Informational Object Nouns and the mass/count distinction[1, 2]

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Abstract. Most theories of the count/mass distinction analyse concrete nouns (denoting physical entities or stuff), and so, implicitly or explicitly, set abstract nouns to one side. We build on a growing number of recent works that address this gap with our analysis of a class of abstract nouns, *Informational Object Nouns* (IONs), such as *information, evidence, belief,* and *statement*. We argue that by incorporating recent work done by Schmitt (2013, 2017) on the development of a domain general mereological sum operation, we can modify theories of the mass/count distinction for concrete nouns and extend their coverage to the set of IONs. As we also argue, an important factor in extending such theories is that they are grounded in the notion that count nouns are interpreted under individuation schemas, relative to a context (Sutton and Filip 2016b; Filip and Sutton 2017, amongst others).

Keywords: mass/count distinction, abstract nouns, countability, individuation.

1. Introduction

Most semantic theories of the mass/count distinction are developed for concrete nouns that denote physical entities (*cat*, *chair*), or stuff (*sand*, *air*), as opposed to abstract nouns such as *love*, *experience*, *information*, *statement*. Although there are some notable exceptions (Grimm 2014; Nicholas 2010; Tovena 2001; Zamparelli 2018, amongst others), most work on the semantics of countability set abstract nouns to one side, not least since there are enough challenges with concrete nouns. However, some have even raised doubts whether extensionally defined theories for concrete nouns can be meaningfully applied to abstract nouns (Barner, 2019). In contrast to this sentiment, theories of the mass/count distinction developed for concrete nouns provide us with useful conceptual tools and formal properties needed to get a handle on characterising countability in more abstract domains (such as the domains of propositions and eventualities). In particular, we focus on abstract nouns such as *evidence, information, belief, statement,* and *fact* that we dub here *Informational Object Nouns* (IONs).

In section 2, we present a means of delimiting the class of IONs based on two grammatical tests: felicitous occurrence with propositional complements and truth/falsity predications. We then argue that the class of IONs shares a number of properties with a well-studied class of concrete nouns, Collective Artefact Nouns (CANs), that includes nouns such as *furniture*, *cutlery*, and *jewellery*. In particular, we highlight that both classes display a large amount of variation in their count/mass lexicalization patterns. When mass, both are individuated with the same

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[2]This paper is a forerunner to Sutton and Filip (2019), a paper already published in the proceedings of the Amsterdam Colloquium in 2019. This paper sets out the groundwork for our analysis of Informational Object Nouns in more detail than in Sutton and Filip (2019), however, in Sutton and Filip (2019) we address more complex data, specifically the impact of pluralities of Agents and Experiencers on the interpretations of IONs.

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classifiers (*item* and *piece*), and resist mass-to-count coercion; and similarly to CANs, across different contexts, IONs overdetermine what counts as one insofar as what counts as, say 'one belief' in one context can count as 'two beliefs' in another. Finally, we identify three classes of IONs based on whether they denote sets of eventualities and on whether these eventualities are STATES or EVENTS.

In section 3, we show that by incorporating recent work done by Schmitt (2013, 2017) on the development of a domain general mereological sum operation, we can modify theories of the mass/count distinction that can explain the distributional patterns of CANs such that their coverage is extended to IONs. The theory for concrete nouns that we modify explains the distributional properties of CANs (that, we will argue, IONs also share) on the basis of the following two claims: (1) An extensional property underpins the distinction between count and mass nouns; (2) The lexical entries of count nouns include a context-indexed individuation schema that ensures that a quantized set is available for grammatical counting operations (see Sutton and Filip 2016a, b; Filip and Sutton 2017, amongst others). The account we propose demonstrates how extensional theories of the mass/count distinction can be extended to cover at least some classes of abstract nouns. It also explains a number of subtle distinctions within the class of IONs, for example whether they can be individuated in terms of propositions or in terms of both propositions and eventualities.

We provide a unified analysis of counting constructions for CANs and IONs in the appendix to this paper.

2. The class of Informational Object Nouns

2.1. Delimiting the class of Informational Object Nouns

We propose that the class of IONs consists of all and only those nouns that pass both of the tests (T1) and (T2) below. So N is an ION if and only if:

(T1) *N that is true/false* is felicitous (truth-evaluability);

(T2) *N that p* is felicitous (propositional complementisers)

For example, *belief* and *statement* take propositional complements: *Alex's belief/statement that it's raining*, and admit of truth/falsity predications: *Alex's belief/statement was true/false*. Further examples of (T1) are given in (1)-(3) for *evidence, information* and *knowledge*, and for (T2) in (4)-(6). (UKWaC = UK Web Annotated Corpus; BNC = British National Corpus.)

(1) a. The war on Iraq was the first one in which Britain has engaged on the basis of intelligence evidence alone, and that evidence has been shown **to be false** or exaggerated. [UKWaC]

b. The Appellant's written evidence which she swore was accurate and **true** clearly suggests that she was present at the time of Dr.Walsh's consultation with Mrs. IM. [UKWaC]

(2) a. he successfully fed Edgar Hoover at the FBI a mixture of tantalising bits of fact and rubbish, including **false** information about Russia's space-rocket programme [BNC]

b. The Candidate agrees to: Provide Information which is in all respects **true** and accurate... [UKWaC]

(3) a. Why bother with research? Anecdotal and expert knowledge and experience are often **false** when tested scientifically. [UKWaC]

b. Thus, knowledge is always **true**, whereas belief admits of both truth and falsehood. [UKWaC]

(4) a. Others, however, point to some evidence that it is encouraging people to become more and more introverted... [UKWaC]

b. there is strong evidence that many women suffer as a consequence of men's poor health. [UKWaC]

(5) a. On [sic.] day we received information that the President would visit the Seventh Street Hospital... [UKWaC]

b. What happens is the seismologist dealing with an earthquake gives out information that the maximum intensity of an earthquake was, say, 9. [UKWaC]

(6) a. This approach is based on the knowledge that people react to drugs differently, partly because of underlying genetic variation. [UKWaC]

b. What I took away from the meeting was the knowledge that our union does an excellent job for us with the Clubs and Leagues we have in Britain. [UKWaC]

In contrast, nouns like *feeling* pass the complementiser test (T2): *the feeling that I have forgotten something*. However, they fail test (T1): *that feeling was true* is odd, if *true* is intended in its truth-value sense, and not in the *genuine, real* sense. Although concrete Ns like *book, article* are attested in collocations such as *this article is true* (understood as meaning that its content is true at a given world/time), they fail test (T2).

