## Minimal sufficiency readings of necessity modals<sup>\*</sup>

Émile Enguehard

ILLC, University of Amsterdam

## 1 Introduction: issues with purpose constructions

#### 1.1 The prejacent problem: where to find cheese in Boston

The combination of *have to* with an infinitive purpose clauses, as in (1), has been called teleological (or goal-oriented) *have-to*.

(1) To get a driving license, you have to be an adult.

A possible analysis of (1) is one where the infinitive clause introduces an ordering on possible worlds, and the modal is a universal quantifier (cf. for instance von Fintel and Iatridou 2005). Then, (1) can be paraphrased as (2).

(2) In all the best worlds w.r.t. getting a driving license, you are an adult.<sup>1</sup>

However, von Fintel and Iatridou (2007) point out that this paraphrase seems incorrect for (3a). According to the analysis of *only* of Horn (1969), (3a) ought to presuppose the truth of (3b). (3b) would in turn have a paraphrase similar to (2).

(3) a. To get good cheese, you only have to go to the North End [a part of Boston].b. To get good cheese, you have to go to the North End.

However, if we know that there are many places in Boston where you can get good cheese, we can accept (3a) but not (3b). The falsity or infelicity of (3b) is expected in such a context given the paraphrase in (2), but (3a) should be rejected as well. This is called the *prejacent problem* by von Fintel and Iatridou (2007) (as (3b) is the *prejacent* of (3a)). The usage of *have to* that (3a) exemplifies is dubbed the "sufficiency modal construction" (SMC) by von Fintel and Iatridou (2007). Given that the difference between (3a) and (3b) lies in the presence of *only*, the analysis that they propose relies chiefly on a novel analysis of *only*.

In this article, we will relate the SMC to another family of examples involving teleological modals, but not necessarily *only*, and we will call the resulting class of examples *Minimal Sufficiency statements*. What brings these modal examples together is that they

<sup>\*</sup>I thank Benjamin Spector, Jakub Dotlačil, Brian Buccola, Sabine Iatridou, Kai von Fintel, Roger Schwarzchild, and the audiences at PALLMYR XII and NELS 50 for discussion.

<sup>&</sup>lt;sup>1</sup>In this paraphrase, we implicitly adopt the *Plural Limit Assumption*: the ordering of worlds is such that there is a set of *best worlds*, i.e. minimal elements for the order, and that all worlds are ranked above at least one of the best worlds. As far as we can tell, our discussion does not hinge on the Limit Assumption.

license a certain set of inferences about propositions lying on a scale, which inferences we will call *Minimal Sufficiency* (MS).

In general, MS statements offer a challenge to existing views on modality, as well as operators like *only*. Indeed, the MS inference is different to what our established understanding predicts, and MS statements are judged to be acceptable in contexts where one would predict them to be false or presuppositional failures. Accounting for this fact while deriving the MS inference in full poses interesting challenges, particularly as concerns the division of labour between semantics and pragmatics. In a sense, the identification of MS statements can be seen as a generalization of the prejacent problem.

We are going to explore here several natural routes towards an account of MS statements. All of them involve a significant weakening of a certain element of the construction, relative to established views, albeit a different one. We will see that all of our attempts encounter serious issues, highlighting the continued difficulty of the problem at hand.

#### **1.2** Some new examples, and an outline

The two examples that follow are very similar to (3a): they also appear to defeat the paraphrase of modality that we started with, in that whatever constraints they set on the world are much weaker than what one would predict. In (4), it is not implied that all situations where you get into the next stage are situations where you get a silver medal. Specifically, (4) is compatible with there being possible situations where you get a gold medal and get into the next stage (in fact (4) licenses a stronger inference: all situations where you get a gold medal are such that you get into the next stage).

(4) To get into the next stage, you have to get a silver medal.

In (5) (where capital letters indicate prosodic prominence), it again fails to be implied that you cannot be over the legal limit without drinking two beers. For instance, you may probably be over the legal limit after having drunk a certain amount of wine and no beer at all. (5) doesn't intuitively contradict this fact, even though we would predict it to do so. This failure of necessity is very reminiscent of (3a) and (4) above.

(5) To be over the legal limit, you have to drink TWO beers.

The similarities in meaning between (3a), (4) and (5) lead us to categorize them together as expressing Minimal Sufficiency (MS). In the next section, we will detail what Minimal Sufficiency is and what is challenging about it. We will then review the analysis of von Fintel and Iatridou (2007) for (3a), consisting in assuming that *only* can be decomposed, and show that it does not provide an adequate characterization of what MS means, although part of the problem can be resolved with some simple modifications to the analysis. Additionally, since this approach relies primarily on a novel account of *only*, it requires additional assumptions to extend to (4) and (5), where *only* is not present.

If the explanation for failures of necessity is not to be found in a weakening of *only*, there are two other options: weakening the embedded clause, and weakening the modal. We are going to explore both options in that order, and show that while the embedded-clause option faces some challenges, the modal option is much more difficult to make viable than one would assume, perhaps pointing towards the embedded-clause solution.

In Section 3, we will start with discussing an analysis of the cheese example (3a) proposed by Alonso-Ovalle and Hirsch (2018), consisting in assuming that the embedded clause is the scope of a covert weakening operator AT-LEAST. We will see why it needs to

be tweaked to properly account for the full range of MS examples, by giving the operator degree-based semantics and making it even weaker than Alonso-Ovalle and Hirsch (2018) assume. We will mention some potential shortcomings of the resulting picture. The analysis discussed in Section 3 is an instance of the second route, where the embedded clause is weakened while the modal is kept strong.

Section 4 explores the third route: weakening the modal. In what we call an alternativesensitive analysis, AT-LEAST is kept as Alonso-Ovalle and Hirsch (2018) define it, and we give alternative-sensitive semantics to modality, following Villalta (2008). While the alternative-sensitive approach is attractive in principle, we will see that it runs into a number of challenges and we will not be able to provide a satisfactory analysis in this vein.

Finally, Section 5 discusses certain empirical aspects of MS constructions in more detail, in particular relating to the exact status of certain inferences when *only* is present in MS examples. We will show that our observations are a challenge for the various analyses that we discuss, and that they might help us gain more insight into the contentious issue of how to analyse *only*.

### 2 Minimal Sufficiency

#### 2.1 A breakdown of Minimal Sufficiency

The reason we categorize (3a), (4) and (5) together is that their meanings have a number of shared components. First, they involve a scale, a set of ranked alternatives. In (4), the scale is the conventional scale of medals shown in (6). In (5), it is the scale of numerals (one, two, three...). In (3a), there is no obvious scale, but we are going to see that we cannot describe its meaning without assuming one is present. That scale features places one might go to to buy things and ranks them by the effort involved; for the sake of exposition we can assume that it looks like (7a), and is ranked as in (7b).

(6) bronze medal  $\prec$  silver medal  $\prec$  gold medal

(7) a. 
$$A = \begin{cases} \phi_C : \text{you cross the street,} \\ \phi_{NE} : \text{you go to the North End,} \\ \phi_{SB} : \text{you go to South Boston} \end{cases}$$
,  
b.  $\phi_C \prec \phi_{NE} \prec \phi_{SB}$ .

Given a scale, MS consists in the conjunction of *Sufficiency* and *Minimality*. Sufficiency is the inference that the alternative being considered is a way to attain the goal, while higher alternatives are unnecessary. (4) implies in some way that getting silver will in fact let you qualify, while getting gold is not necessary. Similarly, (5) implies that drinking two beers is enough to put you over the limit. (3a) also implies that you can get good cheese in the North End.

Minimality is the inference that lower alternatives are not a way to attain the goal. (5) implies that drinking one beer will leave you below the limit, and (4) implies that getting bronze will not let you qualify. In (3a), it is less obvious that Minimality is present, but it can still be detected. Assume that you can get good cheese in many places in Boston; the North End is one, and another one is the cheese shop just across the street from where the speaker resides; the speaker is fully informed about this situation. In a situation like this, it seems that it would be odd for the speaker to say (3a). Intuitively, the element in

this situation that is incompatible with (3a) is that there is another way of getting good cheese that is clearly easier than going to the North End. Thus, while (3a) is compatible with there being good cheese shops outside of the North End, it suggests in some way that the North End is amongst the most accessible places where one can get good cheese, or in other words that other places where you can get good cheese are at least as difficult to access as the North End. This is the meaning component that we call Minimality.

It is important to note that our assumption that (3a) involves a scale is essential to even just describe Minimality. In our description and in our analysis, we need to make reference to a scale to account for the difference between "you can find good cheese in South Boston" (whose truth is perfectly compatible with that of (3a)) and "you can find good cheese across the street" (whose truth seems to make (3a) deviant).<sup>2</sup>

The meaning of our examples does not appear to be any stronger than MS as we just described it. In particular, there is nothing resembling *necessity* as we paraphrase it in (2). The examples fail to implicate necessity of the condition they set in two ways: first, they allow for higher alternatives (going to South Boston, getting gold, drinking exactly three beers<sup>3</sup>) to also let you reach the goal, and in fact implicate that they do. Second, they allow for alternatives not on the scale at all to let you reach the goal. (3a) does not imply that you cannot find good cheese without going anywhere: for instance, you might perhaps ask your neighbour for some. (4) is compatible with a situation where, if you fail to get the silver medal, you may still get into the next stage through some unrelated means, such as being invited. Most clearly, (5) is compatible with the fact that you may drink only wine and end up above the limit.

