Negation, modality, events, and truthmaker semantics

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Overview of the course

- Day 1: Davidsonian event semantics, problems with negation.
- Day 2: Situation semantics, negation as a modality.
- Day 3: Negative events in compositional semantics.
- Day 4: Event semantics as exact truthmaker semantics.
- Day 5: Propositions as sets of events, and negative individuals.

Day 5

Recap from Day 4

- Exact truthmaker semantics is a natural fit for event semantics.
- In bilateral truthmaker semantics, negation flips verifiers and falsifiers.
- In unilateral truthmaker semantics, we use a compatibility negation.
- Because event predicates are not persistent, we cannot use the Goldblatt-Dunn compatibility negation.
- However, a variant of the unilateral negation in Fine (2017) fits the bill.
- The resulting theory is a natural fit for the collective theory of conjunction.
- The Principle of Negation is now a theorem.

Today's contents

- Relation to classical propositional logic.
- Modality in exact truthmaker semantics.
- Negative individuals.

Interpreting a propositional language with events

- In Day 4, we gave a semantics to *or*, *and* and *not* as operators on sets of events.
- We can translate these definitions to define a propositional logic in which propositions are sets of events.

Propositional logic

- $[\![\phi \vee \psi]\!] = \{e \mid e \in [\![\phi]\!] \text{ or } e \in [\![\psi]\!]\}$
- $[\![\phi \wedge \psi]\!] = \{e \mid \exists e_1 \exists e_2,\ e = e_1 \sqcup e_2,\ e_1 \in [\![\phi]\!] \text{ and } e_2 \in [\![\psi]\!]$
- $[\![\neg\phi]\!] = \{e \mid e \text{ precludes } [\![\phi]\!]\}$

E-frames and E-models

E-frame (reminder)

An E-frame is a quadruple $\langle E, \sqsubseteq, \perp, w_0 \rangle$ where:

- E, the *event space*, is a set (understood as the set of events, including possible worlds);
- $\sqsubseteq$, the *parthood relation*, is a binary relation over E such that $\langle E, \sqsubseteq \rangle$ is a complete Boolean lattice;
- $\perp$, the *exclusion relation*, is a binary relation over E which satisfies Symmetry, Cumulativity, Cosmopolitanism, Harmony, and Rashōmon;
- w_0, the *designated world*, is a possible world contained in E (understood as the actual world).

- Adding an interpretation function I that maps each proposition letter to a subset of E gives us an *E-model*.

Equivalence with classical propositional logic

- A model for classical propositional logic is given by an assignment function $\mathcal{I}$ which assigns a truth value to each propositional letter.
- We have the following equivalence:
  - Any classical model can be translated into an E-model in a truth preserving way (i.e., in which exactly the same formulas are true);
  - Any E-model can be translated into a classical model in a truth preserving way.
- But these transformations are not bijections; E-models are more informative than classical models (more on this in a minute).

From classical models to E-model, and vice versa

- From classical models ($\mathcal{I}$) to E-frames:
  - For each letter x, consider a new symbol $\overline{x}$.
  - Let $E = \mathcal{P}(\{p, \overline{p}, q, \overline{q}, \cdots\})$: events are *sets* of xs and $\overline{x}$s.
  - Let $\sqsubseteq$ be $\subseteq$.
  - Let $\{x\} \perp \{\overline{x}\}$ for each letter x, and nothing else.
  - Let $w_0 = \{x \mid \mathcal{I}(x) = T\} \cup \{\overline{x} \mid \mathcal{I}(x) = F\}$.
  - For each letter x, let $I(x) = \{\{x\}\}$.
  - (This is an example of a *canonical model*.)
  - Ex: $[\![p \vee q]\!] = \{\{p\}, \{q\}\}$, $[\![p \wedge q]\!] = \{\{p, q\}\}$ and $[\![\neg p]\!] = \{\{\overline{p}\}\}$
- From E-frames to classical models:
  - Let $\mathcal{I}(x) = T$ if $I(x) \in w_0$, F otherwise.

