Deriving dimensions of comparison

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In English, using the comparative *more* with a mass noun (*more coffee*) allows comparison along various dimensions, including volume and weight, while using *more* with a plural (*more cats*) typically only allows comparison by cardinality.

Wellwood (2019) proposes to capture these facts via a constraint on the measure function μ expressed by the comparative when it is combined with a nominal expression whose denotation P has a parthood relation $\leq$. Wellwood's constraint is that μ must satisfy 'automorphism invariance':

(1) **Automorphism invariance**
$\forall h \in Aut(\langle P, \leq \rangle)\ \forall x \in P\ [\mu(x) = \mu(h(x))]$
'Any automorphism on $\langle P, \leq \rangle$ leaves the value of the measure constant.'

(2) h is an *automorphism* on $\langle P, \leq \rangle$, $h \in Aut(\langle P, \leq \rangle)$, iff h is a bijective function from P onto itself which respects parthood: $\forall x, y \in P\ [x \leq y \text{ iff } h(x) \leq h(y)]$.

For plurals, Wellwood shows that any automorphism respecting parthood must map atomic individuals to atomic individuals. Since two individuals may have different weights or volumes, weight and volume are not automorphism invariant, thus capturing the restriction of μ to cardinality for plurals.

However, the constraint in (1) is too strong: by that criterion, volume and weight would not be admissible measure functions for mass nouns, either! Identify the denotation of *coffee*, reductively, with the closed interval between zero and six — $[0, 6]$ — with mereological parthood understood as set inclusion and μ as interval length.

Define f as follows (cf. Figure 1):

$$f(x) = \begin{cases} 2x + 1 & \text{for } 1 \leq x \leq 2 \\ (x-1)/2 & \text{for } 3 \leq x \leq 5 \\ x & \text{otherwise} \end{cases}$$

Let the function h apply f to each member of a set:

$$h(S) = \{f(x) \mid x \in S\}$$

The function h is an automorphism respecting the subset (i.e. parthood) relation. However, it does not preserve measure: $h([1, 2]) = [3, 5]$, but $\mu([1, 2]) = 1$, while $\mu([3, 5]) = 2$.

An analogous function can be constructed for area or volume, as illustrated in Figure 2, where corresponding points in squares A and B are mapped to each other, and everything else is mapped to itself. If we consider a substance of uniform density, the same mapping shows that mass and weight are also not admissible measure functions.

Is there an alternative? In related work (Schwarzschild 2006), both pseudo-partitives and quantity comparisons have been shown to disallow non-monotonic measure functions like temperature (*10 liters of water* vs. **10 degrees of water*; *more coffee* ≠ *hotter coffee*). For pseudo-partitives, Champollion (2017, p. 92) has argued that this constraint is best captured by 'stratified reference'. We propose that a modification of stratified reference can additionally capture the constraint on plural quantity comparisons: the constraint in (3) requires that the P-parts of x have *the same small* measure.

(3) **Fixed-scale stratified reference**
$\forall x\ [P(x) \rightarrow x \in {}^{*}\lambda y[P(y) \wedge \mu(y) = \varepsilon_x]]$
'Every x satisfying P can be divided into parts that satisfy P and have the same small measure.'

For plurals, it may not always be possible to divide an entity into parts (in P) with the same small volume or weight (e.g. cats have different sizes and weights); hence, neither volume nor weight are admissible. On the other hand, cardinality satisfies fixed-scale stratified reference, since any plurality of cats can be divided into individual cats, whose cardinality, 1, is small. By the same reasoning, one expects that, with 'object' mass nouns like *furniture*, comparison involves cardinality, a generalization with experimental support (Barner and Snedeker 2005 but see also Rothstein 2017). In contrast, any instance of a mass noun like *coffee* can be divided into small parts by volume or weight, while an assignment of cardinality would seem meaningless.

Finally, for Wellwood, automorphism invariance must be supplemented with an additional constraint on monotonicity (Schwarzschild 2006). Here, a single constraint plays both roles. We leave to future work a full comparison of the constraints on pseudo-partitives and quantity comparisons, and if and why they may differ.

References

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This work was supported by ANR-17-EURE-0017 (FrontCog) and ERC H2020 788077 (ORISEM).

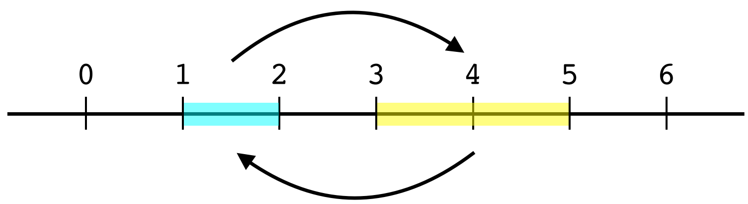

Figure 1: One-dimensional counterexample to automorphism invariance

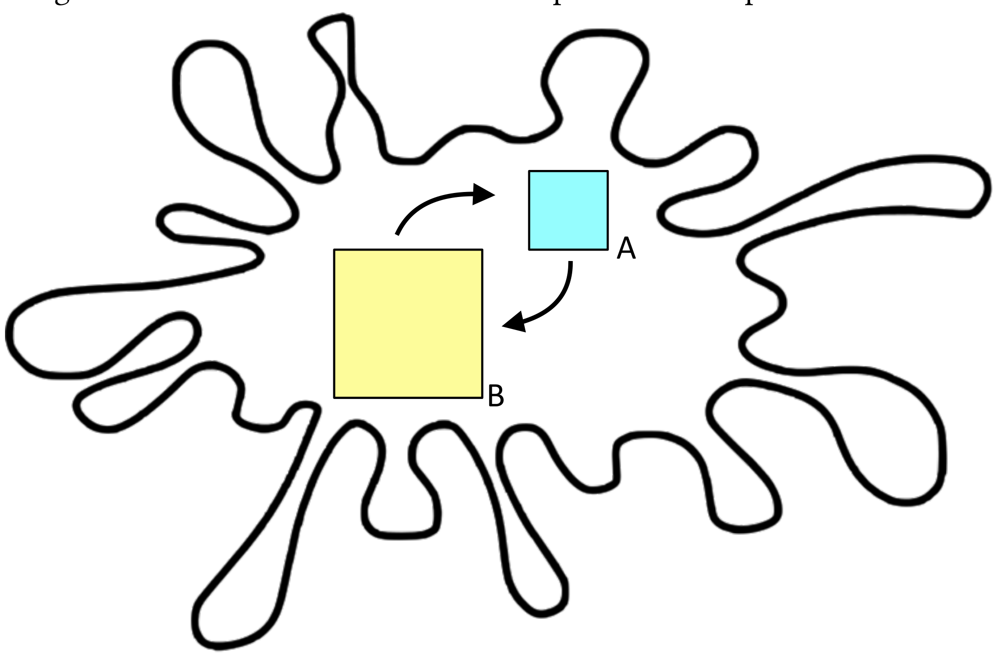

Figure 2: Two-dimensional counterexample to automorphism invariance