Scalarity and additivity in natural language: (III) comparatives (cont.)

Linmin Zhang (NYU Shanghai)
zhanglinmin@gmail.com

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https://lingbuzz.net/lingbuzz/008302
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Recapitulation

- Additivity is a phenomenon of QUD-based anaphoricity, indicating an extension of a previous salient answer in addressing the QUD.
- An additivity/increase-based view of *-er*/*more*
- A new difference-based view of comparatives

	The canonical view	**The new difference-based view**
Assumption	(Ordinal/interval) scales	**Interval scales**
Comparison	Inequality: $M_1 > M_2$	**Subtraction**: $M_1 - M_2 = D$
Representations of & operations on scalar values	Degree points & ordering between degree points	**Intervals (i.e., set of degrees) & interval subtraction**
The semantics of *-er*/*more*	Ordering: $>$	**Additivity a default positive difference**: $(0, +\infty)$

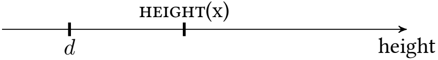
Today

- Day 2 (yesterday) and Day 3 (today): Comparatives and *-er*/*more*
  - How an additivity-based perspective improve our understanding of scalarity-related phenomena?
  - What is additivity?
- Today
  - Formal implementation (see Zhang and Ling 2021 and Zhang and Zhang 2024)
  - Antonyms
  - ~~Cross-linguistic phenomena~~
  - etc.

Outline

The meaning of gradable adjectives (to be revisited)

- Canonical view (See e.g., Cresswell 1976, Hellan 1981, von Stechow 1984, Heim 1985, Schwarzschild 2008, Beck 2011):

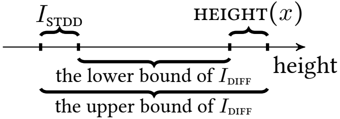


$[\![\text{tall}]\!]$

(1) $[\![\text{tall}]\!]_{\langle d,et\rangle} \stackrel{\text{def}}{=} \lambda d_d.\lambda x_e.\textsc{height}_{\langle e,d\rangle}(x) \geq d$ (i.e., x is d-tall)
On the scale of height, the position of x reaches degree d.

- There are two pieces in this lexical entry
  - A measure function of type $\langle ed\rangle$: $\textsc{height}_{\langle e,d\rangle}(x)$
  - Indicating the direction (of comparison): $\geq d$ (cf. Kennedy 1999)

The meaning of gradable adjectives



$[\![\text{tall}]\!]$

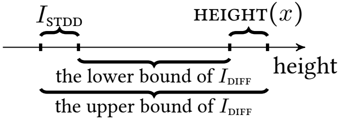
(1) $[\![\text{tall}]\!]_{\langle d,et\rangle} \overset{\text{def}}{=} \lambda d_d.\lambda x_e.\text{HEIGHT}_{\langle e,d\rangle}(x) \geq d$ Canonical view
On the scale of height, the position of x reaches degree d.

(2) $[\![\text{tall}]\!]_{\langle dt,et\rangle} \overset{\text{def}}{=} \lambda I_{\langle dt\rangle}.\lambda x_e.\text{HEIGHT}_{\langle e,dt\rangle}(x) \subseteq I$ (Zhang and Ling 2021)
On the scale of height, the measure of x falls at the position I.

(3) $[\![\text{tall}]\!] \overset{\text{def}}{=} \lambda I_{\text{DIFF}}.\lambda I_{\text{STDD}}.\lambda x.\underbrace{I_{\text{DIFF}} \subseteq [0,+\infty)}_{\text{non-negative presup.}}.\ \text{HEIGHT}(x) \subseteq \iota I[I - I_{\text{STDD}} = I_{\text{DIFF}}]$

(i.e., the height of x reaches the comparison standard, I_{STDD}.
$\rightsquigarrow$ the difference between them, I_{DIFF}, is non-negative) (Zhang and Zhang 2024)

The meaning of gradable adjectives



(2) $[\![\text{tall}]\!]_{\langle dt,et\rangle} \stackrel{\text{def}}{=} \lambda I_{\langle dt\rangle}.\lambda x_e.\textsc{height}_{\langle e,dt\rangle}(x) \subseteq I$ (Zhang and Ling 2021)

