An Algebra of Thought that Predicts Key Aspects of Language Structure

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Abstract

The Meaning First Approach hypothesizes that humans can form complex non-linguistic representations in an 'Algebra of Thought' independent of any language used in communication. Since the Algebra of Thought and language nevertheless must be related, one research program is to reverse engineer the Algebra of Thought from what is known about language. In this paper, we focus on universal structural properties of human languages. We investigate an Algebra of Thought fragment containing logical conjunction, a part-whole relationship and two cognitive efficiency requirements that exclude redundancies. We show that at least three universal structural properties of languages follow from these assumptions: cartographic hierarchies, the obligatory decomposition of non-symmetric binary predicates, and the obligatory lexical content of dependent elements in binding dependencies.

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Abstract. The Meaning First Approach hypothesizes that humans can form complex non-linguistic representations in an 'Algebra of Thought' independent of any language used in communication. Since the Algebra of Thought and language nevertheless must be related, one research program is to reverse engineer the Algebra of Thought from what is known about language. In this paper, we focus on universal structural properties of human languages. We investigate an Algebra of Thought fragment containing logical conjunction, a part-whole relationship and two cognitive efficiency requirements that exclude redundancies. We show that at least three universal structural properties of languages follow from these assumptions: cartographic hierarchies, the obligatory decomposition of non-symmetric binary predicates, and the obligatory lexical content of dependent elements in binding dependencies.

Keywords: structure, efficiency, conjunction, adjectives, decomposition, binding

1. Introduction

Humans are capable of forming complex thoughts and of communicating these to each other using complex sentences. One goal of research on this ability of our species is to understand what the primitive elements and relations forming complex thoughts and sentences are. We approach these questions within the Meaning First Approach (MFA) of (Sauerland and Alexiadou, 2020).

Two assumptions are central to the MFA: For one, structures are built not in a language but in the *Algebra of Thought*. The primitives are concepts; mathematical objects, that only contain information needed for interpretation. In addition, there is the Algebra of Thought (or *Generator* in Sauerland and Alexiadou 2020) which provides complex thought structures closed under a binary algebraic operation. We take the primitive concepts to be nothing but properties of *Models*, while complex nodes are compositionally mapped to properties of Models as discussed in the following. *Models* are abstract mereological structures formed to partially represent perceived or imagined sensory states. In the following, we call a structure generated by the Algebra of Thought

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a *Conceptual Representation*, abbreviated as *CR*. Second, at least some CRs can be articulated with the goal of communication. The articulation process involves linking parts of the CR to the morphemes of a specific language (or sometimes multiple languages) and imposing a linear order on the morphemes. We assume that linearization generally respects the constituency of CRs (see below). Furthermore, we assume that speakers compress as much as possible given what they want to convey if multiple articulations of the CR are possible in a language. Especially logical elements that connect content words tend to be predictable and so wouldn't be pronounced. For example, while in English 'Three or four friends came' is perfectly well-formed, its German counterpart is literally 'Three four friends came' (*Drei vier Freunde sind gekommen*) without an explicit disjunction *oder* ('or'). In other cases, many primitive concepts are bundled into a single articulated morpheme for communication. If speakers choose not to compress, this often results in a manner implicature.

In the MFA model, language provides us quite a direct window to the mind since the structure of a CR and that of the corresponding sentence closely match one another. Only compression makes it difficult to determine the CR of a sentence. If we want to reverse engineer the human mind, we need research strategies to develop models of the Algebra of Thought (AoT) and explore their predictions. Four criteria that can decide between different models of the AoT are listed in (1).

(1) a. Expressivity: AoT should match the expressivity of language.
b. Simplicity: A simple AoT is preferred.
c. Homophonies: An AoT should capture as many homophony relations of logical words as possible.
d. Constituency: The AoT structures should be constrained so as to predict the constituency of natural languages.[2]

A special source of evidence are cases of undercompression—cases like that of *or* above, where you see that one language realizes a concept but another doesn't when you line up two languages. Undercompression is particularly striking when children undercompress in comparison to the adult language in their environment (some cases are mentioned below, also see Guasti, Alexiadou, and Sauerland 2022).

The constituency criterion (1d) is the one we mostly explore in this paper. Specifically, we develop an AoT calculus that derives a constituency without syntactic categories or uninterpreted formal features. In addition, we forego any calculus of semantic types since types are frequently used as an alternative formalization of syntactic categories (Montague 1974 and others). The Algebra of Thought that we explore in this paper consists of operations that have already been in use in formal semantics. The central algebraic operations are conjunction and the part-whole relationship. In addition, we adopt (and adapt) two notions of cognitive efficiency, namely exhaustification and minimality. But we completely avoid other common formal concepts such as function application and even variable binding. Section 2 shows how exhaustification and conjunction derive a restriction to 'cartographic' trees. Section 3 introduces the part-whole relationship and a notion of minimality to derive 'non-cartographic' trees, but also predicate-argument relationships from '∃-Union' and minimization. Section 4 discusses how '∃-Union' can derive non-local copredica-

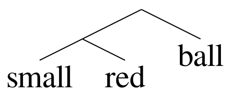
[2]We leave open the possibility that some valid CRs can be ineffable, but expect any language to be able to express almost all valid CRs derived from the core and acquired primitive concepts.

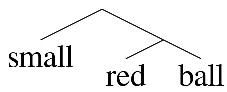
tion in configurations commonly analyzed as involving variable binding. In section 5, we conclude with a review and an outlook on the remaining expressivity gap.

2. Conjunction and Cartography

Conjunction is generally assumed to be one semantic composition principle (Davidson, 1967; Heim and Kratzer, 1998; Pietroski, 2018). There seem to be three good reasons to assume that conjunction is available to the human mind. First of all, conjunction in a broad sense is a necessity for remembering any information—when the immune system remembers to produce antibody A for virus α and antibody B for virus β, we describe that state as a conjunction. Secondly, conjunction is present in animal communication and intersententially. And thirdly, logical systems where conjunction is not a primitive seem not suited for the Algebra of Thought.[3]

But bare conjunction cannot be the sole composition principle of the Algebra of Thought if our goal is to predict phrase structure. Conjunction, as it stands, would not even predict sentence boundaries. More generally, the associativity of conjunction entails that conjunctive composition would not predict any restrictions on constituency at all, as is easy to see. Recall that associativity means that for any p, q, and r that can be conjoined, $p \wedge (q \wedge r) = (p \wedge q) \wedge r$. But the constituent structure of language does not exhibit associativity. For example, evidence from prosodic phrasing (Chomsky and Halle, 1968) and other sources argues that the phrase *small red ball* can only have the structure in (2b).

