nonfinite perception reports:

- (10)
- John sees Mary leave.
 - $\text{closure}((1 \ (\llbracket \text{exp}' \rrbracket) (\llbracket \text{see} \rrbracket) ((2 \ (\llbracket \text{ag}' \rrbracket) (\llbracket \text{leave} \rrbracket) (t_2))) \llbracket \text{Mary} \rrbracket) (t_1))) \llbracket \text{John} \rrbracket)$
 - $\exists e. \text{actual}(e) \wedge \text{exp}(e) = \text{John} \wedge \text{see}(e) \wedge$
 $\exists e'. \text{th}(e) = e' \wedge \text{ag}(e') = \text{Mary} \wedge \text{leave}(e')$

- No Hallucination Principle:

- (11) $\forall e. [\text{actual}(e) \wedge \text{see}(e)] \rightarrow \text{actual}(\text{th}(e))$

- Can *Neg* be used to extend this analysis to cases involving negation?



Negative perception reports

- Extension to cases involving negation:

- (12)
- John sees Mary not leave.
 - $\text{closure}((1 ([\text{exp}'])([\text{see}] ((2 ([\text{not}]([\text{ag}'])([\text{leave}])(t_2)))) ([\text{Mary}]))(t_1))) ([\text{John}])$
 - $\exists e. \text{actual}(e) \wedge \text{exp}(e) = \text{John} \wedge$
 $\text{see}(e) \wedge \exists e'. \text{th}(e) = e' \wedge$
 $e' \in \text{Neg}(\lambda e''. \text{ag}(e'') = \text{Mary} \wedge \text{leave}(e''))$

- In sharp contrast with Krifka (1989)'s negation, the negative events introduced here may vary when the negated predicate varies.



Introducing possible worlds

Assumptions

- Set of possible worlds: W
- Actual world: $w_0 \in W$
- An event may occur in zero, one or more possible worlds ($occursIn : \langle v, \langle s, t \rangle \rangle$);
- *Modal fragility* (Lewis 1986): Most predicates that hold of events do so necessarily.

- (13)
- a. Mary is walking.
 - b. $\lambda e. walk(e) \wedge ag(e) = Mary$
 - c. $\lambda w. \exists e. occursIn(e, w) \wedge walk(e) \wedge ag(e) = Mary$
 - d. $\exists e. occursIn(e, w_0) \wedge walk(e) \wedge ag(e) = Mary$



A Principle of Negation that holds at all worlds

Modal Principle of Negation:

$$\forall w \forall P. [\exists e \in P. \text{occursIn}(e, w)] \leftrightarrow \neg[\exists e \in \text{Neg}(P). \text{occursIn}(e, w)]$$

- (14)
- a. Mary is not walking.
 - b. $\exists e. \text{occursIn}(e, w_0) \wedge e \in \text{Neg}(\lambda e'. \text{walk}(e') \wedge \text{ag}(e') = \text{Mary})$

Modality: the standard picture

- Kripke (1959) introduced the now-standard approach to modal logic as quantification over possible worlds.
- A model is essentially a set of possible worlds, each with an interpretation function. One world is designated as the actual world.
- $\diamond p$ is true iff p is true in some possible world.
- $\Box p$ is true iff p is true in all possible worlds.
- Kripke (1963) adds to this an accessibility relation between worlds.
- The modal quantifiers now range only over worlds that are accessible from the actual world.



The most common approach to modality in semantics

- The standard theory of modality in linguistics is based on Kratzer (1981).
- The accessibility relation is now recast as a set of propositions, the “modal base”.
- Accessible worlds are ranked by another set of propositions, the “ordering source”.
- One can think of the modal base as a hard constraint and the ordering source as a soft constraint.
- The modal quantifiers now range only over the top-ranked worlds.
- In what follows we set aside the ordering source and use Kripke (1963) as a backdrop.

Lexical entries for modals

- We treat modals (*can*, *must*, etc.) similarly to negation: as functions from sets of events to sets of events.
- Events in the output of modals are deontic events (events of having to), epistemic events (events of believing), etc.