These commonalities group our three examples (3a), (4) and (5) together as instances of the MS reading of necessity modals. In contrast, (1) does not involve a scale, does not mean that you automatically get awarded a driving license when you turn 18 (no sufficiency), and does mean that if you're not an adult you cannot get a driving license (necessity).

#### 2.2 The decompositional analysis

Before we move on to our proposals, we must discuss that of von Fintel and Iatridou (2007). The difference between (3a) and (3b) being the presence of *only*, von Fintel and Iatridou (2007) propose that the prejacent problem can be solved by reanalysing *only*. They take it to be decomposable into two elements, a negation NEG and an exceptive QUE paraphrasable as "anything other than".

(8) only = NEG + QUE  

$$\llbracket NEG \rrbracket = \lambda w. \lambda p. [p(w) = 0]$$

$$\llbracket QUE p \rrbracket = \lambda w : [\exists p' \in ALT(p), p'(w) = 1]. [\exists p' \in ALT(p), p' \neq p \land p'(w) = 1]$$

<sup>&</sup>lt;sup>2</sup>The status of Minimality in (3a) is in fact not clear: is it an assertion, a presupposition, a scalar implicature? We will discuss the issue in Section 5. However, nothing we say here depends on what that status is: our point is that Minimality exists in some form, which is seen in the fact that (3a) is deviant *in some way* if you can get good cheese across the street, and that scalarity is needed to derive Minimality *in any form*.

<sup>&</sup>lt;sup>3</sup>Note, though, that as long as we adopt an "at-least" semantics for bare numerals (cf. Spector 2013 for discussion of the issue and arguments in this direction), "drinking exactly three beers" is a subset of "drinking two beers" and the fact that you can drink three beers is not a violation of necessity for (5). We list this case anyway to highlight the parallels between the examples.

In these definitions, ALT(p) denotes the set of formal alternatives to p, in the sense of alternative semantics (Rooth 1992). The decompositional view of *only* opens up the possibility that in (3a), there is a split-scope configuration:

#### (9) NEG $\gg$ have-to $\gg$ QUE

Von Fintel and Iatridou (2007) assume that the split-scope configuration is in fact realized in (3a) under the relevant reading. They further assume, following Horn (1996), that the presupposition of *only* is a mere existential presupposition (some alternative to the prejacent is true), and also that it is in fact que that carries this presupposition. Then, (3a) presupposes that in all the best worlds where you get good cheese, you go somewhere, which is almost trivial, and asserts that in not all of these worlds you go somewhere that is not the North End, i.e. that you can go to the North End and get good cheese.

The analysis just presented predicts very weak truth and usage conditions for (3a). In particular, it does not derive Minimality. Recall that we called Minimality the fact that (3a) is deviant in a situation where you can in fact get good cheese across the street. In the decompositional analysis, however, (3a) would be a perfectly felicitous and true utterance in such a context. Recall also that it is impossible to derive Minimality without making reference to a scale, because we need to find a difference between "you can go across the street and get good cheese" and "you can go to South Boston and get good cheese": only the latter is compatible with the truth of (3a). As there is no scalar component in the decompositional analysis, there is no way to derive that fact.

To be sure, von Fintel and Iatridou (2007, sec. 4.2) remark that in general, "[only is] associated with a scale and the focus is low on the relevant scale". While they do not elaborate on how this fact relates to their non-scalar semantics, they propose that it explains why (3a) suggests that going to the North End is easy; let us call this component of the meaning of (3a) "Easiness." Von Fintel and Iatridou (2007) provide empirical as well as theoretical arguments (based on compositionality) to the effect that what Easiness means is that going to the North End is easy relative to actions in general, and not relative to ways to get good cheese. If that is so, Easiness is perfectly compatible with violations of Minimality: we may very well imagine that going to the North End is an easy thing to do, for instance because the speaker lives a couple of subway stops away from the North End, and yet even in that context, (3a) implies that you cannot find good cheese across the street. Thus, we cannot derive Minimality from Easiness.<sup>4</sup>

Von Fintel and Iatridou (2007, sec. 4.3) also briefly sketch a scalar version of their proposal, and note that it might help explain why "[sufficiency modals] seem to rate the ways of achieving the goal and zero in on the easiest, least-effort-involving way" (this is Minimality). They reject it for unrelated reasons. Since we think that deriving Minimality is important enough that one should pursue the effort, we will attempt here to "fix" the decompositional analysis and make it scalar.

Let us assume, then, that "go to the North End" lies on a pragmatic scale like the one in (7a) for the purposes of interpreting (3a). We can look at existing proposals on the interaction between *only* and pragmatic scales, which has been the subject of a great amount of literature (see in particular Klinedinst 2004). Of particular interest to us is Greenberg's (2019) proposal that *only* presupposes that the prejacent is the lowest alter-

<sup>&</sup>lt;sup>4</sup>Our breakdown of MS did not include Easiness, as the reader will have noticed. This is because no trace of Easiness is detectable in MS examples that do not feature *only*, such as (4) and (5). Thus, we follow von Fintel and Iatridou (2007) in assuming that Easiness is contributed by *only* and in not analysing it further.

native being considered. Adopting this proposal within the decompositional analysis of *only* is most naturally done through the following entry for QUE:

(10) 
$$\llbracket \text{QUE } p \rrbracket = \lambda w : [\exists p' \in \text{ALT}(p), p'(w) = 1] \land [\forall p' \in \text{ALT}(p), p' \succeq p].$$
$$[\exists p' \in \text{ALT}(p), p' \neq p \land p'(w) = 1]$$

We then predict that for (3a) to be felicitous, the alternatives to "go to the North End" must not include anything "easier". The existential presupposition of QUE, applied to these alternatives, will end up being that at least one of them is true. This presupposition projects universally from the scope of the modal, resulting in the overall presupposition for (3a) that in all worlds where you get good cheese, you go somewhere that is at least as inaccessible as the North End.<sup>5</sup> In the problematic situation where you can in fact get good cheese across the street, we predict (3a) to be a presupposition failure.<sup>6</sup> Thus, we do get Minimality.

What we have just seen is that the decompositional analysis can be "fixed" to derive MS for (3a). However, even when fixed, the decompositional account will not extend to examples not containing *only* such as (4) and (5). For this reason, we will explore different ways to account for MS in the next sections. While doing so, we will sometimes make reference to the "decompositional analysis", by which we mean the "fixed" scalar version.

## 3 Getting MS by weakening the condition

#### 3.1 Pragmatic scales, weakening operators, and exhaustification

One of the assumptions made in the decompositional analysis is that *only*'s presupposition is weaker than proposed by Horn (1969). Because they find arguments against this assumption, Alonso-Ovalle and Hirsch (2018) propose a different analysis of the prejacent problem involving a weakening operator AT-LEAST (the introduction of which they attribute to Schwarz (2005)).

The idea behind AT-LEAST is that it acts upon propositions lying on a pragmatic scale, and returns their disjunction with any higher-ranked alternatives. A possible entry is given in (11). It omits the necessary presupposition that the alternatives should form a scale.

(11) 
$$\llbracket \text{AT-LEAST } p \rrbracket = \lambda w. \exists q \in \text{ALT}(p), \ [q \succeq p \land q(w) = 1].^7$$

Alonso-Ovalle and Hirsch (2018) propose that (3a) has an LF similar to (12). Under this analysis, assuming the relevant scale is the one in (7b), what the prejacent asserts is that in all worlds where you get good cheese, you go to the North End or you do something more difficult, such as going to South Boston. It follows that crossing the street won't

<sup>&</sup>lt;sup>5</sup>In general, the combination of Greenberg's scalar presupposition (the prejacent is the lower end point), and an existential presupposition over the same set of alternatives, results overall in an "at least" presupposition for *only*. Beaver and Clark (2008) independently argue for such a presupposition.

 $<sup>^{6}</sup>$ As we already noted, the status of the Minimality inference in (3a) is not clear. We will discuss it some more in Section 5. We can however leave this issue aside for now, as we are going to discuss alternatives to the decompositional analysis anyway.

<sup>&</sup>lt;sup>7</sup>Both AT-LEAST and AT-LEAST-DEG, which we are going to introduce below, can be seen as instantiations of Beaver and Coppock's (2014) MIN operation; the crucial difference between the two will lie in the domain they quantify over.

do (Minimality), but not that going to the North End is necessary (failure of Necessity). Because *only* presupposes the truth of its prejacent, all this is presupposed in (3a), which further asserts that replacing "the North End" by any higher-ranked alternative, and in particular by "South Boston" yields a false sentence, i.e. that it is not the case that in all worlds where you find good cheese you go to South Boston or some more difficult place. It follows, then, that in some worlds where you get good cheese, you go to the North End exactly (Sufficiency).

(12) To get good cheese, you only have to AT-LEAST [PRO go to the NE].