Based on (T1) and (T2), examples of IONs are given below:

> allegation, announcement, assumption, belief, conclusion, declaration, fact, information, intelligence[3], evidence, idea, judgement, knowledge, report, thought, statement, proposition, truth, utterance

2.2. Properties of Informational Object Nouns and parallels with Collective Artefact Nouns

A number of interesting parallels can be observed between the class of IONs and a class of concrete nouns that have been much studied in the literature. When mass these concrete nouns have been called *object mass nouns* (Barner and Snedeker, 2005; Rothstein, 2010), *fake mass nouns* (Chierchia, 2010), and *neat mass nouns* (Landman, 2011) and, in English, include nouns such as *cutlery*, *crockery*, *equipment*, *footwear*, *furniture*, *jewellery*, and *kitchenware*. However, as stressed by Sutton and Filip (2016a, 2016b, amongst others), there are many count counterparts to these nouns cross-linguistically. Sutton and Filip dub this wider class of nouns (both count and mass), *Collective Artefact Nouns*. This brings us onto our first parallel with IONs.

IONs and CANs both exhibit large amounts of cross- and intralinguistic variation in their mass/count lexicalization patterns. As indicated by Table 1, when we look crosslinguis-

[3] As in pieces of military intelligence.

Table 1: Examples of cross- and intralinguistic count/mass variation for Collective Artefact Nouns and Informational Object Nouns

	Mass	Count (NOMINATIVE.PL)
furniture	*furniture*	*huonekalu(t)* (Finnish)
	meubilair (Dutch)	*meubel(s)* (Dutch)
jewellery	*jewellery*	*taxšit(im)* (Hebrew)
	Schmuck (German)	*koru(t)* (Finnish)
footwear	*footwear*	*jalkine(et)* (Finnish)
	Schuhwerk (German)	
information	*information*	*Information(en)* (German)
		tieto (tiedot) (Finnish)
evidence	*evidence*	*Beweis(e)* (German)
		todiste(et) (Finnish)
knowledge	*gnosi* (Greek, dual life)	*gnosi(s)* (Greek, dual life)
	knowledge, *Wissen* (German)	

tically, and also within languages, we find an interesting parallel between CANs and IONs with respect to the variation in their count/mass lexicalization patterns, namely, that we find count counterparts of object mass nouns. It should be stressed that such variation is not highly widespread in concrete nouns. Sutton and Filip (2016b) identify other classes of nouns where we find this pattern, namely granular nouns such as *lentil(-s)* in English and *čočka* ('lentil', mass) in Czech (see also Chierchia, 2010), and also the kinds of context-sensitive object nouns that are important data points within Rothstein's (2010) theory such as *fence*, and *hedge* that have intralinguistic mass-counterparts *fencing*, and *hedging*. Therefore, it is not insignificant that the class of IONs displays a similar kind of variation.

Mass IONs and mass CANs both combine with similar classifiers. Another property shared by CANs and IONs is that, when they are lexicalized as mass, they can only feature in counting constructions with the same limited number of 'unit-extracting' classifiers such as 'item' and 'piece' (7)-(8).

(7) We bought two new items/pieces of furniture/kitchenware/jewellery.

(8) a. All three items of information are shown in the eyepiece display. [UKWaC]
b. The snag is that the prospectus must contain pieces of information which rapidly become dated. [BNC]
c. The third piece of evidence for the big bang is that you can see it. [BNC]
d. He would dispose of it and a key item of evidence would be gone for good. [BNC]
e. ... our reason gets from those ideas to certain items of knowledge which others said were innate. [BNC]
f. ... both pieces of knowledge were essential for survival. [BNC]

Mass IONs and mass CANs both strongly resist mass-to-count coercion. The class of object mass nouns (i.e., mass CANs) has also been observed to strongly resist being coerced into count interpretations, either to obtain a 'unit' reading (Sutton and Filip, 2016a) or a subkind reading (Grimm and Levin, 2017; Sutton and Filip, 2018), a restriction that is not found in other concrete or abstract mass nouns:

(9) a. Two more beers and another fried rice, please.
b. #I went out to buy three furnitures/jewelleries/footwears.

(10) a. I have two loves/passions: wine and cheese.
b. #We just learned of three new informations/evidences/knowledges.

We do note, however, that at least one mass ION, *knowledge*, can be used with the indefinite article, especially when modified (11a). This is not the case for mass CANs, at least in English, or for other mass IONs, which tend to be infelicitous with any determiners that select for count nouns. For example, *information*, *evidence*, *furniture*, and *jewellery* are all infelicitous with indefinite articles (11b)-(11c).

(11) a. She has a good knowledge of chemistry.
b. #He gave a good evidence/information in court.
c. #I saw a good furniture/jewellery while out shopping.

IONs and CANs overdetermine what counts as one. Finally, we observe an interesting semantic property of both CANs and IONs, namely that what counts as one N in one context may count as more than one N in another context. For example, for *kitchenware*, as we see in (12), a pestle and mortar can count as one piece in some contexts, but a proper part, the pestle, can also count as one piece in others. For *opinion* and *statement*, it is perfectly felicitous to use either singular or plural forms with the same propositional complement as shown in the corpus examples in (13a) and (14a), and their minimal pairs in (13b) and (14b); however, the same flexibility is not present for concrete nouns such as *ball* as we see in (15).

(12) a. This $24 ChefSofi Mortar and Pestle is a sturdy piece of kitchenware.[4]
b. I need to buy a few pieces of kitchenware: a chef's knife, a pairing knife, and a new pestle to replace the one I broke.

(13) a. ... the **opinion** that these two German countries belonged together and that the German people should solve their own internal affairs and difficulties. [UKwaC]
b. ... the **opinions** that these two German countries belonged together and that the German people should solve their own internal affairs and difficulties.

