Post-suppositions and uninterpretable questions*

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Abstract. For a sentence like *exactly three boys are between 5 feet 10 inches and 6 feet tall*, why cannot we abstract the height information out and raise a corresponding degree question like #*How tall are exactly three boys*? Inspired by the ideas that (i) there is a connection between *wh*-questions (e.g., *who did Mary kiss*) and definite descriptions (e.g., *the people that Mary kissed*) and (ii) definite descriptions and modified numerals (e.g., *exactly three boys*) bring post-suppositions (i.e., delayed evaluations that lead to relative definiteness, Brasoveanu 2013, Bumford 2017), I propose that when different elements that bring post-suppositions are present, a potential conflict arises in computing relative definiteness, leading to uninterpretability.

Keywords: Dynamic semantics · Post-suppositions · *Wh*-questions · Degree questions · Modified numerals · Cumulative reading · Definiteness · Weak island effects · Intervention effects

1 Introduction

This paper aims to explain the unacceptability of sentences like (2b): a **constituent question** containing a **modified numeral**.

(1) a. Brienne is <u>between 5′10″ and 6′</u> tall.
b. How tall is Brienne?

(2) a. Exactly three boys are <u>between 5′10″ and 6′</u> tall.
b. #How tall are exactly three boys?

For a sentence like (1a), we can naturally abstract the height information (the underlined part) out and raise a corresponding degree question on Brienne's height (see (1b)). Intriguingly, in contrast to (1), for a sentence like (2a), which contains a modified numeral (here *exactly three boys*), abstracting the height information out to form a corresponding degree question does not work, yielding an intuitively unacceptable sentence (see (2b)).

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The uninterpretability of constituent questions like (2b) does not seem like an entirely new observation. Similar unacceptable question phenomena have been reported in the literature on **intervention effects** or **weak island effects** (see, e.g., Szabolcsi and Zwarts 1993, Szabolcsi 2006, Rullmann 1995, Honcoop 1996, Beck 1996, 2006, Fox and Hackl 2007, Abrusán 2014).

As illustrated by the contrast between (3a) and (3b) (i.e., *wh*-in-situ vs. *wh*-movement), **intervention effects** arise when an intervener (here the negation expression *koi nahiiN*) precedes a *wh*-word (here *kyaa*) in a *wh*-question. This kind of intervention effects are often attested in *wh*-in-situ languages (e.g., Hindi, Korean). Cross-linguistically, typical interveners include, but are not limited to, focus particles and downward entailing (DE) quantifiers (e.g., *no*, *few*, *at most*).

(3) **Intervention effects**: examples from Beck (2006)

a. ?? koi **nahiiN** kyaa paRhaa
anyone not what read-Perf.M
Intended: 'What did no one read?' (Hindi: (12a) in Beck 2006)

b. kyaa koi **nahiiN** paRhaa
what anyone not read-Perf.M
'What did no one read?' (Hindi: (12b) in Beck 2006)

Islands refer to domains which prevent the displacement of items contained within them, and **weak islands** are those that are only closed for some kinds of items, but not all kinds of items (see Szabolcsi 2006, Abrusán 2014). As illustrated by (4), negation words or DE quantifiers create **weak islands effects** in the formation of a degree question (see (4a) and (4c)), *how-many* question (see (4b)), or manner question (see (4d)). In contrast, negation words or DE quantifiers do not create islands for the displacement of items like *which book* (see (5)). Elements that create weak island effects are also not limited to negation operators or DE quantifiers.

(4) **Weak island effects**: examples from Abrusán (2014)

a. #How tall isn**'t** John? (§3.4, (32a))
b. ??How many children does **none of these women** have? (§5.3, (19))
c. #How far did **few girls** jump? (§5.3, (24c))
d. #How did **at most 3 girls** behave? (§5.3, (24e)

(5) a. Which book have**n't** you read? (Abrusán 2014: §1.1, (3))
b. Which book did { **no one** / **few girls** / **at most 3 girls** } read?

Within the existing literature, there are already a variety of proposals on intervention effects or weak island effects, sometimes with different empirical coverages. The pattern 'modified numeral + degree question' (see (2b)) seems relevant, but it has not been much studied as a core piece of data. In this paper, I propose to start with the special property of modified numerals that they bring post-suppositions (Brasoveanu 2013) and explore how far this new perspective can advance our understanding of sentences like (2b) as well as empirical data related to intervention effects or weak island effects.

In a nutshell, I adopt and develop existing ideas in the literature on *wh*-questions: there is a connection between the interpretation of *wh*-questions (e.g., *who did Mary kiss*) and definite descriptions (e.g., *the people that Mary kissed*). Then given that definite descriptions and modified numerals are both elements that bring **post-suppositions** (see Brasoveanu 2013, Bumford 2017), i.e., delayed evaluations that result in a deterministic update with relative definiteness, the presence of both these kinds of items in the same sentence potentially yield a conflict with regard to relative definiteness, leading to uninterpretability. I will also address how this potential uninterpretability can be circumvented.

In the following, Section 2 first presents how modified numerals and definite descriptions contribute post-suppositions (Brasoveanu 2013, Bumford 2017). Section 3 argues for a parallel analysis for interpretable *wh*-questions and modified numerals / definite descriptions. Based on this, Section 4 accounts for the uninterpretability of the core data under discussion (see (2b)). Section 5 compares the current proposal with existing approaches developed within the literature on intervention effects and weak island effects and shows advantages of the current proposal. Section 6 concludes.

2 Post-suppositions

2.1 Brasoveanu (2013): Modified numerals as post-suppositions

Modified numerals bring **post-suppositions**: their numerical information is attached to a **non-local, sentence-level maximization** (Brasoveanu 2013).

The maximization effect of modified numerals has been widely reported in the literature (see, e.g., Szabolcsi 1997, Krifka 1999, de Swart 1999, Umbach 2006, Zhang 2018). As illustrated by the contrast in (6), compared to bare numerals like *two dogs*, modified numerals like *at least two dogs* exhibit maximality, as evidenced by the infelicitous continuation *perhaps she fed more*. In other words, while the semantic contribution of *two* in (6a) is **existential**, *at least two* in (6b) conveys the quantity information of the **totality** of dogs fed by Mary.

(6) a. Mary fed <u>two dogs</u>. They are cute. Perhaps she fed <u>more</u>.
b. Mary fed <u>at least two dogs</u>. They are cute. #Perhaps she fed <u>more</u>.

The non-localness of this maximization is best reflected in the **cumulative reading** of sentences like (7). (7) has a distributive reading (7a) and a cumulative reading (7b), and we focus on the cumulative reading (7b) here. (For notation simplicity, cumulative closure is assumed for lexical relations when needed.)

