Scalarity and additivity in natural language: (III) comparatives (cont.)

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https://lingbuzz.net/lingbuzz/008302
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Recapitulation

- Additivity is a phenomenon of QUD-based anaphoricity, indicating an extension of a previous salient answer in addressing the QUD.
- An additivity/increase-based view of *-er*/*more*
- A new difference-based view of comparatives

	The canonical view	**The new difference-based view**
Assumption	(Ordinal/interval) scales	**Interval scales**
Comparison	Inequality: $M_1 > M_2$	**Subtraction**: $M_1 - M_2 = D$
Representations of & operations on scalar values	Degree points & ordering between degree points	**Intervals (i.e., set of degrees) & interval subtraction**
The semantics of *-er*/*more*	Ordering: $>$	**Additivity a default positive difference**: $(0, +\infty)$

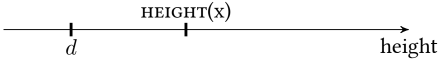
Today

- Day 2 (yesterday) and Day 3 (today): Comparatives and *-er*/*more*
  - How an additivity-based perspective improve our understanding of scalarity-related phenomena?
  - What is additivity?
- Today
  - Formal implementation (see Zhang and Ling 2021 and Zhang and Zhang 2024)
  - Antonyms
  - Cross-linguistic phenomena
  - etc.

Outline

The meaning of gradable adjectives (to be revisited)

- Canonical view (See e.g., Cresswell 1976, Hellan 1981, von Stechow 1984, Heim 1985, Schwarzschild 2008, Beck 2011):

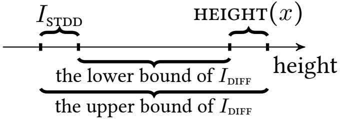


$[\![\text{tall}]\!]$

(1) $[\![\text{tall}]\!]_{\langle d,et\rangle} \overset{\text{def}}{=} \lambda d_d.\lambda x_e.\textsc{height}_{\langle e,d\rangle}(x) \geq d$ (i.e., x is d-tall)
On the scale of height, the position of x reaches degree d.

- There are two pieces in this lexical entry
  - A measure function of type $\langle ed\rangle$: $\textsc{height}_{\langle e,d\rangle}(x)$
  - Indicating the direction (of comparison): $\geq d$ (cf. Kennedy 1999)

The meaning of gradable adjectives



$[\![\text{tall}]\!]$

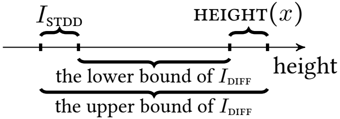
(1) $[\![\text{tall}]\!]_{\langle d,et\rangle} \stackrel{\text{def}}{=} \lambda d_d.\lambda x_e.\text{HEIGHT}_{\langle e,d\rangle}(x) \geq d$ Canonical view
On the scale of height, the position of x reaches degree d.

(2) $[\![\text{tall}]\!]_{\langle dt,et\rangle} \stackrel{\text{def}}{=} \lambda I_{\langle dt\rangle}.\lambda x_e.\text{HEIGHT}_{\langle e,dt\rangle}(x) \subseteq I$ (Zhang and Ling 2021)
On the scale of height, the measure of x falls at the position I.

(3) $[\![\text{tall}]\!] \stackrel{\text{def}}{=} \lambda I_{\text{DIFF}}.\lambda I_{\text{STDD}}.\lambda x.\underbrace{I_{\text{DIFF}} \subseteq [0, +\infty)}_{\text{non-negative presup.}}.\ \text{HEIGHT}(x) \subseteq \iota I[I - I_{\text{STDD}} = I_{\text{DIFF}}]$

(i.e., the height of x reaches the comparison standard, I_{STDD}.
$\rightsquigarrow$ the difference between them, I_{DIFF}, is non-negative) (Zhang and Zhang 2024)

The meaning of gradable adjectives



(2) $[\![\text{tall}]\!]_{\langle dt,et\rangle} \overset{\text{def}}{=} \lambda I_{\langle dt\rangle}.\lambda x_e.\textsc{height}_{\langle e,dt\rangle}(x) \subseteq I$ (Zhang and Ling 2021)

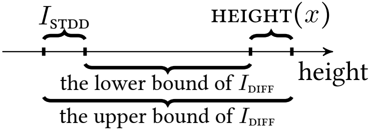
(3) $[\![\text{tall}]\!] \overset{\text{def}}{=} \lambda I_{\text{DIFF}}.\lambda I_{\text{STDD}}.\lambda x.\underbrace{I_{\text{DIFF}} \subseteq [0,+\infty)}_{\text{non-negative presup.}}.\ \textsc{height}(x) \subseteq \iota I[I - I_{\text{STDD}} = I_{\text{DIFF}}]$

(i.e., the height of x reaches the comparison standard, I_{STDD}.
$\rightsquigarrow$ the difference between them, I_{DIFF}, is non-negative) (Zhang and Zhang 2024)

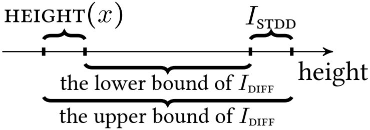
(4) A type shifter $[\![\textsc{compare}]\!]_{\langle\langle dt,et\rangle,\langle dt,\langle dt,et\rangle\rangle\rangle}$ (see also Zhang and Ling 2021)
$\overset{\text{def}}{=} \lambda G_{\langle dt,et\rangle}.\lambda I_{\text{DIFF}}.\lambda I_{\text{STDD}}.\lambda x_e.G\text{-}\textsc{dimension}(x) \subseteq \iota I[I - I_{\text{STDD}} = I_{\text{DIFF}}]$

(5) $[\![\textsc{compare}\ \text{tall}]\!]_{\langle dt,\langle dt,et\rangle\rangle}$
$= \lambda I_{\text{DIFF}}.\lambda I_{\text{STDD}}.\lambda x_e.\textsc{height}_{\langle e,dt\rangle}(x) \subseteq \iota I[I - I_{\text{STDD}} = I_{\text{DIFF}}]$

The meaning of gradable adjectives (Zhang and Zhang 2024)