The entry for deontic *must*, with a meaning postulate

$$(15) \quad \llbracket \text{must} \rrbracket \stackrel{\text{def}}{=} \lambda P \lambda e. e \in \text{must}(P)$$

$$(16) \quad \forall w \forall P.$$

$$\exists e. \text{occursIn}(e, w). e \in \text{must}(P) \leftrightarrow$$

$$\forall w' \in R_{\text{deont}}(w). \exists e'. \text{occursIn}(e', w'). e' \in P$$



How simple modal sentences are derived

- (17)
- Brutus must stab Caesar.
 - $\llbracket \text{must} \rrbracket (\llbracket \text{ag}' \rrbracket (\llbracket \text{th}' \rrbracket \llbracket \text{stab} \rrbracket) \llbracket \text{Caesar} \rrbracket) \llbracket \text{Brutus} \rrbracket$
 - $\lambda e. e \in \text{must}(\lambda e'. \text{ag}(e') = \text{Brutus} \wedge \text{stab}(e') \wedge \text{th}(e') = \text{Caesar})$

- Moving from standard Kripke semantics for modal logic to Kratzer (1981)'s theory is entirely possible and orthogonal to most questions related to events or negation.



Natural interaction between negation and modals

- Negation and modals are here all interpreted as functions from sets of events to sets of events.
- Consequence: They can freely scope over each other (if syntax allows them to do so).

- (18)
- Calpurnia might not convince Caesar.
 - $\llbracket \text{might} \rrbracket (\llbracket \text{not} \rrbracket \llbracket \text{Calpurnia convinces Caesar} \rrbracket)$
 - $\lambda e. e \in \text{might}(\text{Neg}(\lambda e'. \text{ag}(e') = \text{Calpurnia} \wedge \text{convince}(e') \wedge \text{th}(e') = \text{Caesar}))$
- (19)
- Calpurnia cannot convince Caesar.
 - $\llbracket \text{not} \rrbracket (\llbracket \text{can} \rrbracket \llbracket \text{Calpurnia convinces Caesar} \rrbracket)$
 - $\lambda e. e \in \text{Neg}(\text{can}(\lambda e'. \text{ag}(e') = \text{Calpurnia} \wedge \text{convince}(e') \wedge \text{th}(e') = \text{Caesar}))$



Assumptions about attitude verbs

- We model attitude verbs (e.g., *believe*, *know*) with two-place predicates whose arguments are:
 - ① an event, the cognitive state corresponding to the attitude in question;
 - ② a set of events, the proposition towards which this attitude holds.
- Each kind of attitude comes with an accessibility relation; e.g., $R_{believe}(w, x)$ is the set of worlds compatible with x 's beliefs in w (Hintikka 1962).

Attitude verbs: the example of *believe*

The entry for *believe*, with a meaning postulate

$$(20) \quad \llbracket \text{believe} \rrbracket \stackrel{\text{def}}{=} \lambda P \lambda e. \text{believe}(e) \wedge P \in \text{content}(e)$$

$$(21) \quad \forall w \forall x \forall P.$$

$$\exists e. \text{occursIn}(e, w) \wedge \text{believe}(e) \wedge \text{exp}(e) = x \wedge P \in \text{content}(e)$$

$$\rightarrow \forall w' \in R_{\text{believe}}(w, x). \exists e'. \text{occursIn}(e', w') \wedge e' \in P$$



We can model attitudes about (non)negative propositions

- (22) a. Jane believes it is raining.
 b. $\llbracket[\text{exp}']\rrbracket(\llbracket\text{believe}\rrbracket\llbracket\text{rain}\rrbracket)\llbracket\text{Jane}\rrbracket$
 c. $\lambda e. \text{exp}(e) = \text{Jane} \wedge \text{believe}(e) \wedge \text{rain} \in \text{content}(e)$
- (23) a. Jane believes it is not raining.
 b. $\llbracket[\text{exp}']\rrbracket(\llbracket\text{believe}\rrbracket(\llbracket\text{not}\rrbracket\llbracket\text{rain}\rrbracket))\llbracket\text{Jane}\rrbracket$
 c. $\lambda e. \text{exp}(e) = \text{Jane} \wedge \text{believe}(e) \wedge$
 $\text{Neg}(\text{rain}) \in \text{content}(e)$



Day 3: Summary

- *Neg* is a function from sets of events to sets of events.
- The Principle of Negation: P contains an actual event iff $Neg(P)$ doesn't.
- Given this assumption, the desirable linguistic properties of negation follow.
- The negative events obtained through *Neg* can be used to model negative perception reports.
- The Principle of Negation can be modalized to a possible worlds setting in which negation interacts as expected with modals and attitude verbs.
- So, far the Principle of Negation is a stipulation.