(7) Exactly 3 boys saw exactly 5 movies.

a. **Distributive reading**:

$$\underbrace{\sigma x[\mathrm{BOY}(x) \wedge \delta x[\underbrace{\sigma y[\mathrm{MOVIE}(y) \wedge \mathrm{SEE}(x,y)]}_{\text{the mereologically maximal } y} \wedge |y| = 5]]}_{\text{the mereologically maximal } x} \wedge |x| = 3$$

(σ: maximality operator; δ: distributivity operator.)

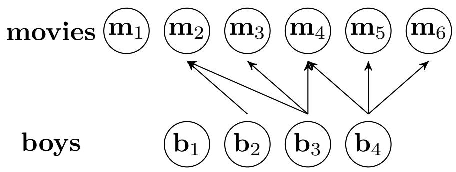


Fig. 1. The **cumulative** reading of *exactly 3 boys saw exactly 5 movies* is **true** under this scenario.

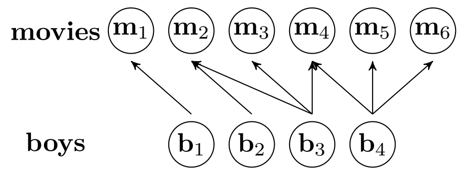


Fig. 2. The **cumulative** reading of *exactly 3 boys saw exactly 5 movies* is **false** under this scenario.

(There are in total three boys, and for each atomic boy, there are in total 5 movies such that he saw.)

b. **Cumulative reading**:

$$\underbrace{\sigma x \sigma y[\text{BOY}(x) \wedge \text{MOVIE}(y) \wedge \text{SEE}(x, y)]}_{\text{the mereologically maximal } x \text{ and } y} \wedge |y| = 5 \wedge |x| = 3$$

(The cardinality of all the boys who saw any movies is 3, and the cardinality of all movies seen by any boys is 5.)

True under the context of Fig. 1, **false** under the context of Fig. 2.

According to the intuition of native speakers, sentence (7) is true under the scenario described by Fig. 1, but false under the scenario described by Fig. 2.

It is worth noting that if we adopt the analysis shown in (8), then sentence (7) should be judged true under the scenario of Fig. 2: there are two such boy-sum witnesses, namely $b_2 \oplus b_3 \oplus b_4$ and $b_1 \oplus b_2 \oplus b_4$, and for each of these two boy-sums, (i) their cardinality is 3, and (ii) the maximal sum of movies seen between them has the cardinality of 5 ($m_2 \oplus m_3 \oplus m_4 \oplus m_5 \oplus m_6$ and $m_1 \oplus m_2 \oplus m_4 \oplus m_5 \oplus m_6$, respectively). There are no larger boy-sums such that they saw in total 5 movies between them. Thus the contrast of intuition (i.e., (7) is true under Fig. 1, but false under Fig. 2) means that (i) the genuine cumulative reading shown in (7b) is distinct from the unattested pseudo-cumulative reading shown in (8), and (ii) there is no scope-taking between the two modified numerals in (7), *exactly 3 boys* and *exactly 5 movies* (see Brasoveanu 2013, Charlow 2017).

(8) **Unattested pseudo-cumulative reading** of (7): Not attested!

$$\underbrace{\sigma x[\text{BOY}(x) \wedge \underbrace{\sigma y[\text{MOVIE}(y) \wedge \text{SEE}(x, y)]}_{\text{the mereologically maximal } y} \wedge |y| = 5]}_{\text{the mereologically maximal } x} \wedge |x| = 3$$

(The maximal plural individual x satisfying the restrictions (i.e., atomic members of x are boys, each atomic boy saw some movies, and the boys in x saw a total of 5 movies between them) has the cardinality of 3.)

True under the context of Fig. 2 (see $b_2 \oplus b_3 \oplus b_4$ and $b_1 \oplus b_2 \oplus b_4$)!

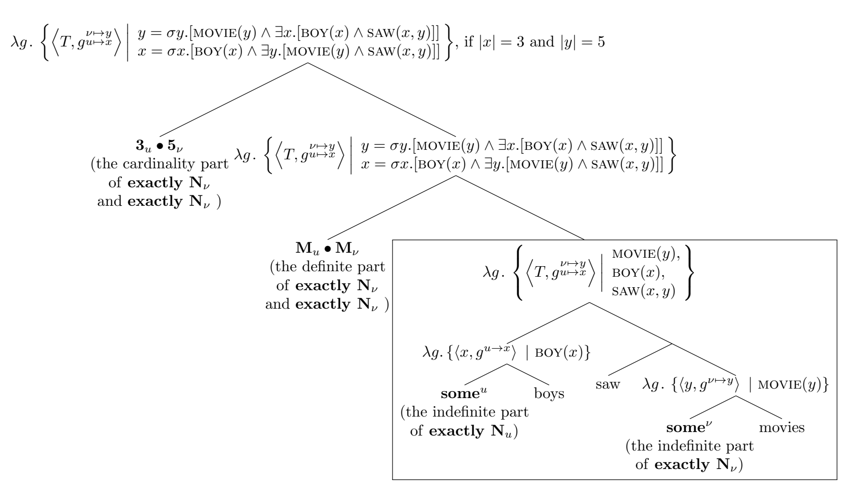
As already pointed out by Krifka (1999), the semantic contribution of both modified numerals in (7), *exactly 3 boys* and *exactly 5 movies*, should take place simultaneously, at the sentential level, beyond their hosting DPs themselves:

> The problem cases discussed here clearly require a representation in which NPs are not scoped with respect to each other. Rather, they ask for an interpretation strategy in which all the NPs in a sentence are somehow interpreted on a par. (Krifka 1999)

Given Fig. 1, in interpreting (7), we count the cardinalities of all boys who saw any movies and all movies seen by any boys, instead of the total cardinalities of all boys and movies in the domain (here in Fig. 1, it's 4 boys and 6 movies). Therefore, the application of maximality operators is subject to more restrictions (here in our context, not just boys, but boy who saw movies; not just movies, but movies seen by boys), leading to a **relativized maximization** effect.

A compositional analysis à la Bumford (2017) is sketched in (9). Within dynamic semantics, meaning derivation is considered a series of updates from an information state to another. The semantic contribution of modified numerals is split. They first introduce discourse referents (drefs), x and y. Restrictions like $\text{MOVIE}(y)$, $\text{BOY}(x)$, and $\text{SAW}(x, y)$ are added onto these drefs. Eventually, it is after all these restrictions are applied that maximality and cardinality tests, $\mathbf{M}_u/\mathbf{M}_\nu/\mathbf{3}_u/\mathbf{5}_\nu$, as delayed evaluations, i.e., post-suppositional tests, come into force. $\mathbf{M}_u$ and $\mathbf{M}_\nu$ check whether u and ν are assigned the mereologically maximal plural individuals x and y that satisfy all the restrictions, and $\mathbf{3}_u$ and $\mathbf{5}_\nu$ check whether the cardinalities of maximal x and y are 3 and 5 respectively.