The fact that what looks like the prejacent of (3a), (3b), doesn't receive an MS reading leads us to conclude that it doesn't have an easily accessible reading where AT-LEAST is present. We can explain the contrast in the following way: AT-LEAST presupposes that its argument lies on a pragmatic scale, but "going to the North End" isn't a scalar item, making AT-LEAST unavailable in (3b). However, *only* presupposes that there is a contextual ranking available, making AT-LEAST licit.<sup>8</sup> Thus *only*'s only role in making (3a) get an MS interpretation consists in forcing us to accommodate a ranking.<sup>9</sup>

This line of explanation leads us to the prediction that if a pragmatic scale is easily accommodated, *only* will not be necessary for AT-LEAST to occur. This prediction is borne out by (4). In (4), the ranking of alternatives is a well-established convention; this makes the insertion of AT-LEAST licit even without *only*, which is why an MS reading obtains. Thus the analysis extends to (4), which we take to have an LF similar to (13).<sup>10</sup>

(13) To get into the next stage, you have to AT-LEAST [PRO get a silver medal.]

One issue at this point is that our derivation of Sufficiency for (3a) involved the contribution of *only*. In (13), *only* is not present. The truth conditions we predict for (13) entail Minimality (getting bronze won't do), but they are compatible with Sufficiency being violated; concretely we predict (13) to be true if what you actually need is to get a

<sup>&</sup>lt;sup>8</sup>Another explanation in the same spirit, but based on a different treatment of *only* goes as follows: *only* does not in fact ever combine with pragmatic scales, but only with logical ones; it asserts that no logically stronger alternatives are true, and presupposes among other things that such alternatives exist. When the prejacent doesn't have obvious stronger alternatives, it is necessary to take it to be part of a scale and to insert AT-LEAST below *only* to satisfy this presupposition.

 $<sup>^{9}</sup>$ As we already noted, *only* also contributes an "easiness" inference: going to the North End is easy. Since that inference is not found in the other examples, we take it to be an effect of *only* and not part of MS.

<sup>&</sup>lt;sup>10</sup>Our explanation of the contrast between (3a) and (3b) also makes the prediction that the presence of other overt scalar operators than *only* should also license AT-LEAST. At first glance, this prediction is borne out. Consider (ia), where the overt scalar operator "at least" is present. Assuming that "at least" means the same thing as its covert counterpart, and that it takes scope above the modal, we predict (ia) to mean something like: to get good cheese, you have to go to the North End, or you have to go to South Boston, or you have to go to New York City, etc. In other words, there is a place at least as inaccessible as the North End such that you have to go there to get good cheese. This is of course not what (ia) means: (ia) does not imply that there is any one place that you absolutely have to go to to get good cheese. What it means is that the most easily reached place where one can find good cheese is at least as far as the North End. Though there might be other ways, we can derive the correct meaning if we just assume that the LF of (ia) is as in (ib). Thus it appears that in (ia) an overt "at least" above the modal licenses a covert AT-LEAST below it.

<sup>(</sup>i) a. To get good cheese, you at least have to go to the North End.

b. ..., you at least have to AT-LEAST [PRO go to the North End.]

gold medal. Intuitively, however, (13) is false in this case.

Fortunately for us, it is standard to assume that upon hearing (13), a hearer would conclude that the stronger sentence whose LF is given in (14) is false, as otherwise the speaker would have said it; in other words Sufficiency should follow from (13) as an implicature. One way to formalize the process by which implicatures are derived is to assume that all sentences can contain a silent counterpart of *only*, often denoted as EXH (after Groenendijk and Stokhof (1984)). Then the LF in (15) should deliver both Sufficiency and Minimality.<sup>11</sup> Since the Sufficiency inference appears to be obligatory here, we have to assume that EXH is obligatory in (4), a stipulation that we do not have an explanation for at this point.<sup>12</sup>

- (14) To get into the next stage, you have to AT-LEAST [PRO get a gold medal.]
- (15) ..., EXH [you have to AT-LEAST [PRO get a silver medal.]]

#### **3.2** Failure of necessity and degrees

It seems that, at this point, we have a satisfactory analysis of MS with pragmatic scales involving a weakening of the condition through the weakening operator AT-LEAST. There is, however, a subtle issue. Recall that we divided "failure of necessity" into two components: failure of necessity w.r.t. scalemates and w.r.t. non-scalemates. The former is provided for by AT-LEAST: since AT-LEAST essentially replaces the condition with its disjunction with higher alternatives, the "have to" statement no longer rules out these alternatives. The latter is not dealt with. The problem is that MS statements also fail to rule out alternatives not found on the scale: (3a) is compatible with it being possible to find good cheese without going anywhere, perhaps if you steal your neighbour's, and (4) is compatible with it being possible to get into the next stage while having lost in the current stage, perhaps because you can be invited by the organizers.

In these two examples, the problematic alternatives can be argued to be remote possibilities, or atypical, or irrelevant, and it sounds like we should look for a solution involving these notions. This will however not work at all for (16) (repeated from (5)). (16) doesn't rule out being over the limit after having drunk some amount of wine, but drinking wine is not intuitively a remote or irrelevant possibility; in fact one may very well utter (16) while drinking wine.<sup>13</sup> Note also that, with an "at-least" semantics for numerals, failure of necessity w.r.t. higher scalemates (i.e. the lack of an inference that drinking three beers is out) is not puzzling in (16) and inserting AT-LEAST would be vacuous.

(16) To be over the legal limit, you have to drink TWO beers.

The problem, then, is that the weakening brought about by AT-LEAST is not sufficient for our purposes: the condition is still too strong. We want to make it so that the condition set in (16) doesn't rule out drinking wine; only lower scalemates, such as drinking exactly one beer, are to be ruled out.<sup>14</sup> A natural move is to also include the possibility of drinking

<sup>&</sup>lt;sup>11</sup>Here we treat EXH as an element of the grammar, but nothing hinges on this at this point, and one may prefer to view it as a purely theoretical device standing in for a pragmatic process instead.

<sup>&</sup>lt;sup>12</sup>Possibly, EXH is obligatory when there is no uncertainty as to what the alternatives are, and they form a non-trivial scale. Evidence in this direction can be found when looking at numerals, as we will discuss in Section 3.4.

<sup>&</sup>lt;sup>13</sup>A potential solution involving some kind domain restriction naturally comes to mind, but there is reason to think it cannot be the end of the story; see Section 4.4 for discussion.

<sup>&</sup>lt;sup>14</sup>The same problem would occur if we had adopted the fixed decompositional analysis, and extended

wine in the grand disjunction that AT-LEAST represents, i.e., to make (16) paraphrasable as (17).

(17) ..., you have to drink two beers or a sufficient amount of something else.

However, recall that in the definition of AT-LEAST(p), we quantify over ALT(p), the set of formal alternatives to p. Following standard assumptions on the effect of prosodic focus (Rooth 1992), the prosody of (16) should constrain the formal alternatives to be as in (18).

(18)  $ALT("you drink TWO beers") = \{"you drink n beers" | n\}$ 

Given that this property appears to be necessary for the MS reading to obtain (cf. discussion in Section 3.4), we cannot assume that the alternatives to "you drink two beers" in (16) include things like "you drink a glass of wine". Thus, we need to divorce AT-LEAST from the set of focus alternatives. We offer here an implementation of this move in terms of degrees.

Bale (2008) introduces the notion of a *universal degree scale*, an abstract scale (isomorphic to part or all of the real line) to which expressions of measure or evaluative expressions may refer. Universal degrees do not have to correspond to any physical or evaluative scale. This suggests that universal degrees may appear even in the semantics of expressions that are not obviously gradable.

Rett (2014) proposes that NPs may be be interpreted as denoting a degree (type d), which allows them to be used in degree constructions like comparatives, and makes them trigger singular agreement; in (19) both phenomena can be seen.

(19) Two bananas is / #?are more than we need.

Notice that comparatives can be used in the prejacent of (3a) as seen in (20). This is a (weak) argument for taking the embedded clause in (3a) to in fact denote a degree.

(20) To get good cheese, you have to do more than go to the North End.

Then, let's assume than "(you) go to the North End", "you drink two beers", etc. each belong to a category of propositions for which a mapping to degrees is available. We denote this mapping as  $\mu$ , and assume it is supplied by the context; the set of rankable propositions will be denoted as $D_{\mu}$ . Because universal degrees are available,  $\mu$  does not have to correspond to a physical measurement. What we want is for the condition (3a) to mean "a degree as high as this one is reached". For the sake of simplicity (since syntax is not the focus of our discussion here) we can assume that this meaning obtains due to presence of the single operator AT-LEAST-DEG (given in (21)) in (3a), as in (22).

- (21)  $[[\text{AT-LEAST-DEG } p]] = \lambda w. \exists q \in D_{\mu}, \ \mu(q) \ge \mu(p) \land q(w) = 1$
- (22) To get good cheese, you only have to AT-LEAST-DEG [PRO go to the North End.]

The difference between AT-LEAST and AT-LEAST-DEG lies in the domain of propositions that they quantify over: instead of the formal alternatives to p, we now disjoin over the entire domain of  $\mu$ , which is a potentially bigger set, and isn't constrained by

it from *only* to EXH: we would predict (16) to presuppose that if you're over the limit, you have drunk two beers. The original decompositional analysis would correctly fail to derive this inference, but would also fail to derive Minimality in any of our examples.

the prosody. In (5),  $D_{\mu}$  will be allowed to contain propositions like "you drink a glass of wine", and (16) will come out as equivalent to (17), thus letting us solve the problem of failure of necessity w.r.t. non-scalemates.<sup>15</sup>

We need some extra assumptions to get MS right. First, note that AT-LEAST-DEG immediately solves the prejacent problem: (3a) doesn't presuppose that you have to go to the NE, but that you have to do something that is ranked at least as high as going to the NE. In this respect AT-LEAST-DEG plays the same role as AT-LEAST in weakening the condition.