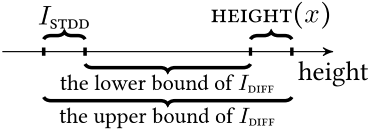
(3) $[\![\text{tall}]\!] \stackrel{\text{def}}{=} \lambda I_{\text{DIFF}}.\lambda I_{\text{STDD}}.\lambda x.\underbrace{I_{\text{DIFF}} \subseteq [0,+\infty)}_{\text{non-negative presup.}}.\ \textsc{height}(x) \subseteq \iota I[I - I_{\text{STDD}} = I_{\text{DIFF}}]$

(i.e., the height of x reaches the comparison standard, I_{STDD}.
$\rightsquigarrow$ the difference between them, I_{DIFF}, is non-negative) (Zhang and Zhang 2024)

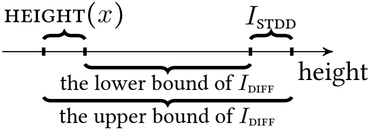
(4) A type shifter $[\![\textsc{compare}]\!]_{\langle\langle dt,et\rangle,\langle dt,\langle dt,et\rangle\rangle\rangle}$ (see also Zhang and Ling 2021)
$\stackrel{\text{def}}{=} \lambda G_{\langle dt,et\rangle}.\lambda I_{\text{DIFF}}.\lambda I_{\text{STDD}}.\lambda x_e.G\text{-}\textsc{dimension}(x) \subseteq \iota I[I - I_{\text{STDD}} = I_{\text{DIFF}}]$

(5) $[\![\textsc{compare}\ \text{tall}]\!]_{\langle dt,\langle dt,et\rangle\rangle}$
$= \lambda I_{\text{DIFF}}.\lambda I_{\text{STDD}}.\lambda x_e.\textsc{height}_{\langle e,dt\rangle}(x) \subseteq \iota I[I - I_{\text{STDD}} = I_{\text{DIFF}}]$

The meaning of gradable adjectives (Zhang and Zhang 2024)



The meaning of *tall*



The meaning of *short*

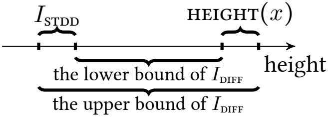
(3) $$[\![\text{tall}]\!] \overset{\text{def}}{=} \lambda I_{\text{DIFF}}.\lambda I_{\text{STDD}}.\lambda x.\underbrace{I_{\text{DIFF}} \subseteq [0, +\infty)}_{\text{non-negative presup.}}.\ \text{HEIGHT}(x) \subseteq \iota I[I - I_{\text{STDD}} = I_{\text{DIFF}}]$$

(i.e., the height of x reaches the comparison standard, I_{STDD}.
$\rightsquigarrow$ the difference between them, I_{DIFF}, is non-negative)

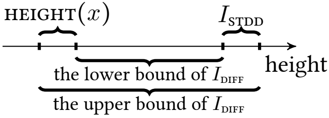
(6) $$[\![\text{short}]\!] \overset{\text{def}}{=} \lambda I_{\text{DIFF}}.\lambda I_{\text{STDD}}.\lambda x.\underbrace{I_{\text{DIFF}} \subseteq [0, +\infty)}_{\text{non-negative presup.}}.\ \text{HGHT}(x) \subseteq \iota I[I_{\text{STDD}} - I = I_{\text{DIFF}}]$$

(i.e., the height of x does not exceed the comparison standard, I_{STDD}.
$\rightsquigarrow$ the difference between them, I_{DIFF}, is non-negative)

Major uses of gradable adjectives: Positive use



The meaning of *tall*



The meaning of *short*

(7) $[\![$Lucy is POS tall$]\!]$

$$\Leftrightarrow \text{HEIGHT}(\text{Lucy}) \subseteq \iota I[I - \underbrace{[d^c{}_{\text{POS}}, d^c{}_{\text{POS}}]}_{I_{\text{STDD}}} = \underbrace{[0, +\infty)}_{I_{\text{DIFF}}}]$$

$$\Leftrightarrow \text{HEIGHT}(\text{Lucy}) \subseteq [d^c_{\text{POS}}, +\infty)$$