(9)



(Here $\mathbf{M}_\nu \overset{\text{def}}{=} \lambda m.\lambda g.\ \{\langle \alpha, h\rangle \in m(g) \mid \neg\exists\langle \beta, h'\rangle \in m(g)\,.\, h(\nu) \sqsubset h'(\nu)\}$.[1] $\bullet$ is used to simplify the notation in bundling two tests together.)

2.2 Bumford (2017): Definite descriptions as post-suppositions

Not only modified numerals bring post-suppositions, Bumford (2017)'s analysis for Haddock (1987)'s example (see Fig. 3) shows that **definite descriptions**

[1] The type of $\mathbf{M}_\nu$ is $(g \to \{\langle \alpha, g\rangle\}) \to (g \to \{\langle \alpha, g\rangle\})$, with g meaning the type for assignment functions, and α standing for the type of the denotation corresponding to the constituent. The usual notation for types $\langle \alpha, \beta\rangle$ is written as $\alpha \to \beta$ here.

like *the rabbit in the hat* also involve post-suppositions, i.e., delayed tests that lead to **relativized definiteness** effects.

Under the scenario shown in Fig. 3, there are multiple rabbits (R1, R2, R3) and multiple hats (H1, H2). Thus, the uniqueness requirement of *the rabbit* or *the hat* cannot be met in an absolute sense. However, *the rabbit in the hat* is still perfectly felicitous in this context.

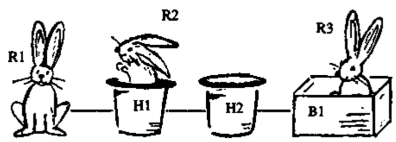


Fig. 3. The rabbit in the hat

Bumford (2017) argues that Haddock (1987)'s definite description is exactly parallel to the case of *exactly 3 boys saw exactly 5 movies*, where maximality tests are applied on drefs satisfying all these restrictions including $\textsc{movie}(y)$, $\textsc{boy}(x)$, and $\textsc{saw}(x, y)$, resulting in relativized maximization.

As shown in (10), under the given scenario in Fig. 3, for *the rabbit in the hat*, uniqueness tests $\mathbf{1}_\nu/\mathbf{1}_u$ are also applied in a delayed, non-local manner, after the introduction of all the drefs (i.e., x and y) and restrictions (i.e., $\textsc{hat}(y)$, $\textsc{rabbit}(x)$, and $\textsc{in}(x, y)$). More specifically, the test $\mathbf{1}_\nu$ first checks whether there is a unique hat in the context such that only this hat contains any rabbits. Then the test $\mathbf{1}_u$ checks whether the rabbit contained in the above-mentioned unique rabbit-containing hat is unique.

(10) The^u rabbit in the$^\nu$ hat $\rightsquigarrow$ rabbit R2 in Figure 3

$$\lambda g.\left\{\left\langle x, g^{\nu\mapsto y}_{u\mapsto x}\right\rangle \,\middle|\, \begin{array}{l} y = \iota y[\textsc{hat}(y) \wedge \exists x[\textsc{rabbit}(x) \wedge \textsc{in}(x, y)]], \\ x = \iota x[\textsc{rabbit}(x) \wedge \textsc{in}(x, y)] \end{array}\right\}$$

$\mathbf{1}_u$ (the definite part of $\mathbf{the}_u$)

$$\lambda g.\left\{\left\langle x, g^{\nu\mapsto y}_{u\mapsto x}\right\rangle \,\middle|\, \begin{array}{l} \textsc{rabbit}(x), \\ \textsc{in}(x, y), \\ y = \iota y[\textsc{hat}(y) \wedge \exists x[\textsc{rabbit}(x) \wedge \textsc{in}(x, y)]] \end{array}\right\}$$

$\mathbf{1}_\nu$ (the definite part of $\mathbf{the}_\nu$)

$$\lambda g.\left\{\left\langle x, g^{\nu\mapsto y}_{u\mapsto x}\right\rangle \,\middle|\, \begin{array}{l} \textsc{hat}(y), \\ \textsc{rabbit}(x), \\ \textsc{in}(x, y) \end{array}\right\}$$

some^u (the indefinite part of $\mathbf{the}_u$)

rabbit

in $\lambda g.\,\{\langle y, g^{\nu\mapsto y}\rangle \mid \textsc{hat}(y)\}$

some$^\nu$ (the indefinite part of $\mathbf{the}_\nu$)

hat

The upshot is that the semantic contribution of modified numerals and definite descriptions can be considered split, (i) introducing drefs at an earlier stage, and (ii) then at a later stage, imposing delayed, post-suppositional tests and leading to a relativized maximization/definiteness effect.

3 A post-suppositional view on *wh*-questions

A post-suppositional view on the interpretability of *wh*-questions can be developed based on the following existing insights.

First, *wh*-expressions are parallel to indefinites (as well as other expressions like proper names, definite descriptions, modified numerals, etc.) in introducing drefs, as evidenced by their parallel behavior in supporting cross-sentential anaphora (see, e.g., Comorovski 1996). As illustrated in (11), the pronoun *he* refers back to the dref introduced by *someone/Kevin/the boy/exactly one boy/who* in all these cases. For (11e), the pronoun *he* can also be considered referring back to the answer to the question *who came?* (see Li 2020).

(11)
- a. Someone^0 came. I heard that he$_0$ coughed a few times.
- b. Kevin^0 came. I heard that he$_0$ coughed a few times.
- c. The^0 boy came. I heard that he$_0$ coughed a few times.
- d. Exactly one^0 boy came. I heard that he$_0$ coughed a few times.
- e. Who^0 came? I heard that he$_0$ coughed a few times.

Second, according to Dayal (1996)'s Maximal Informativity Presupposition, a question presupposes the existence of a maximally informative true answer. This idea can be combined with the Hamblin-Karttunen semantics of questions to reason about the (non-)deterministic updates of propositions.

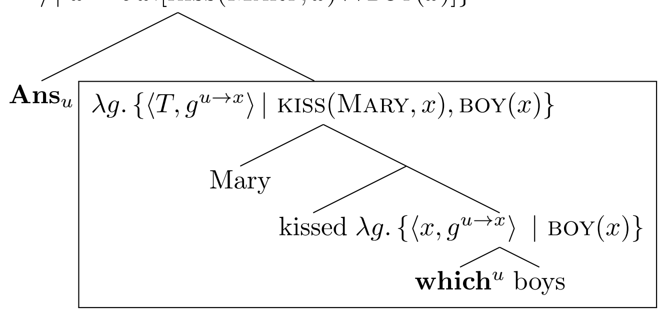
According to Hamblin (1973), a *wh*-question denotes a set of propositions, which are **possible propositional answers** to the question. Then according to Karttunen (1977), a *wh*-question denotes the set of its **true propositional answers**. As illustrated in (12), we can use an answerhood operator to bridge the set of possible answers and the maximally informative true answer. Essentially, this answerhood operator presupposes the existence of a maximally informative true answer p and picks out this p from Q, a set of propositions. What this answerhood operator does is reminiscent of the semantics of definite determiner *the*, which, when defined, contributes definiteness by picking out the unique (e.g., *the dog*) or the mereologically maximal (e.g., *the dogs*) item (see (10)).

