Monotonicity via mereology in the semantics of attitude reports[*]

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Abstract In this paper, we develop a new proposal about how the monotonicity of attitude verbs like *believe* should be modeled. Our empirical focus is on a puzzle discovered by Sharvit (to appear) concerning Negative Polarity Item (NPI) licensing in nominal arguments to monotonic attitude verbs; neither a standard Hintikkan account of attitude verbs, nor Kratzer's (2006) content-based semantics satisfactorily accounts for Sharvit's puzzle. Instead, we pursue the idea that monotonicity is a consequence of the *part-whole structure* of contentful entities; specifically, we achieve monotonicity by constraining how the part-whole structure of attitudinal eventualities relates to the part-whole structure of their *contents*, as well as the part-whole structure of their *theme*s. In order to cash out this idea, we rely on a non-monotonic equality-based semantics for clausal embedding (Moulton 2009, Elliott 2017, Bassi & Bondarenko 2021, Bondarenko 2022). By developing a more flexible account of monotonicity in attitude reports, we thereby resolve Sharvit's puzzle in a way broadly consistent with a content-based semantics for clausal embedding.

Keywords: clausal embedding, attitude reports, monotonicity, negative polarity items, mereology

1 Introduction

This paper is concerned with modeling the monotonicity of certain kinds of attitude reports. Monotonic verbs like *believe* allow us to draw inferences such as in (1). At first blush, it seems that if a proposition p semantically entails a proposition q, then from an attitude holder believing p we can infer that they also believe q. How should such inferences be captured?

(1)　　a.　Katya believes that Anton snowboarded last Friday.
　　　　b.　⤳ Katya believes that Anton snowboarded.

The predominant answer in the literature, originating in the work of Hintikka (1969), is that inferences like the one in (1) follow from a treatment of monotonic

attitude verbs as *universal modals*: e.g., a verb like *believe* states that the proposition expressed by the embedded clause is true in all worlds compatible with the beliefs of the attitude holder. This semantics guarantees that for any propositions p, q, where p semantically entails q, believing p semantically entails believing q.[1]

In this paper, we challenge this orthodox view. We will argue that there are no universal modals in sentences like in (1)—rather, we will capture the monotonicity of attitude verbs in terms of mereological properties of attitudinal eventualities like *believings*. We propose that *believing* eventualities have parts, and that the part-whole structure of *believings* reflects the part-whole structure of the propositional content associated with them. The reason (1a) entails (1b) is because the information conveyed by *Anton snowboarded* is part of the information conveyed by *Anton snowboarded last Friday*. It follows that, since Katya has a belief with content *Anton snowboarded last Friday*, a *part* of her belief is that *Anton snowboarded*.

In arguing for this alternative approach to monotonicity of attitude verbs, we examine patterns of NPI licensing in embedded clauses, as NPIS are known to be sensitive to the monotonicity properties of their local environment (Fauconnier 1975, Ladusaw 1979, and many others). Our main empirical focus will be the contrast in (2), first noticed by Sharvit (to appear): Sharvit observes that in negated belief reports, NPIS cannot be licensed in relative clauses modifying objects like *the rumor*, (2a), but they are licensed in complement clauses composing with such noun phrases, (2b).

(2)   **Sharvit's Puzzle**
   a.   *Katya doesn't believe [the rumor [that Anton has **ever** spread]].
   b.   Katya doesn't believe [the rumor [that Anton has **ever** snowboarded]].

The data in (2) are puzzling given that singular definite descriptions otherwise anti-license NPIS (Lahiri 1998, Guerzoni & Sharvit 2007, Gajewski & Hsieh 2015, Crnič 2019a). So it must be something about the semantics of clausal embedding that makes NPI licensing possible in these sentences. What is it that facilitates NPI licensing in such cases, and can these data help us distinguish between different theories of attitude reports?

Sharvit (to appear) argues that the contrast in (2) cannot be captured by Kratzer's content-based semantics for clausal embedding (Kratzer 2006, 2013a), according to which the complement of a clause-taking predicate is a modifier of that predicate specifying its propositional content (see also Moulton 2009, Bogal-Allbritten 2016, Elliott 2017, Bondarenko 2022). In this paper we will in effect argue against this conclusion by providing a positive proposal for (2) couched within a content-based approach. We show that the issue that arises for Kratzer's semantics turns out to

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1 In section 2 we'll discuss some complications that arise due to this prediction.

be very general, and requires us to rethink how monotonicity should be modeled in attitude reports. Once we have a new approach to modeling monotonicity—one that does away with modal semantics and appeals to mereological properties of contentful eventualities instead—we'll be able to account for the contrast in (2).

This paper is structured as follows: in section 2, we describe the theoretical landscape of approaches to the semantics of clausal embedding: against the backdrop of Hintikka's modal semantics. In doing so, we introduce two variants of the Kratzerian view which we will go on to compare throughout the paper: one that maintains aspects of Hintikka's proposal, which we call *Subset Semantics*, and one which does away with the modal component entirely, which we call *Equality Semantics*. Section 3 introduces our assumptions about the licensing of weak NPIS, and outlines the predictions of subset vs. equality semantics. As we will see, neither version of the Kratzerian approach as stated makes correct predictions regarding Sharvit's puzzle. Section 4 presents our proposal about modeling monotonicity of attitude verbs like *believe*. We argue that that there are certain constraints on the mapping between *believing* eventualities, their propositional contents, and their THEME arguments. Crucially, we'll make use of the idea that both attitudinal eventualities and their contents have a non-trivial part-whole structure. In section 5 we show how our proposal for deriving monotonicity allows us to capture Sharvit's puzzle. Section 6 discusses a possible extension for deriving closure under conjunction for the contents of *believings*. Section 7 is concerned with the notion of parthood for propositions. We discuss issues for defining parthood as the subset relation, and explore possible alternatives. Section 8 concludes the paper.

2 The semantics of clausal embedding

2.1 The Hintikkan view

The standard view of the semantics of the attitude verb *believe* originates in the work of Hintikka (1969) (see also Pearson 2015, Grano 2021 for recent overviews). According to Hintikka, *believe* contributes universal quantification over the attitude holder's *doxastic alternatives* (4). A sample lexical entry is given in (3)—informally, it says that a sentence of the form "*x* believes that *p*" is true iff every world which *x* takes to be a candidate for the actual world is one in which *p* is true.

(3) $[\![\text{believe}]\!]^w = \lambda p_{\langle s,t \rangle} . \lambda x_e . \forall w' \in \text{Dox}_w^x[p(w') = 1]$

(4) *Doxastic alternatives:*
For any individual $x \in D$, possible world $w \in W$,
$\text{Dox}_w^x = \{ w' \mid w' \text{ is compatible with what } x \text{ believes to be true in } w \}$

The semantics in (3) immediately predicts that belief reports should be subject to certain closure properties. One such property is *closure under entailment*, (5), since *believe* contributes universal quantification.

(5) *Closure under entailment:*
     For any sentences S, S', attitude-holder x,
     if S entails S', then "x believes S" entails "x believes S'".

Closure under entailment is responsible for capturing intuitively valid inferences, such as that in (6):

(6) Mitya believes that it's raining heavily.
     $\Rightarrow$ Mitya believes that it's raining.

Another closure property predicted by (3) is *Closure under conjunction*, (7). This follows from universal quantification over the attitude holder's *total* belief state; if $\text{Dox}_w^x \subseteq p$, and $\text{Dox}_w^x \subseteq q$, then $\text{Dox}_w^x \subseteq p \cap q$, for any propositions p, q.

(7) *Closure under conjunction:*
     For any sentences S, S', attitude-holder x,
     "x believes S and x believes S'" entails "x believes [S and S']".

Closure under conjunction is responsible for capturing intuitively valid inferences as in (8).

(8) Mitya believes that Jessica married an American,
     and Mitya believes that Jessica married a philosopher.
     $\Rightarrow$ Mitya believes that Jessica married an American philosopher.

It's worth noting that although the principles in (5) and (7) follow from the standardly-assumed semantics for *believe*, (5) at least is not unproblematic. A standard criticism of the Hintikkan semantics for *believe* is that a rational agent may believe p without believing q, even if p logically entails q, simply by failing to recognize that p entails q. This problem in its full generality came to be known as *the problem of logical omniscience* (Stalnaker 1991). We'll put this problem aside for now (we will return to this issue in section 7), and turn to the content-based semantics of clausal embedding originally developed by Kratzer (2006), which takes into account the distribution of embedded clauses beyond simple attitude reports.

2.2 The Kratzerian view

In an influential series of lectures Kratzer (2006, 2013, 2013, 2014, 2022, etc.) developed a new approach to the semantics of belief reports (and attitude reports

more generally), motivated in part by the distribution of *that*-clauses. The Kratzerian approach is couched within *event semantics* (Davidson 1967), and is in part motivated by the desire to maintain a simple, verbal semantics for attitude verbs. For example, on the Kratzerian view, we can model the semantic contribution of *believe* as simply contributing a *believing* eventuality, as in (9).[2] Note, that we adopt a neo-Davidsonian rendering of the Kratzerian view (following, e.g., Elliott 2017), according to which all arguments are severed from the verb (Castañeda 1967). Argument composition is mediated by *thematic functions*; for example, the attitude holder is introduced via a partial function HOLDER, which maps believing eventualities to their holders.[3]

(9) $[\![\text{believe}]\!] = \lambda e_v \, . \, \text{believe}(e)$

(10) $[\![\text{Mitya believes that} \ldots]\!]^w = 1$ iff

$\exists e \le w[\text{HOLDER}(e) = \text{M} \wedge \text{believe}(e) \wedge \ldots]$
There's an eventuality e located in w,
that's a believing and whose holder is Mitya...

In the remainder of this section, we introduce two different ways of integrating the semantic contribution of an embedded declarative clause: *subset semantics* and *equality semantics*.

2.2.1 Subset semantics

One of Kratzer's innovations was to mediate the composition of embedded *that*-clauses via a (partial) function CONT[4]. The role of CONT is to map *contentful* entities, such as *believings*, *facts*, *stories*, etc. to their informational contents, modeled as propositions. Embedded *that*-clauses themselves are taken to be predicates of contentful entities. One possible implementation of a Kratzerian semantics for *that*-clauses, based on Kratzer's original proposal (see also Hacquard 2006), is given in (11). We call this semantics *subset semantics* (following Bassi & Bondarenko 2021),

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2 We use the term *eventuality* to cover both events and states (Bach 1986).

3 In the following, we make use of technology from both event semantics, and intensional semantics (see, e.g., von Fintel & Heim 2021), therefore we need to be explicit about the relationship between eventualities and possible worlds. Although we don't believe that anything crucial hinges on this, for concreteness we do not assume *transworld* eventualities (cf. Lewis 1987); rather, we assume that possible worlds are maximal eventualities (Fine 2017c). Consequently, eventualities are located in worlds. This justifies the fact that predicates of eventualities are not relativized to a world of evaluation; believe(e) is simply true of any possible eventuality (in any world) that is a *believing*. Following standard assumptions in event semantics, we posit an the existential closure operator $\exists$ responsible for closing off the eventuality variable. We take $\exists$ to be responsible for locating the eventuality in the world of evaluation. $\exists e \le w$ is to be understood as locating eventuality e in world w.

4 See also work by Pietroski (2000) and Moltmann (1989, 2013, 2014, 2020, In press).

since the relationship between the content of the entity argument and the proposition denoted by the *that*-clause is *subsethood* (i.e., semantic entailment). Following Elliott (2017), we assume *that*-clauses themselves have a polymorphic type, which may be instantiated as either $\langle e,t \rangle$ or $\langle v,t \rangle$.[5] This reflects the assumption that both eventualities and individuals may have propositional content, and are therefore in the domain of CONT. This flexibility will be exploited in order to account for the broader distribution of embedded *that*-clauses.

