Disjunctive comparisons

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Disjunction can convey a conjunctive meaning when it occurs in the matrix clause of a comparative sentence. This happens in the absence of expressions that usually accompany, and have been studied in relation to, occurrences of disjunction conveying conjunctive meanings (say, modal expressions). Similar behavior is observed with *any NP*s, which can occur in the matrix clauses of comparative sentences and therein convey meanings akin to conjunctive ones. Dual exhaustification, applying once at the sentence level and once at the embedded level of degree predicates, gains us purchase on these affiliated behaviors.

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1 Conjunctive inferences

The disjunctive sentence in (1) conveys a conjunctive meaning, which corresponds to a conjunction of two permission sentences, provided in the second row of (1). The two conjuncts

are the two alternatives of the initial sentence in which just one of the disjuncts is kept (to give them a name, these are the *'disjunct alternatives'* of the sentence).

(1) Gali is allowed to read *Inland* or *The Plains*.

$\Rightarrow$ Gali is allowed to read *Inland* $\wedge$ Gali is allowed to read *The Plains*

There are many well-known examples of disjunction giving rise to a conjunctive meaning, the best understood of which are the applications of de Morgan's law, exemplified in (2). But while the conjunctive inference in (2) follows from basic logic, its induction in (1) is puzzling.

(2) Gali did not read *Inland* or *The Plains*.

$\Rightarrow$ Gali did not read *Inland* $\wedge$ Gali did not read *The Plains*

The reason for puzzlement is that sentence (1) is not entailment-reversing with respect to the position of the disjunction within it, in contrast to sentence (2): that is, replacing an expression in the position of disjunction in (1) with a stronger expression does not in general lead to a weaker meaning of the sentence. This is illustrated in (3)-(5), where replacing a weaker expression in the object position of the sentence (respectively, an existential quantifier, a single conjunct of a conjunction, and a disjunction with weak disjuncts) with a stronger one (respectively, a universal quantifier, a conjunction, and a disjunction with strong disjuncts) results in a stronger meaning of the sentence. A description of this state of affairs is provided in (6).

(3) Gali is allowed to read some/two/etc. books.

$\not\Rightarrow / \Leftarrow$ Gali is allowed to read every book

(4) Gali is allowed to read *Inland*/*The Plains*.

$\not\Rightarrow / \Leftarrow$ Gali is allowed to read *Inland* and *The Plains*

(5) Gali is allowed to read *Inland* or just *Barley Patch*.

$\not\Rightarrow / \Leftarrow$ Gali is allowed to read both *Inland* and *The Plains* or just *Barley Patch*

(6) **Modal entailment-preservation:** A simple modal sentence is entailment-preserving with respect to the position of any occurrence of disjunction in the scope of the modal.

Unsurprisingly, then, the behavior of disjunction exemplified in (1) has been extensively studied since at least Kamp (1973). The conjunctive inference in (1) has been called a *'free choice inference',* and the occurrences of disjunction that give rise to it occurrences of *'free choice disjunction'* (see Meyer 2020 for a recent overview). Since we will study such inferences in contexts in which the free choice terminology may be confusing, we will call them *'conjunctive inferences'*. A sharper definition of the notion is provided in (7):

(7) **Conjunctive inference:** Sentence S that contains disjunction [p∨q], S[p∨q], induces a conjunctive inference iff it entails S[p∨q/p] ∧ S[p∨q/q] (where S[p∨q/p] is identical to S[p∨q] except that [p∨q] is replaced with p therein and *mutatis mutandis* for S[p∨q/q]).

A similar interpretation of disjunction has been detected in certain other environments, for instance, in generics, in imperatives, in sentences with plural existential quantifiers, and in conditionals (Aloni 2007, Fox 2007, Chemla 2009, Nickel 2010, Bar-Lev and Fox 2020, among others). At the same time, however, conjunctive inferences of disjunction are not freely available (see, e.g., Fox 2007, Singh et al. 2016, Bar-Lev 2018 for discussion). This is demonstrated with unembedded disjunction in (8) and disjunction in the scope of a universal modal in (9) – neither of these sentences gives rise to a conjunctive inference.[1]

(8) Gali read *Inland* or *The Plains.*

⇏ Gali read *Inland* ∧ Gali read *The Plains*

(9) Gali must read *Inland* or *The Plains.*

⇏ Gali must read *Inland* ∧ Gali must read *The Plains*

So much by way of stage setting.

2 The data

Consider the comparative sentences in (10)-(12). Their matrix clauses contain disjunctions that give rise to conjunctive inferences, which are described in the second and third rows of the examples (the examples are shortened variants of sentences found online).[2,3] (Conjunctive

[1] These sentences generate other inferences. For example, (8) gives rise to the ignorance inference that the speaker does not know whether Gali read *Inland* or whether she read *The Plains*; (9) gives rise to the distributive inference that Gali may read *Inland* and that she may read *The Plains.*

[2] The same patterns can be observed in equative sentences, as exemplified in (i). Our discussion and analysis extends immediately to such sentences, which is why we do not discuss them in the main text.

(i) Gali is as angry at the liberals or the progressives as she is at the conservatives.

⇒ Gali is as angry at the liberals as she is at the conservatives ∧
Gali is as angry at the progressives as she is at the conservatives

[3] Disjunction gives rise to conjunctive inferences in the *than*-clauses of comparatives as well, as exemplified in (i). These facts seem to be well-understood, however. This is the case because the *than*-clauses of comparatives constitute an entailment-reversing environment, and so the pattern in (i) may well be expected (see also Sect. 3.5 for discussion). In fact, the conjunctive inference in (i) follows immediately on the analysis of comparatives adopted in the following section. Accordingly, we set such examples aside in this note.

(i) Gali is angrier at the conservatives than she is at the progressives or the liberals.

inferences of comparatives built on the negative antonym adjectives are discussed in Sect. 3.5.)

(10) Gali is angrier at the progressives or the liberals than she is at the conservatives.

⇒ Gali is angrier at the progressives than she is at the conservatives ∧
Gali is angrier at the liberals than she is at the conservatives

(11) The police was more interested in American or Chinese criminals than in French ones.

⇒ The police was more interested in American criminals than in French ones ∧
The police was more interested in Chinese criminals than in French ones

(12) More students read *Inland* or *The Plains* than *War and Peace*.

⇒ More students read *Inland* than *War and Peace* ∧
More students read *The Plains* than *War and Peace*

This behavior is as puzzling as that of disjunction in modal sentences. Simple comparative sentences are, namely, not entailment-reversing with respect to the potential occurrences of disjunction in their matrix clauses. For illustration, if in the above sentences we replace disjunction by pairs of expressions that are ordered by strength, as in (13)-(15), we obtain that the sentences with the stronger expression (respectively, a universal quantifier, a conjunction, a disjunction with strong disjuncts) entail those with the weaker expression (respectively, an existential quantifier, a single conjunct of a conjunction, a disjunction with weak disjuncts). Note that these entailment patterns directly mirror those of disjunction in modal sentences in (3)-(5).

(13) I am angrier at some/two/etc. donors than at the political system.

$\not\Rightarrow_{\Leftarrow}$ I am angrier at every donor than at the political system

(14) I am angrier at the progressives/the liberals than at the conservatives.