(12) $$\textsc{Ans}(Q)(w) = \frac{\exists p[w \in p \in Q \wedge \forall q[w \in q \in Q \to p \subseteq q]].}{\iota p[w \in p \in Q \wedge \forall q[w \in q \in Q \to p \subseteq q]]}$$ Dayal (1996)

With the above two ideas combined, an interpretable *wh*-question can be analyzed in the same dynamic semantics framework as modified numerals and definite descriptions are analyzed in Section 2.

As illustrated in (13) (*wh*-movement and head movement are omitted in the tree), *who* works like *someone* or the indefinite component of *the* in introducing a dref in a non-deterministic way. After all relevant restrictions are added (here $\textsc{boy}(x)$, $\textsc{kiss}(\textsc{Mary}, x)$), the silent operator, $\mathbf{Ans}_u$, plays the same role as a maximality operator, bringing a post-suppositional evaluation and checking in the output information state whether u is assigned the mereologically maximal plural individual x that satisfies $\textsc{boy}(x)$ and $\textsc{kiss}(\textsc{Mary}, x)$. Thus the application of $\mathbf{Ans}_u$ leads to a deterministic update. As far as a *wh*-question satisfies Dayal (1996)'s Maximal Informativity Presupposition and is thus interpretable, the derivation involving the application of **Ans** should not fail.[2]

(13) $\lambda g.\, \{\langle T, g^{u\to x}\rangle \mid x = \sigma x.[\textsc{kiss}(\textsc{Mary}, x) \wedge \textsc{boy}(x)]\}$

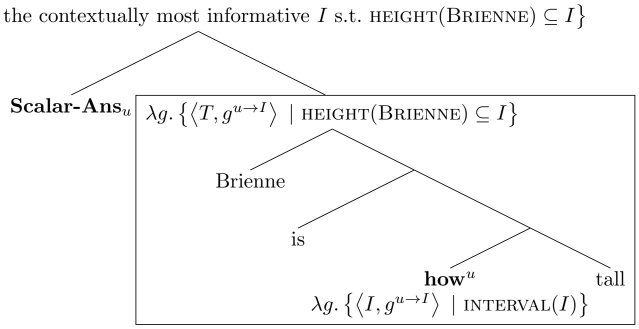


Here $\mathbf{Ans}_u \overset{\text{def}}{=} \lambda m.\lambda g.\ \{\langle \alpha, h\rangle \in m(g) \mid \neg\exists\langle \beta, h'\rangle \in m(g)\,.\, h(u) \sqsubset h'(u)\}$

A similar analysis can be developed for degree questions, with an answerhood operator, $\mathbf{Scalar\text{-}Ans}_u$, which is adjusted for a set of drefs that are scalar values.

(14) $[\![\text{tall}]\!]_{\langle dt,et\rangle} \overset{\text{def}}{=} \lambda I_{\langle dt\rangle} \lambda x.\textsc{height}(x) \subseteq I$ (Zhang and Ling 2021)

(15) $\lambda g.\, \{\langle T, g^{u\to I}\rangle \mid I = \text{the contextually most informative } I \text{ s.t. } \textsc{height}(\textsc{Brienne}) \subseteq I\}$



Here $\mathbf{Scalar\text{-}Ans}_u \overset{\text{def}}{=} \lambda m.\lambda g.\ \{\langle \alpha, h\rangle \in m(g) \mid \neg\exists\langle \beta, h'\rangle \in m(g)\,.\, h'(u) \subset h(u)\}$

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[2] In this short paper, I focus on the most basic data of *wh*-questions (e.g., *who did Mary kiss*) and degree questions (e.g., *how tall is Brienne*). I leave aside for future work cases like mention-some questions that can have multiple complete true answers (see (i)) or higher-order reading questions (see (ii) and Xiang 2021).

(i) Where can I buy an Italian newspaper?

(ii) Which books does John have to read?
The French novels or the Russian poems. The choice is up to him.

I adopt the notion of intervals to represent scalar values (see also Schwarzschild and Wilkinson 2002, Abrusán 2014, Zhang and Ling 2021, a.o.). An interval is a convex set of degrees, e.g., $\{d \mid 5'5'' < d \leq 7'1''\}$, which can also be written as $(5'5'', 7'1'']$. As illustrated in (14), a gradable adjective like *tall* relates an interval I and an atomic individual x, such that the height measurement of x falls within the interval I along a scale of height. For example, the meaning of *Brienne is between* $5'10''$ *and* $6'$ *tall* is analyzed as $\text{HEIGHT}(\text{BRIENNE}) \subseteq [5'10'', 6']$.

As illustrated in (15), I propose that during base generation, how^u non-deterministically introduces an interval dref, I.[3] After relevant restrictions are added (here $\text{HEIGHT}(\text{BRIENNE}) \subseteq I$), the application of $\textbf{Scalar-Ans}_u$ picks out the most informative interval from a set of possible intervals, leading to a deterministic update. Under an ideal context, where measurements don't involve any errors, this most informative interval would be a singleton set of degrees (i.e., the narrowest interval that entails all intervals satisfying relevant restrictions), containing the precise height measurement of Brienne (e.g., $[6'3'', 6'3'']$).

This post-suppositional view on the interpretability of *wh*-questions is also compatible with insights on (i) the cross-linguistic parallelism between *wh*-questions and *wh*-free relatives (Caponigro 2003, 2004, Chierchia and Caponigro 2013), and (ii) the categorial approach to *wh*-questions (see Hausser and Zaefferer 1979).

As illustrated in (16), *wh*-free relatives can be replaced by truth-conditionally equivalent DPs, and in most cases (except for the complement position of existential predicates in some languages, see Caponigro 2004), both *wh*-free relatives and their corresponding DPs exhibit maximality/definiteness.[4] Under the current post-suppositional analysis, the semantics of the free relative in (16a), ⟦what Adam cooked⟧, can be derived by applying the silent maximality operator $\textbf{Ans}_u$ to the meaning of the question *what did Adam cook?*, which yields the maximal sum of things, $\sigma x.[\text{COOK}(\text{ADAM}, x)]$, i.e., the meaning of the DP *the things Adam cooked* (see also Chierchia and Caponigro 2013 for a similar idea).[5]

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[3] (i) shows that *how* is parallel with other *wh*-expressions in introducing drefs and supporting cross-sentential anaphora. In (ia), *6′3″* is similar to definite descriptions or proper names (e.g., *Kevin* in (11b)) in introducing a definite scalar value so that *that* in the subsequent sentence refers back to it. Obviously, the parallelism between (ia) and (ib) is similar to that shown in (11).