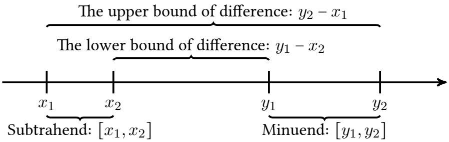
(8) $[\![$Lucy is POS short$]\!]$

$$\Leftrightarrow \text{HEIGHT}(\text{Lucy}) \subseteq \iota I[\underbrace{[d^c_{\text{POS}}, d^c_{\text{POS}}]}_{I_{\text{STDD}}} - I = \underbrace{[0, +\infty)}_{I_{\text{DIFF}}}]$$

$$\Leftrightarrow \text{HEIGHT}(\text{Lucy}) \subseteq (-\infty, d^c_{\text{POS}}]$$

(See Zhang and Zhang 2024)

Subtraction between intervals



(9) $\underbrace{[y_1, y_2]}_{\textbf{minuend: position}} - \underbrace{[x_1, x_2]}_{\textbf{subtrahend: position}} = \underbrace{[y_1 - x_2, y_2 - x_1]}_{\textbf{difference: distance between positions}}$

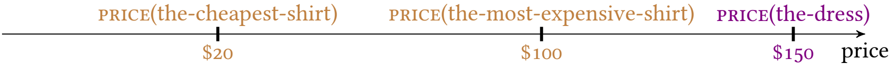
(10) Given the subtrahend position $[a, b]$ and the difference $[c, d]$,
Minuend position = $[b + c, a + d]$ (defined when $b + c \leq a + d$)
$\textsc{height}(\text{Lucy}) \subseteq \iota I[I - [d^c{}_{\text{POS}}, d^c{}_{\text{POS}}] = [0, +\infty)] \Leftrightarrow \textsc{height}(\text{Lucy}) \subseteq [d^c_{\text{POS}}, +\infty)$

(11) Given the minuend position $[a, b]$ and the difference $[c, d]$,
Subtrahend position = $[b - d, a - c]$ (defined when $b - d \leq a - c$)
$\textsc{height}(\text{Lucy}) \subseteq \iota I[[d^c{}_{\text{POS}}, d^c{}_{\text{POS}}] - I = [0, +\infty)] \Leftrightarrow \textsc{height}(\text{Lucy}) \subseteq (-\infty, d^c_{\text{POS}}]$

(See Moore 1979)

Interlude

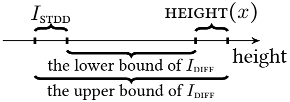
(10) Given the subtrahend position $[a, b]$ and the difference $[c, d]$,
Minuend position = $[b + c, a + d]$ (defined when $b + c \leq a + d$)



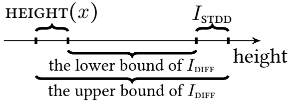
(12) ⟦The dress is up to \$60 more expensive than every shirt is⟧
$\Leftrightarrow$ PRICE(the-dress) $\subseteq$
$\iota I[I - [\text{price of the cheapest shirt, price of the most expensive shirt}] = (0, \$60]]$
The definedness condition for the minuend: the price difference between the most expensive and the cheapest shirt is no more than \$60. Under the above context: undefined!!

(13) The giraffe is exactly 5 inches taller than every tree is.
$\rightsquigarrow$ We have the inference that every tree is of the same height. Why?
HEIGHT(the-giraffe) $\subseteq \iota I[I - I_{\text{STDD}} = [5'', 5'']]$, thus the upper and lower bound of I_{STDD} needs to be the same to meet the definedness requirement

Major uses of gradable adjectives: Measurement sentence



The meaning of *tall*



The meaning of *short*

(14) ⟦Lucy is 6 feet tall⟧ ‘at least’ reading and ‘exactly’ reading

(I_{STDD} is the zero point, the starting point for having a height)

a. $\text{HEIGHT}(\text{Lucy}) \subseteq \iota I[I - \underbrace{[0,0]}_{I_{\text{STDD}}} = \underbrace{[6', +\infty)}_{I_{\text{DIFF}}}] \Leftrightarrow \text{HEIGHT}(\text{Lucy}) \subseteq [6', +\infty)$

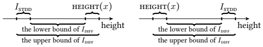
b. $\text{HEIGHT}(\text{Lucy}) \subseteq \iota I[I - \underbrace{[0,0]}_{I_{\text{STDD}}} = \underbrace{[6', 6']}_{I_{\text{DIFF}}}] \Leftrightarrow \text{HEIGHT}(\text{Lucy}) \subseteq [6', 6']$

(15) ⟦Lucy is 5 feet short⟧ Ungrammatical! (⟦short⟧ has no starting point)

$[\![\text{short}]\!] \overset{\text{def}}{=} \lambda I_{\text{DIFF}}.\lambda I_{\text{STDD}}.\lambda x.\underbrace{I_{\text{DIFF}} \subseteq [0, +\infty)}_{\text{non-negative presup.}}.\ \text{HGHT}(x) \subseteq \iota I[I_{\text{STDD}} - I = I_{\text{DIFF}}]$

$\rightsquigarrow$ If Lucy’s height is at the position $[5', 5']$, compared with I_{STDD} that is $[0, 0]$, the non-negative presupposition of I_{DIFF} is violated.