Do we get Minimality and Sufficiency? Going back for instance to the medals example: when (4) is asserted, assuming that it contains AT-LEAST-DEG and EXH, what we predict is that all outcomes that  $\mu$  maps below a silver medal on the scale are ruled out. Furthermore, due to the presence of EXH, if any relevant outcome is mapped by  $\mu$  strictly above a silver medal, then (4) implies that the outcome in question is above the minimal degree one needs to reach. What we want to predict is that getting a bronze medal is ruled out, but getting a gold or silver medal is not (Minimality) and that getting a gold medal is unnecessary (Sufficiency). It can be verified that the following condition is necessary and sufficient for our predictions and our desiderata to be the same:

(23)  $\mu$ (you get bronze) <  $\mu$ (you get silver) <  $\mu$ (you get bronze)

This condition can be called a *monotonicity* constraint: the mapping to degrees has to be monotonic w.r.t. the scale, i.e. it has to respect the scale's ordering. It seems very natural that this condition should hold, for medals as well as for the other cases that we discuss.<sup>16</sup>

Thus, we have Minimality, Sufficiency, and Failure of Necessity w.r.t. scalemates. In this we only replicate what AT-LEAST already does. What is key is that AT-LEAST-DEG also allows for Failure of Necessity w.r.t. non-scalemates, because propositions not part of the scale might be in  $D_{\mu}$ . Note that the monotonicity constraint, i.e. how  $\mu$  ranks the scale, is  $\mu$ 's only specified property, and we do not assume that information about how  $\mu$ ranks other elements of  $D_{\mu}$ , and what  $D_{\mu}$  even is, is available to participants in general.<sup>17</sup> In a sense, AT-LEAST-DEG weakens the condition so as to make only information about the scalar alternatives recoverable from an MS statement.

#### 3.3 Are we really deriving Sufficiency?

There is a potential issue with the degree to which we have just weakened the condition: what we derive for Sufficiency might be somewhat weaker than what we want. Taking (3a) as an example, what we predict it to imply (putting together assertion and presupposition) is something akin to: you can do as much as go to the North End and get good cheese,

<sup>&</sup>lt;sup>15</sup>Instead of making  $\mu$  apply to propositions in a domain, we could have taken it to apply to worlds, or to situations. The general idea, however it is implemented exactly, is that the proposition being apparently expressed actually stands in for a gradable property. A very similar idea is proposed by Magri (2017, sec. 6) to account for the interaction between *only* and pragmatic scales.

<sup>&</sup>lt;sup>16</sup>Note in particular that so far, we have treated pragmatic scales and contextual rankings as some kind of primitive object. Now that we have degree mappings at our disposal, we may keep our ontology lean by deriving pragmatic scales from them, and say that there is a pragmatic scale exactly when there is a salient mapping of a set of alternatives onto degrees. Then the monotonicity "constraint" is just a by-product of the definition.

<sup>&</sup>lt;sup>17</sup>We could make this more explicit by making  $\mu$  an existentially quantified variable over some class of natural metrics in the definition of AT-LEAST-DEG.

and you can do less than go to South Boston and get good cheese. In other words, in some good-cheese worlds, the amount of effort you do is somewhere between going to the North End (included) and going to the South Boston (excluded). Notice that at no point are we specifically saying anything about going-to-the-North-End worlds here; i.e. this could very well be true even though there is no good cheese to be found in the North End, or anywhere else for that matter, and the only way to get good cheese is to order it online (which happens to be as much effort as going to the North End).

The issue is the following: AT-LEAST-DEG constitutes such a weakening that the propositional content that it embeds is lost. There is no role in our analysis for the set of worlds that the speaker chose to refer to to illustrate the degree they intended to communicate. To account for that, we would need to stipulate some sophisticated constraint, such as:

(24)  $\mu$ , the scale and the goal are such that there is a threshold alternative  $p_0$  on the scale such that the goal is attained if and only if a proposition whose degree is at least as high as the degree of  $p_0$  is true.

Then, from Sufficiency we know that the threshold alternative is below South Boston: it is not necessary to reach the degree of South Boston. From Minimality we know that the threshold is at least as high as the North End: it is not sufficient to do any less than go to the North End. It follows that "you go to the North End" must be the threshold alternative, as it is the only alternative in the appropriate interval, and thus when you go to the North End, since "you go to the North End" is true and it ranks as high as the threshold alternative (i.e. itself), you can get good cheese. In addition to being unpleasantly convoluted, our constraint is not compositional, as it refers both to the goal and to the felicity of AT-LEAST-DEG involving a certain  $\mu$ .

To avoid such a move, our only alternative is to accept that MS examples really have meanings as weak as we predict. This is perhaps not so implausible: there are other cases where utterances that we analyse to be truth-conditionally very weak lead listeners to strong inferences, based on world knowledge, typicality, relevance conditions etc.<sup>18</sup> It may be that the leap from our predicted weak sufficiency to the stronger form is an implicature of this kind, prompted by the following question: why would the speaker mention going to places if you could not get good cheese that way?

It must also be pointed out that the issue is in fact more general, and occurs with AT-LEAST already (thus it is not a problem specific to AT-LEAST-DEG). MS readings are easily accessible under "don't have to", even without *only*, as in (25).<sup>19</sup> Crucially, (25)

- (i) a. You don't HAVE to go the North End...
  - ... You can also go to the corner store.
  - ... You can also go to Switzerland.
  - b. You don't have to go to the NORTH END...
    - ... You can also go to the corner store.
    - ... ?? You can also go to Switzerland.

What remains also mysterious is why "go to the North End" can be interpreted as a scalar element in (ia),

<sup>&</sup>lt;sup>18</sup>A specific case of such inferences is the phenomenon known in the literature as *manner implicatures*, where truth-conditionally equivalent sentences may receive different interpretations based on specific choice of words.

<sup>&</sup>lt;sup>19</sup>The MS reading of (25) that we are interested in is one where it licenses the inference that you can get good cheese closer than the NE, perhaps across the street. There is also a literal reading of (25) where it only means that you can get good cheese somewhere that is not the NE. Focus on *have* brings out this reading, as seen in (i).

implies that you can get good cheese in the North End. It would be natural to explain that as a scalar implicature: saying you don't have to do something implies that you can do it. However, one we've weakened "go to the NE" using even just AT-LEAST (which we need to do to account for the fact that (25) entails you can get good cheese somewhere closer), the scalar implicature we derive is "you can get good cheese by doing something at least as effortful as going to the North End", which doesn't entail that you can get good cheese in the North End.

(25) To get good cheese, you don't have to go to the North End.

We will leave open for now the question of how we can have our cake and eat it too, that is, weaken "go to the North End" in (25), while maintaining the inference that going to the North End (as opposed to doing anything of similar or higher effort) is in fact a way of getting good cheese. Hopefully whatever solution we find can also solve the problem of our overly weak Sufficiency condition.

#### 3.4 Prosodic prominence, *only* and the structure of alternatives

Putting aside the problem described in the last section, we now have an analysis of MS whose key idea is to weaken the condition so much that no inference is recoverable about non-scale members. There are two things we still need to address: why the contrast between (3a) and the same sentence without *only*? And why is the marked prosody of (16) necessary?

Let's begin with the latter question. An interaction between the availability of MS readings and prosodic prominence is arguably expected under the analysis we have been developing. Indeed, following Rooth (1992), prosodic prominence is taken to constrain the alternatives set. If we identify the availability of AT-LEAST-DEG to that of a ranking of the alternatives set, then prosody might prevent or force an MS reading in certain cases.

As we have already pointed out, such an interaction can be seen in (16), for which a certain, rather marked prosody is necessary for an MS reading to be accessible. This can be seen in (26).

(26) To be over the limit,

- a. you have to drink TWO beers. You can also drink wine.
- b. ??you have to drink two beers. You can also drink wine.
- c. #you HAVE to drink two beers. You can also drink wine.

Plausibly, the alternatives of (26a) are obtained by replacing the numeral with other numerals. We then obtain a scale that is totally ordered by entailment (if we assume, again, "at-least" semantics for numerals). However, in (26b), the alternatives are plausibly obtained by replacing "two beers" by other drinkable things, as in (27). The resulting set of alternatives is also ordered by entailment, but we do not have a total order this time. Instead, what we have a semi-lattice structure. Finally, in (26c), there is plausibly VERUM focus, or some other form of focus that results in an alternatives set similar to (28) for which no entailment-based order is available. Thus, it appears that the pattern in (26) is consistent with a constraint to the effect that whenever the alternatives are logically related, AT-LEAST-DEG can only occur if entailment defines a total order.<sup>20</sup>

but not in (3b).

<sup>&</sup>lt;sup>20</sup>As an aside, prosodic focus on bare numerals also leads to obligatory "exactly" readings for them in

- (27) ALT("you drink [two beers]<sub>F</sub>") = {"you drink x" | x is a plurality of drinks},
- (28)  $ALT("VERUM_F \text{ you drink two beers"}) =$

{"you drink two beers", "you don't drink two beers"}

Let us move to the second question. As we noted, (3b), without *only*, cannot receive a MS reading. Before we introduced the degree analysis, we took this to be a consequence of the fact that we do not easily accommodate a contextual ranking that would include "go to the North End", but that *only* forces us to. We can mostly maintain this view and rephrase it in terms of degrees: no degree mapping for "(you) go to the North End" is easily accommodated, but *only* has the effect of forcing us to accommodate that one exists anyway. We may adopt here the proposal made for independent reasons by Greenberg (2019) that *only* presupposes that a mapping to degrees is salient for its focus associate.