(2) a. 

b. 

We will refer to this as the *Associativity Problem* of meaning composition by conjunction. Associativity does not arise as a problem if constituency is captured by a syntactic calculus such as a phrase structure (Chomsky, 1957) or a categorial grammar (Ajdukiewicz, 1935). But associativity gets in the way of any attempts to reduce as much of constituency as possible to other properties of grammar. Specifically, the core assumption of the MFA that structure generation takes place in the AoT independent of language.

How can we overcome the associativity problem? We adopt the well-established idea that a type of cognitive efficiency—exhaustification—is obligatorily imposed on certain parts of a complex structure (Magri, 2009; Chierchia, 2013; Meyer, 2013). We understand exhaustification at this point broadly as a requirement that the contribution of a substructure P to the whole must not be replaceable by any alternative equally or less complex substructure Q (Katzir, 2007). Specifically, we propose to impose a requirement to invoke a form of exhaustification on one of the conjuncts, but never the other. This asymmetry between the two conjuncts renders conjunction non-associative, thus solving the associativity problem.

(3) A complex CR $[\alpha\ \beta]$ can be interpreted conjunctively only if exactly one of α or β is

[3]Specifically, lambda calculus with identity can represent conjunction as (i) (Tarski 1923). But even though Tarski's result is mentioned in a classic linguistic paper by Montague (1974), it has remained without influence in linguistics. We cannot address a different thread here of reducing one of conjunction or disjunction to the other (Zimmermann 2000; Meyer 2013; Bowler 2015; Singh et al. 2016; Tieu et al. 2017).

(i) $\lambda x \in D_t\ \lambda y \in D_t\ ((\lambda f \in D_{tt}\ [f{=}f]) = (\lambda f \in D_{tt}\ [x{=}[f(x){=}f(y)]]))$

exhaustified.

The further technical implementation of our proposal we discuss in the context of a concrete case of composition that is frequently understood to be conjunctive: the cartography of adjectives. Dixon (1977), Cinque (1994) and others have argued that across languages the hierarchical order of multiple adjectives exhibits universal preferences. For example, the order in (4a) is preferred in English over the one in (4b).

(4) a. the small red ball
b. #the red small ball

The same preference is present in all other languages, but importantly it is a hierarchical preference, not a linear one. Therefore in languages like Mokilese (Harrison, 1976) where the noun is initial, the preferred order of adjectives is the opposite of that in English:[4]

(5) pwo:la wa:ssa siksikko
ball red small-DET

We follow recent work by Scontras, Degen, and Goodman (2017) and Scontras, Degen, and Goodman (2019) that argues that the preferred hierarchical order of adjectives is determined by semantic properties of the adjectives. Scontras, Degen, and Goodman (2017) establish experimentally for English that the order in (6) holds and that the order preference correlates with the subjectivity of the adjectives as independently tested by faultless disagreement and other subjectivity criteria. The generalization is that the more objective description an adjective provides, the closer to the underlying noun position it occurs.

(6) dimension $\ll$ value $\ll$ age $\ll$ physical $\ll$ shape $\ll$ color $\ll$ material

It is important to note that the English linear order of adjectives is unhelpful for efficient communication. For example, if the listener's task is viewed as identifying the noun phrase referent, the listener would more rapidly identify the correct referent intended by a speaker given the information provided by *red* compared to the information provided by *small*, since speaker and hearer are more likely to agree on which objects are *red* than *small*.[5]

[4]A third type of language reported noun-initial, but the adjective order is that of English as illustrated by Gaelic (Sproat and Shih, 1991, p. 587). We follow the cartographic literature and assume that in such languages the noun is also related to the final position and its initial position is due to the mechanism frequently referred to as *movement* (see also the next footnote).

(i) liathroid bheag bhui
ball small yellow

[5]That the English order is ill-suited for communication may explain why languages like Gaelic exist (see footnote 4), while we don't find any reports of counterparts with the reverse linear order of Gaelic: noun-final noun-phrases, but with the Mokilese adjective order. It is possible to derive this from the assumptions that 1) the position of the noun is determined in the same way as the order of adjectives, but nouns are inherently more objective than adjectives, and 2) linear orders deviating from the universal hierarchical order can be present in a language only if they improve communicative efficiency.

Scontras, Degen, and Goodman (2019) show that the cartography of adjectives can be derived from a mechanism that implements a form of cognitive efficiency, but their mechanism is *ad hoc* for adjective cartography. We suggest instead that the essence of Scontras et al.'s proposal can and should be embedded within general principles of cognitive efficiency, specifically exhaustification.

Recall from above that we assume conjunction is inherently asymmetric in that one of the conjuncts must be exhaustified. Concretely, we assume that exhaustification is marked by **exh** which satisfies the following condition:[6]

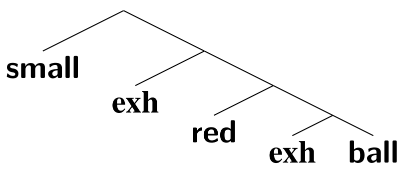
(7) The CR [**exh** p] has the inferences that descriptor p is true and that any *salient, non-worse descriptor* q is false.

What is a *salient, non-worse descriptor*? We assume that at least all properties q occurring in the same algebraic conceptual representation as an occurrence of **exh** are salient (Katzir, 2007). Furthermore, we assume that a property is a *non-worse descriptor* than another if it is more objective and also if it is more informative. Since the likelihood of two speakers choosing the same entity increases with logical strength and also increases with objectivity, we restate this unification slightly more formally as in (8). Adjectives of a semantic class such as shape, color, material and others behave the same. We adope the term *domains of jurisdiction* from (Paillé, 2022) for these classes, and propose that how good a descriptor a predicate is determined only by its domain of jurisdiction to capture the order.[7]

(8) $p \geq q$ (i.e. p is a *non-worse descriptor* than q) if and only if, on average, for any two individuals s_1 and s_2, the expected likelihood of s_1 and s_2 choosing the same entity x given a description from the domain of jurisdiction of p is not lower than when given a description from the domain of jurisdiction of q.