(14) a. The Panel is pleased to note the company's **statement** that the product is no longer available and that it would not form part of its Christmas 2001 gift range. [UKwaC]
b. The Panel is pleased to note the company's **statements** that the product is no longer available and that it would not form part of its Christmas 2001 gift range.

(15) Alex bought two balls/#one ball: a football and a rugby ball

[4]https://www.thrifter.com/chefsofi-mortar-pestle-heavy-granite (Accessed 22.01.2020)

2.3. Three subclasses of IONs

Although there are these four similarities between IONs and CANs, there are, of course, differences. First, their denotations differ. CANs denote physical entities and, when mass, collections thereof. CANs do not denote eventualities, despite the fact that, at least on Grimm and Levin's (2017) analysis, the lexical semantics of object mass nouns specifies the truth conditions of such nouns in terms of related eventualities (e.g., a furnishing event for *furniture*).

In contrast, all IONs denote (sets of) propositions, something which is evidenced by the fact that, by definition, IONs are felicitous with propositional complements and with predications of truth and falsity. For example, we see evidence that IONs can denote (sets of) propositions in (16a)-(16b). In (16a), the content of Pooh's belief, a proposition, is true. In (16b), the reference is to contradictory contents of statements made by witnesses, i.e., *propositions* conveyed by the witnesses.

(16) a. If Pooh's belief is true, his action (going to the cupboard) will succeed: it will get him the honey he wants. [BNC]
b. He was acquitted on May 25, 1990, largely because of contradictory statements by witnesses. [BNC]

On top of this proposition-denoting sense, many IONs also have a sense that denotes an eventuality. Indeed, up to this point, we have focussed on the similarities and differences between the classes of CANs and IONs, but IONs do not form a homogenous class. Some IONs have an eventuality denoting sense, but some do not. Also, of those that do, the type of eventuality (STATE or EVENT) differs. Based on these distinctions, we propose that IONs can be divided into three subcategories in terms of their distributional properties. We name these classes based on exemplars of them: (i) *statement*-like IONs; (ii) *belief*-like IONs; and (iii) *fact*-like IONs.

***Statement*-like IONs**: Nouns in this group also include *utterance, allegation, declaration*. All can be used to denote EVENTS, as shown for *statement* in (17) and (18). In (17), the restriction concerns making statements (acts of *stating*), i.e., tokens of a particular eventuality type, and the use of *during* in (18) indicates that the denotation of *statement* is something with a temporal trace (an eventuality).

(17) She had been restricted ... from taking part in public meetings and from making public statements of any kind. [BNC]

(18) In my opinion now an apology would be wasted, it would be the same crocodile tears as we saw during her statement in December [UKWaC]

Also, the basis of grammatical counting can rest for nouns in this class on either eventualities or on propositions as can be seen by assessing the denotation of 'Alex and Billie's two statements in the following two contexts:

Context 1a: Alex made a statement that p, and Billie that q.
Context 1b: Alex made a statement that p, and Billie did, too.

An utterance of *Alex and Billie's two statements* in Context 1a licenses reference to either the eventualities (the stating events) that Alex and Billie were agents of, or to the contents of their statements (the eventualities). In contrast, the same utterance in Context 1b only licences reference to the eventualities since the contents of Alex and Billie's statements was the same.

***Belief*-like IONs**: This subclass of IONs also includes nouns such as *assumption, idea, judgement, knowledge,* and *thought*. As well has being able to denote propositions, nouns in this class also have a sense that denotes a relation to a proposition characterised as some kind of mental state. For example in (19), Galileo is persecuted for having a relation to a particular proposition (that of believing it, a mental state). He was not persecuted for a proposition itself.

(19) At the time, Galileo had just discovered the Galilean moons (including Europa) in Florence but was being persecuted for his belief that the Earth orbits the sun.[5]

The basis for this extra sense is, we hypothesise, related to but not determined by the fact that they have shared roots with STATE-denoting verbal predicates (*assume, judge, know, think*). The reason for this hedge is that *idea*, insofar as it also has a mental state/attitude-denoting sense, has no cognitive association with a verbal predicate (even if etymologically it is distantly derived from one).

Nouns in this class display a divergence with *statement*-like IONs insofar as this STATE-denoting sense does not seem readily available to grammatical counting functions. In Context 2a, the only natural reading of *Alex and Billie's two beliefs* is that it refers to the contents of their beliefs (the propositions they believe). However, when we make this reading unavailable as in context 2b, the same utterance is anomalous.

Context 2a: Alex has a belief that *p* and Billie has a belief that *q*
Context 2b: Alex has the belief that *p* and Billie does, too.

Indeed, one must do some work to get any reading in which one can access pluralities of beliefs on the eventuality-denoting reading of *belief(s)*. For example, as discussed in Sutton and Filip (2019), if one provides a plurality of experiencers, but only one proposition in the context, it can be possible to anchor mental states to experiencers and so individuate pluralities of mental states as in (20)

(20) A postal strike in France and the resulting delayed arrival of my dad's postcard explained my cousins' beliefs that my he was still in Paris.

***Fact*-like IONs**: IONs such as *fact, information, intelligence, evidence*, and *truth*, do not seem to have a sense pertaining to an eventuality, and so we assume that they do not have an eventuality argument. In turn, for us, this means that the argument structure of their denotations does not have either an agent or an experiencer. For example, *Alex's information/(pieces of) intelligence/evidence/?fact*, if felicitous, seems to evoke a possessor or sender relation, rather than one of an experiencer of a mental state or an agent in an action. An exception to this is possibly *evidence* which, in a legal setting, can be used to refer to the formal giving of evidence. For example, *His evidence lasted for about 40 minutes* refers to the hearing of or giving of evidence.

With these nouns, one cannot refer mental states or attitudes, as we see in (21a)-(21b):

[5] `https://jon-farrow.com/tag/kepler/`

(21) a. persecuted for a belief / an opinion / #a fact / #the truth
b. form a belief / an opinion / #a fact / #a truth

It is not coincidental, furthermore, that these nouns either have no shared root with a verbal predicate or have a meaning which is, in some sense, separated from some related verbal meaning. The noun *information* clearly shares a root with the verbal predicates *inform*, but the modern uses of this noun seems to be independent from any agentive informing event (something can be information without an agent informing someone of it).