Major uses of gradable adjectives: Degree question



The meaning of *tall* The meaning of *short*

- Asking about I_{DIFF}: addressing a distance away from a reference point (I_{STDD}), e.g., *compared to Mary, how tall is Lucy?*
- Two typical references: the zero point, the contextual threshold

(16) ⟦How tall is Lucy⟧

a. $\lambda I_{\text{DIFF}}.\text{HEIGHT}(\text{Lucy}) \subseteq \iota I[I - \underbrace{[0, 0]}_{I_{\text{STDD}}} = I_{\text{DIFF}}]$ No evaluativity!

$\rightsquigarrow$ How far Lucy's height measurement is from / above the zero point

b. $\lambda I_{\text{DIFF}}.\text{HEIGHT}(\text{Lucy}) \subseteq \iota I[I - \underbrace{[d^{c}{}_{\text{POS}}, d^{c}{}_{\text{POS}}]}_{I_{\text{STDD}}} = I_{\text{DIFF}}]$ Evaluativity!

$\rightsquigarrow$ How far Lucy's height is from / above the contextual threshold of being tall

(17) ⟦How short is Lucy⟧

$\lambda I_{\text{DIFF}}.\text{HEIGHT}(\text{Lucy}) \subseteq \iota I[\underbrace{[d^{c}{}_{\text{POS}}, d^{c}{}_{\text{POS}}]}_{I_{\text{STDD}}} - I = I_{\text{DIFF}}]$ Evaluativity!

$\rightsquigarrow$ How far Lucy's height is from / below the contextual threshold of being short

Major uses of gradable adjectives: Degree question

(16a) $[\![\text{How tall is Lucy}]\!] = \lambda I_{\text{DIFF}}.\text{HEIGHT}(\text{Lucy}) \subseteq \iota I[I - \underbrace{[0,0]}_{I_{\text{STDD}}} = I_{\text{DIFF}}]$

(17) $[\![\text{How short is Lucy}]\!] = \lambda I_{\text{DIFF}}.\text{HEIGHT}(\text{Lucy}) \subseteq \iota I[\underbrace{[d^{c}{}_{\text{POS}}, d^{c}{}_{\text{POS}}]}_{I_{\text{STDD}}} - I = I_{\text{DIFF}}]$

- Applying $\textbf{Ans}_{\text{DIFF}}$ returns the most informative interval that addresses how far away Lucy's height is from the reference position I_{STDD}

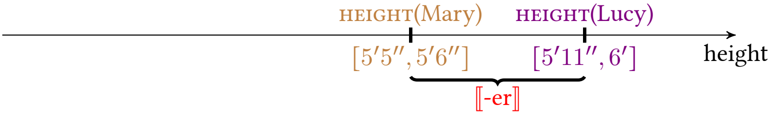
(18) An answerhood operator $\textbf{Ans}_{\text{DIFF}}$ (which returns a maximally informative true answer) is defined for a set of intervals p s.t.
$\exists I[p(I) \wedge \forall I'[[p(I') \wedge I' \neq I] \rightarrow I \subset I']]$ (this maximally informative I exists)
When defined, $\textbf{Ans}_{\text{DIFF}} \overset{\text{def}}{=} \lambda p_{\langle dt,t \rangle}.\iota I[p(I) \wedge \forall I'[[p(I') \wedge I' \neq I] \rightarrow I \subset I']]$

- Two operators are defined to compute the position value from I_{DIFF}

(19) $\textbf{Position-M} \overset{\text{def}}{=} \lambda I_{\text{DIFF}}.\iota I[I - I_{\text{STDD}} = I_{\text{DIFF}}]$ **Minuend position**

(20) $\textbf{Position-S} \overset{\text{def}}{=} \lambda I_{\text{DIFF}}.\iota I[I_{\text{STDD}} - I = I_{\text{DIFF}}]$ **Subtrahend position**