(11) **Subset semantics for *that*-clauses** (after Kratzer 2006):
$$[\![\text{that } S]\!] = \lambda x_\sigma . \text{CONT}(x) \subseteq \{ w' \mid [\![S]\!]^{w'} = 1 \} \qquad\qquad \sigma \in \{ e,v \}$$

How does the propositional content of a believing eventuality relate to Hintikka's notion of *doxastic alternatives*? Minimally, it seems reasonable to assume that an attitude holder's doxastic alternatives are a subset of the contents of any of the attitude holder's individual belief states, as stated in (12). This is because an attitude holder's total beliefs may commit them to more than an individual *believing*.[6]

(12) **Content of *believings*:**
$$e \leq w \wedge \text{believe}(e) \rightarrow \text{Dox}_w^{\text{HOLDER}(e)} \subseteq \text{CONT}(e) \qquad\qquad \forall e \in D_v, w \in W$$

The result is a rather simple semantics for belief reports. Since the attitude verb is of type $\langle v,t \rangle$, and *that*-clauses may be of type $\langle v,t \rangle$, the *that*-clause may compose with the verb via intersective modification, reminiscent of the Davidsonian treatment of adverbials. After a thematic projection introduces the subject DP and relates the individual that it denotes to the believing eventuality by the HOLDER relation, we get the Logical Form in (13).[7]

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5 We take this to be a case of *underspecification* rather than ambiguity per se. Elliott simply assumes no type distinction between eventualities and individuals, but we instead cash this out in terms of polymorphism in order to remain neutral on the question of whether a type distinction is necessary. See also discussion in (Bondarenko 2022: 4.2, p. 244-245).

6 Note that the attitude holder can always be retrieved from the attitudinal eventuality by making reference to the HOLDER of the eventuality, assuming that all believing eventualities are in the domain of HOLDER.

7 There are some non-trivial differences between the compositional regime we assume for embedded *that*-clauses in this paper, versus Kratzer's original proposal. Kratzer (2006) (see also Moulton 2009, 2015) assumes a rigid $\langle e,t \rangle$ type for *that*-clauses. Consequently, in order to compose with attitude verbs like *believe*, Kratzer assumes that the *that*-clause modifies an implicitly assumed individual argument via the composition principle RESTRICT (Chung & Ladusaw 2004). See Elliott (2016, 2017) for arguments that English *that*-clauses combine via the event argument of the verb, and (Bondarenko 2021, Özyıldız 2020, Roberts 2020, Bochnak & Hanink 2022, Bondarenko 2022) for the view that embedded clauses can both be event modifiers and combine via the internal argument path.

(13)　⟦Mitya believes that it's raining⟧^{w} = 1 iff

$$\exists e \leq w \left[\begin{array}{l} \text{HOLDER}(e) = \text{Mitya} \wedge \text{believe}(e) \\ \wedge \, \text{CONT}(e) \subseteq \{ w' \mid \text{it's raining in } w' \} \end{array} \right]$$

If we take an attitude holder x's doxastic alternatives to be a subset of any of x's individual *believings*, then it should be clear that the semantics in (13) delivers something extremely close to the Hintikkan semantics for *believe* discussed in section 2.1. Just as with the Hintikkan semantics, Kratzer's subset semantics entails that the attitude holder's doxastic alternatives are a subset of the proposition denoted by the embedded *that*-clause. Subset semantics therefore inherits both the advantages and disadvantages of the Hintikkan view, predicting both the closure properties in (5) and (7). We informally illustrate closure under entailment in (14), by showing that subset semantics indeed predicts that "Mitya believes that it's raining heavily" entails that "Mitya believes that it's raining".

(14)　Mitya believes that it's raining heavily
　　　p　= *that it's raining heavily*
　　　q　= *that it's raining*
　　　a.　$\exists e \leq w[\text{HOLDER}(e) = \text{M} \wedge \text{believe}(e) \wedge \text{CONT}(e) \subseteq p]$
　　　b.　$p \subseteq q$, therefore if $\text{CONT}(e) \subseteq p$, then $\text{CONT}(e) \subseteq q$
　　　c.　$\therefore \exists e \leq w[\text{HOLDER}(e) = \text{M} \wedge \text{believe}(e) \wedge \text{CONT}(e) \subseteq q]$

One of the virtues of Kratzer's approach to *that*-clauses is that it automatically accounts for their capacity to compose with NPs ranging over contentful entities, as well as their ability to serve as predicates in copular constructions (e.g., "the rumor is that it's raining"). Consider for example the noun "rumor"; assuming that *rumors* are in the domain of CONT, *that*-clauses may compose simply via intersective modification. As a result "rumor that it's raining" denotes a property of individuals that are *rumors*, and whose content entails that it's raining (Kratzer 2006).

(15)　⟦there's a rumor that it's raining⟧^{w} = 1 iff
　　　$\exists x \in w[\text{rumor}(x) \wedge \text{CONT}(x) \subseteq \{ w' \mid \text{it's raining in } w' \}]$

On the face of it, subset semantics preserves many of the advantages of the classical Hintikkan semantics, while providing a uniform analysis of *that*-clauses in different environments. In the next section, we consider a different rendering of the Kratzerian approach to embedded *that*-clauses.

2.2.2　Equality semantics

Equality semantics maintains almost all features of the Kratzerian view discussed in the previous section. The only difference is that instead of treating "that *S*" as a

property of contentful entities whose content *semantically entails* the proposition denoted by *S*, we can instead consider a semantics which *equates* the content of a contentful entity with the embedded proposition (Moulton 2009, Elliott 2017, Bassi & Bondarenko 2021, Bondarenko 2022), (16).

(16) **Equality semantics for *that*-clauses**
(after Moulton 2009, Elliott 2017, Bondarenko 2022, etc.):
$\llbracket \text{that } S \rrbracket = \lambda x_\sigma . \text{CONT}(x) = \{ w' \mid \llbracket S \rrbracket^{w'} = 1 \}$
$\qquad\qquad \sigma \in \{ e, v \}$

This small change results in a semantics that makes radically different predictions. Consider the resulting truth-conditions for a simple attitude report:

(17) $\llbracket \text{Mitya believes that it's raining} \rrbracket^w = 1$ iff

$$\exists e \leq w \begin{bmatrix} \text{HOLDER}(e) = \text{M} \land \text{believe}(e) \\ \land \text{CONT}(e) = \{ w' \mid \text{it's raining in } w' \} \end{bmatrix}$$

The resulting truth-conditions say, informally, that there's a *believing* of Mitya's, whose content is *that it's raining*. There are some immediate disadvantages to this conjecture: we fail to predict *closure under entailment* and *closure under conjunction*. This is because an attitude holder having a *believing e* with content *p* doesn't say anything about whether the attitude holder has a *believing e′* with content *q*, irrespective of any logical relationship between *p* and *q*. For example, under equality semantics, Mitya having a *believing that it's raining heavily*, (18a), is compatible with Mitya having no *believing that it's raining*, (18c). Even though the condition in (12) will guarantee that it's raining in all of Mitya's doxastic alternatives (18b), we need to posit something extra to guarantee from this the existence of an actually *believing* eventuality with content that it's raining.

(18) p = *that it's raining heavily*
q = *that it's raining*
a. $\exists e \leq w[\text{HOLDER}(e) = \text{M} \land \text{believe}(e) \land \text{CONT}(e) = p]$
b. $\text{Dox}_w^\text{M} \subseteq p, \therefore \text{Dox}_w^\text{M} \subseteq q$
c. $\neg \exists e \leq w[\text{HOLDER}(e) = \text{M} \land \text{believe}(e) \land \text{CONT}(e) = q]$

How to enrich equality semantics in a way that derives closure under entailment will be a central concern of this paper, discussed in detail in section section 4. But first, we would like to discuss some reasons for why one might prefer equality semantics over subset semantics despite its aforementioned shortcomings.

2.3 Arguments for equality

2.3.1 Stacking

Probably the most straightforward argument in favor of equality semantics over subset semantics concerns restrictions on the distribution of *that*-clauses. Initially, note that equality semantics draws a close parallel between the semantic contribution of *that*-clauses, and that of bona fide thematic participants. In both cases, a functional relationship is established between the eventuality that the sentence is about, and an individual (in the case of thematic participants), or a proposition (in the case of *that*-clauses). Since the relationship is functional, given a contentful entity we can always retrieve the *unique* proposition that is its contents. This immediately (correctly) predicts that *that*-clauses can never be stacked (Moulton 2009, 2015, Elliott 2016, 2017, Bondarenko 2022). This argument is most forceful for, e.g., *manner-of-speech* verbs, which arguably don't syntactically select a clausal complement.

(19)   a.   Neil whispered.

     b.   Neil whispered [that it's snowing].

     c.   * Neil whispered [that it's snowing] [that it's overcast].

To see why equality, but not subset semantics predicts the unacceptability of (19c), consider the Logical Forms predicted by each approach. Note that the embedded clauses are logically compatible.

(20) $p = $ *that it's snowing*; $q = $ *that it's overcast*

     a.   Equality semantics:
        $\exists e \leq w[\text{THEME}(e) = \text{N} \wedge \text{whisper}(e) \wedge \text{CONT}(e) = p \wedge \text{CONT}(e) = q]$

     b.   Subset semantics:
        $\exists e \leq w[\text{AGENT}(e) = \text{N} \wedge \text{whisper}(e) \wedge \text{CONT}(e) \subseteq p \wedge \text{CONT}(e) \subseteq q]$

Given that p and q aren't semantically identical, equality semantics predicts triviality, since no eventuality e can ever verify the existential statement in (20a). Subset semantics, on the other hand, predicts a completely sensible meaning—whatever the *whispering* conveyed, it entails both p and q. The ban on stacking holds generally for embedded *that*-clauses, including those that compose with nouns, as illustrated below. Equality semantics provides a unified explanation for these facts.[8]

(21)   a.   Neil heard a rumor [that it's raining].

     b.   * Neil heard a rumor [that it's raining] [that it's overcast].

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8 A proviso is necessary for conjoined *that*-clauses. Under equality semantics, additional footwork is necessary to allow for this possibility, but (Bassi & Bondarenko 2021) and (Bondarenko 2022: ch.3) for discussion.

A useful point of comparison is restrictive relative clauses, which do allow stacking. The semantics of restrictive modification doesn't lead to any expectation of infelicity in examples like (22). Subset semantics incorrectly predicts that embedded declaratives should pattern similarly.

(22)   Neil heard a rumor [that Mitya spread] [that Anton later denied].

2.3.2   Interaction with definiteness

In this section, we show that subset semantics makes odd predictions when combined with a standard semantics for definiteness. For concreteness, let us first assume a standard Fregean semantics for the definite article after, e.g., Heim & Kratzer 1998. The definite article composes with a predicate P, presupposes that there is a unique satisfier of P, and (if defined), returns that satisfier.

(23) $[\![\text{the}]\!] = \lambda P_{\langle e,t \rangle} : \exists!x[P(x) = 1] . \iota x[P(x) = 1]$

We'll now examine the predictions of (23) in tandem with a particular content noun—namely, *fact*—and a subset semantics for embedded *that*-clauses. Consider, e.g., the different presuppositions predicted by each theory:

(24)   "the fact that it's raining"
    a.   Presupp. (subset): $\exists!x[\text{fact}(x) \wedge \text{CONT}(x) \subseteq \{w' \mid \text{it's raining in } w'\}]$
    b.   Presupp. (equality): $\exists!x[\text{fact}(x) \wedge \text{CONT}(x) = \{w' \mid \text{it's raining in } w'\}]$

Subset semantics (24a) predicts that the definite description presupposes the existence of a *unique* fact, the content of which entails that it's raining. Importantly, this presupposition is not satisfied if there are two abstract *fact* entities, $\textbf{Fact}_r$ and $\textbf{Fact}_{rh}$ with the following contents, since the contents of both facts entail that it's raining.

(25)   a. $\text{CONT}(\textbf{Fact}_r) = \{w' \mid \text{it's raining in } w'\}$
    b. $\text{CONT}(\textbf{Fact}_{rh}) = \{w' \mid \text{it's raining heavily in } w'\}$

This seems strange, given that in a context where it's common ground that it's raining heavily, a speaker can assert, e.g., "the fact that it's raining doesn't bother me". Nevertheless, perhaps a proponent of subset semantics can insist that either $\textbf{Fact}_r$ and $\textbf{Fact}_{rh}$ exist, but not both. After all, abstract entities such as *facts* are obtuse—it's difficult to be sure whether or not they exist, even if their contents are true. This perspective isn't tenable however, given the acceptability of sentences such as (26). (26) presupposes that *there is a unique fact whose content entails that it's raining*, and *there's a unique fact whose content entails that it's raining heavily*.

For the presuppositions to be satisfied, the definite descriptions must in fact pick out the same fact—call it f. The assertion ascribes contradictory properties to f—that it both bothers and doesn't bother the speaker. Therefore, subset semantics predicts (26) to be trivial.

(26)   [The fact that it's raining] doesn't bother me,
       but [the fact that it's raining heavily] really does.