In summary, the classes of IONs and CANs display some interesting distributional parallels that prompt the following thought: theories of the mass/count distinction that can accommodate CANs can provide a basis for a theory that can also accommodate IONs. That said, any such theory would have to be modified, not least to provide an account of how propositions and eventualities are individuated and grammatically counted. In section 3, we put forward such a unified analysis.

3. Analysis: A theory of countability that extends to IONs

3.1. The count/mass distinction for concrete nouns

The theoretical architecture that we assume and briefly summarise here is developed in Sutton and Filip (2016a, b, 2018) and Filip and Sutton (2017). We assume a simply typed truth conditional semantics enriched with classical extensional mereology such that the domain is structured as a Boolean semilattice meaning that we have not only single entities of type *e*, but also sums of entities of type *e*.

The main tenets of Sutton & Filip's analysis are:

1. An extensional property underpins the distinction between count and mass nouns. The extensional property we assume here is *quantization* (see below).
2. The lexical entries of count nouns include a context-indexed individuation schema that ensures that a quantized set is available for grammatical counting operations.

Quantized sets (Krifka, 1986; Krifka, 1989) are sets for which no two members are proper parts of each other:

(22) $$QUA(P) \leftrightarrow \forall x \forall y[P(x) \wedge P(y) \rightarrow \neg x \sqsubset y]$$

While the interpretations of count nouns specify quantized sets that can be accessed by grammatical counting operations, the interpretations of mass nouns do not. This alone would already be enough to distinguish count nouns, such as *cat*, from mass nouns, such as *air*, on the assumption that the extensions of substance nouns denote cumulative (and so not quantized sets). However, to account for the whole range of data in the mass/count domain, including object mass nouns (mass CANs), such as *furniture*, and context-sensitive count nouns like *fence*, which do not lexically specify what is 'one' entity across all contexts in a uniform way, we need the notion of context-indexed individuation schemas. (This kind of context sensitivity of individuation has its roots in both the *counting contexts* of Rothstein (2010) and the *variants* of Landman (2011).)

Context-indexed individuation schemas, $\mathcal{Q}_c$, are functions from sets to maximally quantized subsets thereof. Therefore, even if a predicate P has a non-quantized extension, applying a context-indexed individuation schema to this predicate ($\mathcal{Q}_c$(P)) at a context c outputs a subset of P that is quantized and is not a subset of a larger quantized subset of of P:

(23) $$X \subseteq_{max.QUA} Y \text{ iff } X \subseteq Y, QUA(X), \forall Z \subseteq Y[Z \supseteq X \wedge QUA(Z) \rightarrow Z = X]$$

(24) $$\forall c.\forall P.\mathcal{Q}_c(P) \subseteq_{max.QUA} P$$

For example, if $A = \{pestle, mortar, pestle \sqcup mortar\}$, then there are two distinguishable functions $\mathcal{Q}_c$ and $\mathcal{Q}_{c'}$, such that $\mathcal{Q}_c(A) = \{pestle, mortar\}$ and $\mathcal{Q}_{c'}(A) = \{pestle \sqcup mortar\}$.

The addition of such individuation schemas allows us to characterise, for example, the difference between the English mass CAN *kitchenware* and the German count CAN *Küchengerät-e* ('piece-s of kitchenware'). We assume that $kitchenware_w$ is a number neutral predicate (denoting items of kitchenware and sums thereof without the imposition of any individuation schema), and that plural morphology is interpreted in terms of Link's (1983) *-operator (such that $^*P$ is the upward closure of P under sum. Following Landman (2011, 2016); Sutton and Filip (2016a); Filip and Sutton (2017); de Vries and Tsoulas (2018), amongst others, we analyze nouns as denoting a bipartite structure that specifies (a) the extension of the noun and (b) a set that is the counting base of the noun. Count nouns have a quantized counting base set, and mass nouns do not. Formally, this is based on the notion of product types in the λ-calculus. Projections of product types are accessed via projection functions. For ease of presentation, we represent this as labelled frames, as schematised in (25). The counting base set specified in (25) is $\lambda y.P_w(y)$. The extension specified in (25) is $\lambda w.\lambda x.P_w(x)$.

(25) $$\lambda w.\lambda x.\begin{bmatrix} ext & = & P_w(x) \\ cbase & = & \lambda y.P_w(y) \end{bmatrix}$$

The lexical entry of a mass CAN like *kitchenware*, given in (26), specifies the semantically number neutral predicate $kitchenware_w$ as both its counting base and extension. This means that the property expressed is not countable since, relative to the world of evaluation w, the set $\lambda y.kitchenware_w(y)$ is not quantized.

(26) $$[\![[_N \text{ kitchenware}]]\!]^c = [\![[_N \text{ kitchenware}]]\!] = \lambda w.\lambda x.\begin{bmatrix} ext & = & kitchenware_w(x) \\ cbase & = & \lambda y.kitchenware_w(y) \end{bmatrix}$$

Singular count CANs are indexed to a schema of individuation at every context so that, at the context of utterance, (27) denotes a function from worlds to a quantized set of items of kitchenware. Crucially, this means that the counting base set is quantized; hence, the property expressed is countable. The interpretation of the plural is given in (28) where the counting base set is the same as the singular noun (a quantized set), but the extension is the closure of the extension of the singular noun under mereological sum (i.e., plural count nouns have cumulative extensions).

(27) $$[\![[_N \text{ Küchengerät}]]\!]^c = \lambda w.\lambda x.\begin{bmatrix} ext & = & \mathcal{Q}_c(kitchenware_w)(x) \\ cbase & = & \lambda y.\mathcal{Q}_c(kitchenware_w)(y) \end{bmatrix}$$

(28) $$[\![[_N \text{ Küchengerät-e}]]\!]^c = \lambda w.\lambda x.\begin{bmatrix} ext & = & ^*\mathcal{Q}_c(kitchenware_w)(x) \\ cbase & = & \lambda y.\mathcal{Q}_c(kitchenware_w)(y) \end{bmatrix}$$

3.2. Pluralities and individuation schemas across semantic types

In order to be able to apply a similar strategy to capture the IONs data, we need a couple of ingredients. First, we will lay out what domains we assume and what kind of structure we assume they have. Minimally, this will require that we can make sense of pluralities of propositions and eventualities. Second, to define the notion of a context-indexed information schema that applies to entities of any of these types.