(i) a. Brienne is $6'3''^{0}$ tall. It seems that Jaime is a bit shorter than that_0.
  b. How^0 tall is Brienne? It seems that Jaime is a bit shorter than that_0.

[4] *Wh*-free choices corresponding to mention-some *wh*-questions are also exceptions (see Chierchia and Caponigro 2013) and don't seem to exhibit maximality:

(i) Mary looked for who can help her.
  = Mary looked for someone that can help her.
  ≠ Mary looked for all the people that can help her.

[5] In addition to *wh*-free relatives, concealed questions also demonstrate the parallelism between definite DPs and *wh*-questions (see e.g., Nathan 2006):

(16) a. Jie tasted what^u Adam cooked. (example from Caponigro 2004)
b. Jie tasted [$_{\text{DP}}$ the^u things Adam cooked].

Within the categorial approach to *wh*-questions (Hausser and Zaefferer 1979), a *wh*-question denotes a function, which takes its short answer as argument to generate a (maximally informative) true proposition, as illustrated in (17).

(17) Categorial approach: $[\![\text{who did Mary kiss}]\!] = \lambda x.$ Mary kissed x
a. Short answer: Kate and Kevin.
b. Propositional answer: Mary kissed Kate and Kevin.

Under the current post-suppositional analysis, as illustrated in (18), this function $\lambda x.$*Mary kissed* x is considered a restriction on the dref introduced by the *wh*-expression, x. Then the short answer, here *Kate and Kevin*, can be considered similar to the cardinality tests in the case of the cumulative-reading sentence *exactly 3 boys saw exactly 5 movies*. The test $\mathbf{(kate{\oplus}Kevin)}_u$ is attached to the application of the maximality test $\mathbf{Ans}_u$, checking whether $\sigma x.\textsc{kiss}(\textsc{Mary}, x)$ is equivalent to the sum 'Kate⊕Kevin'. This amounts to turning a short answer into a corresponding propositional answer to a *wh*-question.

(18) $\lambda g.\{\langle T, g^{u\to x}\rangle \mid x = \sigma x.[\textsc{kiss}(\textsc{Mary}, x)]\}$, if $x =$ Kate $\oplus$ Kevin

$\mathbf{(Kate{\oplus}Kevin)}_u$ $\lambda g.\{\langle T, g^{u\to x}\rangle \mid x = \sigma x.[\textsc{kiss}(\textsc{Mary}, x)]\}$

$\mathbf{Ans}_u$ who did Mary kiss?
$\lambda g.\{\langle T, g^{u\to x}\rangle \mid \textsc{kiss}(\textsc{Mary}, x)\}$

Essentially, based on Dayal (1996)'s Maximal Informativity Presupposition, I propose that for an interpretable *wh*-question, (i) its *wh*-expression introduces a dref non-deterministically, and (ii) a delayed, post-supposition-like maximality operator can bring definiteness to this dref, leading to a deterministic update.

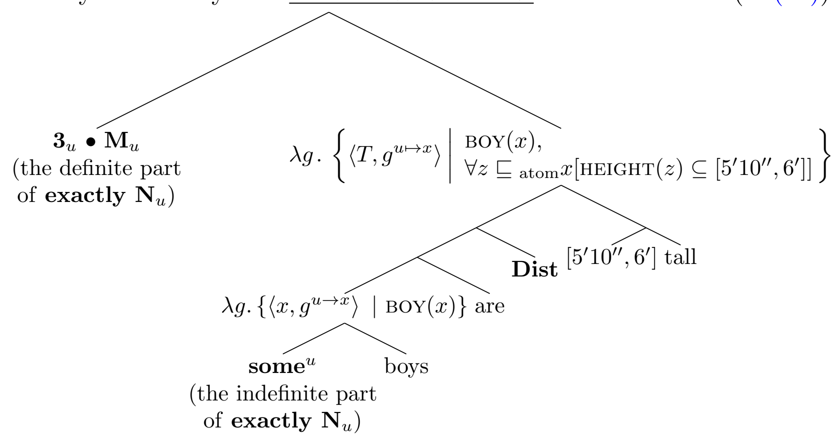
4 Accounting for uninterpretable questions

4.1 Interpreting a modified numeral in a matrix degree question

The interpretation of a declarative degree sentence containing a modified numeral (see (2a), repeated here as (19)) is straightforward. In this sentence, only *exactly three* brings post-suppositional tests. As shown in (19), as post-suppositional tests, $\mathbf{M}_u$ picks out the largest boy-sum x such that for each atomic boy within x, his height falls within the interval $[5'10'', 6']$, and $\mathbf{3}_u$ checks whether the cardinality of this boy-sum x is equal to 3.

(i) a. Jaime knows how tall Brienne is.
b. Jaime knows the height of Brienne.

(19) Exactly three^u boys are <u>between 5′10″ and 6′</u> tall.[6] (= (2a))

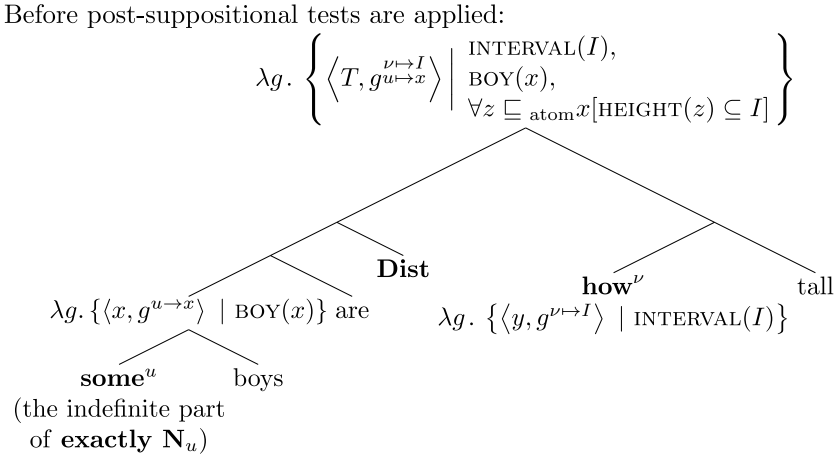


Then I turn to the core data under discussion, a degree question containing a modified numeral (repeated in (20)):

(20) #How$^\nu$ tall are exactly three^u boys? (= (2b))

According to the post-supposition-based analysis addressed in Sections 2 and 3, in sentence (20), both *wh*-expression *how* and modified numeral *exactly three* (*boys*) first introduce a dref, as show in (21):

(21) Before post-suppositional tests are applied:

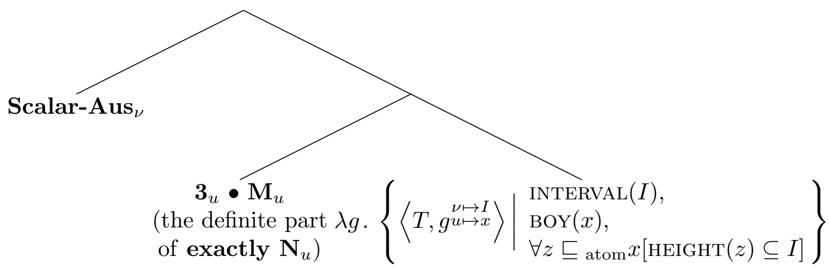


Once all the drefs are introduced and relevant restrictions are added, there are two potential derivation orders: either (i) as shown in (22), the maximality and cardinality tests of *exactly 3* are applied first, letting the deterministic update from **Scalar-Aus**$_\nu$ take place later, or (ii) as shown in (23), the deterministic

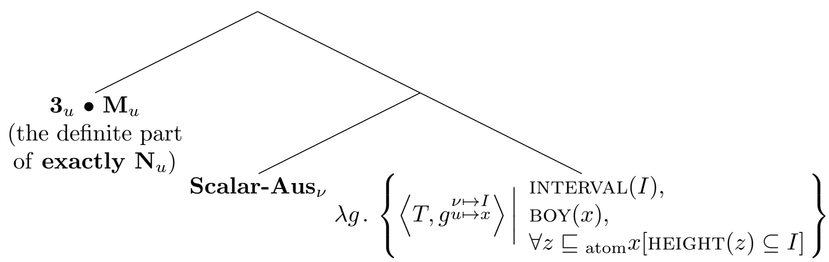
[6] Given that ⟦*tall*⟧ relates an interval and an atomic individual (see (14)), I assume a distributivity operator **Dist** ($\overset{\text{def}}{=} \lambda x.\lambda P_{\langle et\rangle}.\forall z \sqsubseteq_{\text{atom}} x[P(z)]$) here.

update from **Scalar-Aus**$_\nu$ happens first, letting the maximality and cardinality tests of *exactly 3* be checked later.

(22)



(23)



Suppose we adopt the possibility of (22), then $\mathbf{M}_u$ would select out the absolute largest boy-sum in the given context, and $\mathbf{3}_u$ would check whether the cardinality of this absolute largest boy-sum is 3. If the tests $\mathbf{M}_u$ and $\mathbf{3}_u$ don't fail, the application of **Scalar-Aus**$_\nu$ would eventually yield the most informative height interval such that (i) its lower bound is equivalent to the precise height of the shortest boy in the context, and (ii) its upper bound is equivalent to the precise height of the tallest boy in the context. However, given that such a question amounts to requesting the height information of the absolute largest boy-sum in the given context, speakers would use the question *how tall are the* (*three*) *boys* instead. In other words, *exactly three boys* would be ruled out in the competition with *the* (*three*) *boys*.

On the other hand, suppose we adopt the possibility of (23), then **Scalar-Aus**$_\nu$ would select out the absolute most informative height interval such that it includes the height of some boy(s). As far as boys are not of the same height, there cannot be a unique most informative height interval (e.g., suppose the heights of two boys are $[5'10'', 5'10'']$ and $[6', 6']$, respectively. Then there is no unique interval I such that I entails, i.e., is a subset of, both $[5'10'', 5'10'']$ and $[6', 6']$). Thus **Scalar-Aus**$_\nu$ would not fail only if all the boys are of the same height, and when **Scalar-Aus**$_\nu$ does not fail, $\mathbf{M}_u$ would also select out the absolute largest boy-sum in the given context. Obviously, such a question still amounts to requesting the height information of the absolute largest boy-sum in the given context, and *exactly three boys* would be ruled out in the competition with *the* (*three*) *boys*.

Overall, the interpretation of (20) would be problematic because both $\mathbf{M}_u$ and **Scalar-Aus**$_\nu$ need to be checked to result in relative definiteness, i.e., both wait to be applied as the last post-suppositional test in the derivation. Obviously, their requirements cannot be both satisfied, and the unacceptability of the whole sentence thus arises.

4.2 Interpreting a modified numeral in an embedded degree question

As illustrated by (24), in comparative sentences, their *than*-clause can be considered parallel to a degree question (see Fleisher 2020, Zhang 2020).

(24) Brienne is taller than Jaime is ~~tall~~.
$[\![$than Jaime is$]\!]$ $\rightsquigarrow$ addressing a degree question: *how tall is Jaime?*

According to Zhang and Ling (2021), a comparative sentence basically means that the scalar value associated with the subject minus the scalar value associated with the comparative standard results in a positive difference (i.e., an increase). As shown in (25), comparative morpheme *-er* is considered denoting a default positive difference, i.e., an increase. The *than*-clause, i.e., *than Jaime is ~~tall~~* in (24), denotes the short answer to the degree question *how tall is Jaime* and amounts to the most informative interval I' satisfying the restriction $\text{HEIGHT}(\text{JAIME}) \subseteq I'$, written as $\iota I'[\text{HEIGHT}(\text{JAIME}) \subseteq I']$ here. Eventually, as shown in (25d), this short answer to the degree question *how tall Jaime is* plays the role of comparative standard in the derivation of sentential meaning.

(25) a. $[\![\text{tall}]\!]_{\langle dt,et\rangle} \overset{\text{def}}{=} \lambda I_{\langle dt\rangle}\lambda x.\text{HEIGHT}(x) \subseteq I$ (= (14))

b. $[\![\text{-er}]\!] \overset{\text{def}}{=} (0, +\infty)$ $\rightsquigarrow$ a default positive difference
i.e., the most general positive interval that represents an increase
(With a presupposition of additivity: there is a contextually salient scalar value serving as the base of the increase)

c. Assuming a silent operator that performs comparison:
$\textbf{Minus} \overset{\text{def}}{=} \lambda I_{\text{STANDARD}}\lambda I_{\text{DIFFERENCE}}.\iota I[I - I_{\text{STANDARD}} = I_{\text{DIFFERENCE}}]$

d. $[\![(24)]\!] \Leftrightarrow \text{HEIGHT}(\text{BRIENNE}) \subseteq \iota I[I - I_{\text{STANDARD}} = I_{\text{DIFFERENCE}}]$
$\Leftrightarrow \text{HEIGHT}(\text{BRIENNE}) \subseteq$
$\iota I[I - \iota I'[\text{HEIGHT}(\text{JAIME}) \subseteq I'] = (0, +\infty)]$

Intriguingly, although the matrix degree question #*how tall are exactly three boys* (see (2b)/(20)) is uninterpretable, comparative sentence (26), which contains a *than*-clause corresponding to the problematic degree question, is good.

(26) Mary is taller than$^{\nu}$ exactly three^{u} boys are ~~tall~~.

I have proposed an analysis for (26) in Zhang (2020). As mentioned in Section 4.1, for the matrix degree question #*how*$^{\nu}$ *tall are exactly three*^{u} *boys*, $\textbf{Scalar-Aus}_{\nu}$ and $\textbf{M}_{u}$, both tests that bring relative definiteness, require to be applied as the last test, and both requirements cannot be satisfied at the same time.