Major uses of gradable adjectives: Clausal comparative



(21) ⟦ Lucy is taller than Mary is ~~tall~~ ⟧

$\textsc{height}(\text{Lucy}) \subseteq \iota I[I - \underbrace{⟦\text{than Mary is tall}⟧}_{I_{\textsc{stdd}}} = \underbrace{⟦\text{er}⟧}_{I_{\textsc{diff}}}]$

a. ⟦than Mary is ~~tall~~⟧ = **Position-M**[**Ans**$_{\textsc{diff}}$⟦how tall Mary is⟧]
= HEIGHT(Mary) = $[5'5'', 5'6'']$ under the above context

b. ⟦er⟧ $\overset{\text{def}}{=} (0, +\infty)$ $\rightsquigarrow$ extending the value ⟦than Mary is ~~tall~~⟧ in addressing the Current Question 'how tall Lucy is'

c. $\textsc{ht}(\text{Lucy}) \subseteq \iota I[I - [5'5'', 5'6''] = (0, +\infty)] \Leftrightarrow \textsc{ht}(\text{Lucy}) \subseteq (5'6'', +\infty)$

- Without **Position-M**: comparing 2 distances away from a certain reference position: Lucy is farther away from the reference than Mary is
- With **Position-M**: comparing 2 positions along a height scale: Lucy's position involves an increase compared to Mary's position

Comparatives with *than*-clause internal quantifiers

(22) $[\![$ The dress is more expensive than every shirt is ~~expensive~~ $]\!]$

$\text{PRICE}(\text{the-dress}) \subseteq \iota I[I - \underbrace{[\![\text{than every shirt is expensive}]\!]}_{I_{\text{STDD}}} = \underbrace{[\![\text{more}]\!]}_{I_{\text{DIFF}}}]$

a. $[\![$than every shirt is ~~expensive~~$]\!]$ =
Position-M$[$**Ans**$_{\text{DIFF}}[\![$how expensive every shirt is$]\!]]$ =
Position-M$[$**Ans**$_{\text{DIFF}}[\lambda I_{\text{DIFF}}.\forall x[\text{shirt}(x) \rightarrow \text{PRICE}(x) \subseteq \iota I[I - I_{\text{STDD}} = I_{\text{DIFF}}]]]]$, which is
[price of the cheapest shirt, price of the most expensive shirt]

b. $[\![\text{more}]\!] \stackrel{\text{def}}{=} (0, +\infty)$

c. $\text{PRICE}(\text{the-dress}) \subseteq \iota I[I -$ [price of the cheapest shirt, price of the most expensive shirt] $= (0, +\infty)]$
$\Leftrightarrow \text{PRICE}(\text{the-dress}) \subseteq (\text{price of the most expensive shirt}, +\infty)$

Comparatives with *than*-clause internal quantifiers and numerical differentials

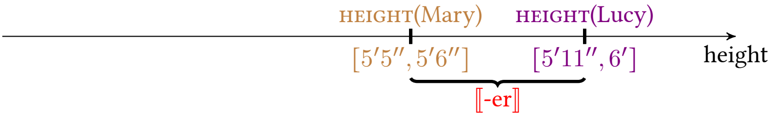
(23) ⟦ The dress is up to \$60 more expensive than every shirt is ~~expensive~~ ⟧
$\textsc{price}(\text{the-dress}) \subseteq \iota I[I - \underbrace{[\![\text{than every shirt is expensive}]\!]}_{I_{\text{STDD}}} = \underbrace{[\![\text{up to \$60 more}]\!]}_{I_{\text{DIFF}}}]$

a. ⟦than every shirt is ~~expensive~~⟧ =
Position-M[$\mathbf{Ans}_{\text{DIFF}}$⟦how expensive every shirt is⟧] =
Position-M[$\mathbf{Ans}_{\text{DIFF}}[\lambda I_{\text{DIFF}}.\forall x[\text{shirt}(x) \rightarrow \textsc{price}(x) \subseteq \iota I[I - I_{\text{STDD}} = I_{\text{DIFF}}]]]$],
which is [price of the cheapest shirt, price of the most expensive shirt]

b. ⟦up to \$60 more⟧ = $(0, +\infty) \cap (-\infty, \$60] = (0, \$60]$

c. $\textsc{price}(\text{the-dress}) \subseteq \iota I[I -$
[price of the cheapest shirt, price of the most expensive shirt] = $(0, \$60]]$
$\Leftrightarrow$
$\textsc{price}(\text{the-dress}) \subseteq$(price of the most expensive shirt, price of the cheapest shirt+ \$60]
(defined when the most expensive shirt is not more than \$60 more expensive than the cheapest shirt is)