For the first part, we assume the notion of type-generalised plural structures developed by Schmitt (2013, 2017). This gives us a generalised mereological sum operation ($\sqcup$) operation over different semantic types. For each domain of type a, $\mathcal{D}_a$, we have a bijection function pl_a on the powerset of $\mathcal{D}_a$ to a plural structure; namely, the set of singularities and pluralities for that domain $\mathbf{PL}_a$ (the inverse of pl_a is pl_a^{-1}):

(29) $$pl_a : (\mathscr{P}(\mathcal{D}_a)\backslash\varnothing) \rightarrow \mathbf{PL}_a$$

Predicates with domains on $\mathbf{PL}_a$ are members of the power set of this domain: $\mathbf{S}_a = \mathscr{P}(\mathbf{PL}_a)$, namely sets of sets of singularities and pluralities for that domain.

With respect to domains, we assume sets of entities, truth values ($\{0,1\}$), worlds, and a domain of eventualities that fall under STATES, PROCESSES, and EVENTS (in the sense of Bach (1981, 1986), i.a.). This automatically gives us plural structures for these domains (we do not assume there is a use for a plural structure for the domain of type t). If we add to this the assumption that the domain of propositions is structured as the powerset of the set of worlds, then we can specify plural structures for this domain. Suppose we have three (atomic) possible worlds in our domain: w_1, w_2, w_3, and so three *atomic functions* of type $\langle s,t\rangle$ characterised by the singleton sets $\{w_1\}$, $\{w_2\}$, and $\{w_3\}$. The set of possible propositions is $\mathscr{P}(\{\{w_1\}, \{w_2\}, \{w_3\}\})\backslash\varnothing$ which is isomorphic to $\mathbf{PL}_{\langle s,t\rangle}$. For example, for some $\mathbf{p},\mathbf{q},\mathbf{r}$, it is the case that $pl_{\langle s,t\rangle}(\{w_1\}) = \mathbf{p}$, $pl_{\langle s,t\rangle}(\{w_2\}) = \mathbf{q}$, and $pl_{\langle s,t\rangle}(\{w_3\}) = \mathbf{r}$. Count and mass predicates that denote sets of propositions can then be distinguished in terms of quantization just as we were able to do for predicates denoting concrete entities. The same strategy also applies to give us plural structures for eventualities.

The second step we need is to generalise our earlier definition of $\mathscr{Q}_c$ to apply to different semantic types. We do this by defining $\mathscr{Q}_c$ to apply to expressions of any type and return a maximally quantized subset thereof (a possibly different set depending on the value of the context, c).

> For all c, for all $\tau \in \mathbf{type}$, $\mathscr{Q}_c$ is a polymorphic function of type $\langle\tau,\tau\rangle$
>
> If X is a set, the members of which are of type a, then $\mathscr{Q}_c$(X) = Y, such that $\{\mathbf{y} : \mathbf{y} = pl_a(y), y \in Y\} \subseteq_{max.QUA} \{\mathbf{x} : \mathbf{x} = pl_a(x), x \in X\}$

For example, applying pl_s to the members of $X = \{\{w_1\},\{w_2\},\{w_1,w_2\}\}$ yields $\{\mathbf{p},\mathbf{q},\mathbf{p}\sqcup\mathbf{q}\}$. Since there are two maximally quantized subsets of this set, $\{\mathbf{p},\mathbf{q}\}$ and $\{\mathbf{p}\sqcup\mathbf{q}\}$, there are two distinguishable contexts c and c' such that:

$$\mathscr{Q}_c(X) \;=\; \{\{w_1\},\{w_2\}\}; \qquad \mathscr{Q}_{c'}(X) \;=\; \{\{w_1,w_2\}\}$$

Our type-generalised individuation schema can now also apply to specify maximally quantized

Table 2: Summary of the semantic subclasses of Informational Object Nouns

Class	Denotation	Examples
Subclass 1	Set of propositions	*fact, information*
Subclass 2.1	Relation between entities, propositions and EVENTS	*statement, utterance*
Subclass 2.2	Relation between entities, propositions and STATES	*belief, knowledge*

subsets for the domains of entities, and eventualities as well; this is all that we will need to model the semantics of IONs.

3.3. Lexical entries for IONs

Based on our observations regarding the sub-classes of IONs in section 2.3, we propose two types of lexical entries for IONs: One denoting a set of propositions, the other denoting a relation between entities, propositions, and eventualities. The latter is divisible into two further subclasses depending on whether the eventualities are STATES or EVENTS. These sub-divisions are summarised in Table 2.

Subclass 1: *fact*-like nouns

Nouns in this group denote sets of propositions (they do not encode an associated eventuality). They can be count (*fact*) or mass (*information*). As with concrete nouns, we assume that singular count nouns denote quantized sets and are interpreted relative to a context-indexed individuation schema. Mass nouns specify non-quantized counting base sets and are not interpreted relative to a context-indexed individuation schema. Where p is a variable over the domain of propositions (of type $\langle s,t \rangle$), the lexical entry for *information* is similar to that of *kitchenware* in (26), except that, relative to a world, the extension of $[\![[_N \text{ information}]]\!]^c$ is a set of propositions, not of entities.

(30) $$[\![[_N \text{ information}]]\!]^c = [\![[_N \text{ information}]]\!] = \lambda w.\lambda p. \begin{bmatrix} ext & = & information_w(p) \\ cbase & = & \lambda p'.information_w(p') \end{bmatrix}$$

Singular count IONs are indexed to a schema of individuation at every context, so that, at the context of utterance, (31) denotes a function from worlds to a quantized set of items of kitchenware. The interpretation of the plural is given in (32).