Consider how these assumptions derive the basic case in (4). A complex conceptual representation conjoining the three concepts **red**, **small**, and **ball** must contain two occurrences of **exh**. Here and in the following we use a special font for concepts that are not fully logical. One possible CR that could be articulated as *small red ball* is the following:

(9)



We now show that CR (9) is predicted to be possible by our approach. By the condition on salience, the alternatives to the sister of each **exh** are at least **small**, **red** and **ball**. In addition, any complex constituent that occurs in the structure is also a salient alternative for exhaustification.

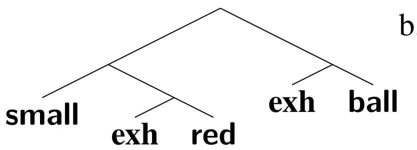
[6]We tacitly assume the presuppositional version of **exh** of Bassi, Del Pinal, and Sauerland 2021 since it makes it easier to handle some case where **exh** might otherwise scopally interact with other operators.

[7]In future work, we hope to derive the role of the domains of jurisdiction from a decomposition of adjectives into logical and experience-based components and the proposal that cognitive efficiency is only sensitive to logical properties (Gajewski, 2002; Chierchia, 2013).

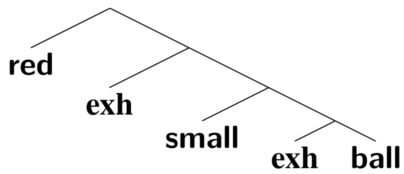
First consider only the three alternatives **small**, **red**, and **ball**. Since we assume that nominal concepts are always better descriptors than adjectives,[8] the constituent '**exh** ball' doesn't exclude the worse descriptors **small** and **red**.[9] For the complex CR '**exh [red exh ball]**', we need to determine how good a description complex CRs are. One corollary of (8) is that if $p \geq q$ then also $p \wedge r \geq q$ since $p \wedge r$ is more informative than p. Therefore, **small** $\leq$ **red** $\wedge$ **ball** holds, and **small** is not excluded by the exhaustification of [**red** [**exh ball**]].

Consider now why the two CRs in (10) are excluded. For (10a), the constituent **exh red** is predicted to exclude the alternative **ball**, which is a better descriptor. At the same time, **exh ball** has the inference that **ball** holds. As a result, the entire CR (10a) is a logical contradiction: **ball** is predicted to not hold and hold simultaneously. Therefore (10a) cannot underlie the articulation of *small red ball*, and only CR (9) is available.

(10) a.



b.



The CR (10b) would in English be articulated with the marked word order in (4). Consider the exclusions arising for the constituent **exh [small [exh ball]]** in (10b). If **red** $\wedge$ **ball** is a better descriptor than **small** $\wedge$ **ball**, **red** $\wedge$ **ball** and consequently **red** is predicted to be excluded by **exh [small [exh ball]]**, making (10b) contradictory. To capture the general cartographic order preference, we assume that the following independence assumption is a general default for complex concepts.

(11) For any three predicates p, q, r from different domains of jurisdiction: If and only if $q \geq r$, also $q \wedge p \geq r \wedge p$ holds.

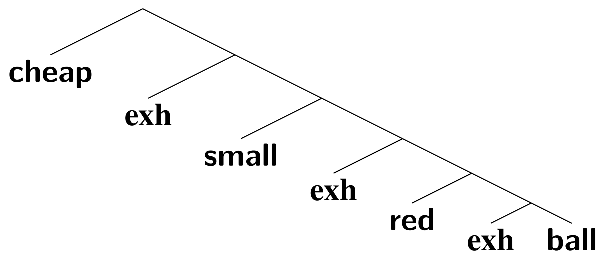
If the independence equivalence is satisfied at least in the rightward 'if' direction, it follows that no contingent CR could lead to the word order *red small ball* in English (4). In sum, the only contingent CR that can be formed from the three concepts **small**, **red**, and **ball** is (9).

The result that only a single CR is possible generalizes also to conjunctions of more than three predicates. Consider the example of adding **cheap** to the previous three concepts. CR (12) is predicted to be contingent, correlating with the possible English phrase *cheap small red ball*.

[8] At this point, we do not know of any prior work claiming this, but it seems intuitive as for example the noun *French* is more specific than the adjective *French*.

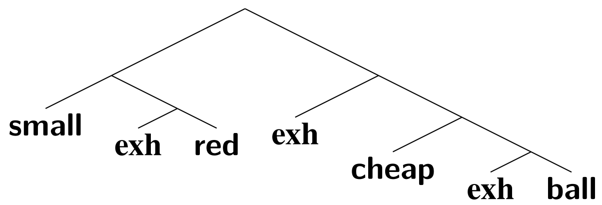
[9] We assume that both nominal and adjectival concepts are decomposed into a idiosyncratic meaning part and at least one core concept characteristic of the category, such as possibly **object** for some nouns (as in current work within Distributed Morphology, cf. Borer 2005).

(12)



But any other structure is predicted to lead a contradictory exclusion. Consider the potential CR in (13), where for example the constituent **[exh cheap [exh ball]]** excludes **red** and **small** leading to a contradiction.

(13)



We refer to CRs like (12) as cartographic structures and contrast them with non-cartographic structures like (13). Generally, cartographic structures are binary trees where any node has at most one branching sub-node. Let us assume furthermore that any two predicate concepts, p and q, are either mutually exclusive when they belong to the same domain of jurisdiction[10] or, if p and q belong to different domains of jurisdiction, either $p > q$ or $q > p$ must hold. Then the system of conjunctive composition we developed in this section can be described as follows: Any CR where all composition is conjunctive must have a cartographic structure; namely the one where the c-command structural order is the inverse of the total order provided by the $>$ relation.