Less



(21) ⟦ Lucy is taller than Mary is ~~tall~~ ⟧
$\textsc{height}(\text{Lucy}) \subseteq \iota I[I - \underbrace{⟦\text{than Mary is } \cancel{\text{tall}}⟧}_{I_{\textsc{stdd}}} = \underbrace{⟦\text{er}⟧}_{I_{\textsc{diff}}}]$

(24) ⟦ Mary is less tall than Lucy is ~~tall~~ ⟧
$\textsc{height}(\text{Mary}) \subseteq \iota I[I - \underbrace{⟦\text{than Lucy is } \cancel{\text{tall}}⟧}_{I_{\textsc{stdd}}} = \underbrace{⟦\text{less}⟧}_{I_{\textsc{diff}}}]$

(25) a. $⟦\text{er}⟧ \stackrel{\text{def}}{=} (0, +\infty)$ an increase based on a contextual salient base
b. $⟦\text{less}⟧ \stackrel{\text{def}}{=} \textsc{little}⟦\text{er}⟧ = [0, 0] - (0, +\infty) = (-\infty, 0)$
a negative increase: a decrease (to be revisited)

Discussion: What is a negative increase

- Additivity is a phenomenon of QUD-based anaphoricity, indicating an extension of a previous salient answer in addressing the QUD.
- In the domain of scalar values, there is not necessarily entailment between a lower and a higher value along a scale.

(26) a. Lucy is exactly 6 feet tall $\not\models$ Lucy is between $5'5$ and $5'8''$ tall
b. Lucy is between $5'5$ and $5'8''$ tall $\not\models$ Lucy is exactly 6 feet tall

- Thus along a scale, both $(0, +\infty)$ (which means moving a distance towards one direction of the scale) and $(-\infty, 0)$ (which means moving a distance towards the other direction of the scale) can be considered extensions of a previous salient answer in addressing the Current Question (i.e., about the measurement of the subject of a comparative).
- However ...

Discussion: Not to negate the increase, but to change the comparison direction

(3) $[\![\text{tall}]\!] \overset{\text{def}}{=} \lambda I_{\text{DIFF}}.\lambda I_{\text{STDD}}.\lambda x.\underbrace{I_{\text{DIFF}} \subseteq [0,+\infty)}_{\text{non-negative presup.}}.\ \textsc{height}(x) \subseteq \iota I[I - I_{\text{STDD}} = I_{\text{DIFF}}]$

(6) $[\![\text{short}]\!] \overset{\text{def}}{=} \lambda I_{\text{DIFF}}.\lambda I_{\text{STDD}}.\lambda x.\underbrace{I_{\text{DIFF}} \subseteq [0,+\infty)}_{\text{non-negative presup.}}.\ \textsc{hght}(x) \subseteq \iota I[I_{\text{STDD}} - I = I_{\text{DIFF}}]$

- Analyzing *less* as $(0,+\infty)$ is at odds with the non-negative presupposition of gradable adjectives.
  - Remedy: decompose $[\![\text{less}]\!]$ into an operator OPPOSITE and $[\![\text{er}]\!]$, then OPPOSITE changes the direction of comparison, not the polarity of I_{DIFF}

(27) $\textsc{opposite}_{\langle\langle dt,\langle dt,et\rangle\rangle,\langle dt,\langle dt,et\rangle\rangle\rangle} \overset{\text{def}}{=} \lambda G_{\langle dt,\langle dt,et\rangle\rangle}.\lambda I_{\text{DIFF}}.\lambda I_{\text{STDD}}.\lambda x.$
$G\text{-}\textsc{dimension}(x) \subseteq \iota I[I - I_{\text{STDD}} = [0,0] - I_{\text{DIFF}}]$

(28) a. $\textsc{opposite}[\![\text{tall}]\!] = [\![\text{short}]\!] \rightsquigarrow [\![\text{less tall}]\!] = [\![\text{shorter}]\!]$
b. $\textsc{opposite}[\![\text{short}]\!] = [\![\text{tall}]\!] \rightsquigarrow [\![\text{less short}]\!] = [\![\text{taller}]\!]$