$\not\Rightarrow_{\Leftarrow}$ I am angrier at the progressives and the liberals than at the conservatives

(15) I am angrier at the progressives or the liberals than at the conservatives.

$\not\Rightarrow_{\Leftarrow}$ I am angrier at the progs and other lefties or the libs than at the conservatives

Similar entailment behavior is observed also in other comparative configurations, two examples of which are in (16)-(17). Example (16) involves a manner modifier (*cherish vigorously* entails *cherish*) and example (17) a temporal modifier (*cases I solved in the last decade* entails *cases I solved*). A description of this state of affairs is provided in (18).

⇒ Gali is angrier at the conservatives than she is at the progressives ∧
Gali is angrier at the conservatives than she is at the liberals

(16) More voters cherish their freedom than they cherish equality.

$\not\Rightarrow_{\not\Leftarrow}$ More voters cherish their freedom vigorously than they cherish equality

(17) I forgot more cases that I solved than you will ever solve.

$\not\Rightarrow_{\not\Leftarrow}$ I forgot more cases that I solved in the last decade than you will ever solve

(18) **Matrix comparative entailment-preservation:** A simple comparative sentence is entailment-preserving with respect to the position of any occurrence of disjunction in its matrix clause (if the comparative is positive, see Sect. 3.5 on their negative variants).

How can we make sense of these data?

3 The analysis

The behavior of disjunction in existential modal sentences has been explained by recourse to exhaustification (Fox 2007). The explanation can be extended to the occurrences of disjunction in the matrix clauses of comparative sentences – though only if a strenghtened meaning of degree predicates in the scope of disjunction is assumed.

3.1 Exhaustification

The conjunctive inferences of disjunction in existential modal sentences can be generated by exhaustification. The exhaustification operator, *exh*, is defined in (19) (Bar-Lev and Fox 2020): it quantifies over a set of formal alternatives to the sentence it attaches to and returns (i) the meaning of its sister, (ii) the negation of (relevant) excludable alternatives, which are defined in (20), and (iii) the affirmation of includable alternatives, which are defined in (21).

(19) $[\![\text{exh}_C\ \text{S}]\!] = [\![\text{S}]\!] \wedge \forall \text{p} \in \text{Excl(S)} \cap \text{C}\text{: } \neg\text{p} \wedge \forall \text{q} \in \text{Incl(S)}\text{: q}$

(20) $\text{Excl(S)} = \bigcap\{\text{M} \mid \text{M is a maximal subset of ALT(S)}$
such that $\{\neg\text{p} \mid \text{p}\in\text{M}\} \cup \{[\![\text{S}]\!]\}$ is consistent$\}$

(21) $\text{Incl(S)} = \bigcap\{\text{M} \mid \text{M is a maximal subset of ALT(S)}$
such that $\{\text{p} \mid \text{p}\in\text{M}\} \cup \{\neg\text{q} \mid \text{q}\in\text{Excl(S)}\}$ is consistent$\}$

The alternatives to disjunctive sentences are the disjunct and the conjunction alternatives (e.g., Sauerland 2004, Fox 2007, Katzir 2007), which we represent as semantic objects:

(22) $\text{ALT(S or S')} = \{[\![\text{S or S'}]\!], [\![\text{S}]\!], [\![\text{S'}]\!], [\![\text{S and S'}]\!]\}$

With this in hand, we can turn to the disjunctive modal sentence in (1), repeated below. The

sentence has the excludable alternative in (23-a) and the includable alternatives in (23-b) (to avoid repetition, we leave out the meaning of the sister of *exh* from the includable alternatives).

(1) Gali is allowed to read *Inland* or *The Plains*.

(23) a. Excl(1) = {◇(Gali reads *In* ∧ Gali reads *TP*)}
b. Incl(1) = {◇(Gali reads *In*), ◇(Gali reads *TP*)}

The exhaustified meaning of the sentence in (1) is computed in (24) (for brevity, we will often assume that the excludable alternatives are not relevant). The meaning of the sister of *exh* is in (24-bsc) (*bsc* stands for 'basic meaning'); it is entailed by the inclusion inferences in (24-inc). The inclusion inferences correspond to the desired conjunctive inference.

(24) ⟦[exh [allowed [Gali read *Inland* or *The Plains*]]]⟧ =
(bsc) ◇(Gali reads *In* ∨ Gali reads *TP*) ∧
(inc) ◇(Gali reads *In*) ∧ ◇(Gali reads *TP*) *(i.e., the conjunctive inference)*

Before we proceed, it is worth pointing out that in the case of unembedded disjunction in (8), *Gali read Inland or The Plains*, no non-trivial alternatives are includable, and hence no conjunctive inference can be generated. In fact, if the conjunctive alternative is taken to be relevant, the negation of the conjunctive inference is generated:

(25) a. Excl(Gali read *Inland* or *The Plains*) = {(Gali read *In* ∧ Gali read *TP*)}
b. Incl(Gali read *Inland* or *The Plains*) = ∅

(26) ⟦[exh$_C$ [Gali read *Inland* or *The Plains*]]⟧ =
(bsc) (Gali read *In* ∨ Gali read *TP*) ∧
(exc) ¬(Gali read *In* ∧ Gali read *TP*)

Similarly, in the case of disjunction scoping below a universal modal in (9), *Gali must read Inland or The Plains*, exhaustification entails the negation of the conjunctive inference as well:

(27) a. Excl([□ [Gali read *Inland* or *The Plains*]]) =
{□(Gali reads *In* ∧ Gali reads *TP*), □(Gali reads *In*), □(Gali reads *TP*)}
b. Incl([□ [Gali read *Inland* or *The Plains*]]) = ∅

(28) ⟦[exh [□ [Gali read *Inland* or *The Plains*]]]⟧ =
(bsc) □(Gali read *In* ∨ Gali read *TP*) ∧
(exc) ¬□(Gali read *In*) ∧ ¬□(Gali read *TP*)

Conjunctive Alternative Condition. How can we differentiate between the disjunctive sentences that give rise to conjunctive inferences (i.e., those with includable disjunct alternatives) and those that do not? They can be distinguished in terms of their alternatives: for example, if the conjunction of the disjunct alternatives to a sentence entails the conjunctive alternative to the sentence, the sentence cannot give rise to a conjunctive inference. In this case, namely, the exclusion of the conjunctive alternative entails the negation of the conjunction of the disjunct alternatives, that is, the negation of the conjunctive inference. And this prevents the disjunct alternatives from being includable. This generalization, which we will call *'Conjunctive Alternative Condition'* or *'CAC'* for short, is stated in (29) (see, e.g., Fox 2007, Meyer 2015, Singh et al. 2016, Bar-Lev 2018, Bar-Lev and Fox 2020 for a thorough discussion).[4]

(29) **Conjunctive Alternative Condition:** A disjunctive sentence, S[p∨q], can give rise to a conjunctive inference (via exhaustification) only if the conjunction of its disjunct alternatives, S[p]∧S[q], does not entail its conjunctive alternative, S[p∧q].