Hyperintensionality of E-models

- E-models are usually hyperintensional, in the sense that two classically equivalent sentences may have two distinct denotations.
- Example, in a canonical model:
  - $[\![\neg(p \wedge q)]\!] = \{\{\overline{p}\}, \{\overline{q}\}, \{\overline{p}, \overline{q}\}\}$
  - $[\![\neg p \vee \neg q]\!] = \{\{\overline{p}\}, \{\overline{q}\}\}$
- Counterfactuals seem to be sensitive to such distinctions (Ciardelli, Zhang & Champollion 2018).

Exact consequence

- Classical entailment: $\phi \models \psi$ iff any model that satisfies ϕ also satisfies ψ.
- We can define alternative notions of entailment.

Exact consequence

ϕ exactly entails ψ iff $[\![\phi]\!] \subseteq [\![\psi]\!]$.

- In a canonical model,
  - tautology $[\![p \vee \neg p]\!] = \{\{p\}, \{\overline{p}\}\}$ is exactly entailed only by propositions that contain at most $\{p\}$ and $\{\overline{p}\}$,
  - contradiction $[\![p \wedge \neg p]\!] = \{\{p, \overline{p}\}\}$ exactly entails only propositions that contain $\{p, \overline{p}\}$.
- See Yablo (2014: pp. 12-14) for applications of alternative notions of entailment.

Kripke's picture of modality: a recap from Day 3

- Kripke (1959, 1963) introduced the now-standard approach to modal logic as quantification over possible worlds.
- A model is essentially a set of possible worlds, each with an interpretation function, and with a relation of accessibility between worlds.
- $\Diamond p$ is true iff p is true in some accessible possible world.
- $\Box p$ is true iff p is true in all accessible possible worlds.
- In the following, we ignore the accessibility relation (as in Kripke 1959).

First steps towards a modal truthmaker semantics

- Here we report on ongoing work that aims at extending our propositional logic to modality.
- Kripke semantics clauses tell us in which possible worlds a formula $\Diamond\phi$ is true: in all those from which some ϕ world is accessible.
- In truthmaker semantics, ϕ is true at a world w iff it holds of an exact truthmaker that is part of w.
- But what are the exact truthmakers for $\Diamond\phi$?

Desiderata for a truthmaker semantics for modals

- We focus on the case of possibility modals here. Necessity modals are analogous.
- $\Diamond\phi$ should be true if and only if ϕ is possible (i.e., is verified by a possible event).
- $\Diamond\phi$ should be valid in E-frames just in case it is valid in classical modal logic.
- The semantics should not collapse hyperintensional distinctions.
  E.g., if ϕ and ψ denote distinct but intensionally equivalent propositions, then it should be possible for $\Diamond\phi$ and $\Diamond\psi$ to do so too.

Idea: e verifies $\Diamond\phi$ iff e admits ϕ

- Think of truthmakers as bodies of information.
- An exact truthmaker for ϕ contains just the right amount of information to establish that ϕ.
- Consider an exact truthmaker e_1 for $\Diamond\phi$. It should contain information that "admits" ϕ.
- We say that e_1 admits e_2 if $e_1 \sqcup e_2$ does not contain any inconsistency not already present in e_1.
- We say that e_1 admits ϕ (it is then a truthmaker for $\Diamond\phi$) if e_1 admits some truthmaker of ϕ.
- This definition allows a proposition to be admitted by an impossible event, which enable some hyperintensional distinctions.

Complements and event subtraction

- In a complemented distributive lattice, each event e_1 has a complement e_1':
  - $e_1 \sqcup e_1' = \blacksquare$ (the full event);
  - $e_1 \sqcap e_1' = \square$ (the null event).

Event subtraction

$$e_0 \setminus e_1 \stackrel{\text{def}}{=} e_0 \sqcap e_1'$$

- $(e_0 \setminus e_1) \sqcup e_1 = e_0 \sqcup e_1$
- $(e_0 \setminus e_1) \sqcap e_1 = \square$
- Intuitively speaking, $e_0 \setminus e_1$ is that portion of e_0 which remains after you take away from e_0 everything that it shares with e_1.