Note that equality semantics doesn't face this problem—the equality semantics in (24b) simply predicts that (26) presupposes the existence of a unique fact whose content is *that it's raining*, and a (distinct!) unique fact whose content is *that it's raining heavily*. Subset semantics on the other hand 'collapses' "the fact that p" and "the fact that q" just in case there is an entailment relationship between p and q. In a sense, the predictions of subset semantics are reminiscent of what we expect for definite descriptions with ordinary intersective modifiers. The unacceptability of (27) is expected, since if the presupposition of "the student" is satisfied, then it and "the French student" must refer to the same individual. This problematic predictions of subset semantics will actually be central to why it can't account for the NPI licensing facts, as we discuss in the next section.

(27)   # The student doesn't bother me, but the French student really does.

Elliott (2017) discusses a particular manifestation of the problematic interaction with definiteness. Elliott notes that "fact" is generally compatible with either a definite or an indefinite article, but as soon as it composes with an embedded *that*-clause, the definite article is obligatory.

(28)   a. The fact (that it's raining).
       b. A fact (#that it's raining).

The unacceptability of (28b) with a *that*-clause is surprising from the perspective of subset semantics. The problematic indefinite DP is predicted to assert that there is a fact whose content entails that it's raining, (29), and in principle there could be many such facts. So it is unclear what would make (28b) infelicitous.

According to the equality semantics, the problematic indefinite DP is predicted to assert that there is a fact whose content *is* that it's raining. Note that there will only ever be one such fact, given the arguably reasonable assumption that there is a one-to-one mapping between true propositions and abstract *fact* entities.

(29)   Subset semantics:
       $\exists x[\text{fact}(x) \wedge \text{CONT}(x) \subseteq \{w' \mid \text{it's raining in } w'\}]$

(30)   Equality semantics:
       $\exists x[\text{fact}(x) \wedge \text{CONT}(x) = \{w' \mid \text{it's raining in } w'\}]$

In order to explain the unacceptability of (28b) under the equality semantics, Elliott exploits Heim's (1991) Maximize Presupposition! (MP) principle. MP is designed to explain certain distributional restrictions on "a" vs. "the". Roughly, it says that given two contextually equivalent alternatives, one must use the alternative with stronger presuppositions, if those presuppositions happen to be met. MP explains, for example, why using the indefinite article in (31) gives rise to infelicity even though the assertion is literally true. Informally, this is because the definite article has stronger presuppositions—it presupposes unique existence, and its presuppositions are contextually satisfied, given that there is exactly one moon orbiting the earth. Exactly the same logic explains the unacceptability of (28b) with an embedded *that*-clause, assuming equality semantics: if there is exactly one fact that it's raining, the presuppositions of the definite alternative are satisfied, and therefore it must be used.

(31) { The | #A } moon shines brightly.

Before we finish this section, let us consider a possible objection to our argument against subset semantics: could it be that the failings are merely an artifact of an overly-simple semantics for the definite article? A salient alternative account of the definite article is provided by von Fintel, Fox & Iatridou 2015, who propose a more nuanced semantics for the definite article based on *maximal informativity*.[9] The definition (after von Fintel, Fox & Iatridou 2015: p. 166) is provided in (32). The idea here is that individuals are ordered relative to the restrictor according to the informativity of the resulting proposition—the definite article picks out the unique most informative individual.

(32) Maximal informativity semantics for "the":
    a. $[\![\text{the } \phi]\!]^w$ is defined iff:
       There is a unique maximal x according to $\leq_\phi$, s.t., $[\![\phi]\!]^w(x) = 1$.
       If defined, it picks out that x.
    b. $x \leq_\phi y$ iff $\{w' \mid [\![\phi]\!]^{w'}(x) = 1\} \subseteq \{w' \mid [\![\phi]\!]^{w'}(y) = 1\}$

Let's consider how this applies to content nouns like *fact*. For a definite description "the fact that p", abstract *fact* entities are ordered based on the proposition schema in (33). Since the content of a *fact* is true, the proposition schema in (33) becomes more informative the more informative the *content* of x is. The semantics in (32) therefore predicts that *the fact that p* should pick out the unique *fact* with the strongest content that entails p.

(33) $\{w' \mid x \in w' \wedge \text{fact}(x) \wedge \text{CONT}(x) \subseteq p\}$

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9 We are grateful to Filipe Hisao Kobayashi (p.c.) for suggesting that we explore this possibility.

This predicts that given **Fact**$_r$ (with content *it's raining*) and **Fact**$_{rh}$ (with content *it's raining heavily*), "the fact that it's raining" and "the fact that it's raining heavily" should both pick out **Fact**$_{rh}$. Maximal informativity therefore doesn't rescue subset semantics from problematic interactions with definiteness.

We'll finish this section by briefly summarizing where we've arrived at: we've considered two implementations of a Kratzerian semantics for clausal embedding: (i) subset semantics, and (ii) equality semantics. Subset semantics is modeled on the classical Hintikkan semantics for attitude reports, and preserves many of its features, such as closure under entailment. While equality semantics might seem unappealing because it does not predict closure under entailment, we've seen that it has some virtues: (a) it accounts for restrictions on stacking embedded *that*-clauses, and (b) it interacts less problematically with the semantics for the definite article.[10]

3 NPIS and monotonicity

3.1 NPI-licensing and singular definite descriptions

We follow an influential approach to NPI licensing based on the idea that whether an NPI is licensed depends on the *monotonicity* of its local environment (Fauconnier 1975, 1978, Ladusaw 1979, 1980b,a, Hoeksema 1986, Kadmon & Landman 1993, Lahiri 1998, von Fintel 1999, Crnič 2019b,a, a.o.). In particular, we adopt von Fintel's (1999) proposal that monotonicity should be computed relative to a kind of entailment that takes presuppositions into account: *Strawson Entailment*. In (34), we give a generalized definition of Strawson entailment for any boolean type τ (von Fintel 1999, Crnič 2019b).[11] The basic idea is as follows: for any boolean functions f and g, f Strawson entails g iff whenever an argument x *satisfies the presuppositions* of g, $f(x)$ entails $g(x)$.

(34) **Strawson Entailment** ($\Rightarrow_s$):

$$p_\tau \Rightarrow_s q_\tau \text{ iff } \begin{cases} p = 0 \text{ or } q = 1 & \tau = t \\ \forall x \in \text{Dom}(g), f(x) \Rightarrow_s g(x) & \tau = \langle \sigma_1, \sigma_2 \rangle \end{cases}$$

With this notion of entailment, we follow Crnič 2019b,a in defining the monotonicity properties of syntactic environments. The two properties we'll need are

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10 This is not an exhaustive summary of arguments in the literature—see, for example, Bondarenko 2022 for additional arguments based on the interpretation of nouns with embedded CPs, and the distribution of polarity-sensitive subjunctive clauses in Russian.

11 A type τ is *boolean* just in case $\tau = t$, or $\tau = \langle \sigma_1, \sigma_2 \rangle$, where σ_1 is any type, and σ_2 is a boolean type (Winter 2001).

Strawson Downward-Entailingness (SDEness), (35), and *Strawson Upward-Entailingness* (SUEness), (36).[12]

(35) **Strawson Downward-Entailing (SDE)** (from Crnič 2019b: p.3, (7)):
A constituent S is SDE wrt. a subconstituent X iff:
For all constituents X', s.t., $[\![X']\!] \Rightarrow_s [\![X]\!]$, $[\![S]\!] \Rightarrow_s [\![S[X / X']\!]]$

(36) **Strawson Upward-Entailing (SUE)**:
A constituent S is SUE wrt. a subconstituent X iff:
For all constituents X', s.t., $[\![X']\!] \Rightarrow_s [\![X]\!]$, $[\![S[X / X']\!]] \Rightarrow_s [\![S]\!]$

To illustrate, let's consider the sentences in (37) and (38). The whole sentence in (37) is SUE and not SDE with respect to its subconstituent [NP linguist], the restrictor of the indefinite article. Any alternative NP whose denotation entails $[\![linguist]\!]$, e.g. $[\![syntactician]\!]$, results in a sentence which entails (37), (37b), but is not entailed by it, (37c). Once we introduce negation into the sentence, (38), the whole sentence is SDE but not SUE with respect to its subconstituent [NP linguist], (38b)-(38c), since if we substitute, e.g., $[\![syntactician]\!]$, we'll get a sentence that does not entail the sentence in (38) but is entailed by it.

(37) [S Nadya met a [NP linguist]].
  a. $[\![syntactician]\!] \Rightarrow_s [\![linguist]\!]$
  b. $[\![Nadya\ met\ a\ syntactician]\!] \Rightarrow_s [\![Nadya\ met\ a\ linguist]\!]$
  c. $[\![Nadya\ met\ a\ linguist]\!] \not\Rightarrow_s [\![Nadya\ met\ a\ syntactician]\!]$

(38) [S Nadya didn't meet a [NP linguist]].
  a. $[\![syntactician]\!] \Rightarrow_s [\![linguist]\!]$
  b. $[\![Nadya\ didn't\ meet\ a\ syntactician]\!] \not\Rightarrow_s [\![Nadya\ didn't\ meet\ a\ linguist]\!]$
  c. $[\![Nadya\ didn't\ meet\ a\ linguist]\!] \Rightarrow_s [\![Nadya\ didn't\ meet\ a\ syntactician]\!]$

With the notions of SDEness and SUEness, we can now formulate the condition for licensing weak NPIS that we'll assume in this paper:

(39) **Licensing condition for weak NPIS**
A sentence containing a weak NPI α is acceptable iff α is dominated by a constituent that is SDE and not SUE with respect to α's restrictor.

This licensing condition correctly predicts that, if we take (37) and (38), and replace the indefinite article *a* with NPI *any*, (37) won't be acceptable but (38) will be:

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12 "S[X/X']" in (35)-(36) should be understood as the structure that is identical to S, except that S's subconstituent X has been replaced with X'.

(40)   a.   *Nadya met any linguist.
       b.   Nadya didn't meet any linguist.

The licensing condition as stated in (39) is actually stronger than is necessary for accounting for simple cases like (40), where simply requiring SDEness would be sufficient. The additional injunction against SUEness is motivated by the fact that weak NPIS are not licensed in singular definite descriptions (Lahiri 1998, Guerzoni & Sharvit 2007, Gajewski & Hsieh 2015, Crnič 2019a), which will be highly relevant to the empirical puzzle we focus on in this paper. Consider (41):

(41)   Nadya met [the linguist who has written any book].

This sentence is SDE with respect to the restrictor of the weak NPI [book], since for any constituent whose denotation entails ⟦book⟧, the sentence in (41) will Strawson entail a sentence in which [book] is substituted with this constituent. For example, if (41) is true, there must be a unique linguist who has written a book. If the presupposition of the sentence "Nadya met the linguist who has written a **new book**" is met, then there must exist a unique linguist who has written a new book. Note that *the linguist who has written any book* and *the linguist who has written any new book* will in this case necessarily pick out the same individual. We call this property of singular definite descriptions *uniqueness collapse*, (43): if two singular definite descriptions have restictors which stand in a (generalized) entailment relation, and their presuppositions are defined, then they co-refer.

(42)   **(41) is SDE with respect to [*book*]**, e.g. ⟦new book⟧ $\Rightarrow_s$ ⟦book⟧, and so:
       Nadya met the linguist who has written any book.
       $\Rightarrow_s$ Nadya met the linguist who has written any new book.
       a.   There is a unique linguist who has written a book, and Nadya met them.
       b.   There is a unique linguist who has written a new book.
       c. $\Rightarrow_s$ The unique linguist who has written a new book is the unique linguist who has written a book, and Nadya met them.

(43)   **Uniqueness collapse:**
       If ⟦the ϕ⟧ and ⟦the ψ⟧ are defined, and ⟦ψ⟧ $\subseteq$ ⟦ϕ⟧, then ⟦the ϕ⟧ $=$ ⟦the ψ⟧.

The property of uniqueness collapse guarantees that the sentence *Nadya met the student who has written a book* Strawson-entails the sentence *Nadya met the student who has written a new book*: since the two singular definites refer to the same individual, the assertive content of the two sentences is identical.

If the NPIS licensing condition only required that a constituent be SDE with respect to the indefinite's restrictor, then it would wrongly predict that the NPI in (41) should be licensed. This is what motivates the stronger requirement that the

constituent additionally not be SUE: due to the uniqueness collapse, (41) is also SUE with respect to the restrictor *[book]*, (44).