Interim summary

- We have developed a new analysis of gradable adjectives and comparatives based on
  - considering *-er*/*more* an additive particle like *another*
  - interval subtraction

	The new difference-based view
Assumption	**Interval scales**
Comparison	**Subtraction:** $M_1 - M_2 = D$
Representations of & operations on scalar values	**Intervals (i.e., set of degrees) & interval subtraction**
The semantics of *-er*/*more*	**Additivity** **a default positive difference:** $(0, +\infty)$

(3) $[\![\text{tall}]\!] \stackrel{\text{def}}{=} \lambda I_{\text{DIFF}}.\lambda I_{\text{STDD}}.\lambda x.\underbrace{I_{\text{DIFF}} \subseteq [0, +\infty)}_{\text{non-negative presup.}}.\ \text{HEIGHT}(x) \subseteq \iota I[I - I_{\text{STDD}} = I_{\text{DIFF}}]$

(6) $[\![\text{short}]\!] \stackrel{\text{def}}{=} \lambda I_{\text{DIFF}}.\lambda I_{\text{STDD}}.\lambda x.\underbrace{I_{\text{DIFF}} \subseteq [0, +\infty)}_{\text{non-negative presup.}}.\ \text{HGHT}(x) \subseteq \iota I[I_{\text{STDD}} - I = I_{\text{DIFF}}]$

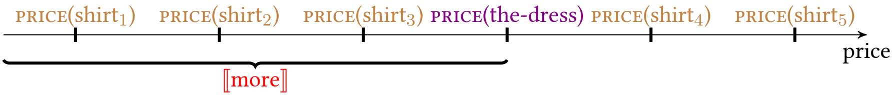
Outline

Outline

How the current additivity/difference-based analysis of comparatives helps solve more puzzles or shed some light on them

- Comparatives with *than*-clause internal modified numerals
- Incomplete comparatives
- Comparison between differences that result from comparisons
- ...(NPI licensing of the *than*-clause, see Zhang 2020a)

Comparatives with *than*-clause internal modified numerals



⟦The dress is more expensive than exactly 3 shirts are ~~expensive~~⟧

(29) ⟦ The dress is more expensive than exactly 3 shirts are ~~expensive~~ ⟧

$\textsc{price}(\text{the-dress}) \subseteq \iota I[I - \underbrace{⟦\text{than exactly 3 shirts are expensive}⟧}_{I_{\text{STDD}}} = \underbrace{⟦\text{more}⟧}_{I_{\text{DIFF}}}]$

- Zhang (2020b): A post-suppositional analysis à la Brasoveanu (2013)
  - The information of the minuend PRICE(the-dress) and the differential ⟦more⟧ is made use of to compute the subtrahend I_{STDD}
  - The cardinality of the maximal sum of shirts s.t., their price falls within I_{STDD} (computed from the step above) is checked (whether it's equal to 3) as post-suppositional requirement.

(See also Schwarzschild 2008)

Incomplete comparatives

- When there is an overt *than*-expression, a numerical measurement can play the role of comparison standard:

(30) a. Lucy is taller than 6 feet. $\textsc{height}(\text{Lucy}) \subseteq (6', +\infty)$

b. Mary is not 6^u feet tall. Lucy is taller than that_u.
$\textsc{height}(\text{Lucy}) \subseteq (6', +\infty)$

- However, in incomplete comparatives (which do not have an overt *than*-expression), it seems that numerical measurements cannot play the role of comparison standard (see Sheldon 1945, Schwarzschild 2010, Li 2023):

(31) a. Mary is not 6 feet tall. Lucy is taller.
$\rightsquigarrow \textsc{height}(\text{Lucy}) \subseteq \iota I[I - \textsc{height}(\text{Mary}) = (0, +\infty)]$
$\not\rightsquigarrow \textsc{height}(\text{Lucy}) \subseteq (6', +\infty)$

b. Mary is not POS tall. Lucy is taller.
$\rightsquigarrow \textsc{height}(\text{Lucy}) \subseteq \iota I[I - \textsc{height}(\text{Mary}) = (0, +\infty)]$
$\not\rightsquigarrow \textsc{height}(\text{Lucy}) \subseteq (d^c_{\text{POS}}, +\infty)$

Incomplete comparatives (Zhang and Zhang 2024)