The general result has some desirable implications as cartographic structures have also been argued for in other domains such as adverbs (Alexiadou 1997, Cinque 1999, and others). But there are also cases where non-cartographic structures must be possible as we discuss in the next section.

3. Parts and Predicate-Argument Relations

In this section, we explore one idea to allow non-cartographic trees which involves introducing the part-whole relationship into the algebra. Let us consider an example that, as far as we know, uncontroversially has a non-cartographic structure:[11]

(14) Small grandmas eat grey wolves.

Our suggestion to capture this is to argue that CRs can contain the part-whole operator $\exists\sqsubseteq$ in (15). Like **exh**, $\exists\sqsubseteq$ is a logical primitive concept. Both **exh** and $\exists\sqsubseteq$ combine with their sister not by conjunction, but by function application.

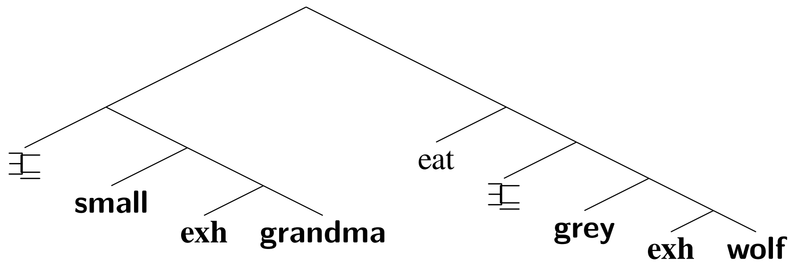
[10] Paillé 2022 argues that the mutual exclusivity is derived from lexical exhaustivization.

[11] To better focus on structural properties of interpretation, we disregard the obligatory expression of nominal number and verbal aspect of English here and in the following.

(15) For any CR X denoting the property p, $[\exists\sqsubseteq X]$ is a CR denoting the property $\lambda x\ \exists y \sqsubseteq x \,.\, p(y)$.

For concreteness, we understand the properties in (15) to be properties of formal models that our cognitive system can form. We assume that, at some level, (14) involves a model of eating that has at least two parts: one that satisfies the properties **small** and **grandma** and another that satisfies the properties **wolf** and **grey**. The introduction of $\exists\sqsubseteq$ allows non-cartographic structures because properties of a part and the whole or another part are not logically related. For example, the possible CR underlying (14) in (16) contains the constituent **small [exh grandma]**. One alternative that **exh** in this constituent excludes is the concept **wolf**. Crucially this exclusion does not lead to a contradiction because it is possible that one part of a model is a wolf, while another part isn't.

(16)



At this point, we are not aware of any motivation to restrict the distribution of $\exists\sqsubseteq$ extrinsically. But $\exists\sqsubseteq$'s distribution is intrinsically restricted if it is correct that total predicates are always better than partial ones, i.e., for any p, q, $p > \exists\sqsubseteq q$. It then follows that **exh** in (17a) will exclude p and therefore always be contradictory. Only (17b) will be generally possible. For (17c), a contradiction arises at least if $\exists\sqsubseteq p \geq \exists\sqsubseteq q$ since $\exists\sqsubseteq p$ would be excluded, while cases with $\exists\sqsubseteq p < \exists\sqsubseteq q$ are predicted to be contingent.[12] In the following, we only make use of the configuration in (17b).[13]

(17) a. $*p \wedge$ **exh** $\exists\sqsubseteq q$
b. $\exists\sqsubseteq p \wedge$ **exh** q
c. $\exists\sqsubseteq p \wedge$ **exh** $\exists\sqsubseteq q$

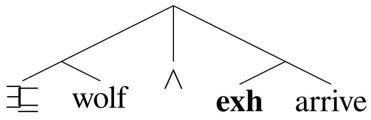
We have seen that adding the $\exists\sqsubseteq$ primitive makes non-cartographic structures possible. But the predicted semantics are at this point too weak: the meaning predicted for (17) requires neither the small grandmas nor the grey wolves to play any particular part in the eating. Even a model of children eating cookies in the presence of small grandmas and grey wolves would be sufficient. The same problem arises even in less complex sentences like (18) for which we show a possible

[12] The prediction changes though if the deactivation (i.e. pruning) of some alternatives is assumed to be possible. (Paillé, 2022) proposes that applying **exh** to a partial property as in (17c) renders it total. This follows from his assumption that all other partial predicates from the same domain of jurisdiction are excluded, and that the domain of jurisdiction is a partition of the possible states and objects. But he allows pruning in coordinations such as *the yellow and black fur* to derive the effect that the describe fur is partially yellow, partially black, and of no other color. His account is however not fully compatible with ours. Specially, we predict that **exh**$\exists\sqsubseteq q$ should always exclude the total predicate q.

[13] The restriction to (17b) raises the possibility to model the Algebra of Thought using *lists* as implemented in the programming language LISP instead of binary trees.

underlying CR to its right. (When writing out CRs, we sometimes indicate conjunctive constituents by $\wedge$ for readability.)

(18) Wolves arrive.



We assume that the concept **arrive** can be semantically specified as in (19); i.e. roughly as true of models in which someone/thing arrives.

(19) **arrive** = $\lambda m\ \exists x \sqsubset m\ .\ x$ arrives in m

The CR in (18) will however be true even of models where grandmas arrive at the wolves place and a wolf just happens to be present at its home. We see that (18)'s meaning needs in some way to require more than just the model containing some wolf and someone/thing arriving: it needs to require that some wolf is arriving.

A straightforward way of strengthening the semantics of (18) in an appropriate way is to require that models be minimal in the way that the following **min** operator captures:

(20) For any CR A, [**min**A] is a valid CR and [**min**A] is true only of those models m that satisfy A and contain the smallest possible number of elements.

We assume that application of **min** is obligatory in some positions, which need to be specified in future work. For now consider the effect **min** exerts when it applies to the CR of (18) as in (21).