It should be noted that the effect of *only* interacts with the constraint on alternatives we just established: *only* may license an MS reading even for cases where one might think the alternatives form a semi-lattice that is not total, as in (29) (contrast with (26b)).

(29) You only have to drink two beers. You may also drink wine.

#### 3.5 Disjunctive cases

To conclude this section, let us mention an interesting special case of MS, that this analysis extends nicely to: that of *disjunctive* MS, where a disjunction can be used to express MS along two dimensions at once. (30) is an example.

(30) To be over the legal limit, you have to drink TWO beers or THREE glasses of wine.

(30) appears to imply minimal sufficiency for both "two beers" and "three glasses of wine" (drinking that will put you over the limit, drinking less than that won't). It also demonstrates failure of Necessity, in the sense that it is compatible with the fact that drinking something else entirely (say, vodka) could put you over the limit. Failure of Necessity extends here to conjunctions of members of both scale: (30) can be true whether drinking one beer *and* two glasses of wine puts you over the limit or not; it implies nothing about what happens then.

Within our degree analysis, it seems natural to take (30) to have an LF equivalent at some level to (31). It can be straightforwardly verified that this delivers the right truth conditions.

(31) ..., you have to AT-LEAST-DEG [PRO drink two beers] or AT-LEAST-DEG [PRO drink three glasses of wine].

unembedded position, as seen in (i). It is tempting to relate this to the fact that (as we had to stipulate) exhaustification is obligatory in MS sentences: a generalization to the effect that exhaustification is obligatory in the presence of totally ordered alternatives would capture both facts.

- (i) a. JOHN drank two beers. In fact, he drank four.
  - b. #John drank TWO beers. In fact, he drank four.

## 4 Getting MS by weakening modality

#### 4.1 Better possibility and *have-to*

While the degree analysis of MS has been mostly satisfactory, we have been forced to rely on someone stipulatory constraints to account for the interaction with alternatives' structure, as described in Section 3.4. Besides, we saw in Section 3.3. that it wasn't entirely clear whether we hadn't derived a form of Sufficiency that is too weak. The latter issue stems from the radical and perhaps excessive weakening of the condition that the operator AT-LEAST-DEG performs. This might suggest a radically different approach: instead of weakening the condition through covert operators, we can make the modal itself directly sensitive to alternatives. This is in fact the only option that we have not explored yet: von Fintel and Iatridou (2007) have tried weakening *only*, and we have discussed in the previous section the option consisting in weakening the condition. Only the modal has been spared so far.

We explore here an approach consisting in giving weaker semantics to *have to*, specifically in the form of making it *alternative-sensitive*. In view of more general considerations on the alternative-sensitivity of modals (Villalta 2008), this line of analysis is quite natural. The role played by prosody in the cheese example, as discussed in Section 3.4, should also come out straightforwardly from an analysis of this sort. However, as we are going to see, the alternative-sensitive approach faces a number of challenges; we will not offer a solution to them here.

As already alluded to, the general idea of giving alternative-sensitive semantics to teleological *have-to* has already been proposed by Villalta (2008) and Krasikova (2010); the latter makes this move precisely to solve the "prejacent problem". Both proposals rely on the notion of "better possibility", which is due to Kratzer (1981).

Better possibility is defined as follows:

- (32) Relative to a set of worlds  $\Omega$  and an accessibility ordering  $\leq$  on these worlds, a proposition p is a better possibility than a proposition q if:
  - a. For any world u in q, there is a world v in p such that  $u \ge v$  (p is at least as good a possibility as q),
  - b. There is a world v in p such that for no world u in q do we have  $v \ge u$  (q is not at least as good a possibility as p).

In particular, if the ordering is based on similarity to the actual world, this intuitively means that p is closer to the actual world than q, that is, that p-worlds are strictly more accessible than q-worlds. It can be checked that if q entails p, then p is at least as good a possibility as q (in particular, anything is at least as good a possibility as the contradiction, and the contradiction is at least as good a possibility as only itself).

Let us then give an alternative-sensitive semantics to *have-to*:

(33) (to be refined) "To p, have-to q" is true iff for any alternative q' to q (other than q itself), w.r.t. to a certain ordering, q is a better possibility than q'.

This is essentially what Villalta (2008) gives for *be necessary*, with the exception that we have left the ordering source unspecified. There are several natural choices. As Villalta (2008) does not discuss teleological modality, she proposes that the ranking should be based entirely on similarity to the actual world. Krasikova (2010) also takes the ordering to be similarity to the actual world, but takes the objects of comparison to be  $p \wedge q$  and  $p \wedge q'$  rather than q and q'. Her proposal is equivalent in many cases to having two ordering sources, a primary one based on p being true (where  $g(w) = \{p\}$ , using the notation of Kratzer (1981)), which we might call the teleological ordering source, and a secondary one based on similarity.<sup>21</sup> Finally, von Fintel and Iatridou (2005) propose to analyse teleological modality with strong modals (as opposed to weak modals such as *ought*) based only on the teleological ordering, with a modal base determined by physical possibility. We will adopt von Fintel and Iatridou's (2005) proposal here because it is simpler, and because the intuition that the actual world plays no particular role in determining the truth of teleological modal statements seems right; however our choice is not crucial as we will discuss.

If we adopt the teleological ordering, we can show that (33) is in fact completely equivalent to (34). Indeed, given our quasi-trivial order, if q' includes any p-world, then q'is at least as good a possibility as any other proposition. For q to be a better possibility than q', it has to be the case that q' should not be at least as good a possibility as q, and therefore that q' should be incompatible with p (within the bounds set by the modal base). Similarly, if q is incompatible with p, then anything other than a contradiction is at least as good a possibility as q; so again for q to be a better possibility than q', it has be the case that either q is compatible with p, or q' is a contradiction. Ignoring the pathological case where all the alternatives to q are contradictions, we can rephrase our statement as follows:

(34) (simplified, to be refined) "To p, have-to q" is true iff q is compatible with p, and for any alternative q' to q (other than q itself), q' is incompatible with p.

With a secondary ordering source based on similarity to the actual world or plausibility, the result we get is that either q is compatible with p and q' is not, or both q and q' are compatible with p, but  $p \wedge q'$  is a more remote possibility than  $p \wedge q$ . We can note that, if q's only alternative is  $\neg q$ , then we are essentially asserting that p is incompatible with  $\neg q$ , or in other words that p entails q, over the domain of best worlds according to the secondary ordering source. This is more or less "normal" necessity.<sup>22</sup> We therefore still allow for "proper" necessity readings in cases where there is VERUM focus.

# 4.2 Adding effort to the ordering source: Krasikova's analysis of the pragmatic cases

Krasikova (2010) claims that the alternative-sensitive view of modality is sufficient to solve the prejacent problem of von Fintel and Iatridou (2007), as well as account for other MS examples not inolving *only*, involving both "pragmatic" scales (as in (4)) and the numeral scale (as in (5)). She argues, as we have done in Section 2.1, that scalarity is an essential feature of these examples. As we do, she assumes that, in (3a), the alternatives to "go to the North End" are a set of places you might think you could go to to buy cheese, and that a pragmatic ordering on these alternatives is contextually available.<sup>23</sup>

 $<sup>^{21}</sup>$ Cf. the discussion of systems of this kind by von Fintel and Iatridou (2005, sec. 6).

<sup>&</sup>lt;sup>22</sup>To be precise, if we drop the Limit Assumption, then defining necessity in our alternative-sensitive way and assuming VERUM focus (ALT(q) = { $\neg q$ }) will yield a notion that Kratzer (1991) calls weak necessity and that is strictly weaker than the notion she calls necessity tout court.

 $<sup>^{23}</sup>$ Krasikova (2010) considers that the prejacent problem, i.e. the lack of an MS reading for the cheese example without *only*, is purely due to the fact that one doesn't accommodate a context where an ordering is salient if that is not made obligatory by *only*; this is also how we have accounted for the contrast.

Second, she assumes that in the face of such pragmatically-ordered alternatives, the modality will be interpreted with respect to an ordering source that involves effort, or equivalently, it will be added to the goal that you want to minimize effort.<sup>24</sup> Thus, the prejacent of (3a) ends up paraphrasable as (35).

(35) To get good cheese and minimize your effort, you have to go to the NE.

(35) asserts that finding good cheese when  $\phi_{NE}$  is true (you go to the NE) is a better possibility than for both  $\phi_C$  (you cross the street) and  $\phi_{SB}$  (you go to South Boston), the former because it is not plausible to find good cheese by crossing the street (this is essentially Minimality) and the latter because it is more effort to go to South Boston than to the North End. In effect, this analysis deals with failure of Necessity w.r.t. higher scalemates by assuming that we do in fact have Necessity, but not under the ordering source that we would expect.<sup>25</sup> Sufficiency also follows: if going to the North End is better than crossing the street even though the latter is more minimal, it has to be the case that going to the North End is compatible with getting good cheese.<sup>26</sup>

There are several concerns to find with this analysis. A first one is that the mechanism by which "minimalism" enters the ordering source is not clear. Presumably it is purely a matter of context. Notice, then, that we predict that depending on contextual variation of what is perceived as most effortful, MS examples could be used to indicate upper bounds on pragmatic scales rather than lower bounds. This might in fact be the case, as seen in (36).