The meaning of *tall*



The meaning of *short*

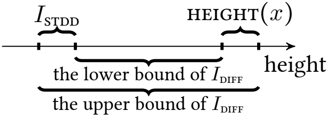
(3) $[\![\text{tall}]\!] \overset{\text{def}}{=} \lambda I_{\text{DIFF}}.\lambda I_{\text{STDD}}.\lambda x.\underbrace{I_{\text{DIFF}} \subseteq [0,+\infty)}_{\text{non-negative presup.}}.\ \text{HEIGHT}(x) \subseteq \iota I[I - I_{\text{STDD}} = I_{\text{DIFF}}]$

(i.e., the height of x reaches the comparison standard, I_{STDD}.
$\rightsquigarrow$ the difference between them, I_{DIFF}, is non-negative)

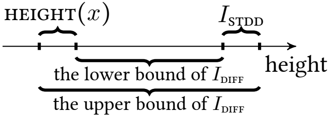
(6) $[\![\text{short}]\!] \overset{\text{def}}{=} \lambda I_{\text{DIFF}}.\lambda I_{\text{STDD}}.\lambda x.\underbrace{I_{\text{DIFF}} \subseteq [0,+\infty)}_{\text{non-negative presup.}}.\ \text{HGHT}(x) \subseteq \iota I[I_{\text{STDD}} - I = I_{\text{DIFF}}]$

(i.e., the height of x does not exceed the comparison standard, I_{STDD}.
$\rightsquigarrow$ the difference between them, I_{DIFF}, is non-negative)

Major uses of gradable adjectives: Positive use



The meaning of *tall*



The meaning of *short*

(7) $[\![\text{Lucy is POS tall}]\!]$

$$\Leftrightarrow \textsc{height}(\text{Lucy}) \subseteq \iota I[I - \underbrace{[d^{c}_{\text{POS}}, d^{c}_{\text{POS}}]}_{I_{\text{STDD}}} = \underbrace{[0, +\infty)}_{I_{\text{DIFF}}}]$$

$$\Leftrightarrow \textsc{height}(\text{Lucy}) \subseteq [d^{c}_{\text{POS}}, +\infty)$$

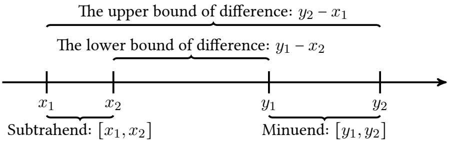
(8) $[\![\text{Lucy is POS short}]\!]$

$$\Leftrightarrow \textsc{height}(\text{Lucy}) \subseteq \iota I[\underbrace{[d^{c}_{\text{POS}}, d^{c}_{\text{POS}}]}_{I_{\text{STDD}}} - I = \underbrace{[0, +\infty)}_{I_{\text{DIFF}}}]$$

$$\Leftrightarrow \textsc{height}(\text{Lucy}) \subseteq (-\infty, d^{c}_{\text{POS}}]$$

(See Zhang and Zhang 2024)

Subtraction between intervals



(9) $$\underbrace{[y_1, y_2]}_{\textbf{minuend: position}} - \underbrace{[x_1, x_2]}_{\textbf{subtrahend: position}} = \underbrace{[y_1 - x_2, y_2 - x_1]}_{\textbf{difference: distance between positions}}$$

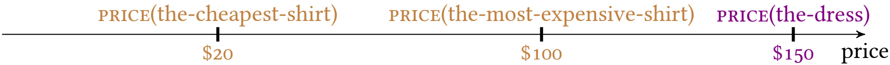
(10) Given the subtrahend position $[a, b]$ and the difference $[c, d]$,
Minuend position = $[b + c, a + d]$ (defined when $b + c \leq a + d$)
$\textsc{height}(\text{Lucy}) \subseteq \iota I[I - [d^c{}_{\text{POS}}, d^c{}_{\text{POS}}] = [0, +\infty)] \Leftrightarrow \textsc{height}(\text{Lucy}) \subseteq [d^c_{\text{POS}}, +\infty)$

(11) Given the minuend position $[a, b]$ and the difference $[c, d]$,
Subtrahend position = $[b - d, a - c]$ (defined when $b - d \leq a - c$)
$\textsc{height}(\text{Lucy}) \subseteq \iota I[[d^c{}_{\text{POS}}, d^c{}_{\text{POS}}] - I = [0, +\infty)] \Leftrightarrow \textsc{height}(\text{Lucy}) \subseteq (-\infty, d^c_{\text{POS}}]$

(See Moore 1979)

Interlude

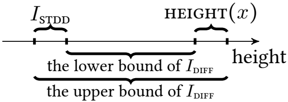
(10) Given the subtrahend position $[a, b]$ and the difference $[c, d]$,
Minuend position = $[b + c, a + d]$ (defined when $b + c \leq a + d$)



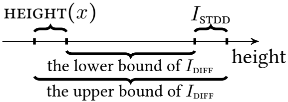
(12) $[\![$The dress is up to $60 more expensive than every shirt is$]\!]$
$\Leftrightarrow$ $\textsc{price}(\text{the-dress}) \subseteq \iota I[I - [\$20, \$100] = (0, \$60]]$
Under the given context, I is undefined!!

(13) The giraffe is exactly 5 inches taller than every tree is.
$\rightsquigarrow$ We have the inference that every tree is of the same height. Why?
$\textsc{height}(\text{the-giraffe}) \subseteq \iota I[I - I_{\textsc{stdd}} = [5'', 5'']]$, thus the upper and lower bound of $I_{\textsc{stdd}}$ needs to be the same to meet the definedness requirement

Major uses of gradable adjectives: Measurement sentence



The meaning of *tall*



The meaning of *short*

(14) ⟦Lucy is 6 feet tall⟧ ‘at least’ reading and ‘exactly’ reading

a. $\textsc{height}(\text{Lucy}) \subseteq \iota I[I - \underbrace{[0,0]}_{I_{\text{STDD}}} = \underbrace{[6', +\infty)}_{I_{\text{DIFF}}}] \Leftrightarrow \textsc{height}(\text{Lucy}) \subseteq [6', +\infty)$