Admission via event subtraction

- Remember: e_1 admits e_2 means that $e_1 \sqcup e_2$ does not contain any inconsistency not already present in e_1.
- In other words, e_1 admits e_2 iff $e_2 \setminus e_1$
  - is consistent (i.e., possible)
  - and does not conflict with e_1.

The semantics of $\Diamond$

$$[\![\Diamond\phi]\!] = \{e_1 \mid \exists e_2 \in [\![\phi]\!].\ \textsc{Poss}(e_2 \setminus e_1) \wedge \neg e_1 \not\simeq (e_2 \setminus e_1)\}$$

- $\Box\phi$ is just $\neg\Diamond\neg\phi$.

Examples of admission

- $e \setminus e = \Box$, which is always a possible event.
- Consequence: Any event that does not conflict with $\Box$ admits itself.
- Consequence: All possible events (and most impossible events as well) from $[\![\phi]\!]$ are also in $[\![\Diamond\phi]\!]$.
- In a canonical model, e_1 admits e_2 iff:
  - for no letter x, both $x, \overline{x} \in e_2 \setminus e_1$;
  - for no letter x, $x \in e_1$ and $\overline{x} \in e_2 \setminus e_1$, or vice versa.

Equivalence with classical modal logic

- A model of classical modal logic is given by a set K of assignment functions with a designated one $G \in K$.
- Similarly to what we did earlier with classical propositional modal logic,
  - there is a truth-preserving transformation from classical models to E-frames,
  - and, conversely, there is a truth-preserving transformation from E-frames to classical models.
- $\Diamond$ and $\Box$ do not, in general, collapse hyperintensional distinctions.

From negative events to negative individuals

- In this course, we have tried to establish the possibility for negative events to serve as respectable and law-abiding citizens in our semantic theories.
- It is natural to wonder at this point whether a similar story can be told in the domain of *entities* by distinguishing between *positive* and *negative individuals*.
- The very idea of a negative individual tends to bring forth ontological worries and hesitation about a mysterious realm of shadow creatures, and may be thought to belong to what David Lewis (1970) calls the “dark ages of logic”. But we think this approach deserves further exploration.

Motivation for negative individuals

Negative individuals might be used for the interpretation of negative nominals like the bolded expressions in (1a)-(1c) and for some non-upward entailing numerical phrases like the bolded expressions in (1d) and (1e):

(1)
a. Alfred but **not Beatrice** is in the counting house.
b. **Not Claribel** but Donatello is in the parlor.
c. **No maid** is in the garden.
d. **At most two blackbirds** were baked in a pie.
e. **An odd number of blackbirds** began to sing.

Motivation for negative individuals

$$\begin{aligned}
[\![\text{Alfred but not Beatrice}]\!] &= \{\mathit{Alfred} \sqcup \neg \mathit{Beatrice}\} \\
[\![\text{Not Claribel but Donatello}]\!] &= \{\neg \mathit{Claribel} \sqcup \mathit{Donatello}\} \\
[\![\text{No maid}]\!] &= \{\neg \mathit{maid1} \sqcup \neg \mathit{maid2} \sqcup \ldots\} \\
[\![\text{At most two blackbirds}]\!] &= \{\neg bb1 \sqcup \neg bb2 \sqcup \neg bb3 \sqcup \ldots, \\
&\quad bb1 \sqcup \neg bb2 \sqcup \neg bb3 \sqcup \ldots \\
&\quad bb1 \sqcup bb2 \sqcup \neg bb3 \sqcup \ldots\} \\
[\![\text{An odd number of blackbirds}]\!] &= \{bb1 \sqcup \neg bb2 \sqcup \neg bb3 \sqcup \ldots, \\
&\quad bb1 \sqcup \neg bb2 \sqcup bb3 \sqcup \ldots\}
\end{aligned}$$

Negative individuals and collective conjunction

Negative individuals bear on the debate between the collective "non-Boolean" theory of conjunction versus the traditional intersective "Boolean" theory based on logical conjunction.