For (26), however, information outside the *than*-clause contributes to settle the deterministic update of ν independent of the update of u.

As mentioned above (see (25c)), the semantics of a comparative addresses the relation among three definite scalar values: (i) the scalar value associated with the subject, which serves as the **minuend**; (ii) the scalar value associated with the *than*-clause, which serves as the **subtrahend**; and (iii) the difference between the minuend and the subtrahend. Given the subtraction relation between these three definite values (see (25c)), we can use two of the three values to reason about the third one.

Thus, for (26), given the minuend (i.e., $\text{HEIGHT}(\text{MARY})$) and the difference (i.e., $(0, +\infty)$), the deterministic update of ν, i.e., the value of the subtrahend, can be settled first: it is the largest interval below $\text{HEIGHT}(\text{MARY})$, which can be written as $(-\infty$, the precise height measurement of Mary$)$.[7] Then similar to the case of (19), $\mathbf{M}_u$ is applied to pick out the largest boy-sum x such that $\forall z \sqsubseteq_{\text{atom}} x[\text{HEIGHT}(z) \subseteq (-\infty,$ the precise height measurement of Mary$)]$, and $\mathbf{3}_\nu$ is applied to check whether the cardinality of x is 3. Therefore, through the derivation of the meaning of (26), the relative maximality of *exactly three boys* is achieved.

It is worth noting that for this x, the interval $(-\infty$, the precise height of Mary$)$ can still be the most informative short answer to the degree question *how tall is* x (i.e., with $\textbf{Scalar-Aux}_\nu$ applied to *how tall is* x). Imagine an extreme case: one of the boys in x is just slightly shorter than Mary is, and another one of the boys in x is extremely short. Then the application of $\textbf{Scalar-Aux}_\nu$ would lead to exactly this interval $(-\infty$, the precise height of Mary$)$. In other words, the above analysis of (26) is not incompatible with the view that the *than*-clause addresses the short answer to a corresponding degree question. It's just that in this case, the information of this short answer (i.e., $(-\infty$, the precise height of Mary$)$) is derived first, and then this definite interval is made use of in checking the post-suppositional requirements of the modified numeral here (i.e., *exactly three boys*).

5 Discussion

In Section 4, I have shown that the uninterpretability of the pattern 'modified numeral + degree question' is essentially due to a conflict between different items that bring post-suppositions (i.e., both need to be applied as the last test to result in relative definiteness) and how this conflict can be circumvented (i.e., additional information is available to resolve the definiteness of one of the items and thus remove the conflict). Here I compare the current proposal with three existing lines of research on intervention effects or weak island effects.

5.1 Intervention effects: Beck (2006) and Li and Law (2016)

Both Beck (2006) and Li and Law (2016) address intervention effects related to focus, but their empirical coverages are different. As shown in (27) and (28), their analyses target different problematic configurations.

[7] In our actual world, the height of a person cannot be a negative value. This should be considered a physical constraint in our world knowledge, not a linguistic constraint. Linguistically, we can imagine characters with a negative height in fantasy works.

(27) The problematic configuration analyzed by Beck (2006):
?* [Q...[focus-sensitive operator [$_{\mathrm{YP}}$... WH...]]]

(28) The problematic configuration analyzed by Li and Law (2016):
?* [...focus-sensitive operator [focus alternatives...ordinary alternatives...]]
(or ?* [...focus-sensitive operator [XP_F...WH...]])

Beck (2006) is based on Rooth (1985)'s focus semantics. A *wh*-expression has its focus semantic value (i.e., a set of alternatives), but lacks an ordinary semantic. A Q operator is needed to turn the focus semantic value of a *wh*-expression into an ordinary semantic value. However, in the problematic configuration in (27), (i) a focus-sensitive operator blocks the association between the Q operator and the *wh*-expression, and moreover, (ii) the focus-sensitive operator needs to be applied on an item that has both a focus semantic value and an ordinary value, which the *wh*-expression lacks. Thus the derivation crashes.

According to Li and Law (2016), given that both XP_F and WH introduce alternatives, embedding WH within the scope of XP_F makes $[\![[\mathrm{XP}_F ...\mathrm{WH}...]]\!]$ a set of sets of alternatives, which becomes an illicit input for the focus-sensitive operator, resulting in a derivation crash.

Both Beck (2006) and Li and Law (2016) explain the uninterpretability of intervention patterns as derivation crash. Different from these approaches, the current account for the uninterpretability of the pattern 'modified numeral + degree question' is based on a potential failure of achieving relative definiteness.

As shown in Section 4, for the pattern 'modified numeral + degree question', the potential failure of achieving relative definiteness exists for matrix degree questions, but not for embedded degree questions (i.e., *than*-clauses of comparatives). Thus empirically, the current account works better than existing approaches that explain uninterpretability as derivation crash.

It is worth investigating whether/how the current approach can be further extended to cover the data of intervention effects. As shown in (29), the matrix *wh*-question (29a) is problematic. Indeed, it has a problematic configuration in both the theories of Beck (2006) and Li and Law (2016). However, once this configuration is embedded in a *wh*-conditional, as shown in (29b), there is no longer uninterpretability. The acceptability contrast between (29a) and (29b) suggests that the problem of (29a) might not be due to a derivation crash.

(29) a. * <u>zhǐyǒu</u> Mary$_F$ dú-le shénme shū?
only Mary read-PFV what book
Intended: 'What book(s) did only Mary$_F$ read?' Chinese

b. Context: Only Mary is interested in the books I read and follows me to read them.

wǒ dú shénme shū, <u>zhǐyǒu</u> Mary$_F$ (yě) gen-zhe wǒ dú <u>shénme</u> shū
I read what book only Mary (also) follow I read what books
'Only Mary follows me to read whatever books I read.' Chinese

Actually, the case of (29b) seems similar to embedded degree questions with a modified numeral (see (26) in Section 4.2). For (29b), suppose both the *wh*-item (i.e., *shénme*) and the focused part (i.e., *zhǐyǒu Mary* 'only Mary') introduce drefs first and bring post-suppositional tests later. Then within a *wh*-conditional, the deterministic update of the *wh*-expression can be resolved independent of the focused part, helping to circumvent the issue of which post-suppositional test need to be applied the last. I leave the details of this analysis for future work.

5.2 Abrusán (2014)'s analysis of weak island effects

Abrusán (2014)'s account for weak island effects is also based on the idea that an interpretable *wh*-question needs to meet Dayal (1996)'s Maximal Informativity Presupposition. As illustrated in (4a) (repeated here in (30)), since there does not exist a maximally informative interval I such that $\neg\text{HEIGHT}(\text{JOHN}) \subseteq I$, (30) does not meet the presuppositional requirement, leading to uninterpretability.