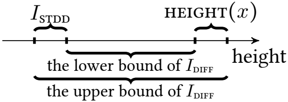
b. $\textsc{height}(\text{Lucy}) \subseteq \iota I[I - \underbrace{[0,0]}_{I_{\text{STDD}}} = \underbrace{[6', 6']}_{I_{\text{DIFF}}}] \Leftrightarrow \textsc{height}(\text{Lucy}) \subseteq [6', 6']$

(15) ⟦Lucy is 5 feet short⟧ **Ungrammatical!**

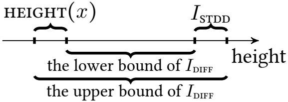
$[\![\text{short}]\!] \overset{\text{def}}{=} \lambda I_{\text{DIFF}}.\lambda I_{\text{STDD}}.\lambda x.\underbrace{I_{\text{DIFF}} \subseteq [0, +\infty)}_{\text{non-negative presup.}}.\ \textsc{hght}(x) \subseteq \iota I[I_{\text{STDD}} - I = I_{\text{DIFF}}]$

$\rightsquigarrow$ If Lucy’s height is at the position $[5', 5']$, compared with I_{STDD} that is $[0, 0]$, the non-negative presupposition of I_{DIFF} is violated.

Major uses of gradable adjectives: Degree question



The meaning of *tall*



The meaning of *short*

(16) ⟦How tall is Lucy⟧

a. $\lambda I_{\text{DIFF}}.\text{HEIGHT}(\text{Lucy}) \subseteq \iota I[I - \underbrace{[0,0]}_{I_{\text{STDD}}} = I_{\text{DIFF}}]$ No evaluativity!

⇝ How far Lucy's height measurement is from / above the zero point

b. $\lambda I_{\text{DIFF}}.\text{HEIGHT}(\text{Lucy}) \subseteq \iota I[I - \underbrace{[d^{c}{}_{\text{POS}}, d^{c}{}_{\text{POS}}]}_{I_{\text{STDD}}} = I_{\text{DIFF}}]$ Evaluativity!

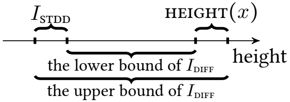
⇝ How far Lucy's height is from / above the contextual threshold of being tall

(17) ⟦How short is Lucy⟧

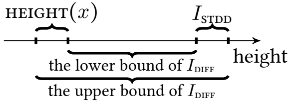
$\lambda I_{\text{DIFF}}.\text{HEIGHT}(\text{Lucy}) \subseteq \iota I[\underbrace{[d^{c}{}_{\text{POS}}, d^{c}{}_{\text{POS}}]}_{I_{\text{STDD}}} - I = I_{\text{DIFF}}]$ Evaluativity!

⇝ How far Lucy's height is from / below the contextual threshold of being short

Major uses of gradable adjectives: Degree question



The meaning of *tall*



The meaning of *short*

(16a) $[\![\text{How tall is Lucy}]\!] = \lambda I_{\text{DIFF}}.\text{HEIGHT}(\text{Lucy}) \subseteq \iota I[I - \underbrace{[0,0]}_{I_{\text{STDD}}} = I_{\text{DIFF}}]$

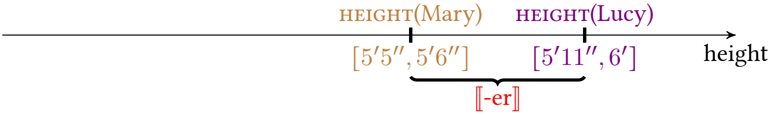
(17) $[\![\text{How short is Lucy}]\!] = \lambda I_{\text{DIFF}}.\text{HEIGHT}(\text{Lucy}) \subseteq \iota I[\underbrace{[d^{c}{}_{\text{POS}}, d^{c}{}_{\text{POS}}]}_{I_{\text{STDD}}} - I = I_{\text{DIFF}}]$

(18) An answerhood operator $\mathbf{Ans}_{\text{DIFF}}$ (which returns a maximally informative true answer) is defined for a set of intervals p s.t.
$\exists I[p(I) \wedge \forall I'[[p(I') \wedge I' \neq I] \rightarrow I \subset I']]$ (this maximally informative I exists)
When defined, $\mathbf{Ans}_{\text{DIFF}} \overset{\text{def}}{=} \lambda p_{\langle dt,t\rangle}.\iota I[p(I) \wedge \forall I'[[p(I') \wedge I' \neq I] \rightarrow I \subset I']]$

(19) **Position-M** $\overset{\text{def}}{=} \lambda I_{\text{DIFF}}.\iota I[I - I_{\text{STDD}} = I_{\text{DIFF}}]$ **Minuend position**

(20) **Position-S** $\overset{\text{def}}{=} \lambda I_{\text{DIFF}}.\iota I[I_{\text{STDD}} - I = I_{\text{DIFF}}]$ **Subtrahend position**

Major uses of gradable adjectives: Clausal comparative



⟦Lucy is taller than Mary is ~~tall~~⟧

(21) ⟦ Lucy is taller than Mary is ~~tall~~ ⟧

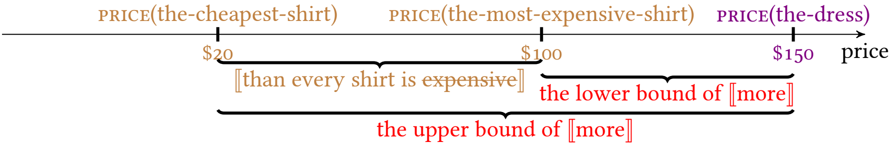
$\textsc{height}(\text{Lucy}) \subseteq \iota I[I - \underbrace{⟦\text{than Mary is } \cancel{\text{tall}}⟧}_{I_{\textsc{stdd}}} = \underbrace{⟦\text{er}⟧}_{I_{\textsc{diff}}}]$

a. ⟦than Mary is ~~tall~~⟧ = **Position-M**[**Ans**$_{\textsc{diff}}$ ⟦how tall Mary is⟧]
   = HEIGHT(Mary) = $[5'5'', 5'6'']$ under the above context

b. ⟦er⟧ $\overset{\text{def}}{=} (0, +\infty)$
   $\rightsquigarrow$ extending the value ⟦than Mary is ~~tall~~⟧ in addressing the Current Question 'how tall Lucy is'

c. $\textsc{height}(\text{Lucy}) \subseteq \iota I[I - [5'5'', 5'6''] = (0, +\infty)]$
   $\Leftrightarrow \textsc{height}(\text{Lucy}) \subseteq (5'6'', +\infty)$