(31) $$[\![[_N \text{ fact}]]\!]^c = \lambda w.\lambda p. \begin{bmatrix} ext & = & \mathcal{Q}_c(fact_w)(p) \\ cbase & = & \lambda p'.\mathcal{Q}_c(fact_w)(p') \end{bmatrix}$$

(32) $$[\![[_N \text{ facts}]]\!]^c = \lambda w.\lambda p. \begin{bmatrix} ext & = & {}^*\mathcal{Q}_c(fact_w)(p) \\ cbase & = & \lambda p'.\mathcal{Q}_c(fact_w)(p') \end{bmatrix}$$

Similarly, the German count counterpart of information, *Information-en* (‘piece-s of information’) has a similar structure to *facts*:

(33) $$[\![[_N \text{ Information-en}]]\!]^c = \lambda w.\lambda p. \begin{bmatrix} ext & = & {}^*\mathcal{Q}_c(information_w)(p) \\ cbase & = & \lambda p'.\mathcal{Q}_c(information_w)(p') \end{bmatrix}$$

Subclass 2.1: *statement*-like nouns

Nouns in this group denote relations between entities, propositions, and EVENTS. At least in our English data, the primary senses of these nouns tends to be count, however, they can sometimes be used as mass nouns as indicated in (34) which contains both a count and a mass use of *utterance* in the same sentence (the mass use is indicated by the bare singular noun).

(34) What's all important is the narrative, the sequence of utterances, rather than the voluptuousness of utterance itself. [BNC]

Recall that, on their count uses, we can individuate the denotations of these nouns on the basis of either the eventualities they relate to or the propositions that are the contents of the eventuality (performed by some agent). Individuation in terms of propositions will be captured in a similar manner as for count *fact*-like IONs; namely, the context-indexed individuation function $\mathcal{Q}_c$ will apply to a (non-quantized) set of propositions and return a maximally quantized subset. To explain why we can also individuate in terms of, e.g., stating- or uttering-events, we must appeal to the nature of these kinds of events, namely, that relative to an agent and a propositional contents, such sets of events are quantized. To see this, suppose that a states that p and that this eventuality, e has a run time t. There is no sub-eventuality in which a also states that p. (Of course, if there are sub-propositions of p, such as q and r, then there may be $e', e'' \sqsubseteq e$ such that e' is a stating q event and e'' is a stating r event). In other words, predicates of such events (more specifically, accomplishments, a subtype of events), restricted in this way, are quantized.

Crucially, when we turn to IONs that denote STATES (such as *belief* and *knowledge*), a similar argument will not apply. That is because states, such as belief-states, hold true for experiencers at relatively long and vague time intervals. If an experiencer has a mental state (e.g., belief) with some propositional contents for some time interval, then they will be in that mental state at any subinterval and moment within that interval. For example, if Ann believes that it is raining during time interval i, then this belief persists at any subinterval and moment of i. Therefore, the set of mental attitude states is not quantized, even if the experiencer and the content of these states are the same.

Turning back to the interpretations of *statement*-like nouns, where p is a variable over the domain of propositions (of type $\langle s,t\rangle$), and e is a variable over EVENTS, the lexical entry for *statement* is as in (35).

(35)

$$[\![[_N \text{ statement}]]\!]^c = \\ \lambda w.\lambda x.\lambda e.\lambda p \begin{bmatrix} ext & = & statement_w(e) \wedge agent(e)(x) \wedge \mathcal{Q}_c(contents(e))(p) \\ cbase & = & \lambda e'.\lambda p'.statement_w(e') \wedge agent(e')(x) \wedge \mathcal{Q}_c(contents(e'))(p') \end{bmatrix}$$

Relative to a world and an agent, we assume that there are two options for how to existentially close the remaining arguments. Either one can existentially close the e (and e') variables, yielding a quantized set of propositions as the extension and the counting base (36), or one can close the p (and p') variables, yielding a set of EVENTS as the extension and the counting base (37). This set of EVENTS is also quantized.[6]

(36)

$$[\![[_N\ a\text{'s statement}]_{proposition\ denoting\ reading}]\!]^{c,w} = \\ \lambda p.\exists e. \begin{bmatrix} ext & = & statement_w(e) \wedge agent(e)(a) \wedge \mathcal{Q}_c(contents(e))(p) \\ cbase & = & \lambda p'.\exists e'.statement_w(e') \wedge agent(e')(a) \wedge \mathcal{Q}_c(contents(e'))(p') \end{bmatrix}$$

[6]For a proposal for the semantics of subjective genitive expressions containing IONs, see Sutton and Filip (2019)

(37) $[\![[_N\ a\text{'s statement}]_{\text{EVENT } denoting\ reading}]\!]^{c,w} =$

$$\lambda e.\exists p.\begin{bmatrix} ext = statement_w(e) \wedge agent(e)(a) \wedge \mathscr{Q}_c(contents(e))(p) \\ cbase = \lambda e'.\exists p'.statement_w(e') \wedge agent(e')(a) \wedge \mathscr{Q}_c(contents(e'))(p') \end{bmatrix}$$

In other words, an expression such as *Alex's statement* is polysemous between denoting a quantized set of events in which Alex stated some proposition and a quantized set of propositions each of which is the contents of some stating event by Alex.

For plural uses of IONs in this class, we assume that the *-operator applies to both the set of eventualities and to the set of propositions. The output of this is shown in (38).

(38) $[\![[_N\ \text{statements}]]\!]^c =$

$$\lambda w.\lambda x.\lambda e.\lambda p\begin{bmatrix} ext = {}^*statement_w(e) \wedge agent(e)(x) \wedge {}^*\mathscr{Q}_c(contents(e))(p) \\ cbase = \lambda e'.\lambda p'.statement_w(e') \wedge agent(e')(x) \wedge \mathscr{Q}_c(contents(e'))(p') \end{bmatrix}$$

The result of this interpretation of plural IONs in this class is that plural uses of nouns such as *statements* should be polysemous between denoting pluralities of propositions and pluralities of events, and this is the pattern we find in the data (see section 2.3).