The CAC is not satisfied with unembedded occurrences of conjunction, as observed in (30), and with occurrences of conjunction in universal modal sentence, as observed in (31). The former violation is trivial, whereas the latter violation follows from the commutativity of universal modals and conjunction. Accordingly, these sentences cannot generate conjunctive inferences.

(30) **Non-Satisfaction of CAC (unembedded):** obvious - $p \wedge q \Leftrightarrow p \wedge q$

(31) **Non-Satisfaction of CAC (universal modality):** $\Box p \wedge \Box q \Leftrightarrow \Box(p \wedge q)$

In contrast, the CAC is satisfied if disjunction occurs in an existential modal sentence. The conjunctive alternative to sentence (1), that Gali is allowed to both read *Inland* and *The Plains*, is not entailed by the conjunction of the disjunct alterantives to the sentence, that Gali is allowed to read *Inland* and that she is allowed to read *The Plains* (Gali may be free to choose which book to read, but she may be prohibited to read more than one book). This is stated in (32). Accordingly, the sentence can generate a conjunctive inference.

(32) **Satisfaction of CAC (existential modality):** $\Diamond p \wedge \Diamond q \not\Rightarrow \Diamond(p \wedge q)$

[4]The same constraint applies to sentences in which disjunction is replaced by an existential quanitifier (say, an *any NP*): an existential quantifier sentence, S[∃D], can convey a conjunctive inference (that is, a universal inference) only if the conjunction of all its (non-empty) subdomain alternatives, $\wedge_{\emptyset \neq D' \subseteq D} S[\exists D']$ does not entail the universal quantifier alternative, S[∀D]. This is satisfied by the occurrences of existential quantifiers in existential modal sentences, but not those that are unembedded or in the scope of a universal modal. See Sect. 4 for discussion.

3.2 The added puzzle of comparatives

The main takeaway from the preceding subsection is that exhaustification can adequately account for the behavior of disjunction in existential modal sentences. Can the behavior of disjunction in comparatives be accounted for similarly? To answer this question, we must introduce some basic assumptions about comaparative semantics. Specifically, we assume that a simple comparative sentence like (33) is analyzed as in (34): comparison involves existential quantification over degrees, conveying that there is a degree of which the matrix degree predicate holds, but not the subordinate degree predicate (see Seuren 1973, Schwarzschild 2008 for details). In the case of (33), this means that there is a degree such that I am angry at the progressives to at least that degree, but not at the conservatives, as stated in (34-b). (We take the 'at-least' semantics of degree predicates as basic, that is, 'Gali is $\geq$d-angry at X' stands for 'the anger of Gali at X measures at least to degree d'; see Beck 2011 for an overview.)

(33) Gali is angrier at the progressives than the conservatives.

(34) a. [er$_d$ Gali is d-angry at the conservatives]$_{d'}$ [Gali is d'-angry at the progressives]
b. $\exists$d: (Gali is $\geq$d-angry at P) $\wedge$ $\neg$(Gali is $\geq$d-angry at C)

As desired, this analysis captures the intuitive entailment-preserving nature of the matrix clauses of comparative sentences: any degree that a strong degree predicate holds of (a degree predicate based on one disjunct of a disjunction, say) is such that a weak degree predicate holds of it as well (a degree predicate based on a disjunction, say).

But, *ipso facto*, the analysis fails to predict the observed conjunctive inferences on its own. The basic meaning of the disjunctive comparative sentence in (10), provided in (35-b), is strictly weaker than the observed conjunctive inference of the sentence, provided in (36): (35-b) is true whenever (36) is true, but it may also be true if Gali is angrier at the conservatives than at the liberals (as long as the opposite holds for the progressives), that is, even if (36) is false.

(10) Gali is angrier at the progressives or the liberals than she is at the conservatives.

(35) a. [er$_d$ than Gali is d-angry at the conservatives]$_{d'}$
[Gali is d'-angry at the progressives [or [Gali is d'-angry at the liberals]]]
b. $\exists$d: (Gali is $\geq$d-angry at P $\vee$ Gali is $\geq$d-angry at L) $\wedge$ $\neg\big($Gali is $\geq$d-angry at C$\big)$

(36) **Conjunctive inferences of (10):**
$\exists$d: (Gali is $\geq$d-angry at P) $\wedge$ $\neg\big($Gali is $\geq$d-angry at C$\big)$ $\wedge$
$\exists$d: (Gali is $\geq$d-angry at L) $\wedge$ $\neg\big($Gali is $\geq$d-angry at C$\big)$
Fact: (35-b) $\not\Rightarrow / \not\Leftarrow$ (36)

Conjunctive inferences must thus be derived by other means.

The added puzzle. The structure in (35) not giving rise to conjunctive inferences on its own is similar to what we observed with disjunctive modal sentences, where the conjunctive inferences were then derived by exhaustification. In contrast to disjunctive modal sentences, however, exhaustification of the structure in (35) cannot yield a conjunctive inference. This is because the CAC is not satisfied in (35): the conjunctive alternative to the structure in (35) is equivalent to the conjunction of its disjunct alternatives – if the degree of my anger at both the progressives and the liberals is greater than some threshold, then so is my anger at the progressives/liberals, and *vice versa*, as stated in (37). And since the CAC is not satisfied, the conjunctive inference cannot be generated.

(37) **Non-Satisfaction of CAC (weak matrix degree predicates):**

$\exists$d: (Gali is $\geq$d-angry at P $\wedge$ Gali is $\geq$d-angry at L) $\wedge$ $\neg$(Gali is $\geq$d-angry at C)

$\Leftrightarrow$ $\exists$d: (Gali is $\geq$d-angry at P) $\wedge$ $\neg$(Gali is $\geq$d-angry at C) $\wedge$
$\exists$d: (Gali is $\geq$d-angry at L) $\wedge$ $\neg$(Gali is $\geq$d-angry at C)

What gives?

3.3 Comparatives and strong degree predicates

The non-satisfaction of the CAC just observed is obviated if the disjoined matrix degree predicates have strong meanings, that is, 'exactly' rather than 'at-least' meanings. Such a construal is provided in (38), where we mark strength in syntax by a superscripted *'str'* and by '=' in semantics. The structure in (38-a) expresses that there is a degree such that either Gali is angry at the progressives or at the liberals to exactly that degree (=*d-angry*), but Gali is not angry at the conservatives to at least that degree ($\geq$*-angry*). This meaning is equivalent to (35).

(38) a. [er$_d$ than G is *d*-angry at the conservatives]$_{d'}$
[G is d'-angry^{str} at the progressives [or [G is d'-angry^{str} at the liberals]]]

b. $\exists$d: (Gali is =d-angry at P $\vee$ Gali is =d-angry at L) $\wedge$ $\neg$(Gali is $\geq$d-angry at C)

This construal effects a change in the meaning of the conjunctive alternative to the sentence. The alternative, provided in (39-a), has now an additional entailment that is stated at the bottom of (39-b): the conjoined degree predicates have the same maximal element (that is, there is a degree such that Gali is angry at the progressives and she is angry at the liberals to exactly that degree). No such entailment obtained on a weak construal of degree predicates.