Comparatives with *than*-clause internal quantifiers



⟦The dress is more expensive than every shirt is ~~expensive~~⟧

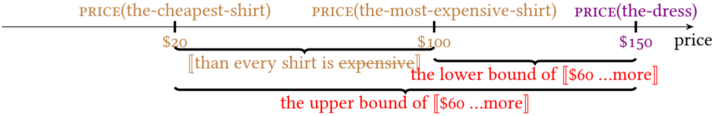
(22) ⟦ The dress is more expensive than every shirt is ~~expensive~~ ⟧
$\textsc{price}(\text{the-dress}) \subseteq \iota I[I - \underbrace{⟦\text{than every shirt is } \cancel{\text{expensive}}⟧}_{I_{\textsc{stdd}}} = \underbrace{⟦\text{more}⟧}_{I_{\textsc{diff}}}]$

a. ⟦than every shirt is ~~expensive~~⟧ =
Position-M$[$**Ans**$_{\textsc{diff}}$⟦how expensive every shirt is⟧$]$ =
Position-M$[$**Ans**$_{\textsc{diff}}[\lambda I_{\textsc{diff}}.\forall x[\text{shirt}(x) \rightarrow \textsc{price}(x) \subseteq \iota I[I - I_{\textsc{stdd}} = I_{\textsc{diff}}]]]]$,
which is $[\$20, \$100]$ under the current context

b. ⟦more⟧ $\stackrel{\text{def}}{=} (0, +\infty)$

c. $\textsc{price}(\text{the-dress}) \subseteq \iota I[I - [\$20, \$100] = (0, +\infty)]$
$\Leftrightarrow \textsc{price}(\text{the-dress}) \subseteq (\$100, +\infty)$

Comparatives with *than*-clause internal quantifiers and numerical differentials



⟦The dress is up to $60 more expensive than every shirt is ~~expensive~~⟧ (false here)

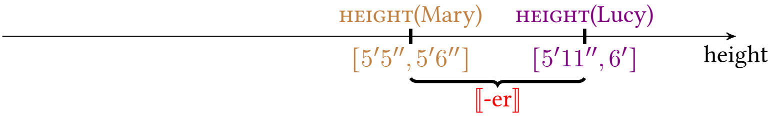
(23) ⟦ The dress is up to $60 more expensive than every shirt is ~~expensive~~ ⟧
$\text{PRICE}(\text{the-dress}) \subseteq \iota I[I - \underbrace{[\![\text{than every shirt is } \cancel{\text{expensive}}]\!]}_{I_{\text{STDD}}} = \underbrace{[\![\text{up to \$60 more}]\!]}_{I_{\text{DIFF}}}]$

a. ⟦than every shirt is ~~expensive~~⟧ =
Position-M[$\textbf{Ans}_{\text{DIFF}}$⟦how expensive every shirt is⟧] =
Position-M[$\textbf{Ans}_{\text{DIFF}}[\lambda I_{\text{DIFF}}.\forall x[\text{shirt}(x) \rightarrow \text{PRICE}(x) \subseteq \iota I[I - I_{\text{STDD}} = I_{\text{DIFF}}]]]]$,
which is $[\$20, \$100]$ under the current context

b. ⟦up to $60 more⟧ $= (0, +\infty) \cap (-\infty, \$60] = (0, \$60]$

c. $\text{PRICE}(\text{the-dress}) \subseteq \iota I[I - [\$20, \$100] = (0, \$60]]$ undefined!
(Given the subtrahend $[a, b]$ and the difference $[c, d]$,
the minuend $= [b + c, a + d]$, which is defined when $b + c \leq a + d$)

Less



(19) $[\![$ Lucy is taller than Mary is ~~tall~~ $]\!]$

$\textsc{height}(\text{Lucy}) \subseteq \iota I[I - \underbrace{[\![\text{than Mary is } \cancel{\text{tall}}]\!]}_{I_{\textsc{stdd}}} = \underbrace{[\![\text{er}]\!]}_{I_{\textsc{diff}}}]$

(24) $[\![$ Mary is less tall than Lucy is ~~tall~~ $]\!]$

$\textsc{height}(\text{Mary}) \subseteq \iota I[I - \underbrace{[\![\text{than Lucy is } \cancel{\text{tall}}]\!]}_{I_{\textsc{stdd}}} = \underbrace{[\![\text{less}]\!]}_{I_{\textsc{diff}}}]$

(25) a. $[\![\text{er}]\!] \stackrel{\text{def}}{=} (0, +\infty)$ an increase based on a contextual salient base

b. $[\![\text{less}]\!] \stackrel{\text{def}}{=} \textsc{little}[\![\text{er}]\!] = [0, 0] - (0, +\infty) = (-\infty, 0)$

a negative increase: a decrease (to be revisited)

Discussion: What is a negative increase

- Additivity is a phenomenon of QUD-based anaphoricity, indicating an extension of a previous salient answer in addressing the QUD.
- In the domain of scalar values, there is not necessarily entailment between a lower and a higher value along a scale.

(26) a. Lucy is exactly 6 feet tall $\not\models$ Lucy is between $5'5$ and $5'8''$ tall
b. Lucy is between $5'5$ and $5'8''$ tall $\not\models$ Lucy is exactly 6 feet tall

- Thus along a scale, both $(0, +\infty)$ (which means moving a distance towards one direction of the scale) and $(-\infty, 0)$ (which means moving a distance towards the other direction of the scale) can be considered extensions of a previous salient answer in addressing the Current Question (i.e., about the measurement of the subject of a comparative).
- However ...