(44)   **(41) is SUE with respect to [*books*]**, e.g. ⟦new books⟧ $\Rightarrow_s$ ⟦books⟧, and so:
Nadya met the linguist who has written a new book.
$\Rightarrow_s$ Nadya met the linguist who has written a book.

   a. There is a unique linguist who has written a new book, and Nadya met them.

   b. There is a unique linguist who has written a book.

   c. $\Rightarrow_s$ The unique linguist who has written a new book is the unique linguist who has written a new book, and Nadya met them.

Thus, the condition in (39) successfully captures NPI licensing in singular definite descriptions—the definite article creates an environment that is both SDE and SUE, due to uniqueness collapse. Now if we return to the example with a relative clause from the Sharvit's paradigm, (2a), repeated below as (45), we can see why the sentence with an NPI is ungrammatical: it is SUE due to uniqueness collapse.

(45)   *Katya doesn't believe [the rumor [that Anton has **ever** spread]].

Since for any w and any time interval τ, ⟦rumor that Anton has spread during τ⟧^{w} $\subseteq$ ⟦rumor that Anton has spread⟧^{w}, ⟦the rumor that Anton has spread during τ⟧^{w} and ⟦the rumor that Anton has spread⟧^{w} will pick out the same rumor in w. This means that for any τ, the sentence *Katya doesn't believe the rumor that Anton spread during τ* will Strawson-entail the sentence *Katya doesn't believe the rumor that Anton spread*. Hence, examples like (45) are correctly predicted to be ungrammatical.

3.2   Predictions of subset vs. equality semantics

Sharvit's puzzle is, at its heart, about the licensing of NPIS in singular definite descriptions: why is an NPI possible in (47b), whereas NPIS are otherwise unacceptable in singular definite descriptions? In the previous section, we showed how the standard analysis of NPIS accounts for cases such as (46a) and (46b), due to the property of *uniqueness collapse*. This account can be maintained irregardless of whether one chooses to adopt a subset semantics for embedded *that*-clauses or an equality semantics, since the examples in (46) make no reference to content. Therefore, the examples in (46) don't discriminate between theories of clausal embedding. The cases in (47) are significantly more interesting—with embedded CPs, the presence of negation *does* make a difference to whether or not an NPI is licensed.

(46)   a.   * Mitya believes the rumor that anyone spread.

     b. * Mitya doesn't believe the rumor that anyone spread.

(47)   a. * Mitya believes the rumor that anyone was late.

     b.   Mitya doesn't believe the rumor that anyone was late.

It's important to emphasize that Sharvit's puzzle doesn't just concern negation; we find a similar contrast between relative clauses and embedded *that*-clauses in other SDE environments, e.g., in the restrictor of *every* (48a), the scope of *only* (48b) and *few* (48c), among others—in all these cases NPIS can be licensed in embedded clauses that combine with a singular definite noun phrase that is the object of *believe*.

(48)   a. Every student who believes…

        i. …[the claim that Mary **ever** left] is easily fooled

       ii. *…[the claim that Mary **ever** spread] is easily fooled

     b. Only John believes…

        i. …[the claim that Mary **ever** left].

       ii. *…[the claim that Mary **ever** spread].

     c. Few believe…

        i. …[the claim that Mary **ever** left].

       ii. *…[the claim that Mary **ever** spread].

As we'll show in detail in the following section, both subset semantics and equality semantics incorrectly predict *both* (47a) and (47b) to be unacceptable, but for different reasons. The predictions will be the same for the environments in (48), but we'll focus on negation from here on out, for the sake of exposition.

3.2.1   Subset semantics and uniqueness collapse

First, we'll show that subset semantics predicts both (47a) and (47b) to be unacceptable, since it predicts that the singular definite descriptions create environments that are both SDE and SUE due to uniqueness collapse—the logic, in fact, is identical to the account of the examples in (46). To demonstrate this prediction, let us consider definite descriptions of the form "the rumor that Q was late". Let's take the relevant descriptions to be "the rumor that a syntactician was late", and "the rumor that a linguist was late", and consider what is predicted by subset semantics.

(49)   **Subset semantics: predictions**

     a. Mitya doesn't believe the rumor that a syntactician was late.

     b. Mitya doesn't believe the rumor that a linguist was late.

     c. If the presupposition of (49a) is satisfied, then:
        $\exists!x[\text{rumor}(x) \wedge \text{CONT}(x) \subseteq \{\, w' \mid \text{a syntactician was late in } w' \,\}]$

    d. If the presupposition of (49a) is satisfied, then:
$\exists!x[\text{rumor}(x) \wedge \text{CONT}(x) \subseteq \{\, w' \mid \text{a linguist was late in } w' \,\}]$

    e. If both presuppositions are satisfied, then:
⟦the rumor that a syntactician was late⟧^{w}
$=$ ⟦the rumor that a linguist was late⟧^{w}

    f. ∴ (49a) and (49b) are Strawson-equivalent.

The source of this problematic prediction echoes the problematic interaction of subset semantics with definiteness that we discussed in section 2.3.2. At the heart of the problem is the fact that subset semantics collapses "the rumor that P" and "the rumor that P'", just in case $P' \subseteq P$. This leads to wrong predictions both about meanings of expressions like *the fact that p*, and about NPI licensing.

Despite making problematic predictions for certain singular definite descriptions, subset semantics naturally accounts for NPI-licensing in a *that*-clause embedded directly under *believe*, by rendering the local environment monotonic.

(50)   * Mitya believes that anyone is coming to the party.

(51)   Mitya doesn't believe that anyone is coming to the party.

To see why, consider the simplified logical form of a belief report according to subset semantics, and it's negated counterpart, (52)-(53).

(52) $\exists e \leq w[\text{believe}(e) \wedge \text{HOLDER}(e) = x \wedge \text{CONT}(e) \subseteq p]$

(53) $\neg\exists e \leq w[\text{believe}(e) \wedge \text{HOLDER}(e) = x \wedge \text{CONT}(e) \subseteq p]$

(52) preserves the entailingness of p: given an arbitrary q s.t., $p \subseteq q$, any believing eventuality whose content entails p will also entail q. The reverse does not hold—given an arbitrary p' s.t., $p \supseteq p'$, it isn't guaranteed that if a believing eventuality's content entails p it will also entail p'. (53), on the other hand, reverses the entailingness of p. Given an arbitrary p' s.t., $p \supseteq p'$, if there is no believing eventuality whose content entails p, it follows that there is no believing eventuality whose content entails p'. The reverse does not hold—given an arbitrary q s.t., $p \subseteq q$, just because there is no believing eventuality whose content entails p, does not mean that there is no believing eventuality whose content entails q. Therefore, the licensing condition in (39) predicts the attested judgments in (50) and (51).[13]

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13 One might have a potential concern that neg-raising is crucially implicated in the licensing of NPIS in the embedded clause: a salient reading of (51) conveys that *Mitya believes that nobody is coming to the party*. However, NPIS can be licensed in the complements of doxastic attitude predicates which don't allow for neg-raising, such as *to be certain* (see also Sharvit to appear), and so licensing NPIS cannot be contingent on neg-raising.

  (1)   Mitya { * is | isn't } certain that anyone is coming.

3.2.2 Equality semantics and non-monotonicity

Equality semantics also doesn't have the resources to capture the contrast between the pertinent examples in (47), but for a different reason: it creates an environment that is *neither* SDE *nor* SUE. Unlike subset semantics, it does not give rise to uniqueness collapse. This is a consequence of the fact that, given any function f, f cannot by definition map the same individual x to distinct values, (54).

(54) Functionality guarantees disjoint reference:
  If $\exists!x \in D_\sigma[f(x) = p]$ and $\exists!y \in D_\sigma[f(y) = q]$, and $p \neq q$
  then $\iota x[f(x) = p] \neq \iota y[f(y) = q]$ $\forall f \in D_{\langle \sigma, \tau \rangle}, p, q \in D_\tau$

Under equality semantics, CONT plays the part of f. This blocks the uniqueness collapse: "the rumor that a syntactician was late" and "the rumor that a linguist was late" necessarily pick out distinct entities, (55). Because of this, the sentences in (55a) and (55b) are truth-conditionally independent: neither entails the other one, even though $[\![\text{syntactician}]\!] \Rightarrow_s [\![\text{linguist}]\!]$. Thus, singular definite descriptions of the form "the rumor that P" create *non-monotonic* environments: sentences containing them are neither SDE nor SUE with respect to indefinites inside of the embedded CP.

(55) **Equality semantics: predictions**
  a. Mitya doesn't believe the rumor that a syntactician was late.
  b. Mitya doesn't believe the rumor that a linguist was late.
  c. If the presupposition of (55a) is satisfied, then:
    $\exists!x[\text{rumor}(x) \wedge \text{CONT}(x) = \{w' \mid$ a syntactician was late in $w'\}]$
  d. If the presupposition of (55b) is satisfied, then:
    $\exists!y[\text{rumor}(y) \wedge \text{CONT}(y) = \{w' \mid$ a linguist was late in $w'\}]$
  e. If both presuppositions are satisfied, then given (54):
    $[\![\text{the rumor that a syntactician was late}]\!]^w$
    $\neq [\![\text{the rumor that a linguist was late}]\!]^w$
  f. $\therefore$ (55a) $\not\Rightarrow_S$ (55b), and (55b) $\not\Rightarrow_S$ (55a).

Note that these predictions of equality semantics qua (non-)monotonicity are connected to the predictions for belief reports more generally. Consider the schematic Logical Forms below—if p and q are distinct, then, by functionality (56a) and (56b) assert the non-emptiness of disjoint sets. Therefore, (56a) and (56b) are independent.

(56) a. $\exists e \leq w[e$ is a believing of Mitya's and $\text{CONT}(e) = p]$
  b. $\exists e \leq w[e$ is a believing of Mitya's and $\text{CONT}(e) = q]$

3.2.3 Interim summary

To sum up, we have seen that neither subset semantics nor equality semantics can account for the contrast in (57a)-(57b).

(57) a. * Mitya believes the rumor that anyone was late.

  b.   Mitya doesn't believe the rumor that anyone was late.

The two theories fail for different reasons. Subset semantics models monotonicity via universal quantification, and because of that it predicts that the examples in (57a)-(57b) should pattern with any other singular definite descriptions, and both sentences should be bad as the environment will be both SDE and SUE due to universal collapse. Equality semantics, on the other hand, fails due to the functionality of CONT, making the environment neither SDE nor SUE. These results are summarized in table 1.

	x believes the rumor that [____]	x doesn't believe the rumor that [____]
Subset	✓ SUE, ✓ SDE	✓ SUE, ✓ SDE
Equality	✗ SUE, ✗ SDE	✗ SUE, ✗ SDE
Goal	✓ SUE, ✗ SDE	✗ SUE, ✓ SDE

Table 1   Predictions of theories for complement clauses within definite DPs

Despite the fact that it makes better predictions for NPI licensing in simple belief reports like (58)-(59), it is difficult to see how subset semantics might be rescued in such a way that NPI licensing in cases like (57b) is accounted for. In what follows, we take a different path. We adopt equality semantics and propose a way to re-introduce monotonicity in both (58)-(59) and (57a)-(57b), while still avoiding the issue of the uniqueness collapse which plagued subset semantics.

(58)   * Mitya believes that anyone is coming to the party.

(59)   Mitya doesn't believe that anyone is coming to the party.

4 Proposal

The informal intuition behind our proposal is that, while eventualities like those described in (60a) and (60b) are distinct, they stand in a systematic relationship. The eventuality described in (60a) is part of the eventuality described in (60b), because part of what it means to believe that it's raining heavily, is to believe that its raining. At this point, we use the term "part" pre-theoretically, but we will formalize this idea using some standard notions. First of all, we will introduce our assumptions regarding the mereology of eventualities, before moving on to propositions.

(60) a. *Mitya's believing that it's raining.*

   b. *Mitya's believing that it's raining heavily.*

4.1   The mereological structure of eventualities

Mereology is the study of part-whole relationships in mathematics and logic, with broad applications in natural language semantics across domains such as plurality and aspect.[14] A mereology is simply a domain of entities, equipped with a *parthood* relation $\leq$, which satisfies the axiomatic principles in (61). Our proposal will ultimately exploit the idea, common in the literature on event semantics, that eventualities have a rich mereological structure (Bach 1986).