(21) **min**[[$\exists\sqsubseteq$**wolf**] $\wedge$ [**exh arrive**]]

Above we considered as problematic a scenario of a grandma arriving at a location where a wolf happens to be present. Any model of such a scenario contains at least three entities: a grandma, a wolf and arrival. But a model with only two entities can also satisfy CR (18); namely, one containing only a wolf and an arrival. But since no other entities are contained in such a model, the wolf must be responsible for the arrival. For this reason, the models satisfying (21) will all be models where a wolf arrives.

We will use the term $\exists$-*Union* for the effect that minimization by **min** has on existential quantification in its scope. We will describe this effect for (21) in a different algebraic system—standard first order logic—as follows. In (21), there are two existential inferences made: one explicit by $\exists\sqsubseteq$ and another that is implicit in the concept **arrive** (cf. (19)). Using first order logic, we can display the effect of **min** as in (22). It amounts to replacing two existential quantifiers with narrow scope, with a single existential quantifier that takes scope at the position of **min** and binds all the variables the two single quantifiers bound.

(22) **min**[$\exists x$ wolf$(x) \wedge \exists y$ arrive(y)] $\Longleftrightarrow \exists z$ [wolf$(z) \wedge$ arrive(z)]

Importantly, $\exists$-Union does not rely on the use of indexed variables in CRs even though it derives the effect that coindexation has in first order predicate logic.

Wherever **min** applies, $\exists$-Union affects almost all existential quantifiers in its scope for the conjunctive CRs considered so far. The only exception are two existential quantifiers that express logically inconsistent claims. For example, if we were to replace **arrive** with the negation of **wolf**, then the minimal model containing both a wolf and a non-wolf necessarily contains two entities. Therefore $\exists$-Union would not have the effect it has in (22) with two inconsistent descriptions.

In sum, the introduction of **min** and, with it, $\exists$-Union makes a number of interesting predictions that have consequences for how CRs must be structured to capture different meanings. We are ready at this point to explore some of these predictions and their linguistic consequences. In the remainder of this section we will discuss predictions related to local predication and in the following section we will discuss predictions for non-local predication (or dependencies).

The first consequence we discuss concerns transitive verbs. Specifically, we derive that transitive verbs must be decomposed, as has been proposed in much work within lexical semantics as well as syntactic approaches to the lexicon (see, e.g., Alexiadou, Borer, and Schäfer 2014 for an overview). This follows from the obligatory reflexivization of binary predicates which we demonstrate using example (23).

(23) Grandmas eat wolves.

We want to show that the meaning of a transitive verb like *eat* cannot be captured by means of a single concept such as (24) within the current set of assumptions.

(24) $\textbf{eat}_1 = \lambda m\ \exists x \sqsubset m\ \exists y \sqsubset m\ .\ x \text{ eats } y \text{ in } m$

Consider first the CR in (25) (we omit obligatory **exh**-operators here and in the following, unless they play an important role). The second **min**-operator in (25) has the effect of reflexivizing the concept $\textbf{eat}_1$: the constituent **min** $[\textbf{eat}_1 \wedge \exists\sqsubseteq \textbf{wolf}]$ can only be true in models where a wolf eats itself.

(25) $\textbf{min}[\exists\sqsubseteq\textbf{grandma} \wedge \textbf{min}[\textbf{eat}_1 \wedge \exists\sqsubseteq\textbf{wolf}]]$

Therefore (25) doesn't capture the meaning of (23). But the same holds for other conceivable CRs involving the concept $\textbf{eat}_1$. Since wolves can also be grandmas, (26) is predicted to also be reflexivized and can then only be true in models where a wolf-grandma eats itself.

(26) $\textbf{min}[\exists\sqsubseteq\textbf{grandma} \wedge \textbf{eat}_1 \wedge \exists\sqsubseteq\textbf{wolf}]$

If we assume that **exh** can apply as in (27), a different but equally unsuitable meaning results. The subscripts on **exh** in (27) notate a salient, non-worse alternative that **exh** excludes. While **exh** thereby prevents the $\exists$-Union of $\exists\sqsubseteq\textbf{wolf}$ and $\exists\sqsubseteq\textbf{grandma}$, (27) is predicted to allow models where wolves eat grandmas or themselves or grandmas eat wolves or themselves.

(27) $\mathbf{min}[\underset{\sqsubseteq}{\exists}\mathbf{exh}_{\mathsf{wolf}}\textsf{\textbf{grandma}} \wedge \textsf{\textbf{eat}}_1 \wedge \underset{\sqsubseteq}{\exists}\mathbf{exh}_{\mathsf{grandma}}\textsf{\textbf{wolf}}]$

Reflexivization could also be prevented within the meaning of the transitive verb by the adoption of the concept $\textsf{\textbf{eat}}_2$ in (28).

(28) $\textsf{\textbf{eat}}_2 = \lambda m\ \exists x \sqsubset m\ \exists y \sqsubset m\ .\ x$ eats y in m and $x \neq y$

Obviously, $\textsf{\textbf{eat}}_2$ would struggle to explain actual reflexive uses of the verb *eat*. But even putting that aside, $\textsf{\textbf{eat}}_2$ would also not provide an account of the meaning of (23). For example, the CR in (29) is true of either models where a grandma eats a wolf or ones where a wolf eats a grandma.[14]

(29) $\mathbf{min}[\underset{\sqsubseteq}{\exists}\textsf{\textbf{grandma}} \wedge \textsf{\textbf{eat}}_2 \wedge \underset{\sqsubseteq}{\exists}\textsf{\textbf{wolf}}]$

In sum, non-symmetric transitive verbs are predicted to be impossible as primitive concepts.We will suggest as a path forward to decompose transitive verbs. Before we do that, we briefly mention another case of reflexivization that supports our contention that minimization can lead to reflexivization. Namely, impersonal existential and reflexive pronouns can be homophonous. This has been reported for Italian *si* (Cinque, 1996), Polish *się* and Slovenian *se* (Rivero and Sheppard, 2003).