Discussion: Not to negate the increase, but to change the comparison direction

(3) $[\![\text{tall}]\!] \stackrel{\text{def}}{=} \lambda I_{\text{DIFF}}.\lambda I_{\text{STDD}}.\lambda x.\underbrace{I_{\text{DIFF}} \subseteq [0,+\infty)}_{\text{non-negative presup.}}.\ \text{HEIGHT}(x) \subseteq \iota I[I - I_{\text{STDD}} = I_{\text{DIFF}}]$

(6) $[\![\text{short}]\!] \stackrel{\text{def}}{=} \lambda I_{\text{DIFF}}.\lambda I_{\text{STDD}}.\lambda x.\underbrace{I_{\text{DIFF}} \subseteq [0,+\infty)}_{\text{non-negative presup.}}.\ \text{HGHT}(x) \subseteq \iota I[I_{\text{STDD}} - I = I_{\text{DIFF}}]$

- Analyzing *less* as $(0,+\infty)$ is at odds with the non-negative presupposition of gradable adjectives.
  - Remedy: decompose $[\![\text{less}]\!]$ into an operator OPPOSITE and $[\![\text{er}]\!]$, then OPPOSITE changes the direction of comparison, not the polarity of I_{DIFF}

(27) $\text{OPPOSITE}_{\langle\langle dt,\langle dt,et\rangle\rangle,\langle dt,\langle dt,et\rangle\rangle\rangle} \stackrel{\text{def}}{=} \lambda G_{\langle dt,\langle dt,et\rangle\rangle}.\lambda I_{\text{DIFF}}.\lambda I_{\text{STDD}}.\lambda x.$
$G\text{-DIMENSION}(x) \subseteq \iota I[I - I_{\text{STDD}} = [0,0] - I_{\text{DIFF}}]$

(28) a. $\text{OPPOSITE}[\![\text{tall}]\!] = [\![\text{short}]\!] \quad \rightsquigarrow [\![\text{less tall}]\!] = [\![\text{shorter}]\!]$
b. $\text{OPPOSITE}[\![\text{short}]\!] = [\![\text{tall}]\!] \quad \rightsquigarrow [\![\text{less short}]\!] = [\![\text{taller}]\!]$

Interim summary

- We have developed a new analysis of gradable adjectives and comparatives based on
  - considering *-er*/*more* an additive particle like *another*
  - interval subtraction

	The new difference-based view
Assumption	**Interval scales**
Comparison	**Subtraction:** $M_1 - M_2 = D$
Representations of *&* operations on scalar values	**Intervals (i.e., set of degrees) *&* interval subtraction**
The semantics of *-er*/*more*	**Additivity** **a default positive difference:** $(0, +\infty)$

(3) $$[\![\text{tall}]\!] \stackrel{\text{def}}{=} \lambda I_{\text{DIFF}}.\lambda I_{\text{STDD}}.\lambda x.\underbrace{I_{\text{DIFF}} \subseteq [0, +\infty)}_{\text{non-negative presup.}}.\ \textsc{height}(x) \subseteq \iota I[I - I_{\text{STDD}} = I_{\text{DIFF}}]$$

(6) $$[\![\text{short}]\!] \stackrel{\text{def}}{=} \lambda I_{\text{DIFF}}.\lambda I_{\text{STDD}}.\lambda x.\underbrace{I_{\text{DIFF}} \subseteq [0, +\infty)}_{\text{non-negative presup.}}.\ \textsc{hght}(x) \subseteq \iota I[I_{\text{STDD}} - I = I_{\text{DIFF}}]$$

Outline

Languages with morphemes like *-er*/*more*

- Many languages (e.g., English, French) require the use of a comparative morpheme in the comparative use of gradable adjectives:

(29) a. Lucy is tall. **Positive**: tall
b. Lucy is taller than Mary is. **Comparative**: taller

(30) a. Lucy has many books. **Positive**: many
b. Lucy has more books than Mary does. **Comp.**: more

(31) **French data**

a. Jean est grand.
John be.3SG tall
'John is tall.' **Positive**: grand 'tall'

b. Jean est plus grand que Pierre.
John be.3SG more tall what Peter.
'John is taller than Peter.' **Comp.**: plus+grand 'taller'

Languages without morphemes like *-er*/*more*

- However, many other languages (e.g., Chinese, Japanese) don't make a distinction between the comparative vs. non-comparative use:

(32) **Chinese data**

a. Lèlè gāo ma?
Lèlè tall Q
'Is Lèlè tall?' **Positive**: gāo 'tall'

b. Lèlè bǐ Mǐmǐ gāo ma?
Lèlè STDD Mǐmǐ taller Q
'Is Lèlè taller than Mǐmǐ?' **Comp.**: gāo 'taller'

(33) **Japanese data**

a. Rika-wa (se-ga) taka-i.
Rika-TOP back-NOM tall-PRES
'Rika is tall.' **Positive**: taka- 'tall'

b. Rika-wa Makoto-yori (se-ga) taka-i.
Rika-TOP Makoto-STDD back-NOM tall-PRES
'Rika is taller than Makoto.' **Comp.**: taka- 'taller'

English comparatives vs. Chinese comparatives

(34) a. Lucy is taller than Mary is. $\textsc{height}(L) > \textsc{height}(M)$

b. Lèlè bǐ Mǐmi gāo.
Lèlè STDD Mǐmi taller
'Lèlè is taller than Mǐmi.' $\textsc{height}(L) > \textsc{height}(M)$

(35) a. $[\![$Lucy is POS tall$]\!]$ $\Leftrightarrow$ $\textsc{height}(\text{Lucy}) \geq d^c{}_{\text{POS}}$ **Positive use**

b. $[\![$Lucy is $5'8''$ inches tall$]\!]$ $\Leftrightarrow$ $\textsc{height}(\text{Lucy}) \geq 5'8''$ **Measure**

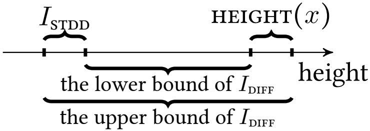
c. $[\![$how tall is Lucy$]\!]$ $\Leftrightarrow$ $\lambda d.\textsc{height}(\text{Lucy}) \geq d$ **Degree Q.**

d. $[\![$Lucy is as tall as Bill (is)$]\!]$ $\Leftrightarrow$ $\textsc{height}(\text{Lucy}) \geq \textsc{height}(\text{Bill})$ **Equative**

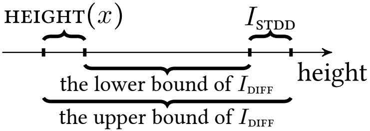
e. $[\![$Lucy is taller than Mary (is)$]\!]$ $\Leftrightarrow$ $\textsc{height}(L) > \textsc{height}(M)$ **Comparative**

- Zhang and Zhang (2024)'s proposal:
  - English gradable adjectives encode a non-strict inequality, and with the use of *-er*/*more*, comparatives express a strict inequality.
  - Chinese gradable adjectives directly encode a strict inequality.