(61) **Axioms of classical extensional mereology**

   a. **Reflexivity:** $\forall x, x \leq x$

   b. **Transitivity:** $\forall x, y, z, (x \leq y \wedge y \leq z) \rightarrow x \leq z$

   c. **Antisymmetry:** $\forall x, y (x \leq y \wedge y \leq x) \rightarrow x = y$

   In this section, we will argue that from the perspective of equality semantics, this is independently a necessary move for attitudinal eventualities like *believings*.[15] Consider the schematic Logical Form again that equality semantics posits for a simple belief report like "Mitya believes that it's raining":

$$(62)\quad \exists e \leq w \left[\begin{array}{l} \text{HOLDER}(e) = \text{M} \wedge \text{believe}(e) \\ \wedge\, \text{CONT}(e) = \{\, w' \mid \text{it's raining in } w' \,\} \end{array} \right]$$

   (62) should of course be compatible with Mitya simultaneously having distinct beliefs; indeed, as we discussed in section 2.2.2, the only constraint that (62) places on the worlds in Mitya's doxastic alternatives is that they are all worlds where it is raining. Intuitively, Mitya's believing that it's raining is merely *part* of Mitya's believing. For a given rational agent x, we can posit an eventuality corresponding to x's *total beliefs*. The content of this total believing is exactly x's doxastic alternatives.

   We assume, therefore, that *believing* eventualities may have non-trivial parts. Without saying more, this won't contribute much — the action will lie in specifying exactly how the part-whole structure of eventualities relates to other properties they may have, such as (crucially for our purposes) their contents. In order to do this, we need to have a notion of parthood that can be applied to propositional contents—and defining such a notion is what we turn to in the next section.

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14 See Champollion & Krifka 2016 for an overview.

15 See also Pasternak 2018a, which posits mereological structure for attitudinal eventualities for independent reasons.

4.2 Parts of propositions

At the heart of our proposal is the idea that we can talk about parts of pieces of information qua classical propositions. A strong conceptual motivation for this move comes from the fact that, at least in English, the same vocabulary is used to talk about part-whole structures in the domain of concrete individuals, as well as in the more abstract domain of content (Moltmann 2013, 2020). Consider first the sentences in (63), which involve so-called "sub-atomic quantification" over *parts* of flags. In English, existential quantification over a sub-atomic part can be expressed using the noun "part" or the adverbial "partly" (see, e.g., Wągiel 2021).

(63)   a. The flag is partly red.
        b. Part of the flag is red.
            ⇒ *There is an x, s.t., x is part of the unique flag, and x is red.*

Moltmann (2013, 2020) observes that the same language is used to talk about parts of abstract, contentful entities, such as *rumors* (64), as well as parts of propositions (65).[16] There are two interesting things to observe about the examples below: (i) we can talk about *parts* of informational objects, and (ii) the information conveyed by the part seems to stand in a systematic relation to the information conveyed by the whole (as reflected by the provided contexts). In general, it seems to be the case that the content of the whole is *more informative* than its parts. In other words, part of the information *that Anton snowboarded on Friday* is the (less informative) information *that Anton snowboarded.*

(64)   Context: *Mitya believes that Anton snowboarded on Friday.*
        a. What Mitya believes is partly that Anton snowboarded.
        b. Part of what Mitya believes is that Anton snowboarded.

(65)   Context: *there's a rumor that Anton snowboarded on Friday.*
        a. Part of the rumor is that Anton snowboarded.
        b. The rumor is partly that Anton snowboarded.

If we assume a classical perspective on propositions as sets of worlds, or their characteristic functions—as we have throughout this paper—this suggests the notion of parthood for propositions based on informativity, (66) (cf. Brown 2022).

(66)   **Parthood for propositions:**
        $p \leq q$ iff p is semantically entailed by q (i.e., $p \supseteq q$).

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16 Here, we assume that "what Mitya believes" is a free relative picking Mitya's total beliefs, i.e., his doxastic alternatives.

There are a couple of things to note at this juncture. First, the definition in (66) straightforwardly captures the intuition that, e.g., the proposition *that it's raining* is part of the proposition *that it's raining and it's cold*, and more generally predicts that the individual conjuncts are parts of conjunctive propositions. It also has some less intuitive consequences. For example, the proposition denoted by a sentence of the form "*P or Q*" will be part of the proposition denoted by *P*. It certainly doesn't seem right to say that *it's raining or snowing* conveys *part* of what is conveyed by *it's raining*. As it so happens, any problematic predictions of (66) will find their counterpart in more general problems for closure under entailment. We'll come back to this issue in section 7.

Relatedly, although supersethood qualifies as a partial order on the set of propositions, the set of propositions isn't a mereology as typically construed—this is due to the presence of the tautological proposition W, which acts as a bottom element, i.e., every proposition has W as a part. Classical extensional mereology typically rules out the existence of a bottom element (Champollion & Krifka 2016). Nevertheless, we can make use of the closely related notion of a *lattice*. Let's say that we model the set of classical propositions as the powerset of logical space $\mathcal{P}(W)$. As mentioned, the superset relation $\supseteq$ qualifies as a *partial order* $(\leq)$ on $\mathcal{P}(W)$ since it is reflexive, anti-symmetric, and transitive (i.e., it satisfies the axiomatic principles in (61)). The structure on propositions induced by $\supseteq$ is illustrated by the Hasse diagram in fig. 1, which provides a comparison with a (more familiar) mereology presented as an atomic join semilattice. Thus, the lattice structure we assume for propositions is, in effect, the reverse of the typical example of the subset lattice on a powerset.
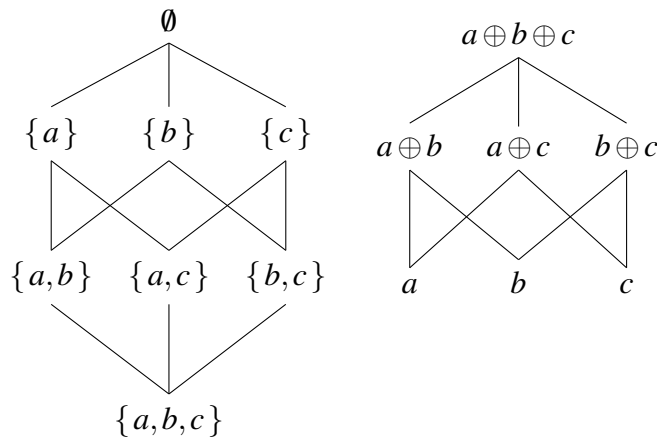
Figure 1 $\mathcal{P}(\{a,b,c\})$ as a superset lattice; $\oplus\{a,b,c\}$ as a mereology

In order to develop intuitions about this notion of parthood, let us consider a concrete example with a toy model involving a game played by Mitya (m) and Neil (n). The game can result in a tie, but only one person may win. The resulting possibilities are exhausted by w_m (Mitya is the winner), w_n (Neil is the winner), and $w_\emptyset$ (the game was a tie). The superset lattice on $\mathcal{P}(\{w_n, w_m, w_\emptyset\})$ gives rise to a notion of parthood that can be applied to complex sentences made up of the contrary statements "Mitya is the winner" and "Neil is the winner". This is illustrated in fig. 2.
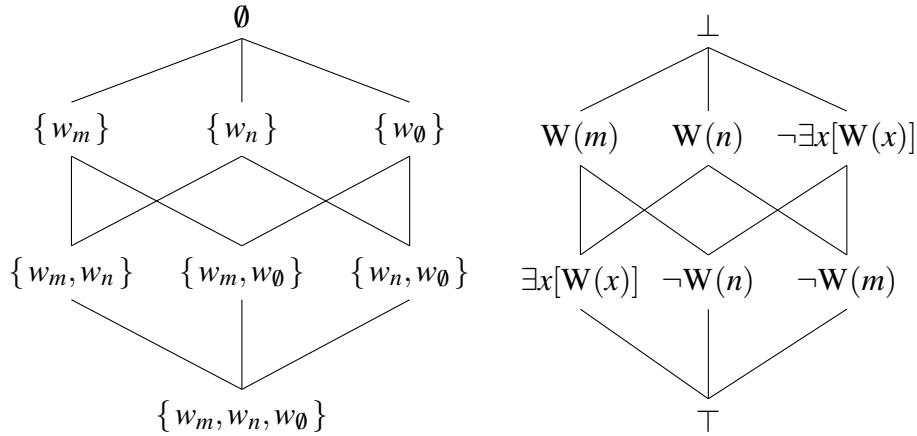
Figure 2 $W(m) := $ *Mitya is the winner*; $W(n) := $ *Neil is the winner*

The notion of parthood we assume based on informativity makes some welcome predictions. For example, it seems intuitively correct to say that *someone is the winner* $\{w_m, w_n\}$ is part of what is conveyed by both *Mitya is the winner* $\{w_m\}$ and *Neil is the winner* $\{w_n\}$. Similarly, it seems right to say that *Mitya isn't the winner* $\{w_n, w_\emptyset\}$ and *Neil isn't the winner* $\{w_m, w_\emptyset\}$ are both parts of what is conveyed by *Nobody is the winner* $\{w_\emptyset\}$. We'll take this notion to be sufficient for the purposes of our analysis, but we will revisit it in section 7.

4.3   Mapping from *beliefs* to propositions

Now that we've established a partial ordering on eventualities, based on mereological parthood, and a partial ordering on the set of propositions, based on informativity, we can state our proposal for reinstating monotonicity in equality semantics. Let us first illustrate the intuition informally. Let's say that Mitya holds a belief e with content p, and furthermore that p has proper parts (i.e., propositions it properly entails) q and r. This necessitates the existence of a belief of Mitya's $e' < e$ with content q, and a belief of Mitya's $e'' < e$ with content r. We formalize this as the condition

Mapping to Subparts of the Input (MSI), defined in (67), which demands that the content function preserve proper parthood.

(67) **CONT satisfies Mapping to Subparts of the Input (MSI):**
$$p' < \text{CONT}(x) \rightarrow \exists x' < x[\text{CONT}(x') = p'] \qquad \forall x \in \text{Dom}(\text{CONT}), p' \in D_{st}$$

The condition in (67) is sufficient to reinstate monotonicity in the semantics of *believe*. In order to see this, we'll reason through cases involving arbitrary propositions p and p', where p is strictly more informative than p', i.e., $p' \supset p$. From the perspective developed in the previous section, this means that p' is *a proper part of* p, $p' < p$. With this in mind, let us show that a positive belief report is upward (and not downward) entailing. Assume that *Mitya believes p*, (68).[17] This is sufficient to guarantee that Mitya holds a belief e' with content p'. No such guarantee holds for any proposition that entails p.[18]

(68) Mitya believes p.
    a. $\exists e \leq w[\text{HOLDER}(e) = \text{M} \wedge \text{believe}(e) \wedge \text{CONT}(e) = p]$
    b. $p' < \text{CONT}(e)$
    c. $\exists e' < e[\text{CONT}(e') = p']$ by MSI
    d. $\therefore \exists e' \leq w[\text{HOLDER}(e') = \text{M} \wedge \text{believe}(e') \wedge \text{CONT}(e') = p']$

Conversely, it's easy to see that negated belief reports are downward (and not upward) entailing, in light of (67). This is demonstrated (by contradiction) in (69). Asserting that *Mitya has no belief with content p'* is incompatible with the existence of a belief of Mitya's with content p. This is because positing such a p-belief, guarantees the existence of a p'-belief, by the logic of MSI.

(69) Mitya doesn't believe p'.
    a. $\neg\exists e' \leq w[\text{HOLDER}(e') = \text{M} \wedge \text{believe}(e') \wedge \text{CONT}(e') = p']$
    b. $\exists e \leq w[\text{HOLDER}(e) = \text{M} \wedge \text{believe}(e) \wedge \text{CONT}(e) = p]$
    c. By the reasoning in (68),
       $\exists e' \leq w[\text{HOLDER}(e') = \text{M} \wedge \text{believe}(e') \wedge \text{CONT}(e') = p']$
    d. $\therefore$ (69c) contradicts (69a).

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17 Note that we also importantly assume that if e is a believing of Mitya's, then every part of e is also a believing of Mitya's; this is analogous to the property of *divisive reference* often discussed in the literature on mass nouns (Cheng 1973).

18 Since Davidsonian Logical Forms are existential statements, our proofs will frequently involve multiple steps of existential instantiation/generalization. In order to keep the proofs readable, we will adopt the convention (where it doesn't lead to ambiguity) that existentially-quantified variables are implicitly instantiated and subsequently generalized over for the purposes of subsequent proof steps.

So, now that equality semantics has been supplemented with the condition in (67), monotonicity is reinstated, and closure under entailment is captured. This addresses one of the main objections to equality semantics, and accounts for the possibility of NPIS in negated belief reports. The next step will be to generalize this mereological approach to monotonicity to contentful THEMEs of *believings*.