(30) a. Tutaj się pracuje sporo. (POLISH)
here REFL work-3s much
'Here people work a lot.' (Rivero and Sheppard, 2003, p. 92)

b. Janek ubiera się.
John dresses self
'John gets dressed.' (Rivero and Sheppard, 2003, p. 99)

Work on Italian child language by Silleresi et al. (2023) indicates that both the existential and the reflexive use of *si* emerge in Italian children's production at the same age (namely 1;8 years). The simultaneous emergence argues further that the homophony of impersonal *si* and reflexive *si* is not accidental. As Silleresi et al. (2023) argue, the homophony can be explained if we assume that the reflexive meaning can be derived from the existential via minimization.

The prediction that transitive verbs must generally be decomposed matches findings from the study of argument and event structure. Stechow (1996) and Beck and Johnson (2004) argue that particles like *wieder* ('again') provide evidence for a decomposition of transitive and ditransitive verb meanings. Different lines of work propose that each argument must be introduced by a single predicate drawn from a universal inventory (e.g., Parsons 1990; Pylkkänen 2008). Rappaport Hovav and Levin (2001, p. 779) also conclude that 'There must be at least one argument XP in the syntax per subevent in the event structure.' Pietroski (2018) also states the empirical generalization that only unary predicates exist. What is novel in the present approach is that it derives from a theoretical framework that binary predicates are unavailable, with the possible exception of

[14] Concepts with an inequality requirement like $\textsf{\textbf{eat}}_2$ may provide an account of symmetric relations other than identity such as *similar* and *sister* (Schwarz, 2006).

symmetric predicates (see footnote 14).

In the following, we focus only on causation. For causation, evidence from undercompression in child language further supports the decomposition of verbs. Martin et al. (2022) report that *faire* ('make') is used with causative verbs redundantly by French children as in (31):

(31) va le faire couper (Marilyn, 2;9)
go it CAUSE cut
'(I'm) going to cut it.'

Causation is neither an experience-based concept like **wolf** nor is it solely a logical concept like **exh**, **min**, or $\exists\sqsubseteq$. Carey (2009) classifies causation as a core concept rooted in a specialized cognitive system not directly related to language. We use a different font for the concept CAUSE to mark this distinction. The contribution of CAUSE to a CR can be captured by non-conjunctive composition as follows:[15]

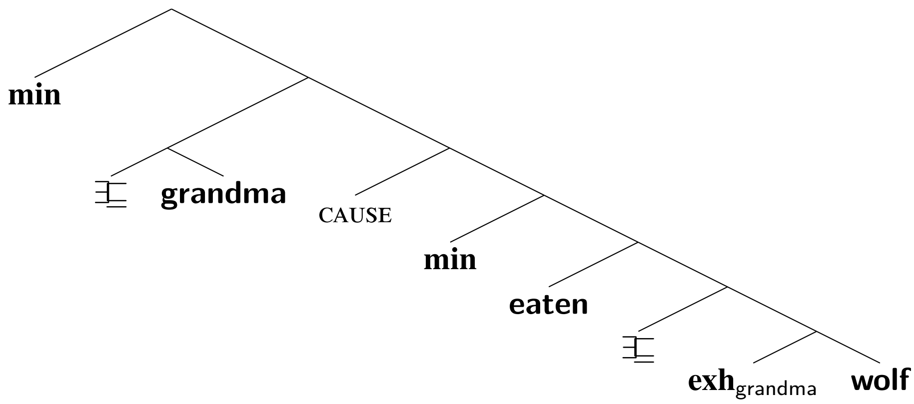
(32) For any CR A, $[$CAUSE $A]$ is a valid CR and $[$CAUSE $A]$ is true only of those models m such that there exist $m', m'' \sqsubset m$ such that m'' makes A true and m' causes m'' in m.

In addition, we assume that the root meaning of *eat* is captured by the following concept **eaten**:

(33) **eaten** $= \lambda m \exists x \sqsubset m \,.\, x$ is eaten in m

Using the light verb CAUSE, the meaning of (23) can now be captured by the following CR:

(34)



The constituent **min**[**eaten**[$\exists\sqsubseteq$**wolf**]] in (34) is true only of models containing exactly an eaten

[15] It may also be conceivable to capture causation fully by conjunctive composition on the basis of a lexical entry such as (i). But this would require an understanding of how CAUSE$_1$ differs from **eat**$_1$ and **eat**$_2$. To prevent reflexivization, the assumption that the cause x and the caused entity y are different is sufficient and this assumption is plausible for causation. This would lead to a CAUSE$_2$ analogous to **eat**$_2$. Still the problem of restricting CAUSE$_2$ to one direction of the cause leading to the caused entity would remain.

(i) CAUSE$_1 = \lambda m \exists m', m'' \subseteq m \,.\, m'$ causes m'' in m

wolf and **exh** blocks ∃-union with **grandma**. Therefore, the minimal models containing the causation relation are ones where the concept **grandma** is ∃-unified with the existential quantification over the cause that CAUSE introduces.

The last prediction we mention in this section concerns modification. Consider how a modifier structure as expressed by the sentence in (35) can be captured by a CR.

(35) Wolves similar to grandmas arrive.

Note first that the CR in (36) does not correctly capture the meaning conveyed by (35); we omit **exh** in (36) for perspicuity. There are a number of reasons for this, but one is that there is no asymmetry between **wolf** and **grandma** in (36). Assume for the purposes of this argument that **wolf** and **grandma** were logically inconsistent properties.[16] Then any minimal model for (36) would necessarily contain both a wolf and a grandma. But since the existential quantification in **arrive** is consistent with either **wolf** or **grandma**, (36) is not predicted to require the wolf to arrive. Instead it could also be the grandma.

(36) $\mathbf{min}[[\exists\!\sqsubseteq[\exists\!\sqsubseteq\mathsf{\mathbf{wolf}} \wedge \exists\!\sqsubseteq\mathsf{\mathbf{grandma}} \wedge \mathsf{\mathbf{similar}}]][\mathsf{\mathbf{arrive}}]]$

To capture the meaning of (35), a CR akin to (37) is therefore necessary, again omitting any required occurrences of **exh**.