(36) Context: John is completely dominating his sport, and hasn't failed to win a race in years. He couldn't lose if he tried.
John has to get a bronze medal in the semis to not make the finals.
→ He'll make the finals iff he gets a silver or gold medal.

However, (36) and other attempts at getting upper-bound readings of teleological *have-to* have a certain emphatic flavour. Here, it seems to only be acceptable if we accept that John will never get anything below bronze (in which case there is no failure of Necessity), as is shown by the perceived incoherence of the discourse in (37a) (compare (37b)).

- (37) a. Context: same as above.
   #?John has to get a bronze medal in the semis to not make the finals. Perhaps he will not get a medal at all.
  - b. Context: John is not particularly favourite in this competition. John has to get a silver medal in the semis to make the finals. Perhaps he will ?(in fact) get gold.

Thus, we might want our analysis not to allow these reversals, since the lower-bound examples do not have this emphatic flavour and do not set such strong conditions on the context. Besides, it seems problematic to assume that (3a) involves a ranking of worlds such that going to South Boston is considered remote, given the validity of the discourse in (38); if we assume that the possibility modal receives its standard semantics and that

<sup>&</sup>lt;sup>24</sup>If we analyse this interpretation as proceeding from an additional ordering source, then presumably this ordering source should rank after the teleological ordering source, but before considerations of plausibility in the hierarchy of orderings.

<sup>&</sup>lt;sup>25</sup>Failure of Necessity w.r.t. non-scalemates is not dealt with, a point we will come back to.

 $<sup>^{26}</sup>$ This is not quite logical Sufficiency, which we take to be desirable; cf. discussion in Footnote 29.

the necessity and possibility modals are evaluated relative to the same ordering, then (38) should be a contradiction.

(38) To get good cheese, you only have to go to the North End; you can also go to South Boston.

#### 4.3 The case of the numeral scale

The problem becomes clearer when one at looks at numeral examples such as (39) (repeated from (5)). Krasikova (2010) assumes that numerals are generally interpreted under an "exactly" reading. Under the alternative-sensitive analysis, (39) then essentially asserts that drinking exactly two beers and being over the limit is a better possibility than being after the limit after having drunk exactly one beer or exactly three beers. This follows from the meaning of *have-to* adopted in (33) together with the natural assumption that the alternatives to the condition in (39) are those given in (40).

- (39) To be over the legal limit, you have to drink TWO beers.
- (40)  $A = \{\text{"you drink } n \text{ beers"} | n \in \mathbb{N} \}$

Once again, we predict that reversal of the scale should be possible in appropriate contexts, that is, there should be a reading of examples like (39) asserting that to meet the goal, it is sufficient to drink two beers or less than that, in a context where drinking less is perceived as less of an effort. This prediction does not appear to be borne out, as seen in (41a) (compare (41b)). Even as part of a discourse that would make an "at-most" reading of the condition natural, the reading is not observed. Instead, inasmuch as the first sentence of (41a) is acceptable, it appears to presuppose that at least two beers will be drunk.<sup>27</sup>

- (41) a. #To stay under the limit, you have to drink TWO beers. For instance, if you drink just one beer you will be under the limit.
  - b. To be over the limit, you have to drink TWO beers. For instance, if you drink three beers you will be over the limit.

The difficulty in "reversing the scale" makes another approach towards numeral examples attractive: if we instead assume that numerals are interpreted in an "at least" fashion, then we no longer expect the direction of the scale to be context-dependent. Indeed, as we have seen, even with a traditional, strong account of *have to*, we do not expect (39) to entail that drinking three beers does not put you over the limit as long as the numeral is interpreted as "at least two". In other words, the problem of failure of Necessity w.r.t. higher scalemates is at first glance avoided. We therefore do not need to postulate an additional mechanism that tinkers with the ordering source. Additionally, there are independent reasons to think that numerals are generally interpreted with an "at least" reading (cf. Spector 2013 for an overview of the issue).

The problem with the "at least" semantics for numerals is that it makes our analysis of *have to*, as given in (33) break down:

• We predict that drinking at least two beers is a better possibility than drinking at

 $<sup>^{27}</sup>$ This is compatible with an analysis of the first sentence of (41a) where the numeral is interpreted as "exactly" and *have to* means regular Necessity. Indeed, in a context where it is presupposed that you will drink at least two beers, there is no failure of Necessity in this example.

least one. Yet the altter proposition entails the former, and as already mentioned, if q entails q', then q' is always at least as good a possibility as q, and thus q is not a better possibility than q'. Then (39) should be a trivial falsehood.

• We predict that drinking at least two beers is a better possibility than drinking at least three. Given our world knowledge on how drinking relates to being over the limit, it is not plausible that drinking at least two beers should be compatible with being over the limit but drinking at least three should not. Thus, assuming a teleological ordering source, the only way this can be true is if the modal base excludes drinking at least three beers. With a secondary realistic ordering source, another way for it to be true would be if drinking at least three beers were a more remote possibility than drinking exactly two. There is no particular reason why either of these should be the case in many natural contexts where (39) is acceptable.

It is feasible to fix these two issues by tweaking the analysis. First, we want that the objects of comparison include some proposition equivalent to "you drink exactly one beer". This is indeed the set of worlds that we want to indicate represent a bad possibility, and not the set of worlds where you drink at least one beer. Second, we want that alternatives that lead to tautologies and contradictions not enter the computation. We may accomplish this by modifying our analysis of the teleological modal:

(42) (refined) "To p, have-to q" is true iff for any alternative q' to q, w.r.t. an ordering based on p being true, q is a better possibility than  $q' \wedge \neg q$ .<sup>28</sup>

If we restrict our attention to the teleological ordering source, this can be simplified as follows:

(43) (refined and simplified) "To p, have-to q" is true iff q is compatible with p and for any alternative q' to q,  $q' \land \neg q$  is not compatible with p.

With the new condition, the statement that (39) makes with respect to the *n*-alternative for  $n \ge 2$  is trivial. Indeed, for  $n \ge 2$ , "you drink (at least) two beers" (q) is entailed by "you drink (at least) *n* beers" (q'), and therefore  $\neg q \land q'$  is a contradiction, and as we already noted, any non-contradiction is trivially a better possibility than a contradiction. This is how we get failure of Necessity w.r.t. higher scalemates. As far as n = 1 is concerned, what (39) says is that "you drink two beers" is a better possibility than "you drink one beer but not two", i.e. that drinking at least two beers is compatible with being over the limit but drinking exactly one beer isn't. This is close to Minimality (though we will see that it is slightly wrong). As with the degree-based analysis, we can get Sufficiency through *exhaustification*: if the minimal amount of beers you had to drink was three, (39) would be true, but so would the same sentence with "three" instead of "two". As we desire, we do not derive Necessity: all what we know about drinking (exactly) two beers is that

<sup>&</sup>lt;sup>28</sup>There are potential alternatives to this particular refinement that would do as well for our purposes. One consists in redefining the "at least as good a possibility as" relation so that it only quantifies over worlds that are in certain subsets of q and p. Kratzer (2012), for independent reasons, modifies the definition in this way, replacing q by q - p and p by p - q in the restrictor of the quantifiers. While her specific definition turns out to behave badly in our case, similar changes could allow us to stay closer to the "simple" version of the analysis outlined in (33). Alternatively, we could use exhaustification mechanisms to generate an object of comparison equivalent to "you drink exactly one beer" rather than build it in in the semantics. We discuss an idea along these lines in Section 4.4.

it is *compatible* with being over the limit, leaving open whether drinking something else entirely is a possibility.<sup>29</sup>

As an interim conclusion to our discussion of the numeral cases, what we have seen here is that the alternative-sensitive approach can deliver failure of Necessity w.r.t. nonscalemates and Sufficiency in two ways. If, following Krasikova (2010), we assume "exactly" semantics for the numeral, and we allow for some notion of minimalism to make it to the ordering source, we make the unwelcome prediction that the scale could be reversed in the appropriate context. If we instead adopt "at least" semantics for the numeral, and we modify the specific analysis of teleological modals, we can avoid the problematic prediction, although the entry for the modal that we end up with is perhaps *ad hoc*. Under either approach, the issue described in Section 3.3 does not occur: we have kept the condition strong enough that we are still clearly talking about beer-drinking worlds, and our (enriched) semantics properly entails that drinking exactly two beers is compatible with being over the limit.

#### 4.4 Why we are still getting MS wrong

Unfortunately, the analysis of the beer example (5)/(16)/(39) we have been developing in this section is faced with the following major problem, in both of its incarnations: it does not actually deliver failure of Necessity w.r.t. non-scalemates. Indeed, we end up predicting that drinking exactly one beer is incompatible with being over the limit, where "exactly one beer" means "one beer but not two beers". However, this cannot be true in any sensible context: someone may very well be over the limit after having drunk one but not two beers, as long as they have been drinking wine as well, and we may still judge (39) true.

Put another way, the problem is that the worlds we wanted to rule out were those where you drink exactly one beer *and no wine*. Instead, we ruled out worlds where you drink one beer and not two.