4.4 Mapping from eventualities to objects

Sharvit's puzzle involves sentences in which *believe* composes with a DP of the form *the rumor that p*. We assume that *rumors*, much like more concrete entities, have a mereological structure. In fact, MSI for CONT (67), *guarantees* that contentful entities like rumors have (proper) parts. The reasoning is the same as for *believing* eventualities, as discussed in the previous section. When we apply MSI to the content of a *rumor* in (70), we derive that for any part p' of the content of the rumor r, there must be a corresponding *sub-rumor* r' which has p' as its content.

(70)   There's a rumor that p.
   a. $\exists x[\text{rumor}(x) \land \text{CONT}(x) = p$
   b. $p' < p$
   c. $\therefore \exists x' < x[\text{CONT}(x') = p']$ By MSI

Given that both *believing* eventualities, and contentful individuals like *rumors* have a non-trivial mereological structure, we can ask how to constrain the mapping between these two domains, given that *believings* can have *rumors* as their THEMEs. We posit a similar constraint to MSI, which we call Mapping to Subparts of the Output (MSO). Informally, it says that if an eventuality e has a THEME, then for any subeventuality $e' < e$, there must be an x', which is part of THEME(e), and is e''s THEME. Essentially, it requires that all subevents be mapped onto sub-THEMEs.

(71)   **THEME satisfies Mapping to Subparts of the Output (MSO):**
   $e' < e \rightarrow \exists x' < \text{THEME}(e)[\text{THEME}(e') = x']$ $\forall e \in \text{Dom}(\text{THEME}), e' \in D_v$

We do not intend (71) to be specific to *believing* eventualities and their themes, but rather we take this to be a special case of so called *incremental theme* verbs. In the following section, we zoom out and explore the parallels between MSO, MSI and Krifka's (1998) work on incrementality.

4.5 The connection with incrementality

There is a compelling parallel between MSI, MSO, and Krifka's (1998) approach to *incrementality*. One of the goals of Krifka 1998 is to characterize the relationship

between the eventuality and the THEME expressed by sentences such as (72). The sentence in (72) describes Mitya's *eating* of a plum. A common intuition in the literature is that there is a systematic relationship between the temporal parts of Mitya's eating, and the mereological parts of the plum: Mitya's eating 'uses up' parts of the apple, as it temporally progresses. Krifka characterizes this intuition formally in (73), where f is taken to be a thematic function, such as THEME. *Mapping to sub-events* is a property of a thematic function f just in case for any event e in its domain and any individual x' which is a proper sub-part of the individual $f(e) = x$, $x' < x$, there exists e', which is a proper subpart of e, with x' as its participant.

(72) Mitya ate the plum.

(73) **Mapping to sub-events** (Krifka 1998):
      A partial function $f : D_v \mapsto D_e$ shows *mapping to sub-events* iff:
      $x' < f(e) \rightarrow \exists e' < e[f(e') = x']$ $\hspace{2cm}$ $\forall e \in \text{Dom}(f), x' \in D_e$

We can apply (73) to (72), assuming the Logical Form in (74). Since *the plum* has a non-trivial mereological structure, (73) will guarantee the corresponding proper parts of e (*Mitya's eating*) to have proper parts of *the plum* as their THEMEs.

(74) $\exists e \leq w[\text{AGENT}(e) = \text{M} \wedge \text{eating}(e) \wedge \text{THEME}(e) = \iota x[\text{plum}(x)]]$

Krifka (1998) also postulates the reverse property, *mapping to sub-objects*. Applied to the sentence in (72), (75) ensures that for each proper subevent of Mitya's eating there is a proper subpart of the apple that is its THEME.

(75) **Mapping to sub-objects** (Krifka 1998):
      A partial function $f : D_v \mapsto D_e$ shows *mapping to sub-objects* iff:
      $e' < e \rightarrow \exists x' < f(e)[f(e') = x']$ $\hspace{2cm}$ $\forall e \in \text{Dom}(f), e' \in D_v$

There is a tight connection between Krifka's *mapping to sub-events* and *mapping to sub-objects* and our principles MSI and MSO introduced in the previous sections. These four properties are just particular instantiations of the two general principles in (76) and (77), which govern how a function maps one structured domain to another.

(76) A function $f : D \mapsto D'$ satisfies MSI iff:
      $y' < f(x) \rightarrow \exists x' < x[f(x') = y']$ $\hspace{2cm}$ $\forall x \in D, \forall y' \in D'$

(77) A function $f : D \mapsto D'$ satisfies MSO iff:
      $x' < x \rightarrow \exists y' < f(x)[f(x') = y']$ $\hspace{2cm}$ $\forall x, x' \in D$

Krifka's proposal then says that the INCREMENTAL THEME function that applies to *eating* events exhibits both of the properties in (76)-(77). As for our proposal, it amounts to saying that the property in (76) holds of the CONT function, and

that the property in (77) holds of the THEME function when applied to *believing* eventualities.[19] Thus, the principles that we proposed in (67) and (71) are not specific to *believe*, but exemplify more general principles for mapping between structured domains.

Despite clear similarity, there is an important difference between our account of the relationship between *believings* and their contents and Krifka's account of incrementality of predicates like *eat*. The notion of parthood that Krifka assumes for events tracks their temporal structure: e.g., for an eating eventuality *e*, the run-time of any proper part of *e* is included within the run-time of *e* itself. It is however crucial that the parthood relation that applies to believing eventualities would *not* necessarily track their temporal structure: if Mitya believes *that it's raining and it's hot* over the course of a time interval *t*, that doesn't entail that Mitya believes *it's raining* only over the course of a proper sub-interval. Similarly, if Mitya partly believes a conjunctive proposition, it doesn't mean that he is part way through the duration of a believing event with the conjunctive content. Instead, we interpret **??** as saying that the content of Mitya's belief is part of the proposition *it's raining and it's hot*.

(78)   Mitya partly believes that it's raining and it's hot.

Thus, the notion of parthood has to be different for predicates like *eat* and for attitudinal eventualities like *believings*. For *believings*, it must be more abstract than what is commonly assumed in work on incrementality, and is crucially not grounded in the spatio-temporal properties of the eventualities in question. We leave a more thorough investigation of the interaction between attitude verbs and incrementality for future work (but see, e.g., Zuchewicz 2020, Zuchewicz & Szucsich 2020).

4.6   Theme-Event Content Matching (TECM)

The final ingredient that we will need to account for Sharvit's puzzle involves the argument-structural properties of *believe*. Many attitude verbs, including *believe*, may embed both CPs and DPs. When the DP is headed by a content noun, which itself embeds a CP, some attitude verbs license an entailment pattern schematized in (79), which we call DP-to-CP entailment (Prior 1971, Vendler 1972, Ginzburg 1995, Pietroski 2000, King 2002, Moltmann 2013, Elliott 2016, Uegaki 2016, Djärv 2023).

(79)   **DP-to-CP entailment:**
       $[\![x \text{ V } [_{\text{DP}} \text{ the NP CP}]]\!] \Rightarrow [\![x \text{ V CP}]\!]$

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19 A natural question is whether (76) holds of THEME, and relatedly whether (77) holds of CONT for *believings*. We think that it is plausible that the answer to these questions is positive, but leave this issue for future research.

Believe is one of the verbs that exhibits the DP-to-CP entailment, (80). Many attitude verbs however do not exhibit the DP-to-CP entailment, (81).[20]

(80) **Verbs exhibiting the DP-to-CP entailment**

    a. Katya *believes/denied/accepted/doubted/memorized/trusted/ (dis)proved/validated/repeated* [the claim that Anton snowboarded].

    b. $\Rightarrow$ Katya *believes/denied/accepted/doubted/memorized/trusted/ (dis)proved/validated/repeated* [that Anton snowboarded].

(81) **Verbs not exhibiting the DP-to-CP entailment**

    a. Katya *knew/recognized/recalled/forgot/discovered/imagined/judged/ regretted/praised/saw* [the claim that Anton snowboarded].

    b. $\nRightarrow$ Katya *knew/recognized/recalled/forgot/discovered/imagined/judged/ regretted/praised/saw* [that Anton snowboarded].

It is conceivable to derive the DP-to-CP entailment compositionally (see Uegaki 2016, Djärv 2023, a.o., for discussion). Since our proposal doesn't hinge on a particular compositional approach, we will simply assume the meaning postulate Theme-Event Content Matching (TECM) for the verbs that exhibit the DP-to-CP entailment. TECM is intended to capture the intuition that in cases like (80), the source of the DP-to-CP entailment is the requirement that the content of attitudinal eventuality being described matches the content of the contentful THEME.

(82) **Theme-Event Content Matching (TECM):**
TECM holds of $P \in D_{\langle v,t \rangle}$ iff:
$P(e) \rightarrow (\text{CONT}(e) = \text{CONT}(\text{THEME}(e)))$
$$\forall e \in \text{Dom}(\text{CONT}) \cap \text{Dom}(\text{THEME})$$

To illustrate how TECM derives the entailment, consider the case of *believe*. If Katya has a belief whose THEME is *the rumor that p*, (83a), the content of that rumor is p. By TECM, this must also be the content of Katya's *believing*, (84). Thus, the truth of (83b) follows directly.

(83)   a. Katya believes the rumor that p.

        b. Katya believes that p.

(84)   a. $\exists e \leq w \begin{bmatrix} \text{HOLDER}(e) = \text{K} \wedge \text{believe}(e) \\ \wedge \text{THEME}(e) = \iota x[\text{rumor}(x) \wedge \text{CONT}(x) = p] \end{bmatrix}$

        b. $\text{CONT}(e) = \text{CONT}(\text{THEME}(e)) = p$ By TECM

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20 Note that this distinction does not align with (non)-factivity/presuppositionality: e.g., *imagine* is not presuppositional, but nevertheless lacks the DP-to-CP entailment.

c. $\exists e \leq w \left[\text{HOLDER}(e) = \text{K} \wedge \text{believe}(e) \wedge \text{CONT}(e) = p \quad \right]$

Since, by assumption, TECM doesn't hold for the verbs in (81), we cannot infer anything about the content of the eventuality that they describe even if we know the content of their THEME. In the following section, we will see that TECM turns out to be a crucial ingredient in our account of the Sharvit's puzzle, alongside with the MSI and MSO principles defined in the previous sections.

5   Accounting for Sharvit's puzzle

We now turn to the resolution of Sharvit's puzzle. The necessary ingredients, repeated below, are: (i) a non-monotonic equality semantics for clausal embedding, (85); (ii) MSI of CONT for *believing* eventualities, (86); (iii) MSO of THEME, (87), and (iv) TECM, to link up the content of a *believing* eventuality with its THEME, (88).

(85)   **Equality semantics for *that*-clauses**
(after Moulton 2009, Elliott 2017, Bondarenko 2022, etc.):
$[\![\text{that } S]\!]^w = \lambda x_\sigma . \text{CONT}(x) = \{ w' \mid [\![S]\!]^{w'} = 1 \}$ $\qquad \sigma = e$, or $\sigma = v$

(86)   **CONT satisfies MSI:**
$p' < \text{CONT}(x) \rightarrow \exists x' < x[\text{CONT}(x') = p']$ $\qquad \forall x \in \text{Dom}(\text{CONT}), p' \in D_{st}$

(87)   **THEME satisfies MSO:**
$e' < e \rightarrow \exists x' < \text{THEME}(e)[\text{THEME}(e') = x']$ $\quad \forall e \in \text{Dom}(\text{THEME}), e' \in D_v$

(88)   **TECM:**
TECM holds of $P \in D_{\langle v,t \rangle}$ iff:
$P(e) \rightarrow (\text{CONT}(e) = \text{CONT}(\text{THEME}(e)))$
$\qquad\qquad\qquad\qquad \forall e \in \text{Dom}(\text{CONT}) \cap \text{Dom}(\text{THEME})$

Recall that Sharvit's paradigm shows that embedded clauses in singular definite descriptions can be monotonic environments: i.e., positive sentences are SUE and not SDE with respect to restrictors of indefinites inside of the embedded clause. When entailment-reversing operators like negation are introduced, the sentences become SDE and not SUE. This captures the contrast between (89c) and (90c) with respect to NPI licensing. It is also borne out by our intuitions about entailment: (89b) entails (89a) but not vice versa, and (90a) entails (90b) but not vice versa.