Lexical semantics of gradable adjective *tall*/*gāo*



The meaning of *tall*/*gāo*



The meaning of *short*/*ǎi*

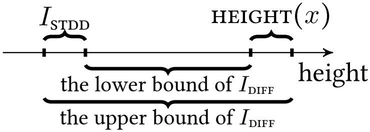
(3) $[\![\text{tall}]\!] \stackrel{\text{def}}{=} \lambda I_{\text{DIFF}}.\lambda I_{\text{STDD}}.\lambda x.\underbrace{I_{\text{DIFF}} \subseteq [0, +\infty)}_{\text{non-negative presup.}}.\ \text{HEIGHT}(x) \subseteq \iota I[I - I_{\text{STDD}} = I_{\text{DIFF}}]$ English

(i.e., the height of x reaches the comparison standard, I_{STDD}.
$\rightsquigarrow$ the difference between them, I_{DIFF}, is non-negative)

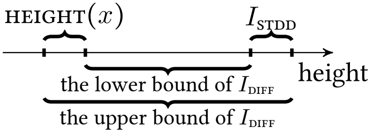
(36) $[\![\text{gāo}]\!] \stackrel{\text{def}}{=} \lambda I_{\text{DIFF}}.\lambda I_{\text{STDD}}.\lambda x.\underbrace{I_{\text{DIFF}} \subseteq (0, +\infty)}_{\text{positive presup.}}.\ \text{HEIGHT}(x) \subseteq \iota I[I - I_{\text{STDD}} = I_{\text{DIFF}}]$ Chinese

(i.e., the height of x exceeds the comparison standard, I_{STDD}.
$\rightsquigarrow$ the difference between them, I_{DIFF}, is positive)

Lexical semantics of gradable adjective *short*/*ǎi*



The meaning of *tall*/*gāo*



The meaning of *short*/*ǎi*

(6) $[\![\text{short}]\!] \overset{\text{def}}{=} \lambda I_{\text{DIFF}}.\lambda I_{\text{STDD}}.\lambda x.\underbrace{I_{\text{DIFF}} \subseteq [0, +\infty)}_{\text{non-negative presup.}}.\ \text{HGHT}(x) \subseteq \iota I[I_{\text{STDD}} - I = I_{\text{DIFF}}]$ English

(i.e., the height of x does not exceed the comparison standard, I_{STDD}.
$\rightsquigarrow$ the difference between them, I_{DIFF}, is non-negative)

(37) $[\![\text{ǎi}]\!] \overset{\text{def}}{=} \lambda I_{\text{DIFF}}.\lambda I_{\text{STDD}}.\lambda x.\underbrace{I_{\text{DIFF}} \subseteq (0, +\infty)}_{\text{positive presup.}}.\ \text{HEIGHT}(x) \subseteq \iota I[I_{\text{STDD}} - I = I_{\text{DIFF}}]$ Chinese

(i.e., the height of x is below / does not reach the comparison standard, I_{STDD}.
$\rightsquigarrow$ the difference between them, I_{DIFF}, is positive)

The positive use of gradable adjectives

- In the positive use, neither I_{STDD} nor I_{DIFF} is overtly uttered (though I_{DIFF} can be restricted by degree modifiers like *very*, *quite*, *a bit*, *extremely*). Thus the subtle truth-conditional difference between 'reaching a threshold' and 'exceeding a threshold' cannot be detected.

(7) ⟦Lucy is POS tall⟧ English

$$\Leftrightarrow \textsc{height}(\text{Lucy}) \subseteq \iota I[I - \underbrace{[d^{c}{}_{\text{POS}}, d^{c}{}_{\text{POS}}]}_{I_{\text{STDD}}} = \underbrace{[0, +\infty)}_{I_{\text{DIFF}}}]$$

$$\Leftrightarrow \textsc{height}(\text{Lucy}) \subseteq [d^{c}_{\text{POS}}, +\infty)$$

(i.e., the height of Lucy reaches the contextual threshold of being tall)

(38) ⟦Lucy hěn POS gāo⟧ Chinese

$$\Leftrightarrow \textsc{height}(\text{Lucy}) \subseteq \iota I[I - \underbrace{[d^{c}_{\text{POS}}, d^{c}_{\text{POS}}]}_{I_{\text{STDD}}} = \underbrace{(0, +\infty)}_{I_{\text{DIFF}}}]$$

$$\Leftrightarrow \textsc{height}(\text{Lucy}) \subseteq (d^{c}_{\text{POS}}, +\infty)$$

(i.e., the height of Lucy exceeds the contextual threshold of being tall)

Measurement sentences

- In measurement sentences, there is always a numerical expression specifying I_{DIFF}, leading to the same truth conditions for these sentences in English and Chinese.

(39) $[\![$Lucy is 5 feet 8 inches tall$]\!]$ English

$\Leftrightarrow \textsc{height}(\text{Lucy}) \subseteq \iota I[I - [0,0] = [5'8'', +\infty) \cap [0, +\infty)]$

$\Leftrightarrow \textsc{height}(\text{Lucy}) \subseteq [5'8'', +\infty)$

(40) $[\![$Lucy (yǒu) 1.7272 m gāo$]\!]$ Chinese

$\Leftrightarrow \textsc{height}(\text{Lucy}) \subseteq \iota I[I - [0,0] = [1.7272m + \infty) \cap (0, +\infty)]$

$\Leftrightarrow \textsc{height}(\text{Lucy}) \subseteq [1.7272m, +\infty)$

Comparatives

- English comparatives need the use of *-er*/*more* to turn a non-negative I_{DIFF} into a positive one, while in Chinese, I_{DIFF} is already positive by default.