- ▶ Barker, Chris & Chung-chieh Shan. 2014. *Continuations and natural language*. Oxford, UK: Oxford University Press.
- ▶ Bentham, Jeremy. 1789. *An introduction to the principles of morals and legislation*. London, Great Britain: T. Payne & Son.
- ▶ Bernard, Timothée & Lucas Champollion. 2018. Negative events in compositional semantics. *28th Semantics and Linguistic Theory Conference (SALT 28)* 28. 512–532.
<https://doi.org/10.3765/salt.v28i0.4429>.
- ▶ Champollion, Lucas. 2015. The interaction of compositional semantics and event semantics. *Linguistics and Philosophy* 38(1). 31–66.
<https://doi.org/10.1007/s10988-014-9162-8>.
- ▶ Hintikka, Jaakko. 1962. *Knowledge and belief - an introduction to the logic of the two notions*. (Contemporary Philosophy). Ithaca, NY: Cornell University Press.
- ▶ Jacobson, Pauline. 2012. Direct compositionality. In Wolfram Hinzen, Edouard Machery & Markus Werning (eds.), *Oxford handbook of compositionality*, chap. 5, 109–128.



Oxford University Press. <https://doi.org/10.1093/oxfordhb/9780199541072.013.0005>.

[//doi.org/10.1093/oxfordhb/9780199541072.013.0005](https://doi.org/10.1093/oxfordhb/9780199541072.013.0005).

- ▶ Koopman, Hilda & Dominique Sportiche. 1991. The position of subjects. *Lingua* 85(2-3). 211–258.
[https://doi.org/10.1016/0024-3841\(91\)90022-w](https://doi.org/10.1016/0024-3841(91)90022-w).
- ▶ Kratzer, Angelika. 1981. The notional category of modality. In Hans-Jürgen Eikmeyer & Hannes Rieser (eds.), *Words, worlds, and contexts: new approaches in word semantics*, vol. 6 (Research in text theory), 38–74. Berlin, Germany: de Gruyter.
<https://doi.org/10.1002/9780470758335.ch12>.
- ▶ Krifka, Manfred. 1989. Nominal reference, temporal constitution and quantification in event semantics. In Renate Bartsch, Johan van Benthem & Peter van Emde Boas (eds.), *Semantics and contextual expression*, 75–115. Dordrecht, Netherlands: Foris. <http://amor.cms.hu-berlin.de/~h2816i3x/Publications/Krifka%201989%20Nominal%20Reference.pdf>.



- ▶ Kripke, Saul A. 1959. A completeness theorem in modal logic. *Journal of Symbolic Logic* 24(1). 1–14. <https://doi.org/10.2307/2964568>.
- ▶ Kripke, Saul A. 1963. Semantical analysis of modal logic I: normal modal propositional calculi. *Zeitschrift für mathematische Logik und Grundlagen der Mathematik* 9(5-6). 67–96. <https://doi.org/10.1002/malq.19630090502>.
- ▶ Lewis, David K. 1986. *On the plurality of worlds*. Basil Blackwell.
- ▶ Mossel, Benjamin. 2009. Negative actions. *Philosophia* 37(2). 307–333. <https://doi.org/10.1007/s11406-008-9163-3>.
- ▶ Muskens, Reinhard. 2007. Intensional models for the theory of types. *Journal of Symbolic Logic* 72(1). 98–118. <https://doi.org/10.2178/jsl/1174668386>.
- ▶ Tian, Ye & Richard Breheny. 2019. Negation. In *The oxford handbook of experimental semantics and pragmatics*, 194–207. Oxford University Press. <https://doi.org/10.1093/oxfordhb/9780198791768.013.29>.