(89)   **Positive sentences: ✓SUE, ✗SDE**

a.   Katya believes the rumor that Anton has snowboarded.

b.   Katya believes the rumor that Anton has snowboarded last Friday.

c.   *Katya believes the rumor that Anton has ever snowboarded.

(90)   **Negative sentences: ✗SUE, ✓SDE**

    a. Katya doesn't believe the rumor that Anton has snowboarded.

    b. Katya doesn't believe the rumor that Anton has snowboarded last Friday.

    c. Katya doesn't believe the rumor that Anton has ever snowboarded.

With all of the necessary pieces now in place, we'll first walk through how our proposal predicts that the positive sentences are SUE but not SDE. Let p and p' stand for propositions such that p is strictly more informative than p': $p' \supset p$. The reader can take p' to stand in for *Anton has snowboarded*, and p to stand in for *Anton has snowboarded last Friday*. First, let's derive that (91b) Strawson-entails (91a), and the environment is thus SUE. The proof is given in (92).[21]

(91)   a. Katya believes the rumor that p'.
       (e.g., p' = Anton has snowboarded)

      b. Katya believes the rumor that p.
       (e.g., p = Anton has snowboarded last Friday)

(92)   **Positive sentences like (89) are SUE**

    a. *Assume (91b)'s presupposition is true:*
      $\exists! r[\text{rumor}(r) \land \text{CONT}(r) = p]$

    b. *Assume (91b) is true:*
      $\exists e[\text{believe}(e) \land \text{HOLDER}(e) = \text{K} \land \text{THEME}(e) = r]$

    c. *Assume (91a)'s presupposition is true:*
      $\exists! r'[\text{rumor}(r') \land \text{CONT}(r') = p']$

    d. $\text{CONT}(e) = p$ via TECM, (92a)-(92b)

    e. $\exists e' < e[\text{CONT}(e') = p']$ via MSI, (92d), given $p' < p$

    f. $\exists r'' < r[\text{rumor}(r'') \land \text{THEME}(e') = r'']$ via MSO, given (92b), (92e)

    g. $\text{CONT}(r'') = p'$ via TECM, (92e)-(92e)

    h. $r'' = r'$ via uniqueness; (92c)

    i. $\exists e'[\text{believe}(e') \land \text{HOLDER}(e') = \text{K} \land \text{THEME}(e') = r']$
                                       generalization over e'

Let us see how this proof works. First, assume that (91b) is true and that the presuppositions of (91a) are true. In order to demonstrate Strawson entailment, we need to show is that this necessitates the truth of (91a). If (91b) is true, then there must be a unique rumor r with content p, and Katya must be a holder of a belief state e with THEME r. Due to TECM, the content of e is also p, (92d). Thanks to MSI, and the fact that $p' < p$, there must be a sub-belief e', whose content is p'. Now, MSO of

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21 Recall that in order to keep our proofs relatively concise, we adopt the convention that existentially-quantified variables are implicitly instantiated.

THEME tells us that there must be a sub-rumor of the rumor r, let's call it r'', which is the THEME of the e' sub-event. Due to TECM, the content of r'' is p'. Since the presupposition of (91a) is true, there is a unique rumor r' with content p'—from this we can conclude that r'' just *is* this rumor r' (92h). Thus, we arrive at the conclusion that (91a) must be true: there exists a believing e' whose attitude holder is Katya and whose THEME is the unique rumor r' with content p'. Thus, we successfully capture the intuition that (91b) semantically entails (91a).

Importantly, we also predict 'positive' sentences not to be SDE, (93).

(93)   **'Positive' sentences like (89) are not SDE**

     a. *(91a)'s presupposition is true:*
        $\exists! r'[\text{rumor}(r') \wedge \text{CONT}(r') = p']$

     b. *(91a) is true:*
        $\exists e' \leq w[\text{believe}(e') \wedge \text{HOLDER}(e') = \text{K} \wedge \text{THEME}(e') = r']$

     c. *(91b)'s presupposition is true:*
        $\exists! r[\text{rumor}(r) \wedge \text{CONT}(r) = p]$

     d. $r \neq r'$ from functionality

     e. $\text{CONT}(e') = p'$ via TECM
        N.b.: no guarantee of a belief with content p!

Due to equality semantics, if there is a unique rumor r' with content p' (93a), and a unique rumor r with content p (93c), r' and r are necessarily distinct (93d). Thus, Katja believing r' does not tell us anything about whether Katya also believes r. All we can infer is that Katya believes the content of r', i.e., p'. Because $p \not< p'$, no further conclusions about Katya's beliefs can be established.

Now let us see what happens when we embed the sentences under negation, (94). Because our 'positive' sentence was monotonic with respect to restrictors of indefinites inside the embedded CP, adding negation will reverse the entailment, making the sentence SDE and not SUE. The proof that the sentences like in (94) are SDE under our proposal is presented in (95). It is a proof by contradiction: we're assuming that (94a) is true, the presupposition of (94b) is true, but (94b) itself is false. We arrive at a contradiction.

(94)   a. Katya doesn't believe the rumor that p'.
        (e.g., p' = Anton has snowboarded)

     b. Katya doesn't believe the rumor that p.
        (e.g., p = Anton has snowboarded last Friday)

(95)   **Sentences with negation like (90) are SDE**

     a. *(94a)'s presupposition is true:*
        $\exists! r'[\text{rumor}(r') \wedge \text{CONT}(r') = p']$

b. *(94a) is true:*
$\neg\exists e' \le w[\text{believe}(e') \wedge \text{HOLDER}(e') = \text{K} \wedge \text{THEME}(e') = r']$

c. *(94b)'s presupposition is true:*
$\exists!r[\text{rumor}(r) \wedge \text{CONT}(r) = p]$

d. *Assume that (94b) is false:*
$\exists e \le w[\text{believe}(e) \wedge \text{HOLDER}(e) = \text{K} \wedge \text{THEME}(e) = r]$

e. *By proof (92)*
$\exists e' \le w[\text{believe}(e') \wedge \text{HOLDER}(e') = \text{K} \wedge \text{THEME}(e') = r']$

f. *The conclusion in (95e) contradicts (95b)*
Assumption (95d) must be incorrect:
$\neg\exists e \le w[\text{believe}(e) \wedge \text{HOLDER}(e) = \text{K} \wedge \text{THEME}(e) = r]$

If (94b) is false, Katya must have a belief whose THEME is the rumor that p. As we have already seen in (92), existence of such a belief implies existence of a belief by Katya whose THEME is the rumor that p', (95e). This contradicts our assumption that (94a) is true: that Katya doesn't believe the rumor that p'. Hence, (94b) must be true whenever (94a) is, and the sentence is correctly predicted to be SDE.

Note that our proposal avoids predicting that the environment would be SUE, (96). This is again due to the lack of the uniqueness collapse, (96d), which prevents the truth of (94b) automatically making (94a) true.

(96) **Sentences with negation like (90) are not SUE**

a. *(94b)'s presupposition is true:*
$\exists!r[\text{rumor}(r) \wedge \text{CONT}(r) = p]$

b. *(94b) is true:*
$\neg\exists e \le w[\text{believe}(e) \wedge \text{HOLDER}(e) = \text{K} \wedge \text{THEME}(e) = r]$

c. *(94a)'s presupposition is true:*
$\exists!r'[\text{rumor}(r') \wedge \text{CONT}(r') = p']$

d. $r' \ne r$ due to functionality

e. *(96a)-(96c) are consistent with Katya believing r', because $p \not< p'$:*
$\exists e' \le [\text{believe}(e') \wedge \text{HOLDER}(e') = \text{K} \wedge \text{THEME}(e') = r']$

The fact that Katya does not have a belief whose THEME is the rumor that p, (96b), does not tell us anything about whether Katya believes the rumor that p', (96e): because the $p' < p$ but not vice versa. For example, Katya could believe *the rumor that Anton snowboarded* (p') without believing *the rumor that Anton snowboarded last Friday* (p). Thus, we predict negative sentences to not be SUE.

To sum up, subset semantics can capture some of the basic entailment facts associated with *believe*, but it suffers due to the property of uniqueness collapse, repeated below as (97). This issue arises because on the subset semantics, for any

p', p such that $p' \supset p$, meaning of an embedded clause *that p* is a subset of the meaning of an embedded clause *that p'*. This predicts that there should be no difference in licensing NPIS in embedded CPs compared to relative clauses, contra to the fact.

(97) **Uniqueness collapse:**
      If $[\![\text{the } \phi]\!]$ and $[\![\text{the } \psi]\!]$ are defined, and $[\![\psi]\!] \subseteq [\![\phi]\!]$, then $[\![\text{the } \phi]\!] = [\![\text{the } \psi]\!]$.

Equality semantics on the other hand does not face the issue of the uniqueness collapse: for any p', p such that $p' \supset p$, the meaning of an embedded clause *that p* is disjoint with the meaning of an embedded clause *that p'*—no entity with content p can be an entity with content p' and vice versa, unless the two propositions are the same. This is guaranteed by functionality of CONT. This was crucial in allowing us to construct a semantics for *x believes the rumor that p* that renders the embedded clause a monotonic environment even when it is inside a singular definite description.

Thus, we conclude that it's better *not* to build monotonicity into the semantics of the embedded clause itself. On our view, embedded clauses are fundamentally non-monotonic environments. For simple belief reports, monotonicity was reinstated via the natural assumption that a systematic relationship exists between the part-whole structure of *believings* and the part-whole structure of their contents, formalized via MSI. Monotonicity was reinstated for clauses embedded within singular definite descriptions by additionally taking into account how the part-whole structure of believing events is related to the part-whole structure of their THEMEs (MSO), and appealing to an independently needed constraint on how the propositional content of *believings* and their THEMEs are related (TECM). Taken together, these ingredients allowed us to solve Sharvit's puzzle, while maintaining Kratzerian approach to the semantics of clausal embedding.

In the remainder of the paper, we discuss two outstanding questions which naturally arise from the perspective on monotonicity we argue for in this paper: the issue of closure under conjunction (section 6), and possible refinements of the notion of parthood for propositions (section 7).

6 Closure under conjunction

While MSI derives closure under entailment for *believe*, equality semantics supplemented with MSI still fails to derive *closure under conjunction*, (98).

(98) a. Mitya believes that Jessica married an American,
      b. ...and Mitya believes that Jessica married a philosopher.
      c. ⇒ Mitya believes that Jessica married an American philosopher.

Essentially, the problem lies in the fact that (98a) and (98b) are still about distinct believing eventualities. (98a) guarantees the existence of (sub-)believings corresponding to every proposition strictly weaker than *Jessica married an American*, and (98b) guarantees the existence of (sub-)believings corresponding to every proposition strictly weaker than *Jessica married a philosopher*, but MSI fails to guarantee the existence of believing with content *Jessica married an American philosopher*. Fortunately, there is an extremely natural extension of our mereological perspective which will derive closure under conjunction—concretely, we suggest the principle in (99) for summing contentful entities. (99) says that, given two contentful entities *x*, *y*, the content of their *sum* is the conjunction of their individual contents.

(99) **Summing contentful entities:**
$$\text{CONT}(x \oplus y) = \text{CONT}(x) \cap \text{CONT}(y) \qquad\qquad \forall x, y \in \text{Dom}(\text{CONT})$$

Once we assume that an attitude holder's believings are closed under mereological summation, we guarantee that the existence of a believing with content p, and the existence of a believing with content q, guarantees the existence of a believing with content $p \cap q$, irrespective of the relationship between p and q, (100).

(100)   a. $\exists e \leq w[\text{HOLDER}(e) = \text{M} \wedge \text{believing}(e) \wedge \text{CONT}(e) = p]$
        b. $\exists e' \leq w[\text{HOLDER}(e') = \text{M} \wedge \text{believing}(e') \wedge \text{CONT}(e') = q]$
        c. $\text{HOLDER}(e \oplus e') = \text{M} \wedge \text{believing}(e \oplus e')$ by closure under summation
        d. $\text{CONT}(e \oplus e') = \text{CONT}(e) \cap \text{CONT}(e') = p \cap q$ by (99)
        e. $\therefore \exists e''[\text{HOLDER}(e'') = \text{M} \wedge \text{believe}(e'') \wedge \text{CONT}(e'') = p \cap q]$

An interesting consequence of this proposal is that, without further ado, it can be used to account for (certain cases of) what Pasternak (2018a,b) calls *non-distributive belief ascriptions* (see also Haslinger & Schmitt 2021, Flor 2023). Non-distributive belief ascriptions involve cases where a belief can be attributed to a plural attitude holder, without necessarily being attributed to the atomic individuals that make up the plurality. Consider the following example from Pasternak 2018a: p. 548.