(19) $[\![\text{Lucy is tall } \underbrace{\text{er}}_{(0,+\infty)} \underbrace{\text{than Mary is}}_{I_{\text{STDD}}}]\!]$

$\Leftrightarrow \textsc{height}(\text{Lucy}) \subseteq \iota I[I - \textsc{height}(\text{Mary}) = \underbrace{(0,+\infty)}_{[\![\text{er}]\!]} \cap [0,+\infty)]$

$\Leftrightarrow \textsc{height}(\text{Lucy}) \subseteq \iota I[I - \textsc{height}(\text{Mary}) = (0,+\infty)]$

(41) [[Lèlè bǐ Mǐmi gāo]]

$\Leftrightarrow \textsc{height}(\text{Lèlè}) \subseteq \iota I[I - \textsc{height}(\text{Mǐmi}) = (0,+\infty)]$

Comparison in English vs. Chinese

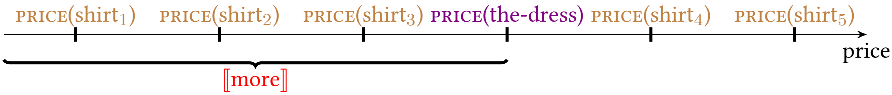
- Within our proposed view,
  - For languages that require the use of *-er* in comparatives (e.g., English): gradable adjectives encode a non-strict inequality
    - ★ In terms of I_{DIFF}, there is a non-negative requirement
  - For languages that use the same form for the comparative and non-comparative uses (e.g., Chinese):
    gradable adjectives encode a strict inequality
    - ★ In terms of I_{DIFF}, there is a positive requirement

Outline

How the current additivity/difference-based analysis of comparatives helps solve more puzzles or shed some light on them

- Comparatives with *than*-clause internal modified numerals
- Incomplete comparatives
- Comparison between differences that result from comparisons
- ...(NPI licensing of the *than*-clause, see Zhang 2020a)

Comparatives with *than*-clause internal modified numerals



⟦The dress is more expensive than exactly 3 shirts are ~~expensive~~⟧

(42) ⟦ The dress is more expensive than exactly 3 shirts are ~~expensive~~ ⟧

$\textsc{price}(\text{the-dress}) \subseteq \iota I[I - \underbrace{[\![\text{than exactly 3 shirts are } \cancel{\text{expensive}}]\!]}_{I_{\textsc{stdd}}} = \underbrace{[\![\text{more}]\!]}_{I_{\textsc{diff}}}]$

- Zhang (2020b): A post-suppositional analysis à la Brasoveanu (2013)
  - The information of the minuend PRICE(the-dress) and the differential ⟦more⟧ is made use of to compute the subtrahend $I_{\textsc{stdd}}$
  - The cardinality of the maximal sum of shirts s.t., their price falls within $I_{\textsc{stdd}}$ (computed from the step above) is checked (whether it's equal to 3) as post-suppositional requirement.

(See also Schwarzschild 2008)

Incomplete comparatives

- When there is an overt *than*-expression, a numerical measurement can play the role of comparison standard:

(43) a. Lucy is taller than 6 feet. $\textsc{height}(\text{Lucy}) \subseteq (6', +\infty)$

b. Mary is not 6^u feet tall. Lucy is taller than that_u.
$\textsc{height}(\text{Lucy}) \subseteq (6', +\infty)$

- However, in incomplete comparatives (which do not have an overt *than*-expression), it seems that numerical measurements cannot play the role of comparison standard (see Sheldon 1945, Schwarzschild 2010, Li 2023):

(44) a. Mary is not 6 feet tall. Lucy is taller.
$\rightsquigarrow \textsc{height}(\text{Lucy}) \subseteq \iota I[I - \textsc{height}(\text{Mary}) = (0, +\infty)]$
$\not\rightsquigarrow \textsc{height}(\text{Lucy}) \subseteq (6', +\infty)$

b. Mary is not POS tall. Lucy is taller.
$\rightsquigarrow \textsc{height}(\text{Lucy}) \subseteq \iota I[I - \textsc{height}(\text{Mary}) = (0, +\infty)]$
$\not\rightsquigarrow \textsc{height}(\text{Lucy}) \subseteq (d^c_{\text{POS}}, +\infty)$

Incomplete comparatives (Zhang and Zhang 2024)

- Comparative morpheme *-er*/*more*, as an additive particle, extends a previous salient answer in addressing the Current Question.
  - A previous salient answer: a position along a relevant scale (here a height scale)

(44) a. Mary is not 6 feet tall. Lucy is taller.
$\rightsquigarrow$ $\textsc{height}(\text{Lucy}) \subseteq \iota I[I - \textsc{height}(\text{Mary}) = (0, +\infty)]$
$\not\rightsquigarrow$ $\textsc{height}(\text{Lucy}) \subseteq (6', +\infty)$

b. Mary is not POS tall. Lucy is taller.
$\rightsquigarrow$ $\textsc{height}(\text{Lucy}) \subseteq \iota I[I - \textsc{height}(\text{Mary}) = (0, +\infty)]$
$\not\rightsquigarrow$ $\textsc{height}(\text{Lucy}) \subseteq (d^c_{\text{POS}}, +\infty)$

- Under the current analysis, in a measurement sentence, the numerical measurement plays the role of I_{DIFF}, meaning the distance away from the zero point. Thus this numerical measurement cannot be a salient position for the use of *-er*/*more*.
- Then the contextual threshold in the positive use is probably never a salient value in a discourse. Thus it cannot be the antecedent for *-er*/*more*.