We motivate this double application of * in the following way. Plural morphology in English, semantically speaking, encodes sum formation. However, this sum formation presupposes a quantized set (i.e., a singular count noun). Where the noun expresses a relation and where this noun is extensionally polysemous between two of its relata (e.g., between EVENTS and propositions), the meaning of plural morphology is to generate sums on both of these sets provided that they are both quantized sets. If only one of these sets is quantized, then sum generation will only be on the basis of the quantized set. Where neither of the sets are quantized, application of plural morphology will be infelicitous. As we will argue below, this assumption about the application of plural morphology can explain the different distributional patterns we observed for nouns like *statement*, *belief*, and *knowledge*.

Subclass 2.2: *belief*-like nouns

Nouns in this class also denote relations, however, unlike *statement*-like nouns that relate agents, EVENTS, and propositions, they relate experiencers, STATES, and propositions. Nouns in this class can be count (*belief, opinion*) or mass (*knowledge*). The default reading for nouns in this class is one that denotes propositions, especially when used in the plural or in counting constructions (see section 2). As we argued above, this divergence from the grammatical behaviour of *statement*-like IONs can be explained on the basis of the fact that, for *statement*-like IONs, the relevant EVENTS relative to an agent and a propositional contents form a quantized set, whereas for *belief*-like IONs the relevant STATES, relative to an experiencer and a propositional contents, do not form a quantized set.

Where p is a variable over the domain of propositions (of type $\langle s,t\rangle$), and s is a variable over STATES, and *exp* is the Experiencer thematic relation, the lexical entry for the count ION *belief* is as in (39).

(39) $[\![[_N\ \text{belief}]]\!]^c =$

$$\lambda w.\lambda x.\lambda s.\lambda p.\begin{bmatrix} ext = belief_w(s) \wedge exp(s)(x) \wedge \mathscr{Q}_c(contents(s))(p) \\ cbase = \lambda s'.\lambda p'.belief_w(s') \wedge exp(s')(x) \wedge \mathscr{Q}_c(contents(s'))(p') \end{bmatrix}$$

Crucially, given an experiencer, when the s and s' arguments are existentially closed, we get a quantized set of propositions, but when the p and p' arguments are existentially closed, we get a non-quantized set of states. Also, given our assumptions about plural morphology above, when such IONs are pluralised, as shown in (40), the *-operator only applies to the set of propositions. This explains why in most circumstances, when we individuate beliefs, we do so on the basis of their contents and not on the basis of different mental states of the same experiencer.[7] Hence, sentences like *Alex's beliefs that p and q* cannot get a reading in which the plurality denoted is a plurality of Alex's mental states, and sentences such as *Alex's beliefs that p* are infelicitous, precisely because there is no plurality of propositions made available, and the set of Alex's mental states is not quantized leading to an unavailability of any plurality of states generated from a quantized set.

(40) $[\![[_N \text{ beliefs}]]\!]^c =$

$$\lambda w.\lambda x.\lambda s.\lambda p.\begin{bmatrix} ext & = & belief_w(s) \wedge exp(s)(x) \wedge {}^*\mathscr{Q}_c(contents(s))(p) \\ cbase & = & \lambda s'.\lambda p'.belief_w(s') \wedge exp(s')(x) \wedge {}^*\mathscr{Q}_c(contents(s'))(p') \end{bmatrix}$$

For a mass noun in this class such as *knowledge*, just as we saw for mass nouns in the class of *fact*-like IONs, there is no context-indexed individuation schema in the lexical entry. This is shown in (41), where we have additional included a veridicality constraint on the meaning of knowledge under the widespread philosophical assumption that knowledge cannot be false. (CG_w is the set of worlds in the common ground accessible from the world of evaluation.)

(41) $[\![[_N \text{ knowledge}]]\!]^c =$

$$\lambda w.\lambda x.\lambda s.\lambda p.\begin{bmatrix} ext & = & knowledge_w(s) \wedge exp(s)(x) \wedge contents(s)(p) \wedge \\ & & \forall w' \in CG_w.contents(s)(p)(w') = 1 \\ cbase & = & \lambda s'.\lambda p'.knowledge_w(s') \wedge exp(s')(x) \wedge contents(s')(p') \wedge \\ & & \forall w' \in CG_w.contents(s')(p')(w') = 1 \end{bmatrix}$$

Because there is no context-indexed individuation schema in (41), when provided with an experiencer and a world of evaluation, we get neither a quantized set of knowledge-states nor a quantized set of propositions. This predicts that not only will *knowledge* be infelicitous in counting constructions (see Appendix), it will also be infelicitous with plural morphology since there is no quantized set from which to generate sums of either propositions or knowledge states.

Finally, we turn to a puzzle about mass IONs raised in section 2. There we noted that mass IONs such as *information* and *evidence* are highly resistant to any kind of mass-to-count coercion: If placed in any syntactic environment that selects for count nouns, the result is infelicitous. However, *knowledge* is different. Whereas combining *knowledge* with numerical expressions (*# three knowledges*) is not felicitous, *knowledge* can be used in at least one count noun environment, namely with an indefinite determiner (11a). While we cannot, here, provide a full analysis of these data, we would like to highlight a possible explanation that is available to us, given our analyses of the different sub-classes of IONs. What distinguishes nouns such as *knowledge*, on the one hand, from nouns such as *information* and *evidence*, on the other, is that only the former specify eventualities as part of their lexical semantics. Hence, one path

[7]Where a plurality of experiencers is specified, but only a simple sentence is given as in (20), one can get a reading where states are individuated in terms of experiencers. For an account of such cases, see Sutton and Filip (2019).

for explaining the fact that *knowledge* can be used with the indefinite article but *information* and *evidence* cannot is that the individuation of *knowledge* in this context turns on anchoring some set of knowledge states in some way (for a proposal regarding anchoring of psych nouns to events and agents, see Grimm (2014)). Hence, *a good knowledge of chemistry* would denote some sum of mental states that is selected in terms of an experiencer and the topic to which the contents of these mental states pertains. If this sort of explanation is on the right track, then it would automatically predict the infelicity of *a good information/evidence*, since neither *information* nor *evidence* make available an eventuality in their lexical semantics that can be anchored to something.