(101)   Context: *Paul just got married, and his cousins Arnie and Beatrice, who have never met, just caught wind of it. Arnie suspects that Paul's husband is rich, and has no other relevant opinions. Beatrice thinks he's a New Yorker, and has no other relevant opinions.*
        Paul's cousins think that he married a rich New Yorker.

As Pasternak points out, when a belief report has a plural holder, the content of the believing can be understood as the conjunction of what the atomic individuals

believe. This is exactly what we expect if (a) believings are closed under summation, and (b) on the propositional side, summation corresponds to conjunction.[22]

7 Informational parthood revisited

In this paper, we adopted a simple notion of *parthood* for propositions based on informativity, repeated below in (102).

(102) **Parthood for propositions:**
$p' \leq p$ iff p' is semantically entailed by p (i.e., $p' \supseteq p$).

We chose to define parthood in this way essentially to cleave as closely as possible to the predictions of a classical, Hintikkan approach to clausal embedding—in particular to derive *closure under entailment*, repeated below in (103).

(103) *Closure under entailment:*
For any sentences S, S', attitude-holder x,
if S entails S', then "x believes S" entails "x believes S'".

The notion of informational parthood in (102) however, does not always match up with our intuitions. It seems that there are certain cases where, despite the fact that $p' \supseteq p$, it doesn't not seem right to say that p' is *part of* p. Consider, for example, the contrast between the felicity of (104a) vs. (104b) in the provided context. Parthood *qua* classical entailment makes good predictions for (104a), but poor predictions for (104b), which is expected to be equally natural in the provided context, due to the fact that *Jessica married an American philosopher* (classically) entails that *Jessica married a linguist or a philosopher*.

(104) Context: *Mitya believes that Jessica married an American philosopher*.
    a. What M. believes is partly that J. married an American.
    b. #What M. believes is partly that J. married a linguist or a philosopher.

Intuitively, it seems that what distinguishes the two cases is that (104a)—the good case—involves reasoning via conjunction elimination, whereas (104b)—the bad case—involves reasoning via disjunction introduction. Classical entailment, however, provides no way of distinguishing between these two cases. This problem is highly reminiscent of the problem of logical omniscience, and (we suspect) intrinsically linked to it. Both the classical Hintikkan account of closure under

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22 Pasternak (2018a) argues that when the beliefs of the members of a plurality are incompatible, their plural belief can be reported using a disjunction (see also Pasternak 2018b and Marty 2019 for critical discussion). If this is correct, the principle in (99) will need refinement, as it predicts that if x and y have contrary beliefs then their plural belief should be trivial.

entailment, and our mereological account, predict that (105a) should entail (105b), however it does not seem that we really infer (105b) from (105a).[23]

(105)   a. Mitya believes that Jessica married an American philosopher.

          b. Mitya believes that Jessica married a linguist or a philosopher.

Although we will leave a more thorough exploration of this problem to future work, we will tentatively suggest the possibility that informational parthood can be defined via a more restrictive notion of entailment, which we call *conjunctive parthood*, following Fine 2017c,b (see also Yablo 2014,Moltmann In press, a.o.). Formulating this notion of entailment minimally requires shifting to a non-classical semantics for disjunction, according to which disjunctive sentences semantically introduces *alternatives*. There are various prominent frameworks which provide a suitable semantics for disjunction, such as truthmaker semantics, whence the notion of conjunctive parthood originates (Fine 2017c,a,b, Yablo 2014), and inquisitive semantics (Ciardelli 2009, Ciardelli, Groenendijk & Roelofsen 2019). Here we will make use of *alternative semantics* (Hamblin 1973, Kratzer & Shimoyama 2002) for simplicity of exposition.

In alternative semantics, sentences express *sets* of classical propositions. In (106) we give a toy example of how simple sentences may be mapped to their alternative-semantic values (via $[.]$) in terms of their ordinary semantic values. In general, the denotation of a simple sentence is just the singleton set containing the proposition it expresses (106a). Conjunctive sentences follow this general rule (106b). The crucial move is the idea that disjunctive sentences denote non-trivial sets of *alternative* propositions, corresponding to each of the disjuncts (106c). Negation 'closes off' any alternatives, by first taking their union, and returning the singleton set of the negation of the resulting proposition (106d).

(106)   **Alternative semantics**

        a. $[\text{S}] := \{\, [\![\text{S}]\!] \,\}$

        b. $[\text{S and S'}] := \{\, [\![\text{S and S'}]\!] \,\}$

        c. $[\text{S or S'}] := \{\, [\![\text{S}]\!] , [\![\text{S'}]\!] \,\}$

        d. $[\text{not S}] := \{\, W - \bigcup[\text{S}] \,\}$

With the additional structure afforded by alternatives, we can define a more restrictive notion of entailment alongside the classical notion. We'll use $Q, Q', \ldots$ as variables over *sets of classical propositions*, which provide the relevant notion

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23 There is a ready alternative explanation for the perceived lack of entailment in (105). Via Gricean reasoning reason, (105b) plausibly implicates that *it's not the case that Mitya believes that J. married a philosopher*, which contradicts (104a), as Rothschild (2017) points out. It's however not clear how this reasoning could be extended to the case in (104b).

of content in alternative semantics. Classical entailment is defined as in (107): we render any additional structure provided by alternative sets inert by taking the grand union of each alternative set. Conjunctive parthood is sensitive to the additional structure afforded by alternative sets: an alternative-set Q' is a *conjunctive part* of an alternative set Q, just in case every alternative in Q classically entails an alternative in Q', and every alternative in Q' is classically entailed by an alternative in Q.[24]

(107) **Classical entailment in alternative semantics:**
Q semantically entails Q' iff $\bigcup Q \subseteq \bigcup Q'$ $\qquad\qquad \forall Q, Q' \subseteq \mathrm{Pow}(W)$

(108) **Conjunctive parthood:**
Q' is a conjunctive part of Q iff
$\forall p \in Q, \exists p' \in Q', p \subseteq p'$ and $\forall p' \in Q', \exists p \in Q, p \subseteq p'$ $\quad \forall Q, Q' \subseteq \mathrm{Pow}(W)$

Let's see what this accomplishes. It's easy to see that *it's raining* is a conjunctive part of *it's raining heavily*, since when Q and Q' are singleton sets, conjunctive parthood is equivalent to classical entailment. Conjunctive parthood does some interesting work once disjunction is involved—concretely, in alternative semantics, a sentence of the form *S or S'* is not a conjunctive part of *S*, even though it is semantically entailed by *S*. To see this, we walk through in (109) why *Jessica married a linguist or philosopher* (Q') is not a conjunctive part of *Jessica married an American linguist* (Q). The reason is simple—the alternative introduced by the *philosopher* disjunct isn't entailed by any alternative in Q.

(109) $Q' := [\text{Jessica married a linguist or philosopher}]$
$\qquad = \{ \underbrace{[\![\text{J. married a linguist}]\!]}_{p_1}, \underbrace{[\![\text{J. married a philosopher}]\!]}_{p_2} \}$
$Q := [\text{Jessica married an American linguist}]$
$\qquad = \{ \underbrace{[\![\text{J. married an American linguist}]\!]}_{p_3} \}$

a. $p_3 \subseteq p_1$
b. $p_3 \nsubseteq p_2$
c. $\exists p' \in Q', p \in Q, p \nsubseteq p'$
d. $\therefore Q'$ is not a conjunctive part of Q

We can now reformulate *parthood for propositions*, assuming that alternative semantics provides the relevant notion of content. This has the advantage of immediately capturing the infelicity of (104b).

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24 This is Fine's notion of *conjunctive parthood* translated into the alternative-semantic framework (see also Yablo 2014).

(110) **Parthood for propositions (alternative semantics):**
$Q' \leq Q$ iff Q' is a *conjunctive part* of Q.

It also captures the intuitive lack of entailment in (105). Recall that at the heart of our proposal is the condition repeated in (111), here modified to accommodate the shift in perspective to alternative semantics: we assume that the content of a contentful event or individual is an alternative set, rather than a set of worlds.

(111) **CONT satisfies MSI (alternative semantics):**
$Q' < \mathrm{CONT}(x) \rightarrow \exists x' < x[\mathrm{CONT}(x') = Q']$
$$\forall x \in \mathrm{Dom}(\mathrm{CONT}), Q' \subseteq \mathrm{Pow}(W)$$

(111) introduces the requirement that, given a contentful x, every *proper conjunctive part Q'* of its content is the content of some sub-part of x. Since the alternative set expressed by *Jessica married a linguist or philosopher* is not a proper conjunctive part of *Jessica married an American linguist*, having a belief with the content *Jessica married an American linguist* does not necessitate having a (sub)-belief with the content that *Jessica married a linguist or a philosopher*.[25]

Thus, the notion of conjunctive parthood seems like a promising refinement that could address some of the problems that the classical notion of entailment suffers from. We leave its more detailed exploration to future work.

8 Conclusion

In this paper, we developed a new perspective on monotonicity of attitude verbs, focusing on the case of *believe*. Our account naturally implies that what distinguishes monotonic attitude verbs like *believe* from non-monotonic attitude verbs, is whether or not CONT is subject to MSI. For *believings*, CONT is subject to MSI, which means that for any proper part of the content of a *believing* there is a corresponding proper sub-*believing*. For example, if Katya has a *belief that Anton snowboarded last Friday*, she also must have *a sub-belief that Anton snowboarded*, since *Anton snowboarded* is part of *Anton snowboarded last Friday*. This account does away with the idea that monotonicity is linked to the presence of a universal modal in the semantics

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[25] Yablo (2014) already made the connection between conjunctive parthood and propositional attitudes when discussing *agreement*. He argues that x and y *agree* on p iff p is a conjunctive part of the content of each of their beliefs. This captures the fact that in (1) we judge (1a) as true but (1b) as false.

(1) Context: *Mitya thinks Jessica married an American linguist, Neil thinks Jessica married an American philosopher.*

    a. [True]: Mitya and Neil agree that Jessica married an American.

    b. [False]: Mitya and Neil agree that Jessica married a linguist or a philosopher.

of sentences with clausal embedding. Instead, it arises as a consequence of how the part-whole structures of attitudinal eventualities is related to the part-whole structures of their contents.

If this proposal is on the right track, then the Kratzerian approach to clausal embedding (Kratzer 2006), and in particular the implementation in terms of equality (Moulton 2009, Elliott 2017, Bassi & Bondarenko 2021, Bondarenko 2022) seems like a very viable approach to the semantics of attitude reports and clausal embedding more generally. Among its virtues are: (i) a uniform semantics for *that*-clauses that occur in different syntactic environments, (ii) good predictions about the impossibility of stacking embedded CPs, and (iii) good predictions concerning how embedded clauses interact with definiteness. In this paper we have shown that it also allows for a straightforward account of the puzzling NPI licensing facts about CPs that modify nouns inside of singular definite descriptions under *believe* (Sharvit to appear), once we incorporate independently motivated principles governing properties of the THEME of a *believing*.

One open question for future research is how well our account can be extended to predicates beyond *believe*. Here is some tentative evidence that it is on the right track. We predict that if an MSI of CONT holds for some predicate of events *e*, and if $e \in Dom(\text{THEME})$, and MSO and TECM hold, then NPIS should be licensed in CPs combining with nouns in the following configuration: *not [x Vs [the N CP]]*. There are indeed other verbs that follow this pattern: e.g. *accept* and *trust*, (112)-(113).

(112)   a. John accepts/trusts that it's raining heavily,
              #but he doesn't accept/trust that it's raining.

          b. John accepts/trusts the claim that it's raining.
              ⤳ John accepts/trusts that it's raining.

(113)   Mitya doesn't accept/trust the rumor that Anton has ever been skiing.

We also expect that if verbs denote predicates of events whose CONT exhibits MSI, but whose THEME arguments do not have the same content as they do—in other words, cases where TECM doesn't hold—then NPIS shouldn't be licensed in the Sharvit's configuration. Two verbs which display this pattern are e.g. *remember* and *imagine*, (114)-(115).

(114)   a. John remembers/is imagining that it's raining heavily, #but he doesn't
              remember/isn't imagining that it's raining.

          b. John remembers/is imagining the claim that it's raining.
              ⇝̸ John remembers/is imagining that it's raining.

(115)   *John doesn't remember/isn't imagining the claim that Anton has ever been snowboarding.

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