(39) a. [er$_d$ than G is *d*-angry at the conservatives]$_{d'}$
[G is d'-angry^{str} at the progressives [and [G is d'-angry^{str} at the liberals]]]

b. $\exists$d: (Gali is =d-angry at P $\wedge$ Gali is =d-angry at L) $\wedge$ $\neg$(Gali is $\geq$d-angry at C)

$\Rightarrow$ $\exists$d: Gali is =d-angry at P $\wedge$ Gali is =d-angry at L

A consequence of this new meaning of the conjunctive alternative is that it is not entailed by the conjunction of the disjunct alternatives anymore: Gali may be both angrier at the progressives, and angrier at the liberals, than she is at the conservatives, but still be angry at the progressives and the liberals to different degrees. This is represented schematically in (40):

(40) **Satisfaction of CAC (strong matrix degree predicates):**
$\big(\exists$d: p(=d) $\wedge \neg$r($\geq$d)$\big) \wedge \big(\exists$d: q(=d) $\wedge \neg$r($\geq$ d)$\big) \not\Rightarrow / \not\Leftarrow \big(\exists$d: p(=d) $\wedge$ q(=d) $\wedge \neg$r($\geq$d)$\big)$

As the structure in (39-a) satisfies the CAC, a conjunctive inference can be generated. More to the point, the sentence has the excludable and includable alternatives in (41). Exhaustification yields the meaning in (42): the inclusion inferences, computed in (42-inc), are equivalent to the conjunctive inference; they also entail the meaning of the sister of *exh* in (42-bsc).[5]

(41) a. Excl(39-a) =
{$\exists$d: (G is =d-angry at P $\wedge$ G is =d-angry at L) $\wedge \neg$(G is $\geq$d-angry at C)}

b. Incl(39-a) =
{$\exists$d: (G is =d-angry at P) $\wedge \neg$(G is $\geq$d-angry at C),
$\exists$d: (G is =d-angry at L) $\wedge \neg$(G is $\geq$d-angry at C)}

(42) $[\![$ [exh [er$_d$ than Gali is *d*-angry at the conservatives]$_{d'}$
[Gali is d'-angry^{str} at the progressives [or [Gali is d'-angry^{str} at the liberals]]] $]\!]$ =

(bsc) $\exists$d: (G is =d-angry at P $\vee$ G is =d-angry at L) $\wedge \neg$(G is $\geq$d-angry at C) $\wedge$

(inc) $\exists$d: (G is =d-angry at P) $\wedge \neg$(G is $\geq$d-angry at C) $\wedge$
$\exists$d: (G is =d-angry at L) $\wedge \neg$(G is $\geq$d-angry at C)

The observed conjunctive inferences of disjunctive comparatives like (10) have thus been derived – what was needed was an application of matrix exhaustification and a strengthened construal of the matrix degree predicates. However, the preceding presentation was schematic, and we skimped on the details on how the strengthening of the matrix degree predicates is achieved. While one does not want to assume that the strong construals are encoded in the lexicon (e.g., Heim 2006a,b), at least two other routes for fleshing out *str* are available.

[5]The optional exclusion of the conjunctive alternative in (39-a) reduces to the negation of the sameness inference, provided in (i): if the sameness inference were true, we would obtain that it is false that either disjunct holds to a degree to which the *than*-clause does not hold, in contradiction to the basic meaning of the sentence.

(i) $\neg\exists$d: (Gali is =d-angry at P $\wedge$ Gali is =d-angry at L)

3.4 Maximality or embedded exhaustification?

We present two routes to strengthening the meaning of degree predicates. Both utilize a strengthening mechanism external to the comparison operator and degree predicates. The first one uses a mechanism employed independently in degree semantics, a maximality operator. The second one uses exhaustification. Only the second route can account for the data in this note without *ad hoc* stipulations. The discussion of the first route, furthermore, shows that the puzzle of conjunctive inferences of disjunctive comparatives is not an artifact of our choice of comparative semantics, but rather emerges on any prevailing comparative semantics.

Maximality? We have seen that the derivation of conjunctive inferences requires degree predicates in the matrix clause to have an 'exactly' meaning. This 'exactly' meaning can be arrived at by means of a maximality operator – that is, an operator that takes us from a weak degree predicate to its maximal degree. Such an operator is the mainstay of recent approaches to comparative semantics (Heim 2006b, Beck 2010, 2014, Dotlačil and Nouwen 2016, and others).

Keeping our analysis of comparison fixed, we can define a maximality operator, *max*, in a way compatible with our preceding derivation: it takes a degree and a degree predicate as its arguments, and conveys that the degree is the maximal element in the degree predicate (cf. Crnič and Fox 2019 for a similar definition, though they treat it merely as a place-holder).

(43) $[\![\text{max}]\!](d)(D) = 1$ iff $d = \iota d\text{: } D(d) \wedge \forall d'\text{: } D(d') \rightarrow d' \leq d$

The degree argument of *max* is abstracted over in the computation of the meaning of the comparative due to the movement of the *er*-phrase (e.g., Bhatt and Pancheva 2004). The interpretation of a *max*-prefixed degree predicate is strong, as demonstrated in (44):

(44) $[\![\text{OP}_d\ [\text{max}\ d]_{d'}$ Gali is d'-angry at the progressives$]\!] =$

$\lambda d.\ \big(d = \iota d\text{: Gali is} \geq d\text{-angry at the progressives} \wedge$
$\forall d'\text{: Gali is} \geq d'\text{-angry at the progressives} \rightarrow d' \leq d\big) =$

$\lambda d.$ Gali is $=d$-angry at the progressives

A full LF of disjunctive sentence (10), repeated below, is provided in (45).[6] As above, the comparison introduces existential quantification that combines with two degree predicates: one now corresponds to the set containing the maximal degrees of the disjoined matrix degree predicates, which was obtained by an application of *max* in the two disjoined degree predicates and abstraction over their degree arguments; the other consists of degrees of which the subordinate degree predicate does not hold. The meaning of (46) is, unsurprisingly, the one in (42) above:

[6] If *max* operator is introduced above disjunction, rather than in its scope as in (45), this does not help with the satisfaction of the CAC – exhaustification in the scope of disjunction would have to be employed to achieve it.

the CAC is satisfed and exhaustification yields the desired conjunctive inference.

(10) Gali is angrier at the progressives or the liberals than she is at the conservatives.

(45) [exh [er$_d$ than Gali is d-angry at the conservatives]$_{d'}$
[[max d']$_{d''}$ Gali is d''-angry at the progressives [or
[[max d']$_{d''}$ Gali is d''-angry at the liberals]]]]

But there are obvious problems with taking this route. Although the maximality operator in (43) and the structure in (45) can derive the desired interpretation of the sentences under discussion, they constitute a setup that is more or less custom-made for the examples at hand. On the one hand, the maximality operator in (43) cannot be employed to derive various facts that originally motivated its introduction (as emphasized in Crnič and Fox 2019). In particular, no matter where you plug in *max* in sentences with universal quantifiers, it fails to yield the meanings we observe for such sentences (see, e.g., Schwarzschild and Wilkinson 2002, Heim 2006b, Beck 2010 for an extensive discussion). This is exmeplified in (46), where the degree predicate is supposed to contain the degree of anger of the angriest student, while allowing the students to be angry to different degrees. No parse with *max* yields such a degree predicate.