(37) $\mathbf{min}[\mathbf{min}[\exists\!\sqsubseteq[\exists\!\sqsubseteq\mathsf{\mathbf{wolf}} \wedge \exists\!\sqsubseteq\mathsf{\mathbf{grandma}} \wedge \mathsf{\mathbf{similar}}]] \wedge \mathbf{min}[\exists\!\sqsubseteq\mathsf{\mathbf{wolf}} \wedge \mathsf{\mathbf{arrive}}]]$

At this point, the empirical consequences of this prediction need to be explored in further work. It is noteworthy that the modification structure sketched in (37) resembles the structure of correlatives.

This section started as an exploration of one addition to the inventory of logical concepts, the part-whole relation $\exists\!\sqsubseteq$, with the goal of allowing non-cartographic structures. We saw that though $\exists\!\sqsubseteq$ makes non-cartographic CRs possible, to derive the right interpretation of such CRs requires further additions: the minimization operator **min** and light verbs like CAUSE. The major novel result accomplished in this section was to derive the almost obligatory decomposition of non-unary predicates. The ratio between assumptions and results in this section therefore does not clearly validate the path we have chosen to explore. But in addition to the decomposition result, the system developed so far has a second major consequence that comes entirely for free that we have so far only hinted at, namely the treatment of dependencies via ∃-Union, which we discuss in the following section.

4. Dependencies with ∃-Union

We mentioned in the previous section that ∃-Union derives an effect on the interpretation of CRs that in predicate logic can be expressed by coindexation of variables. But the Algebra of Thought we are proposing derives this effect without the use of variables or similar mechanisms. In this section, we argue that ∃-union in conjunction with exhaustification provides an empirically superior account of three phenomena frequently analyzed as variable binding: donkey anaphora,

[16] Actually the omitted **exh** brings the inconsistency about.

bound pronouns, and syntactic movement chains.

Recall the effect of ∃-Union shown in the formalism of predicate logic in (38), repeated from (22).

(38) $\textbf{min}[\exists x\ \text{wolf}(x) \wedge \exists y\ \text{arrive}(y)] \iff \exists z\ [\text{wolf}(z) \wedge \text{arrive}(z)]$

In (38), it is possible for the same entity to fulfil the scope of both the existential $\exists x$ and the existential $\exists y$. The **min**-operator therefore requires the two existentials to be verified by the same entity, which derives the equivalence to the wide scope existential with coindexation of variables across the two scopes.

If, however, **exh** applies in both scopes as in (39), ∃-Union is blocked.[17] This follows from the assumption that **exh** excludes all non-worse alternative concepts occurring in the same CR in conjunction with the assumption that **arrive** must be non-worse than **wolf** or vice-versa.

(39) $\textbf{min}[\exists x\ \textbf{exh}_{(\textsf{arrive})}\text{wolf}(x) \wedge \exists y\ \textbf{exh}_{(\textsf{wolf})}\text{arrive}(y)]$

There are two exceptions, though—cases where even if **exh** applies to parts of the scope of two existentials, ∃-Union will nevertheless unify the two. In the first case, the two arguments of **exh** are identical, e.g. both are **wolf**. In the second case, one of the arguments of **exh** with CR α entails the other argument with CR β; α is also more complex than β and the CR of α doesn't contain any subconstituents α' that are of equal or lower structural complexity than a subconstituent β' of β where furthermore α' is a not-worse description than β'. The two examples in (40) illustrate the second case. In both cases, the first occurrence of **exh** does not result in an exclusion of the scope of the second occurrence of **exh**. In (40a), **exh** also causes no exclusion because the full scope, **wolf** $\wedge$ **grey**, is more complex than the first subconstituent, **wolf**, and the second subconstituent, **grey**, is a worse descriptor than the scope of the second **exh**. But in (40b), the second occurrence of **exh** will exclude **wolf** if **wolf** is not more complex than **grey**[18] since **wolf** is a better descriptor than **grey**.

(40) a. $\textbf{exh}[\textbf{wolf} \wedge \textbf{grey}]$, $\textbf{exh}_\emptyset$ **wolf**
b. $\textbf{exh}[\textbf{wolf} \wedge \textbf{grey}]$, $\textbf{exh}_{\textsf{wolf}}$ **grey**

Note that the first case above is reducible to a special case of the second. (41) illustrates a case where there is an entailment relation between two scopes that at least seem to be of equal structural complexity, but where ∃-union is blocked. In this case, we need to consider **wolf** as a subconstituent of **wolf** that is not a worse descriptor than **grandma** is.

(41) **exh wolf**, $\textbf{exh}_{\textsf{wolf}}$ **grandma**

The types of structures where long dependencies are uncontroversially attested mostly go beyond

[17] The way **exh** blocks dependencies is reminiscent of the proposal of Chomsky (1980).

[18] Recall than both **wolf** and **grey** may have more internal structure than shown here, though it also remains to be seen whether this affects structural complexity in the relevant sense.

the current fragment of the Algebra of Thought. For presentational purposes, we introduce restricted universal quantification into our present model of the Algebra of Thought by means of two novel syncategorematic concepts. The first, ∂ in (42), converts a truth-condition on models into a presupposition adopting a standard trivalent perspective of model properties with the truth values 1, #, and 0.

(42) $[\partial\, p] = \lambda m : \begin{cases} \# & \text{if } p(m) \neq 1 \\ 1 & \text{otherwise} \end{cases}$

Furthermore, we add universal quantification on the assumption that bound elements are presupposed by ∂ as follows:[19]

(43) $[\forall\, p] = \lambda m : \forall m' \sqsubseteq m \, . \, p(m') \neq \# \;\rightarrow\; p(m') = 1$

We can now give CRs that capture the meaning of some core examples of binding dependencies. We illustrate donkey anaphora with (44).

(44) Always if grandmas eat wolves, they burp.