Comparison between differences

(19) $[\![$ Lucy is taller than Mary is ~~tall~~ $]\!]$
$\textsc{height}(\text{Lucy}) \subseteq \iota I[I - \underbrace{[\![\text{than Mary is } \cancel{\text{tall}}]\!]}_{I_{\text{STDD}}} = \underbrace{[\![\text{er}]\!]}_{I_{\text{DIFF}}}]$

$[\![\text{than Mary is } \cancel{\text{tall}}]\!] = \textbf{Position-M}[\textbf{Ans}_{\text{DIFF}}[\![\text{how tall Mary is}]\!]]$

(45) Mona is more happy than Jude is sad.

(Kennedy 1999, Zhang and Ling 2021)

a. **Comparison 1** – along a scale of happiness:
Mona's happiness vs. the threshold of happiness
$\rightsquigarrow$ Mona is happy

b. **Comparison 2** – along a scale of sadness:
Jude's sadness vs. the threshold of sadness $\rightsquigarrow$ Jude is sad

c. **Comparison 3** – along a scale of deviation / difference size
difference from Comparison 1 vs. difference from Comparison 2

- The comparison between differences should be derived without the operator **Position-M**.

Comparison between differences

(45) ⟦ Mona is much+er happy than Jude is sad ⟧

$\text{HAPPINESS}(\text{Lucy}) \subseteq$

$\iota I[I - [d^c_{\text{POS-HAPPY}}, d^c_{\text{POS-HAPPY}}] = \iota I[I - \underbrace{⟦\text{than Jude is sad}⟧}_{I_{\text{STDD}}} = \underbrace{⟦\text{er}⟧}_{I_{\text{DIFF}}}]]$

Here ⟦than Jude is sad⟧ = $\mathbf{Ans}_{\text{DIFF}}$⟦how sad Jude is⟧

$= \mathbf{Ans}_{\text{DIFF}}[\lambda I_{\text{DIFF}}.\text{SADNESS}(\text{Jude}) \subseteq \iota I[I - [d^c_{\text{POS-SAD}}, d^c_{\text{POS-SAD}}] = I_{\text{DIFF}}]]$

Today's take-home messages

- Day 2 (yesterday) and Day 3 (today): Comparatives and *-er/more*
  - How an additivity-based perspective improve our understanding of scalarity-related phenomena?
  - What is additivity?
- Today
  - Formal analysis of gradable adjectives, including
    - antonyms
    - *-er/more*
    - *less*
    - various uses of gradable adjectives
    - *than*-clause internal quantifiers
    - numerical differentials
  - Cross-linguistic phenomena: languages without morphemes like *-er/more*
  - etc.

Tomorrow

- Day 1: Basics of scales and degrees; how they are relevant to natural language
  - What are scales? What are their formal properties? What operators do they support?
- Day 2 and Day 3: Comparatives and *-er*/*more*
  - How an additivity-based perspective improve our understanding of scalarity-related phenomena?
  - What is additivity?
- Day 4 and Day 5: *Even* and its cross-linguistic siblings
  - How a scalarity-based perspective improve our understanding of additivity-related phenomena?

Selected references I

Beck, Sigrid. 2011. Comparative constructions. In *Semantics*, ed. Claudia Maienborn, Klaus von Heusinger, and Paul Portner, volume 2, 1341–1390. Berlin, Boston: de Gruyter. https://doi.org/10.1515/9783110255072.1341.

Brasoveanu, Adrian. 2013. Modified numerals as post-suppositions. *Journal of Semantics* 30:155–209. https://doi.org/10.1093/jos/ffs003.

Cresswell, Max J. 1976. The semantics of degree. In *Montague grammar*, ed. Barbara Partee, 261–292. New York: Academy Press. https://doi.org/10.1016/B978-0-12-545850-4.50015-7.

Heim, Irene. 1985. Notes on comparatives and related matters. https://semanticsarchive.net/Archive/zc0ZjY0M/Comparatives%2085.pdf, unpublished ms., University of Texas, Austin.

Hellan, Lars. 1981. *Towards an integrated analysis of comparatives.* Tübingen: Narr.

Kennedy, Christopher. 1999. *Projecting the adjective.* New York: Routledge. https://doi.org/10.4324/9780203055458.

Li, Ang. 2023. Comparing alternatives. Doctoral Dissertation, Rutgers University. https://doi.org/10.7282/t3-gkqz-6152.

Moore, Ramon E. 1979. *Methods and applications of interval analysis.* SIAM. https://epubs.siam.org/doi/10.1137/1.9781611970906.

Schwarzschild, Roger. 2008. The semantics of comparatives and other degree constructions. *Language and Linguistics Compass* 2:308–331. https://doi.org/10.1111/j.1749-818X.2007.00049.x.

Selected references II

Schwarzschild, Roger. 2010. Comparative markers and standard markers. In *Proceedings of the MIT workshop on comparatives*, volume 69, 87–105. https://web.mit.edu/schild/www/papers/public_html/CMsSMs2011.pdf.

Sheldon, Esther K. 1945. The rise of the incomplete comparative. *American Speech* 20:161–167. https://doi.org/10.2307/486719.

von Stechow, Arnim. 1984. Comparing semantic theories of comparison. *Journal of semantics* 3:1–77. https://doi.org/10.1093/jos/3.1-2.1.

Zhang, Linmin. 2020a. Comparatives bring a degree-based NPI licenser. In *Monotonicity in Logic and Language: Proceedings of TLLM 2020*, ed. Dag Westerståhl and Mingming Liu, 217–238. Springer. https://doi.org/10.1007/978-3-662-62843-0_12.

Zhang, Linmin. 2020b. Split semantics for non-monotonic quantifiers in *than*-clauses. In *Syntax and Semantics Vol. 42: Interactions of Degree and Quantification*, ed. Peter Hallman, 332–363. Brill. https://doi.org/10.1163/9789004431515_011.

Zhang, Linmin, and Jia Ling. 2021. The semantics of comparatives: A difference-based approach. *Journal of Semantics* 38:249–303. https://doi.org/10.1093/jos/ffab003.

Zhang, Linmin, and Florence Zhang. 2024. Comparative morphemes are additive particles: English *-er*/*more* vs. Chinese *gèng*. https://ling.auf.net/lingbuzz/008122, manuscript.