(46) **Potential configurations with a universal quantifier and their inferences:**

a. [OP$_d$ [every student$_x$ [max d]$_{d'}$ x is d'-angry at the progessives]]
If $[\![$(46-a)$]\!] \neq \emptyset$, then $\exists$d$\forall$x: (x student) $\rightarrow$ (x is =d-angry at P). *(too strong)*

b. [OP$_d$ [max d]$_{d'}$ [every student is d'-angry at the progessives]]
$[\![$(46-b)$]\!] = \lambda$d. d = the degree of anger of the least angry student at P. *(too weak)*

On the other hand, if an alternative implementation of maximality is adopted, one which does not generate the sameness inferences (e.g., Heim 2006b, Beck 2010, Dotlačil and Nouwen 2016), it would *ipso facto* fail to obviate the violation of the CAC on its own (see fn. 7 below for a discussion of one alternative implementation). A different route must be pursued.

Exhaustification. The strengthened meaning of degree predicates in comparative sentences can be achieved by means of exhaustification as well. The comparative sentence in (10), repeated below, may be assigned the structure in (47), which includes an application of exhaustification at two levels, one at the matrix level and one at the level of the disjuncts.

(10) Gali is angrier at the progressives or the liberals than she is at the conservatives.

(47) [exh [er$_d$ than Gali is d-angry at the conservatives]$_{d'}$
[exh Gali is d'-angry at the progressives [or [exh Gali is d'-angry at the liberals]]]]

Exhaustification in the two disjuncts strengthens their 'at least' interpretation to an 'exactly' interpretation by the exclusion of all stronger (degree) alternatives to the disjuncts: these are that Gali is $\geq$d'-angry at the progressives/liberals, where d'>d, as provided in (48). Unsurprisingly, the meaning that is obtained matches the meaning that was obtained by the application of maximality. (Note that the embedded *exh* in the matrix clause does not require a parallel *exh* in the *than*-clause, as it is generated outside the degree predicate projection.)

(48) $[\![\text{OP}_d$ [exh Gali is *d*-angry at the progressives]$]\!]$ =

λd. (Gali is $\geq$d-angry at P $\wedge$ $\forall$d'>d: $\neg$(Gali is $\geq$d'-angry at P)) =

λd. Gali is =d-angry at the progressives

The meaning of (47), accordingly, corresponds to the targeted meaning of the sentence derived in the preceding subsection. This is expected since both the meaning of the sister of *exh* in (47) as well as all the alternatives to it are identical to those in the preceding subsection. In particular, the structure satisfies the CAC and entails the conjunctive inference. And no departure from the standard assumptions about exhaustification or comparison is required.[7]

3.5 Comparatives with antonyms

All the comparatives discussed in the preceding sections were built on positive adjectives (e.g., *angry-angrier, many-more*). If these positive comparatives are replaced with their negative counterparts (containing *less angry, fewer*, e.g.), the entailment-preservation and -reversal properties of the comparative sentences are swapped: the sentences become entailment-reversing with respect to certain expressions in the matrix clause, and entailment-preserving with respect to the expressions in the *than*-clause. This is illustrated in (49)-(50), where the (a)-examples recapitulate the entailment patterns of comparatives built on positive adjectives, and the (b)-examples show the patterns of those built on their antonyms.

[7] If we assume that the matrix clause of a comparative sentence must contain a maximality operator, this does not significantly affect our derivation in case the maximality operator is interpreted above disjunction. If the operator is interpreted below disjunction, however, the derivation will have to be a variant of the one described in the main text. For illustration, building on Heim 2006b and Beck 2010, we could assign the matrix clause of (10) the structure in (i-a): the Π operator conveys that the maximum of its degree predicate argument is in the respective set of degrees; the minimality operator *min* plays a role comparable to that of exhaustification in the main text – it should filter out any sets of degrees that are not singletons. The meaning of the clause, provided in (i-b), can combine with rest of the sentence via the type-shift BE (Partee 1987). This setup makes the same predictions about the conjunctive inferences of disjunction as are derived in the main text.

(i) a. $[\text{OP}_D$ [[min D]$_{D'}$ [Π D']$_d$ Gali is d-angry at P] [or [[min D]$_{D'}$ [Π D']$_d$ Gali is d-angry at L]]]]

b. λD. {max(λd. I like increased pay =d-much} = D $\vee$ {λd. I like inceeased pay =d-much} = D

(49) a. I forgot more cases that I solved than NYPD cases.
$\not\Rightarrow_{\Leftarrow}$ I forgot more cases that I solved in the last decade than NYPD cases

b. I forgot fewer cases that I solved than you will ever solve.
$\Rightarrow_{\not\Leftarrow}$ I forgot fewer cases that I solved in the last decade than NYPD cases

(50) a. I forgot more NYPD cases than cases that I solved.
$\Rightarrow_{\not\Leftarrow}$ I forgot more NYPD cases than cases that I solved in the last decade

b. I forgot fewer NYPD cases than cases I solved.
$\not\Rightarrow_{\Leftarrow}$ I forgot fewer NYPD cases than cases I solved in the last decade

Accordingly, negative antonym comparatives might not on their own entail conjunctive inferences if disjunction occurs in their *than*-clauses. Nonetheless, conjunctive inferences can be observed for such occurrences of disjunction as well. This is exemplified in (51):

(51) Gali is less angry at the conservatives than she is at the progressives or the liberals.
$\Rightarrow$ Gali is less angry at the conservatives than she is at the progressives $\wedge$
Gali is less angry at the conservatives than she is at the liberals

In contrast to comparative semantics, however, assumptions about the semantics of antonyms play a critical role in our conclusions. On some plausible assumptions, there is a non-exhaustified parse of (51) that yields the conjunctive inference. On some other assumptions, exhaustification is necessary. We discuss both types of derivation for the sake of completeness.

Antonym negation: low scope. We assume that negative antonyms are in both the matrix and the subordinate clause accompanied by a c-commanding negation, which we represent with NEG (cf. Seuren 1973, Heim 2006a). An LF of sentence (51) is provided in (52), where negation accompanying the negative antonyms is generated in both disjuncts – that is, antonym negation takes low scope relative to the disjunction. This is one possible parse of the sentence.

(52) [er$_d$ [NEG Gali is d-angry at P [or [NEG Gali is d-angry at L]]]]$_{d'}$
[NEG Gali is d'-angry at C]

The meaning of (52) is provided in (53). Recall that the two negations in the meaning of the *than*-clause are due to the comparison, as before, and the negative adjective.

(53) $\exists$d: $\neg$(Gali is $\geq$-angry at C) $\wedge$ $\neg$($\neg$(Gali is $\geq$d-angry at L) $\vee$ $\neg$(Gali is $\geq$-angry at P))

The negations in the *than*-clause are eliminated in (54) by de Morgan's rule. Clearly, (54) entails the conjunctive inference, as stated in the second and third rows of (54): if there is a degree such that Gali is angry at both the progressives and the liberals to at least that degree,

but not at the conservatives, then there is a degree such that Gali is angry at the progressives to at least that degree, but not at the conservatives (and the same holds for the liberals).