(30) #How tall isn't John? (= (4a))

The current analysis is essentially in the same spirit as Abrusán (2014). Although Abrusán (2014) focuses on weak island effects, she raises the issue of how intervention effects and weak island effects can be connected. As addressed in Section 5.1, the current analysis has the potential of explaining intervention effects as well. It is also worth investigating whether the current analysis can eventually be extended to bridge between the phenomena of intervention effects and those of weak island effects.

6 Conclusion

In this paper, I have adopted a dynamic semantics perspective to explain why a degree question like #*how tall are exactly three boys?* is unacceptable. The account crucially relies on the ideas that (i) both *wh*-items (e.g., *how*) and modified numerals (e.g., *exactly three boys*) introduce drefs and bring post-suppositonal tests that result in relative definiteness, and (ii) when different post-suppositional tests are present, their relative definiteness cannot be all achieved, leading to uninterpretability.

Presumably, the current account will bring new insights on more empirical phenomena, in particular, intervention effects and weak island effects. How the current account will influence our understanding on the scope-taking issue within a *wh*-question is also left for future research.

References

Abrusán, M.: Weak island effects. OUP (2014)

Beck, S.: Wh-constructions and transparent logical form. Ph.D. thesis, Universtät Tübingen (1996)

Beck, S.: Intervention effects follow from focus interpretation. Nat. Lang. Semant. **14**, 1–56 (2006) https://doi.org/10.1007/s11050-005-4532-y

Brasoveanu, A.: Modified numerals as post-suppositions. J. Semant. **30**, 155–209 (2013) https://doi.org/10.1093/jos/ffs003

Bumford, D.: Split-scope definites: Relative superlatives and Haddock descriptions. Linguist. Philos. **40**, 549–593 (2017) https://doi.org/10.1007/s10988-017-9210-2

Caponigro, I.: Free not to ask: On the semantics of free relatives and *wh*-words cross-linguistically. Ph.D. thesis, UCLA (2003)

Caponigro, I.: The semantic contribution of *wh*-words and type shifts: Evidence from free relatives crosslinguistically. In: Young, R. B. (eds) Proc. SALT **14**, 38–55 (2004) https://doi.org/10.3765/salt.v14i0.2906

Charlow, S.: Post-suppositions and semantic theory. J. Semant. In press (2017) https://ling.auf.net/lingbuzz/003243

Chierchia, G., Caponigro, I.: Questions on questions and free relatives. Handout of Sinn und Bedeutung **18** (2013) https://scholar.harvard.edu/chierchia/publications/demo-presentation-handout

Comorovski, I.: Interrogative phrases and the syntax-semantics interface. Kluwer (1996)

Dayal, V.: Locality in Wh Quantification. Kluwer. (1996)

Fleisher, N.: Nominal quantifiers in *than* clauses and degree questions. Syntax Semant. **42**, 364–381 (2020) https://doi.org/10.1163/9789004431515

Fox, D., Hackl, M.: The universal density of measurement. Linguist. Philos. **29**(5), 537–586 (2007) https://doi.org/10.1007/s10988-006-9004-4

Haddock, N.: Incremental interpretation and combinatory categorial grammar. The 10th international joint conference on artificial intelligence **2**, 661–663 (1987) Morgan Kaufmann Publishers Inc.

Hamblin, C.: Questions in Montague grammar. Foundations of Language **10**, 41–53 (1973)

Hausser, R., Zaefferer, D.: Questions and answers in a context-dependent Montague grammar. In Formal semantics and pragmatics for natural languages, 339–358. Springer (1979)

Honcoop, M.: Towards a dynamic semantics account of weak islands. In: Galloway, T., Spence, J. (eds) Proc. SALT **6**, 93–110 (1996) http://dx.doi.org/10.3765/salt.v6i0.2773

Karttunen, L.: Syntax and semantics of questions. Linguist. Philos. **1**, 3–44 (1977) https://www.jstor.org/stable/pdf/25000027.pdf

Krifka, M.: At least some determiners aren't determiners. In: Tuner, K. (eds) The semantics/pragmatics interface from different points of view, vol. 1, 257–291. Elsevier (1999)

Li, H., Law, J. H.-K.: Alternatives in different dimensions: A case study of focus intervention. Linguist. Philos. **39**, 201–245 (2016) http://dx.doi.org/10.1007/s10988-016-9189-0

Li, H.: A dynamic semantics for *wh*-questions. Ph.D. thesis, NYU (2020)

Nathan, L. E.: On the interpretation of concealed questions. Ph.D. thesis, MIT (2006)

Rooth, M.: Association with focus. Ph.D. thesis, UMass Amherst (1985)

Rullmann, H.: Maximality in the semantics of *wh*-constructions. Ph.D. thesis, University of Massachusetts-Amherst (1995)

Schwarzschild, R., Wilkinson, K.: Quantifiers in Comparatives: A Semantics of Degree Based on Intervals. Nat. Lang. Semant. **10**, 1–41 (2002) https://doi.org/10.1023/A:1015545424775

de Swart, H.: Indefinites between predication and reference. Proc. SALT **9**, 273–297 (2006) http://dx.doi.org/10.3765/salt.v9i0.2823

Szabolcsi, A., Zwarts, F.: Weak islands and an algebraic semantics for scope taking. Nat. Lang. Semant. **1**(3), 235–284 (1993) http://www.jstor.org/stable/23748421

Szabolcsi, A.: Strategies for scope taking. In: Szabolcsi, A. (eds) Ways of scope taking, 109–154. Springer (1997)

Szabolcsi, A.: Strong vs. weak island. The Blackwell Companion to Syntax. Vol. **4**, 479–531. Blackwell (2006)

Umbach, C.: Why do modified numerals resist a referential interpretation. Proc. SALT **15**, 258–275 (2006) http://dx.doi.org/10.3765/salt.v15i0.2931

Xiang, Y.: Higher-order readings of *wh*-questions. Nat. Lang. Semant. **29**, 1–45 (2021) https://doi.org/10.1007/s11050-020-09166-8

Zhang, L.: Modified numerals revisited: The cases of *fewer than 4* and *between 4 and 8*. Proc. Sinn und Bedeutung **21(2)**, 1371–1388 (2018) https://doi.org/10.18148/sub/2018.v21i2.204

Zhang, L.: Scopelessness and uninterpretable degree questions. In: Bekki, D. (eds.) Proc. International Workshop of Logic and Engineering of Natural Language Semantics (LENLS) **16** (2019) https://www.academia.edu/43799952

Zhang, L.: Split semantics for non-monotonic quantifiers in *than*-clauses. Syntax Semant. **42**, 332–363 (2020) https://doi.org/10.1163/9789004431515

Zhang, L., Ling, J.: The semantics of comparatives: A difference-based approach. J. Semant. **38**, 249–303 (2021) https://doi.org/10.1093/jos/ffab003