- Comparative morpheme *-er/more*, as an additive particle, extends a previous salient answer in addressing the Current Question.
  - A previous salient answer: a position along a relevant scale (here a height scale)

(31) a. Mary is not 6 feet tall. Lucy is taller.
$\rightsquigarrow$ $\textsc{height}(\text{Lucy}) \subseteq \iota I[I - \textsc{height}(\text{Mary}) = (0, +\infty)]$
$\not\rightsquigarrow$ $\textsc{height}(\text{Lucy}) \subseteq (6', +\infty)$

b. Mary is not POS tall. Lucy is taller.
$\rightsquigarrow$ $\textsc{height}(\text{Lucy}) \subseteq \iota I[I - \textsc{height}(\text{Mary}) = (0, +\infty)]$
$\not\rightsquigarrow$ $\textsc{height}(\text{Lucy}) \subseteq (d^c_{\text{POS}}, +\infty)$

- Under the current analysis, in a measurement sentence, the numerical measurement plays the role of I_{DIFF}, meaning the distance away from the zero point. Thus this numerical measurement cannot be a salient position for the use of *-er/more*.
- Then the contextual threshold in the positive use is probably never a salient value in a discourse. Thus it cannot be the antecedent for *-er/more*

Comparison between differences

(21) ⟦ Lucy is taller than Mary is ~~tall~~ ⟧
$\textsc{height}(\text{Lucy}) \subseteq \iota I[I - \underbrace{⟦\text{than Mary is tall}⟧}_{I_{\text{STDD}}} = \underbrace{⟦\text{er}⟧}_{I_{\text{DIFF}}}]$

⟦than Mary is ~~tall~~⟧ = **Position-M**[$\mathbf{Ans}_{\text{DIFF}}$⟦how tall Mary is⟧]

(32) Mona is more happy than Jude is sad.

(Kennedy 1999, Zhang and Ling 2021)

a. **Comparison 1** – along a scale of happiness:
Mona's happiness vs. the threshold of happiness
⇝ Mona is happy

b. **Comparison 2** – along a scale of sadness:
Jude's sadness vs. the threshold of sadness ⇝ Jude is sad

c. **Comparison 3** – along a scale of deviation / difference size
difference from Comparison 1 vs. difference from Comparison 2

- The comparison between differences should be derived without the operator **Position-M**.

Comparison between differences

(32) $[\![$ Mona is much+er happy than Jude is sad $]\!]$
$\textsc{happiness}(\text{Lucy}) \subseteq$
$\iota I[I - [d^c_{\text{POS-HAPPY}}, d^c_{\text{POS-HAPPY}}] = \iota I[I - \underbrace{[\![\text{than Jude is sad}]\!]}_{I_{\text{STDD}}} = \underbrace{[\![\text{er}]\!]}_{I_{\text{DIFF}}}]]$

Here $[\![\text{than Jude is sad}]\!] = \mathbf{Ans}_{\text{DIFF}}[\![\text{how sad Jude is}]\!]$
$= \mathbf{Ans}_{\text{DIFF}}[\lambda I_{\text{DIFF}}.\textsc{sadness}(\text{Jude}) \subseteq \iota I[I - [d^c_{\text{POS-SAD}}, d^c_{\text{POS-SAD}}] = I_{\text{DIFF}}]]$

Today's take-home messages

- Day 2 (yesterday) and Day 3 (today): Comparatives and *-er*/*more*
  - How an additivity-based perspective improve our understanding of scalarity-related phenomena?
  - What is additivity?
- Today
  - Formal analysis of gradable adjectives, including
    - ★ antonyms
    - ★ *-er*/*more*
    - ★ *less*
    - ★ various uses of gradable adjectives
    - ★ *than*-clause internal quantifiers
    - ★ numerical differentials
  - ~~Cross-linguistic phenomena: languages without morphemes like *-er*/*more*~~
  - etc.

Tomorrow

- Day 1: Basics of scales and degrees; how they are relevant to natural language
  - What are scales? What are their formal properties? What operators do they support?
- Day 2 and Day 3: Comparatives and *-er*/*more*
  - How an additivity-based perspective improve our understanding of scalarity-related phenomena?
  - What is additivity?
- Day 4 and Day 5: *Even* and its cross-linguistic siblings
  - How a scalarity-based perspective improve our understanding of additivity-related phenomena?

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