A somewhat natural move consists in assuming that the domain of worlds that the modal quantifies over is reduced to worlds where nothing else than beer is drunk. This could be taken to be a case of loose association with focus as described by Beaver and Clark (2008). The embedded clause's focus structure is anaphoric on a salient situation where beer is drunk (as this is what the disjunction of all alternatives amounts to). The modal's domain variable (or its implicit modal base, as one will have it) is identified to the corresponding set of worlds containing a situation of this kind through a higher-level

(i) Students have to submit TWO squibs to pass.

<sup>&</sup>lt;sup>29</sup>One may be find it concerning that we do not derive logical sufficiency either: our semantics does not entail that drinking exactly two beers *will* reliably put you over the limit. We argue that this result is correct. In this particular example, it proceeds from world knowledge that, if drinking two beers can put you over the limit, then it will generally do so. In the more general case, MS does not have to come with logical sufficiency. (i), for instance, exhibits failure of necessity in the sense that it may accurately describe a situation where students who do a presentation do not have to submit anything to pass. It also exhibits failure of (logical) sufficiency in the sense that it does not necessarily imply that students who submit two squibs will pass. It may be that students who pass through submitting squibs, rather than doing a presentation, also have attendance requirements. It may also be that the squibs have to be up to a certain standard. What (i) does imply is that students do not need to submit three squibs, and that submitting one squib won't help them pass, which we take to be all what MS means.See also Section 4.1 of von Fintel and Iatridou 2007 for a closely related point.

pragmatic process.

It should be noted that the focus structure we're dealing with does not in fact presuppose or suggest in any way that no wine is drunk. (44) is perfectly compatible with John having drunk some wine, and does not even suggest otherwise. On the contrary, narrow focus suggests that John may very well have drunk wine, but that whether he did is irrelevant for the speaker's purposes.

(44) John drank TWO beers yesterday.

Thus, if we rely on domain restriction, we cannot say that it proceeds from loose association with focus. We have to assume that non-systematic domain restriction occurs until the utterance is not trivially false, with worlds where no wine is drunk being perhaps more salient, accessible or prototypical than other beer-drinking worlds.<sup>30</sup>

#### 4.5 Local conclusion

Let us sum up our discussion of the alternative-sensitive approach. We can implement it in two ways: either we let some consideration of minimal effort, or something of this kind, enter the ordering source, and we adopt "exactly" semantics for numerals, as Krasikova (2010) does, or we slightly complicate the modal and adopt "at least" semantics for numerals.<sup>31</sup> In both cases, we account for failure of Necessity w.r.t. higher scalemates and for the appropriate notion of Sufficiency (the condition stated is compatible with the goal). In both cases, we avoid the problem described in Section 3.3. In only the former case, we make a possibly unwanted prediction that the direction of the scale can be reversed by context. Most importantly, in both cases, we derive a notion of Minimality that is too strong: we predict that the "bad" propositions and the goal are actually incompatible, even though we have seen that this is not actually tenable in our examples. We are essentially missing failure of Necessity w.r.t. non-scalemates.

We might try to weaken the predicted meaning by assuming that considerations of physical likelihood or typicality can affect the ordering source or induce domain restriction.

Recall that disjunctive MS consists in cases where a disjunction can be used to express MS along two scales at once, as in (i) (repeated from (30)).

(i) To be over the legal limit, you have to drink TWO beers or THREE glasses of wine.

Crucially, (i) does not specify whether drinking one beer *and* two glasses of wine puts you over the limit or not. Then, we want to analyse it in such a way that we do not predict that a world where you have drunk both beer and wine is ruled out. If all we have is domain restriction, we are going to need the domain to be a set of worlds where beer or wine are drunk, but not both at the same time. It is implausible that this disconnected domain would be implicitly accommodated through a pragmatic process.

 $^{31}{\rm The}$  second option can be extended to pragmatic cases if we assume that AT-LEAST is present in the relevant examples.

 $<sup>^{30}</sup>$ It may appear that once we have such domain restriction, failure of necessity is no longer a problem. Indeed, because we are only quantifying over worlds where no wine is drunk, it becomes actually true that you have to drink two beers to be over the limit. Then, since the fact that this was false was our motivation for introducing AT-LEAST-DEG or changing the semantics we assign to the modal, it appears that there is nothing remarkable going on any more in the beer case, and that for the other examples we may stick to the analysis of Alonso-Ovalle and Hirsch (2018). Here is the problem: we already had to stretch domain restriction beyond what prosodic presuppositions have to offer. Extending a domain-restriction analysis to cases of disjunctive MS is going to require an even more *ad hoc* notion of domain restriction.

However, in our examples, the situations we want to restrict ourselves to (drinking beer but nothing else, submitting squibs but not doing a presentation) do not seem to be especially likely or prototypical, and as far as I can tell, the context dependency that one would expect if likelihood entered into consideration is not there.<sup>32</sup>

Thus, our problem does not appear to be easily solved through established views on how modals might be alternative-sensitive, which tends to suggest that the "second route", where we weaken the embedded clause, was the more promising one. However, it might be the case that another implementation of the alternative-sensitive intuition, possibly based on a different definition of better possibility, would prove more successful and let us avoid the problems that we identified for the degree-based account.

## 5 Issues with the presupposition of *only* and the status of Minimality

The issue we discuss in this section is a puzzling feature of MS examples using *only*; since the rest of our discussion mostly focussed on examples without *only*, this section is relatively orthogonal to it. It does, however, make the various theories come apart on a specific point: whether Minimality is linked to the prejacent inference of *only*.

Recall that we stated that the cheese example (3a) implicates in some way that you cannot get good cheese across the street, a meaning component we call Minimality, and that we took this fact as evidence against the decompositional analysis due to von Fintel and Iatridou (2007). In both the analysis based on degrees, and, a priori, an analysis along the alternative-sensitive line, Minimality is an entailment of MS statements without only. Thus, Minimality in MS statements with *only* can be seen as an instance of the inference to the prejacent. If, following Horn (1969), we take the inference to the prejacent to be a presupposition, then Minimality should be a presupposition. In contrast, under the decompositional analysis as originally proposed by von Fintel and Iatridou (2007), an existential presupposition conspires with the at-issue meaning of *only* to produce the inference to the prejacent in the general case. In an MS statement, this does not occur and Minimality is not generated at all. If we adopt the "fixed" decompositional analysis as described in Section 2.2, Minimality will be a presupposition that is triggered below the modal. An alternative view truer to the spirit of von Fintel and Iatridou's would consist in generating Minimality as an implicature: to do so, we can take (3a) only entails Sufficiency and has a trivial presupposition, for instance through the entry for QUE in (45), and additionally assume that it has as an alternative (46). When (3a) is uttered, a scalar implicature to the effect that (46) is false is triggered, which is how Minimality obtains.

(45) 
$$[\![QUE \, p]\!] = \lambda w : [\exists p' \in ALT(p), \, p'(w) = 1]. \, [\exists p' \in ALT(p), \, p' \succ p \land p'(w) = 1]^{33}$$

<sup>&</sup>lt;sup>32</sup>Possibly, a solution would involve the notion of minimal situations in the sense of situation semantics (Kratzer 2019). Intuitively, the reason that, for instance, in the scenario described in Footnote 29, (i) is true is that a greater share of the sufficient conditions for passing are met when you write two squibs than when you write just one. In particular, if we know that a student wrote two squibs, the information that they meet the attendance requirements lets us conclude that they will pass, while otherwise it does not. From this intuition we can derive a notion of comparative possibility over situations, where a situation is better than another in view of a goal if adding extra information to it will let us conclude that the goal is attained in more cases. Unfortunately, as far as I can tell, this approach does not lead to a notion of alternative-sensitive necessity that reduces to the traditional view in simple cases, which seems like a problem.

(46) To get good cheese, you only have to cross the street.

To summarize, under both the degree-based and the alternative sensitive approaches, Minimality in (3a) proceeds from the inference to the prejacent (traditionally seen as a presupposition). In the decompositional analysis, Minimality will not exactly be an instance of the inference to the prejacent, and in particular we can generate it as a scalar implicature under a specific version of the analysis.

In view of the debate on *only*'s prejacent, it is therefore interesting to look at properties of the Minimality inference in MS examples, and see whether we can ascertain its status as a presupposition or an implicature. The main fact speaking in favour of the view that Minimality is a presupposition is that there is a contrast between the Minimality inference that MS statements trigger, and the similar inference deriving from an actual statement of sufficiency using *enough*, as in (47a). To begin with, we already discussed the fact that (3a) is somewhat degraded in a context where you can in fact get good cheese across the street. It appears there is a contrast with (47a), which seems to be more universally accepted (from an informal survey). In a similar fashion, the "enough" statement might be more easily defeasible, as seen in (48), though the judgement is not clear.

- (47) Context: you can in fact get good cheese in the neighbourhood.a. ?To get good cheese, it is enough to go to the North End.b.#?To get good cheese, you only have to go to the North End.
- (48) a. ?To get good cheese, it is enough to go to the North End. In fact, you can also find some across the street.
  - b. ??To get good cheese, you only have to go to the North End. In fact, you can also find some across the street.

The two constructions also contrast in questions, where MS Minimality appears to project, as judged by the degraded status of (49b), while "enough" Minimality disappears. All this points to an analysis of the Minimality inference triggered by *enough* statements as a *bona fide* scalar implicature: it is deafeasible and disappears in questions. In contrast, the MS statement is of dubious defeasibility, and projects from questions, suggesting it is a presupposition. All this would speak against the view that Minimality is an implicature, and therefore against the decompositional account as outlined above.