> *The hardest nut to crack for anyone wishing to pursue the collective theory is probably coordination of non-upward entailing quantifiers such as* **John and nobody else** *or* **John and an odd number of women**. *Not only do Heycock & Zamparelli (2005) not give a satisfying account of these conjunctions, it also does not seem easy to give one under any approach that takes the basic meaning of 'and' to be collective. For this reason alone, it seems preferable to make the intersective theory work if one is interested in using generalized quantifier denotations for at least some non-upward entailing noun phrases. (Champollion 2016, p. 612)*

Composing negative individuals: challenges

- Peter T. Geach 1962 and P. F. Strawson 1974 raised a number of arguments against the logical possibility of negating names or subject terms.
- Their arguments anticipate some of the difficulties involved in getting negative individuals to compose properly.
- Basic difficulty: ensuring that the negation contributed by a negative individual has the right scope-taking behavior.

(2) Nobody *P*ed or *Q*ed.
$\rightsquigarrow$ (P or Q)($\neg$Alfonso) and (P or Q)($\neg$Claribel) and...
$\rightsquigarrow$ (P($\neg$Alfonso) or Q($\neg$Alfonso)) and ...
$\rightsquigarrow$ ($\neg P$(Alfonso) or $\neg Q$(Alfonso)) and ...

- But we want: nobody Ped and nobody Qed. The negation contributed by each negative individual needs to scope above the disjunction.

Composing negative individuals: a solution

- Bledin (work in progress) proposes a polarity-sensitive composition rule by which negative individuals can pass negation through semantic derivation in a well-behaved way.
- Let E range over sets of entities.
- Let P range over properties of entities.
- Let E_e^+ be the set of (non-null) positive parts of an atomic or plural entity e, and E_e^- be the set of (non-null) negative parts of e.
- For instance:
  $[\![\text{John and nobody else}]\!] = \{John \sqcup \neg Mary \sqcup \neg Peter \sqcup ...\}$
  $E^+_{\{John \sqcup \neg Mary \sqcup \neg Peter \sqcup ...} = \{\text{John}\}$
  $E^-_{\{John \sqcup \neg Mary \sqcup \neg Peter \sqcup ...} = \{\neg\text{Mary}, \neg\text{Peter}, ...\}$

Cracking the hard nuts for collective conjunction

(3) **Polarity-sensitive composition**

$$\ggg \quad := \quad \lambda E \lambda P. \bigcup_{e \in E} (\bigsqcup_{e' \in E_e^+} P(e') \sqcup \bigsqcup_{e' \in E_e^-} Neg(P(\neg e')))$$

- $[\![$John and nobody else sang$]\!] = \ldots$
- Composition with $\ggg$:
  - Applies VP to the positive part of the subject, John, which returns states of John singing.
  - Applies VP to the negative counterparts of the negative parts of subject, $\neg$*Mary*, $\neg$*Peter*, etc., and then applies the *Neg* function to this result. This returns negative states of everybody else not singing.
  - The outputs of these computations are then fused together, delivering the correct kind of truthmaker for this example, namely a positive state of John singing fused with negative states of everybody other than John not singing.

Day 5: Summary

- We can define a modal propositional logic in which propositions are sets of events.
- The clauses for $\wedge$ and $\vee$ are similar to the ones in unilateral truthmaker semantics.
- The clause for $\neg$ is the one we have introduced for *Neg* yesterday.
- The clause for $\Diamond$ is defined based on a notion of admission ("not introducing any new inconsistency").
- The resulting logic is compatible in terms of truth with classical logic, yet richer; it enables different notions of entailment.
- Negative individuals may also serve as respectable citizens in semantic theory, offering new treatments of negative DPs (which also bears on the choice between intersective vs. collective conjunction).

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