(54) $\exists$d: $\neg$(Gali is $\geq$d-angry at C) $\wedge$ (Gali is $\geq$d-angry at P $\wedge$ Gali is $\geq$-angry at L)
$\Rightarrow$ $\exists$d: $\neg$(Gali is $\geq$d-angry at C) $\wedge$ (Gali is $\geq$d-angry at P) $\wedge$
$\exists$d: $\neg$(Gali is $\geq$d-angry at C) $\wedge$ (Gali is $\geq$d-angry at L)

Thus, the conjunctive inference has been generated in the absence of exhaustification.[8]

Antonym negation: wide scope. A different LF of the sentence in (51) is provided in (55), where NEG accompanying negative adjectives takes wide-scope relative to the disjunction. On this parse, the two negations introduced in the *than*-clause cancel each other out, and we effectively obtain comparatives that match those from the preceding sections. Accordingly, dual exhaustification is required to get conjunctive inferences: the exhaustification below disjunction is required to allow for the satisfaction of the CAC, and the matrix exhaustification is required to generate the conjunctive inference.

(55) [exh [er$_d$ [NEG [exh Gali is d-angry at P [or [exh Gali is d-angry at L]]]]]$_{d'}$
[NEG Gali is d'-angry at C]]

The meaning of (55) is provided in (56), where the conjunctive alternative is excludable (and taken not to be relevant) and the disjunct alternatives are includable. The inclusion inferences, again, correspond to the conjunctive inference and entail the meaning of the sister of *exh*, which is for this reason not represented. The meaning of (55), just like that of (52), matches the meaning of sentence *Gali is angrier at the progressives or the liberals than at the conservatives*.

(56) $[\![$ [exh [er$_d$ [NEG [exh Gali is d-angry at L [or [exh Gali is d-angry at P]]]]]$_{d'}$
[NEG Gali is d'-angry at C]] $]\!]$ =

(inc) $\exists$d: $\neg$(Gali is $\geq$d-angry at C) $\wedge$ (Gali is =d-angry at L) $\wedge$
$\exists$d: $\neg$(Gali is $\geq$d-angry at C) $\wedge$ (Gali is =d-angry at P)

[8]On an analogous parse of sentences with *any NP*s, the *Any* Licensing Condition can be checked at the level of the matrix sentence. For illustration, the sentence in (i-a) is on the parse in (i-b), where the antonym negation takes scope below the *any NP*, downward-entailing with respect to the domain of the *any NP*; the parse also yields the desired interpretation that every leftist is such that Gali is angrier at them than she is at the conservatives.

(i) a. Gali is less angry at the conservatives than any leftists.
b. [er$_d$ [any leftists$_x$ [NEG Gali is d-angry at x]]]$_{d'}$ [NEG Gali is d'-angry at C]
c. $\exists$d: $\neg$(Gali is $\geq$d-angry at C) $\wedge$ $\neg$($\exists$y: y leftists $\wedge$ $\neg$(Gali is $\geq$d-angry at y))
$\Leftrightarrow$ $\exists$d: $\neg$(Gali is $\geq$d-angry at C) $\wedge$ $\forall$y: y leftist $\rightarrow$ Gali is $\geq$d-angry at y

A directly analogous state of affairs to this last derivation obtains if we adopt a non-decompositional analysis of negative adjectives on which the positive and negative adjectives pick out complementary parts of a scale of degrees (e.g., Seuren 1984, von Stechow 1984, Kennedy 2001). Hence, on all such analyses, dual exhaustification is required.

4 Polarity items in comparatives

The distribution of *any NP*s tracks the ability of disjunction to give rise to conjunctive inferences. Insofar as there is no intervention, we expect this to extend to comparative sentences.

4.1 The data

An important advance in our understanding of the behavior of *any NP*s was the recognition that (i) the environments in which *any NP*s are acceptable even though there is no entailment-reversal with respect to them align with (ii) the environments in which disjunction gives rise to conjunctive inferences (e.g., Chierchia 2013; see Sect. 4.4 for qualifications).

Existential modal sentences. The prototypical examples of this alignment are existential modal sentences like (57). In such sentences, *any NP*s are acceptable and give rise to a universal inference pertaining to the domain of *any*, which is represented in the second row of (57). This 'free choice' inference corresponds to the conjunctive inference of disjunction, not least because it is equivalent to a conjunction of permission sentences, each one of which pertains to a single book, as represented in the third row of (57). In order to hold on to the above terminology, we will call these inferences *'conjunctive inferences'* (see fn. 4 for related discussion).

(57) Gali is allowed to read any book.

$\Rightarrow$ Every book is such that Gali is allowed to read it

$\Leftrightarrow$ Gali is allowed to read *Inland* $\wedge$ Gali is allowed to read *The Plains* $\wedge$ ...

Data in (57) run afoul of some prominent descriptions of the behavior of *any NP*s – in particular, the condition that they must occur in environments that are entailment-reversing with respect to them (as observed already by the progenitors of these descriptions, esp., Ladusaw 1979). Namely, as discussed in Sect. 1, modal sentences are not entailment-reversing with respect to the position of the *any NP* in examples like (57) – hence, *any NP*s should not be grammatical in them. This led to a revision of the description of their distribution: *any NP*s must occur in environments that are entailment-reversing with respect to their NP complements, rather than with respect to the *any NP*s themselves, as stated in (58) (cf. the Strenghtening Condition in Kadmon and Landman 1993; see Crnič 2021, 2022 on the different statuses of the condition in the theories of *any NP*s).

(58) ***Any* Licensing Condition:** An occurrence of an *any NP* is acceptable only if it is in a sentence that is entailment-reversing with respect to its NP complement.

Given the *Any* Licensing Condition, the acceptability of sentences like (57) is underwritten by the conjunctive inferences: namely, if every book is such that Gali is allowed to read it – which corresponds to the conjunctive inference of (57) – then every subset of books is such that Gali is allowed to read every book in it – which corresponds to the meanings of the sentence in which the complement of *any* is replaced by a stronger expression. This allows the *Any* Licensing Condition to be satisfied in (57), and hence the *any NP* to be acceptable.

The alignment between the acceptability of *any NP*s and the availability of conjunctive inferences of disjunction does not stop here. *Any NP*s are unacceptable when unembedded, as exemplified in (59), or when they occur merely in the scope of a universal modal, as exemplified in (60) (but see Dayal 2013, Chierchia 2013, Crnič 2019, 2022 for a discussion of apparent counterexamples). We have seen in (8) and (9) above that neither of these environments licenses conjunctive inferences of disjunction. The absence of conjunctive inferences, and the attendant lack of entailment-reversal, causes the *Any* Licensing Condition to be violated in (59)-(60).

(59) #Gali read any book.

(60) #Gali must read any book.

These parallels between the acceptability of *any NP*s and disjunction giving rise to conjunctive inferences suggest that, all else equal, *any NP*s may be acceptable in matrix clauses of comparatives as well – namely, disjunction can give rise to conjunctive inferences therein.

Comparative sentences. The suggestion seems to be borne out. Some acceptable occurrences of *any NP*s in comparatives, together with the conjunctive inferences they give rise to, follow (some of these examples are shortened variants of examples found online):

(61) Gali is closer to any one of my relatives than she is to anyone in her family.