- (49) I have no idea where you can get good cheese in Boston.
  - a. Is it enough to go to the North End?
  - b. #Do you only have to go to the North End?

There are, however, also arguments in favour of the implicature analysis. To begin with, as we have already mentioned, the Minimality inference still seems to be easily defeasible, as speakers surveyed generally accept (48b) even when they perceive a contrast. Presuppositions are generally not defeasible, at least when they are generated in unembedded environments (presumably because what is presupposed is also entailed). Note also that in the degree-based analysis and the alternative-sensitive analysis, Minimality proceeds from the prejacent inference; yet as discussed by Beaver and Clark (2008, chap. 9), the prejacent inference of *only* is not cancellable in simple positive sentences like (50).<sup>34</sup>

<sup>&</sup>lt;sup>33</sup>Note the use of  $\succ$  rather than  $\succeq$ .

 $<sup>^{34}</sup>$ Beaver and Clark (2008) in fact argue that the defeasibility patterns of the prejacent inference in un-

#### (50) Muriel only likes Hubert. (#In fact, she doesn't like Hubert.)

Another puzzling fact is that when an MS statement with *only* occurs in the antecedent of conditionals, Minimality does not constrain either the overall context or the antecedent. The reported and predicted behaviour of presuppositions (cf. e.g. Heim 1983) is that they should "project" from the antecedent of a conditional, that is to say, they should constrain the overall context. One may easily construct examples such as (51) where the prejacent presupposition of *only* fails to project, and is instead accommodated into the antecedent (as if it were entailed): (51) asserts that Muriel will accept if the world is such that she likes Hubert (prejacent) and noone else (assertion). In similar MS cases, Minimality neither projects (as we might expect, if it is a presupposition) nor is locally accommodated (as we might expect, if it is a prejacent inference); it just disappears. Example (52a) does not imply that you in fact have to drink two beers to go to jail (this would make it a contextual contradiction), and it also does not imply that (52b) is true. (52b) is in fact judged to be a plain falsehood. This is exactly what one would expect under the implicature analysis: implicatures are known to disappear in the antecedent of a conditional (because it is a downwards-entailing environment). In fact, we observe exactly the same thing when we look at the "enough" statement and its well-behaved Minimality implicature.

- (51) If Muriel only likes Hubert, she will accept, but I doubt she likes Hubert.
- (52) a. If you only have to drink two beers to go to jail, then you might still go to jail after just one beer.
  - b. #If you have to drink two beers to go to jail, then you might still go to jail after just one beer.
  - c. If it is enough to drink two beers to go to jail, then you might still go to jail after just one beer.

(Brian Buccola, p.c.)

To summarize, the Minimality inference in MS contrasts with the well-behaved implicature of *enough* statements. This first set of facts suggest it should be seen as a presupposition, which could be derived from a presuppositional prejacent inference. Yet, it also exhibits certain behaviours that are abnormal for a presupposition, and in this respect it does not behave like the prejacent inference in simpler sentences. This second set of facts speaks in favour of a view of Minimality as an implicature and is compatible with a version of the decompositional analysis. MS statements with *only* therefore constitute a challenge for analyses of MS along all routes discussed in this article, as well as for various views on the prejacent inference of *only*. We do not have a solution to this puzzle to offer here.

## 6 Conclusion

This article discusses three routes towards accounting for MS statements with minimal disruption to Montagovian composition. They consist in, respectively, weakening *only* (the decompositional analysis), weakening the condition (the degree-based analysis), and weakening the modal (the alternative-sensitive approach). All three routes have arguments

modalized positive and negative sentences are fully consistent with neither an analysis as a presupposition nor as an implicature, but that they can be understood under a refined version of the presuppositional view.

in their favour *a priori*. Yet, they quickly run into a number of challenges.

The decompositional analysis ties in well with other observations of unexpected weakness of *only* (Horn 1996, a.o.) as well as interactions between modals and negative elements (von Fintel and Iatridou 2007). Yet, the weakening of *only* might be problematic in some other ways (Alonso-Ovalle and Hirsch 2018). Additionally, in the implementation of von Fintel and Iatridou (2007), the Minimality inference is wholly absent and at best seen as a side-effect of another component, Easiness, whose origin is not accounted for. Alternative implementations can reintroduce Minimality proper, but only if we are willing to make the novel assumption that silent comparative elements exist. Finally and most problematically, MS statements not involving *only* (where Minimality is clearly part of the truth conditions) are unexplained.

The degree-based approach "keeps *only* strong" (Alonso-Ovalle and Hirsch 2018), does not require any unusual assumption about the composition of the sentence, and can account for MS statements without *only*. Its main defect is that it results in extremely weak truth conditions for MS statements, from which we cannot recover our intuition as to what Sufficiency is (Section 3.3). This gap can plausibly be explained away by invoking manner implicatures or "leaving it to the pragmatics" in some other way.

The alternative-sensitive approach is very promising *a priori* in that it builds up on a more general account of alternative-sensitivity in modals (Villalta 2008). It also keeps *only* strong and eschews the main problem of the degree-based approach. However, if paired with "exactly" numerals, it makes the incorrect prediction that the direction of the scale is a matter of context. If paired with "at least" numerals, it does not weaken the truth conditions enough, and we still predict that MS statements should be plain false in contexts where we judge them to be true.

Finally, the fact that in MS examples involving *only*, the Minimality inference does not behave exactly like a presupposition nor like a scalar implicature, is problematic for all approaches, inasmuch as they can only make one or the other prediction.

In the end, the issue of how to integrate MS statements into our understanding of modal semantics remains as pressing as ever.

## References

- Alonso-Ovalle, Luis and Aron Hirsch (2018). "Keep only strong". Semantics and Linguistic Theory 28. Ed. by Sireemas Maspong et al., pp. 251–270.
- Bale, Alan C. (2008). "A universal scale of comparison". Linguistics and Philosophy 31.1, pp. 1–55.
- Beaver, David I. and Brady Z. Clark (2008). Sense and sensitivity: How focus determines meaning. Oxford: Wiley-Blackwell.
- Coppock, Elisabeth and David I. Beaver (2014). "Principles of the Exclusive Muddle". Journal of Semantics 31.3, pp. 371–432.
- von Fintel, Kai and Sabine Iatridou (2005). "What to do if you want to go to Harlem: anankastic conditionals and related matters". Ms. Massachussetts Institute of Technology.
- von Fintel, Kai and Sabine Iatridou (2007). "Anatomy of a modal construction". *Linguistic Inquiry* 38.3, pp. 445–483.
- Greenberg, Yael (2019). "A scalar presupposition with universal force: *only* as (only) a scalar mirror image of *even*". Ms., Bar Ilan University.

- Groenendijk, Jeroen and Martin Stokhof (1984). "Studies on the Semantics of Questions and the Pragmatics of Answers". PhD thesis. Univ. Amsterdam.
- Heim, Irene (1983). "On the projection problem for presuppositions". In: Formal semantics: the essential readings, pp. 249–260.
- Horn, Laurence R. (1969). "A presuppositional analysis of only and even". Papers from the fifth regional meeting of the Chicago Linguistic Society. Ed. by Robert I. Binnick et al., pp. 98–107.
- Horn, Laurence R. (1996). "Exclusive company: Only and the dynamics of vertical inference". Journal of semantics 13.1, pp. 1–40.
- Klinedinst, Nathan (2004). "Only scalar only". Unpublished work.
- Krasikova, Sveta (2010). "Sufficiency inference in anankastic conditionals". *Proceedings* of SALT 20. Ed. by Nan Li and David Lutz, pp. 91–108.
- Kratzer, Angelika (1981). "The notional category of modality". In: Words, worlds, and contexts: New approaches in word semantics. Ed. by Hans J. Eikmeyer and Hannes Rieser. De Gruyter, pp. 38–74.
- Kratzer, Angelika (1991). "Modality". In: Semantics: An international handbook of contemporary research. Ed. by Arnim von Stechow and Dieter Wunderlich. Berlin: de Gruyter, pp. 639–650.
- Kratzer, Angelika (2012). "The Notional Category of Modality". In: Modals and Conditionals: New and Revised Perspectives. Oxford University Press.
- Kratzer, Angelika (2019). "Situations in Natural Language Semantics". In: *The Stanford Encyclopedia of Philosophy*. Ed. by Edward N. Zalta. Summer 2019.
- Magri, Giorgio (2017). "Blindness, Short-sightedness, and Hirschberg's contextually ordered alternatives: a reply to Schlenker (2012)". In: *Linguistic and psycholinguistic approaches on implicatures and presuppositions*. Ed. by Salvatore Pistoia-Reda and Filippo Domaneschi. London: Palgrave Macmillan, pp. 9–54.
- Rett, Jessica (2014). "The polysemy of measurement". Lingua 143, pp. 242–266.
- Rooth, Mats (1992). "A theory of focus interpretation". Natural Language Semantics 1.1, pp. 75–116.
- Schwarz, Bernhard (2005). "Scalar additive particles in negative contexts". Natural Language Semantics 13.2, pp. 125–168.
- Spector, Benjamin (2013). "Bare numerals and scalar implicatures". Language and Linguistics Compass 7.5, pp. 273–294.
- Villalta, Elisabeth (2008). "Mood and gradability: an investigation of the subjunctive mood in Spanish". *Linguistics and Philosophy* 31.467.