The CR in (45) contains the constituent $[\exists\!\sqsubseteq\!\mathbf{exh}_{\mathsf{wolf}}\ \mathsf{\mathbf{grandma}}]$ in a position corresponding to that of the pronoun *they* in (44). The **min**-operator in the immediate scope of $\forall$ $\exists$-unifies the existential quantification introduced by this constituent with the other occurrence of the concept **grandma**.[20]

(45) $[\forall[\mathbf{min}[$
$\quad [\partial[\exists\!\sqsubseteq\!\mathsf{\mathbf{grandma}}[\textsc{cause}[\mathsf{\mathbf{eaten}}\exists\!\sqsubseteq\!\mathbf{exh}_{\mathsf{grandma}}\mathsf{\mathbf{wolf}}]]]]$
$\wedge$
$\quad [[\exists\!\sqsubseteq\!\mathbf{exh}_{\mathsf{wolf}}\ \mathsf{\mathbf{grandma}}]\mathsf{\mathbf{burp}}]]]]$

As far as we can see there is no way of capturing the intended meaning of (44) without involving two occurrences of the **grandma** concept within the Algebra of Thought we proposed here. Heim (1990) and Elbourne (2006) have provided some arguments that donkey anaphora require an account involving compressed content related to the pronoun.

The account for bound pronouns is similar, which we illustrate by means of (46). The interpretation we target is one that requires any grandma x to eat any wolf y in case x and y are similar.

(46) All grandmas eat wolves similar to them.

[19] We put aside for now that the universal quantifier defined here is not persistent in the sense of Kratzer, 1989 and therefore would not exert universal force in the scope of **min**.

[20] The exclusion by **exh** must have a modal component to it for the account to be fully satisfactory (see also Sauerland 2007). As it stands, grandmas who are also wolves are predicted to be irrelevant for the truth conditions of (45) because **exh** excludes them.

In the CR in (47), the two entities involved in the concept **similar** are specified further by the concepts **wolf** and **grandma**. The application of **exh** ensures that ∃-Union is blocked from requiring the relevant grandmas to also be wolves. Sauerland (2000) and Sauerland (2008) argues that bound variable pronouns have silent lexical content.

(47) $$[\forall[\mathbf{min}[[\partial[\exists\!\sqsubseteq\mathbf{grandma}]] \wedge [\textsc{cause}[[\mathbf{min}[\mathbf{eaten}[\exists\!\sqsubseteq\mathbf{wolf}]]] \\ \wedge\ \mathbf{min}[\exists\!\sqsubseteq[\mathbf{exh}_{\mathsf{grandma}}\mathbf{wolf}] \wedge \exists\!\sqsubseteq\mathbf{grandma} \wedge \mathbf{similar}]]]]]]$$

The final application of ∃-Union we discuss involves dependencies frequently referred to as syntactic movement chains. There are many subcases of syntactic movement chains, and we can only selectively address the phenomenon here. The most frequently discussed case of syntactic movement chains, constituent questions, is beyond the expressive power of the current proposal because questions are usually modeled as sets of propositions. As discussed above, modification structures may already illustrate one case where syntactic movement chains are claimed to arise, namely the case of relative clauses. For example, only one of the two occurrences of the concept **wolf** in (47) is articulated, as is typical of syntactic movement chains.

We focus now on the relative clause in (48).

(48) Grey wolves that grandmas eat arrive.

The CR in (49) captures the interpretation of (48). In (49), the constituent **grey wolf** occurs twice, and as in the cases discussed above, the present model of the Algebra of Thought requires at least some repetition in order to capture the meaning of (48).

(49) $$\mathbf{min}[[\mathbf{min}\ [\exists\!\sqsubseteq[\mathbf{grey\ wolf}] \wedge \mathbf{arrive}]] \\ \wedge\ [\mathbf{min}\ [[\exists\!\sqsubseteq\mathbf{grandma} \wedge [\textsc{cause}\ [\mathbf{eaten} \wedge \exists\!\sqsubseteq\mathbf{exh}_{\mathsf{grandma}}[\mathbf{grey\ wolf}]]]]]]]$$

The way movement chains need to be represented in the present system is consonant with evidence that has been given for the syntactic copy theory of traces (Chomsky, 2015; Fox, 1999; Sauerland, 2004; Romoli, 2015). But these accounts invoke indexed variables as in predicate calculus and, if a syntactic requirement to create copies was not invoked, dependent elements could also be represented simply as indexed variable. The present account does not invoke any syntactic theory. The need for dependent elements to be something close to copies is a consequence of the Algebra of Thought we propose.

In (40) we showed that ∃-union allows the unified elements to differ from one another with predictable limits. For instance, an alternative CR for (48) is (50), where one occurrence of **grey wolf** is reduced to **wolf**. As we argued above, though, a reduction to **grey** instead of **wolf** is predicted to be impossible. Related asymmetries have been discussed with the term *late adjunction* in the literature.

(50) $$\mathbf{min}[[\mathbf{min}\ [\exists\!\sqsubseteq[\mathbf{grey\ wolf}] \wedge \mathbf{arrive}]] \\ \wedge\ [\mathbf{min}\ [[\exists\!\sqsubseteq\mathbf{grandma} \wedge [\textsc{cause}\ [\mathbf{eaten} \wedge \exists\!\sqsubseteq\mathbf{exh}_{\mathsf{grandma}}[\mathbf{wolf}]]]]]]]$$

In sum, we have seen in this section that the central case of binding dependencies can be reduced to $\exists$-Union. The resulting mechanism is different from two existing proposals in mathematical logic (predicate logic and combinatorial logic) to model dependencies/co-argument relations.

Our proposal predicts that dependent elements are required to share conceptual content with one another. In the case of movement chains, shared content is often articulated in one place in the adult language. Child languages, however, exhibit undercompression phenomena that support the view developed in this section (Labelle 1990, Hu, Cecchetto, and Guasti 2018, Yatsushiro and Sauerland 2018, and others). Example (51) illustrates this type of evidence:

(51)	Ich	möchte	das Mädchen	sein,	das	der Opa	das Mädchen	umarmt.
	I	want	the girl	be	who	the granddad	the girl	hugs

I want to be the girl who the granddad hugs. (Yatsushiro and Sauerland, 2018)

5. Conclusions

We presented a sketch of an Algebra of Thought based on the Meaning First Approach of Sauerland and Alexiadou (2020). We focused specifically on modeling some of what is known about the constituent structure of language. The model that we presented accounts, at least in part, for three important universal properties of language: cartographic hierarchies as presented in Section 2, the obligatory decomposition of non-symmetric binary predicates as presented in Section 3, and the requirement of lexical content for dependent elements as presented in Section 4.

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