4. Conclusion

Providing an adequate theory of the count/mass distinction, even for concrete nouns, is no mean feat, and so it is not surprising that the majority of semantic analyses of countability have focused on concrete nouns to the exclusion of abstract nouns. However, we have argued that it is not a futile endeavour to modify our theories of countability for concrete nouns and so extend their coverage to abstract domains. Central notions in extensional mereological approaches to countability such as quantization can be meaningfully applied to the domains of abstract nouns, such as over propositions. Furthermore, as we have argued, there are interesting and mostly unexplored, parallels between classes of abstract nouns, such as IONs, and classes of concrete nouns, such as CANs, that hint at the possibility of further unifying our semantic analyses of nouns that denote the concrete and the abstract.

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Appendix: Counting constructions

Here, we give a unified semantics for counting constructions for abstract IONs and concrete nouns. Our analysis can derive interpretations for *zwei Informationen* ('two pieces of information', German), *zwei Küchengeräte* ('two pieces of Kitchenware', German), but rule out *two information(s)* and *two kitchenwares*. We select English and German to show how our analysis captures crosslinguistic facts, however, as we will make clear, crosslinguistic pairs of numerical expressions such as *two* in English and *zwei* ('two') in German will be given the same interpretation (so any infelicity in counting constructions will be explained via properties of the noun). We leave the analysis of counting constructions for relational nouns for further work, and so do not, here, extend this analysis to *statement*- or *belief*-like IONs.

Numerical expressions

The interpretations of numerical expressions such as *two* in English and *zwei* ('two') in German are based on a cardinality function that we define in (42). The cardinality function is relative to a set P, and is only defined if this set is quantized. For quantized sets, the cardinality of an entity x with respect to P is the cardinality of the P parts of x:

(42) $$\forall P.\forall x.\mu(x,P) = \begin{cases} |\{y : y \sqsubseteq x, P(y)\}| & \text{if } QUA(P) \\ \bot & \text{otherwise} \end{cases}$$

In the interpretations of numerical expressions, the relevant set will be the counting base set in the lexical entries of nouns. This gives us the right selectional restrictions for numerical expressions, namely that they can only felicitously combine with count nouns (barring coercion).

We assume that numerical expressions in English and German are of the equivalent of an adjectival (modifier) type (the equivalent in this system of type $\langle\langle s,\langle et\rangle\rangle,\langle s,\langle et\rangle\rangle\rangle$). This is represented in (46). Where $\mathcal{P}$ is a variable over common noun interpretations such as $[\![$information$]\!]$, $[\![$Information(en)$]\!]$ ('piece(s) of information', German), $[\![$kitchenware$]\!]$ and $[\![$Küchengerät-e$]\!]$ ('piece-s of Kitchenware', German). This means that we must assume that $\mathcal{P}$ is of an underspecified type, i.e., underspecified between the type for entity denoting noun interpretations and proposition denoting noun interpretations. The variables $\chi, \mathfrak{y}$ are underspecified between type e and type $\langle s,t\rangle$. The schema for a noun lexical entry is given in (43).

(43) $$\lambda w.\lambda \chi.\begin{bmatrix} ext & = & P_w(\chi) \\ cbase & = & \lambda \mathfrak{y}.P_w(\mathfrak{y}) \end{bmatrix}$$

In addition, we use two projection functions *CBASE* and *EXT* such that:

(44) $$\lambda w.\lambda \chi.CBASE(43)(\chi)(w) = \lambda \mathfrak{y}.P_w(\mathfrak{y})$$

(45) $$\lambda w.\lambda \chi.EXT(43)(\chi)(w) = P_w(\chi)$$

The interpretation of *two* (or equivalently *zwei* in German) can now be stated as follows in (46). This is a function that applies to a common noun and returns an entry with the same counting base set and with an extension that is restricted to only include entities with a cardinality of 2 with respect to the counting base set.

(46) $$[\![[_{Num}\text{ two}]\!] = [\![[_{Num}\text{ zwei}]\!] = \lambda \mathcal{P}\lambda w.\lambda \chi.\begin{bmatrix} ext & = & EXT(\mathcal{P}(\chi)(w)) \wedge \mu_{\#}(\chi, CBASE(\mathcal{P}(\chi)(w))) = 2 \\ cbase & = & CBASE(\mathcal{P}(\chi)(w)) \end{bmatrix}$$

Counting constructions

This semantics for numerical expressions, i.e. modifiers that restrict the extension of a noun to entities that have a cardinality of n with respect to the counting base set, automatically selects for count nouns, given the definition of $\mu_{\#}$. This means that we can straightforwardly account for the interpretation and felicity of counting constructions with both concrete count nouns and count IONs such as *zwei Küchengeräte* ('two pieces of Kitchenware', German) and *zwei Informationen* ('two pieces of information', German), respectively. The derivations for these are based on (46) and the entries for *Küchengeräte* (28) and *Informationen* (33) in section 3

(47) $$[\![[_{NumP}\text{ zwei Küchengeräte}]\!]^c = [\![[_{Num}\text{ zwei}]\!]([\![[_N\text{ Küchengeräte}]\!]^c) = \lambda w.\lambda x.\begin{bmatrix} ext & = & {}^*\mathcal{Q}_c(kitchenware_w)(x) \wedge \mu_{\#}(x, \mathcal{Q}_c(kitchenware_w)) = 2 \\ cbase & = & \lambda y.\mathcal{Q}_c(kitchenware_w)(y) \end{bmatrix}$$

(48) $$[\![[_{NumP}\text{ zwei Informationen}]\!]^c = [\![[_{Num}\text{ zwei}]\!]([\![[_N\text{ Informationen}]\!]^c) = \lambda w.\lambda p.\begin{bmatrix} ext & = & {}^*\mathcal{Q}_c(information_w)(p) \wedge \mu_{\#}(x, \mathcal{Q}_c(information_w)) = 2 \\ cbase & = & \lambda q.\mathcal{Q}_c(information_w)(q) \end{bmatrix}$$

For the English mass CAN and mass ION cases (*#two kitchenwares/informations*), composition with $[\![[_{Num}$ two$]\!]$ is ruled out since the counting base sets of $[\![[_N$ kitchenware$]]\!]$ and $[\![[_N$ information$]]\!]$ are not quantized, and so the use of these sets with $\mu_{\#}$ is undefined.