⇒ Every relative of mine is s.t. Gali is closer to them than to her family

(62) Gali is prouder of any student of hers than she is of herself.

⇒ Every student of Gali's is s.t. she is prouder of them than she is of herself

(63) The police was more interested in any petty thief than they were in real criminals.

⇒ Every thief is s.t. the police were more interested in them than in real criminals

(64) More seniors solved any of the assignments than juniors did.

⇒ Every assignment is s.t. more seniors than juniors solved it

Before we proceed to the analysis of these data, one important consequence of the above observations can already be noted at this point: these observations are unexpected on the accounts of the distribution of *any NP*s in non-entailment-reversing environments that connect this distribution to modality and modal notions (e.g., Dayal 1998, Giannakidou 1999, Aloni 2007) – the comparative sentences above lack any modal elements that could distinguish them from simple episodic sentences, in which *any NP*s are unacceptable. Therefore, a satisfactory theory of *any NP*s should not privilege modality.

4.2 The analysis

Since neither the complements of existential modals nor the matrix clauses of comparatives constitute entailment-reversing environments by themselves, the entailment-reversal required by the *Any* Licensing Condition must come from elsewhere – exhaustification.

Existential modal sentences. Let's first review how the *Any* Licensing Condition is satisfied in (57) (see Crnič 2019 for details, and Chierchia 2013, Dayal 2013 for alternative derivations). The sentence in (57) has the excludable and includable alternatives in (65). The exhaustified meaning of the sentence is provided in (66), where we again assume that the excludable alternatives are not relevant; (66-inc) represents the inclusion inferences, which correspond to the conjunctive inference of the sentence.[9]

(57) Gali is allowed to read any book.

(65) a. Excl(57) = {◇(Gali read every book in D) | $D \subseteq$ book $\wedge$ card(D)>1}
b. Incl(57) = {◇(Gali read a book in D) | $\emptyset \neq D \subseteq$ book}

(66) ⟦[exh [◇ [Gali read any book]]]⟧ =
(bsc) ◇(Gali read a book) $\wedge$
(inc) $\forall$D: $\emptyset \neq$ D $\subseteq$ book $\rightarrow$ ◇(G read a book in D)

If we replace the NP complement of the *any NP* in (66) (*book*) with an expression that denotes a proper, non-empty subset of the NP (say, *long book*), the resulting sentence has a weaker meaning, as illustrated in (67): if you are allowed to read any book, you are allowed to read any long book. This means that the sentence is entailment-reversing with respect to the NP complement of the *any NP*, and so the *any NP* is correctly predicted to be acceptable in (57).

[9]The exclusion inferences, which we have taken not to be relevant (hence pruned from the domain of *exh*) is provided in (i). See Crnič 2019, 2022 for further details.

(i) $\forall$D: D $\subseteq$ book $\wedge$ card(D)>1 $\rightarrow \neg$◇(G read every book in D)

(67) $\forall D: \emptyset \neq D \subseteq \underline{\text{book}} \rightarrow \Diamond(\text{Gali read a book in D})$

$\Rightarrow \forall D: \emptyset \neq D \subseteq \underline{\text{long book}} \rightarrow \Diamond(\text{Gali read a book in D})$

Comparative sentences. Let us now extend the above analysis to comparative sentences. As was the case with existential modal sentences, the conjunctive inferences of *any NP*s allow the *any NP*s to satisfy the *Any* Licensing Condition in comparatives. Specifically, a sentence like (61), repeated below, may be assigned the LF in (68), where the degree predicate in the scope of *any NP* has a strong, 'exactly' meaning due to an embedded *exh*.

(61) Gali is closer to any one of my relatives than she is to anyone in her family.

(68) [exh [er$_d$ than Gali is d-close to anyone in her family]$_{d'}$

[[any one of my relatives]$_x$ [exh Gali is d'-close to x]]]

The dual exhaustification structure in (68) mirrors the dual exhaustification structures with disjunction, such as (47), in the relevant aspects: it has a matrix *exh* operator, and an *exh* operator embedded below an existential quantifier/disjunction. In particular, the structure satisfies the CAC. Accordingly, the universal quantifier alternatives to the sister of *exh* in (68) are excludable (as long as their domain is not a singleton set) and the existential quantifier alternatives in which *any NP* is replaced with a stronger *any NP'* are includable. The meaning of (68) is provided in (69), where the inclusion inferences correspond to the observed conjunctive inferences.

(69) $[\![$ [exh [er$_d$ than Gali is d-close to anyone in her family]$_{d'}$

[[any one of my relatives]$_x$ [exh Gali is d'-close to x]]] $]\!]$ =

(bsc) $\exists$d: $\exists$x: (x relative of mine $\wedge$ Gali is =d-close to x) $\wedge$
$\neg(\exists$y: y relative of Gali $\wedge$ Gali is $\geq$d-close to y)

(inc) $\forall$x: x relative of mine $\rightarrow \exists$d: (Gali is =d-close to x) $\wedge$
$\neg(\exists$y: y relative of Gali $\wedge$ Gali is $\geq$d-close to y)

The sentence in (61) satisfies the *Any* Licensing Condition on the above analysis. If we replace the NP complement of the *any NP* with a stronger expression, the resulting sentence has a weaker meaning. This is illustrated in (70): if all <u>relatives of mine</u> are such that Gali is closer to them than her own relatives, then all <u>old relatives of mine</u> are such that Gali is closer to them than she is to her own relatives.[10]

[10]We again set the exclusion inferences aside. These have been shown not to affect the satisfaction of the *Any* Licensing Condition (Crnič 2019, 2022). In the cases at hand, the exclusion inferences reduce to the following: the pertinent matrix degree predicate does not hold of any two elements in the domain of *any NP* to the same degree – this is, again, the negation of the sameness inference. This exclusion inference feels at best optional, however. But this is not a problem. On the one hand, the inference can be treated as exceedingly weak (at least if one can adjust the granularity relative to which the sentence is evaluated – at some level of precision Gali might well be close to

(70) $\forall$x: x relative of mine $\rightarrow$ $\exists$d: (Gali is =d-close to x) $\wedge$
$\neg(\exists$y: y relative of Gali $\wedge$ Gali is $\geq$d-close to y)
$\Rightarrow \forall$x: x old relative of mine $\rightarrow$ $\exists$d: (Gali is =d-close to x) $\wedge$
$\neg(\exists$y: y relative of Gali $\wedge$ Gali is $\geq$d-close to y)

4.3 Other polarity items

There is substantial variation in the domain of polarity items. For example, not all polarity items are subject to an NP-complement-centered condition like the *Any* Licensing Condition. The distribution that we predict for such items may come apart from that of *any NP*s when it comes to comparative sentences. On the other hand, if polarity items are subject to a condition similar to the *Any* Licensing Condition, and can occur in existential modal sentences, we may exepect that they can also occur in comparatives.

Minimizers. There are expressions whose distribution can be described as being restricted to environments that are entailment-reversing with respect to them – rather than to their potential NP complements, as is the case for *any NP*s. Two prominent examples of such expressions are the *even*-prefixed phrases in which *even* associates with the weakest element on a scale (e.g., *even ONE book*; Lahiri 1998, Guerzoni 2003, 2004, and others), and the minimizer negative polarity items (e.g., *lift a finger*; Schmerling 1971, Lahiri 1998, Eckardt 2005, Chierchia 2013, and others). These expressions are acceptable in sentences that are entailment-reversing with respect to them – e.g., they are acceptable when they occur in the scope of negation, as in (71-a)-(72-a), but they are unacceptable in existential modal sentences, as in (71-b)-(72-b):

(71) a. Gali did not read even ONE book.
b. #Gali is allowed to read even ONE book.

(72) a. Gali did not lift a finger to help.
b. #Gali is allowed to lift a finger to help.

Given this distribution, which demonstrates that minimizers cannot be rescued in existential modal sentences by exhaustification, we expect them to also be unacceptable in the matrix clauses of simple comparative sentences. This expectation is borne out, as shown in (73)-(74).

(73) #Gali is angrier at even one progressive than at the conservatives.

(74) #More students lifted a finger to help than professors did.

all my relatives to different degrees). On the other hand, the excludable alternatives can be pruned by the context, as we do in the main text, and as is wont to happen with exclusion inferences that contradict the common ground.

Polarity items in Slovenian. Slovenian bare *wh* phrases and *wh-koli* phrases have a distribution similar to that of *any NP*s. Specifically, while they are unacceptable in simple episodic sentences, they can appear in modal sentences, particularly, in existential modal sentences, where they generate conjunctive inferences. This is exemplified in (75)-(76):

(75) #Gali je prebrala katero(koli) od teh knjig.
Gali aux read which(koli) of these books
'#Gali read any one of these books.'

(76) Gali lahko prebere katero(koli) od teh knjig.
Gali may-prt read-3sg which(koli) of these books
'Gali may read any one of these books.'
⇒ Everyone of these books is such that Gali may read it

If the acceptability and semantic contribution of Slovenian *wh*-phrases in (76) is governed by exhaustification, as argued by Crnič (2019, 2022), we can expect both types of expressions to occur in matrix clauses of comparatives and give rise therein to universal inferences pertaining to the domain of the *wh*-phrases. The expectation is confirmed, as exemplified in (77).

(77) Več študentov je prebralo katero(koli) od teh knjig kot pa *Vojno in Mir*.
more students aux read which(koli) of these books that prt *War and Peace*
'More students read any one of these books than *War and Peace*.'

The two predictions about the distribution of polarity items other than *any NP*s have thus been borne out: (i) polarity items that cannot be rescued in modal environments by exhaustification are unacceptable in the matrix clauses of simple comparatives, while (ii) polarity items that can be rescued in modal sentences may also be rescuable in comparatives. But it is well-known that the distribution of polarity items can vary in complex ways (e.g., Alonso-Ovalle and Menéndez-Benito 2010, Gajewski 2011, Chierchia 2013, among many others). Accordingly, our scraping of the surface of this variation in comparatives is only the first of many steps.

4.4 Intervention

Exhaustification is known not to rescue all occurrences of *any NP*s that could in principle be rescued by it (e.g., Chierchia 2013, Crnič 2022:fn.7). Most blatantly, whereas exhaustification can rescue *any NP*s in modal, generic, and imperative sentences, as well as in the matrix clauses of comparatives, as we argued above, it fails to do so in some other structurally comparable environments, such as the scope of existential nominal and temporal quantifiers. This is demonstrated with a sentence in which an existential nominal quantifier c-commands an *any NP*: the sentence in (78) has the same logical structure as the existential modal sentences in (57). Accordingly, the CAC is satisfied in (78) – but the sentence is nonetheless unacceptable.

(78) #Some student(s) read any book.

What distinguishes examples like (78) from the modal and comparative sentences discussed above? We suggest that the unacceptability in (78) is due to intervention, that is, it should be described and derived like the run-of-the-mill cases of intervention (as was also suggested in Crnič 2022, following Chierchia 2013). To appreciate this, we must switch to a fuller version of the *Any* Licensing Condition (cf. Gajewski 2011, Chierchia 2013, Buccola and Crnič 2021):

(79) ***Any* Licensing Condition (revised, incl. intervention):** An *any NP* is acceptable only if it occurs in a sentence (i) that is entailment-reversing with respect to its NP complement, and (ii) whose scalar alternatives do not entail the sentence (more precisely, the scalar alternatives derived by manipulating material that c-commands the *any NP*).

Now, (78) violates (79). The culprit is the universal quantifier alternative, which entails the existential sentence, as indicated in (80) (this holds also when both sentences are exhaustified). In the case of the comparative sentences above, in contrast, there are no stronger alternatives, hence the condition is satisfied (note that a superlative sentence is morphologically more complex than a comparative one, and hence does not count as an alternative, cf. Bobaljik 2012).

(80) Every student read a(ny) book. $\Rightarrow$ Some student(s) read a(ny) book.

There is a wrinkle, however. The odd man out given the revised *Any* Licensing Condition are the existential modal sentences, where their universal modal alternative fails to trigger a violation of the condition. This is yet another instance in a long line of exceptional behaviors that modals exhibit with respect to intervention (see, esp., Chierchia 2013, Crnič et al. 2015, Bar-Lev and Fox 2020). Another such instance is illustrated by a contrast in entailment-reversing environments in (81): while a nominal quantifier (*everyone*) intervenes in the licensing of the *any NP* in (81-a) (since there is a stronger alternative: *It is not required that anyone fire any of their assistants*), in line with the expectations raised by (79), a modal quantifier (*require*) does not in (81-b) (although there is a stronger alternative: *It is not allowed that you fire anyone*). For our purposes, the independent existence of the contrast between modal and other kinds of interveners suffices (cf. Chierchia 2013 for an explanation). The issues pertaining to intervention in non-entailment-reversing environments more generally, however, require further investigation.

(81) a. #It is not required that everyone$_i$ fire any of their$_i$ assistants.
b. It is not required that you fire anyone.

5 Conclusion

The ability of disjunction to give rise to conjunctive inferences in the matrix clauses of comparatives can be explained by means of dual exhaustification: in addition to matrix exhaustification, embedded exhaustification has to be employed in order to derive strong meanings of degree predicates, without which conjunctive inferences are underivable.

The dual exhaustification account also correctly predicts that *any NP*s should be able to surface in the matrix clauses of comparatives and give rise to conjunctive inferences therein. In contrast, this distribution of *any NP*s is unexpected on the approaches to *any NP*s that explicitly privilege modality and modal notions.

While our focus was solely on comparative sentences – and solely on their matrix clauses at that – the proposal could be extended to constructions that are either built on comparatives or employ comparison operations. Two constructions offer themselves as candidates: (i) the various expressions of preference (constructions with *prefer, want, be glad*, etc.) and (ii) superlative constructions. Both types of constructions are built on comparatives or, at least, employ comparison operations (see, e.g., Heim 1992 on (i), and Bobaljik 2012 on (ii)). Moreover, the distribution of polarity items in relation to them remains ill-understood (e.g., Linebarger 1980, Kadmon and Landman 1993, von Fintel 1999, Crnič 2019 on (i), and von Fintel 1999, Howard 2014, Bumford and Sharvit 2022 on (ii)). But these extensions, as well as many other challenges and tasks, must be left to a future occasion.

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