

Scalarity and additivity in natural language: (III) comparatives (cont.)

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Slides are available on lingbuzz:

<https://lingbuzz.net/lingbuzz/008302>

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Recapitulation

- Additivity is a phenomenon of QUD-based anaphoricity, indicating an extension of a previous salient answer in addressing the QUD.
- An additivity/increase-based view of *-er/more*
- A new difference-based view of comparatives

	The canonical view	The new difference-based view
Assumption	(Ordinal/interval) scales	Interval scales
Comparison	Inequality: $M_1 > M_2$	Subtraction: $M_1 - M_2 = D$
Representations of ⌘ operations on scalar values	Degree points ⌘ ordering between degree points	Intervals (i.e., set of degrees) ⌘ interval subtraction
The semantics of <i>-er/more</i>	Ordering: >	Additivity a default positive difference: $(0, +\infty)$

Today

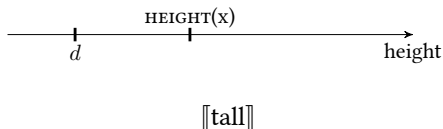
- Day 2 (yesterday) and Day 3 (today): Comparatives and *-er/more*
 - How an additivity-based perspective improve our understanding of scalarity-related phenomena?
 - What is additivity?
- Today
 - Formal implementation (see [Zhang and Ling 2021](#) and [Zhang and Zhang 2024](#))
 - Antonyms
 - Cross-linguistic phenomena
 - etc.

Outline

- 1 Formal analysis of comparatives
- 2 Comparatives in *-er*-less languages
(to be discussed on Day 5 along with Chinese cousins of English *even*)
- 3 Further discussion

The meaning of gradable adjectives (to be revisited)

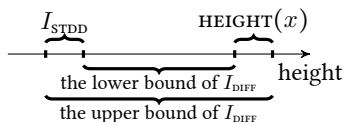
- Canonical view (See e.g., Cresswell 1976, Hellan 1981, von Stechow 1984, Heim 1985, Schwarzschild 2008, Beck 2011):



- (1) $[[\text{tall}]]_{\langle d, et \rangle} \stackrel{\text{def}}{=} \lambda d_d. \lambda x_e. \text{HEIGHT}_{\langle e, d \rangle}(x) \geq d$ (i.e., x is d -tall)
On the scale of height, the position of x reaches degree d .

- There are two pieces in this lexical entry
 - ▶ A **measure function** of type $\langle ed \rangle$: $\text{HEIGHT}_{\langle e, d \rangle}(x)$
 - ▶ Indicating the **direction (of comparison)**: $\geq d$ (cf. Kennedy 1999)

The meaning of gradable adjectives



[[tall]]

(1) $[[\text{tall}]]_{\langle d, et \rangle} \stackrel{\text{def}}{=} \lambda d_d. \lambda x_e. \text{HEIGHT}_{\langle e, d \rangle}(x) \geq d$ Canonical view
On the scale of height, the position of x reaches degree d .

(2) $[[\text{tall}]]_{\langle dt, et \rangle} \stackrel{\text{def}}{=} \lambda I_{\langle dt \rangle}. \lambda x_e. \text{HEIGHT}_{\langle e, dt \rangle}(x) \subseteq I$ (Zhang and Ling 2021)
On the scale of height, the measure of x falls at the position I .

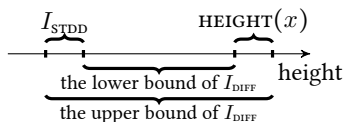
(3) $[[\text{tall}]] \stackrel{\text{def}}{=} \lambda I_{\text{DIFF}}. \lambda I_{\text{STDD}}. \lambda x. \underbrace{I_{\text{DIFF}} \subseteq [0, +\infty)}_{\text{non-negative presup.}}. \text{HEIGHT}(x) \subseteq \iota I [I - I_{\text{STDD}} = I_{\text{DIFF}}]$

(i.e., the height of x reaches the comparison standard, I_{STDD} .

\rightsquigarrow the difference between them, I_{DIFF} , is non-negative)

(Zhang and Zhang 2024)

The meaning of gradable adjectives



$$(2) \quad \llbracket \text{tall} \rrbracket_{\langle dt, et \rangle} \stackrel{\text{def}}{=} \lambda I_{\langle dt \rangle} \cdot \lambda x_e \cdot \text{HEIGHT}_{\langle e, dt \rangle}(x) \subseteq I \quad (\text{Zhang and Ling 2021})$$

$$(3) \quad \llbracket \text{tall} \rrbracket \stackrel{\text{def}}{=} \lambda I_{\text{DIFF}} \cdot \lambda I_{\text{STDD}} \cdot \lambda x \cdot \underbrace{I_{\text{DIFF}} \subseteq [0, +\infty)}_{\text{non-negative presup.}} \cdot \text{HEIGHT}(x) \subseteq \iota I [I - I_{\text{STDD}} = I_{\text{DIFF}}]$$

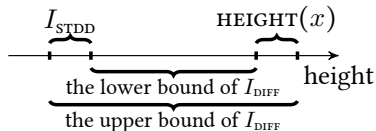
(i.e., the height of x reaches the comparison standard, I_{STDD} .

\rightsquigarrow the difference between them, I_{DIFF} , is non-negative) (Zhang and Zhang 2024)

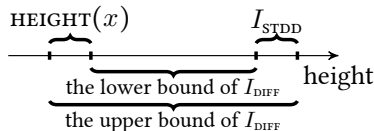
$$(4) \quad \text{A type shifter } \llbracket \text{COMPARE} \rrbracket_{\langle \langle dt, et \rangle, \langle dt, \langle dt, et \rangle \rangle \rangle} \quad (\text{see also Zhang and Ling 2021}) \\ \stackrel{\text{def}}{=} \lambda G_{\langle dt, et \rangle} \cdot \lambda I_{\text{DIFF}} \cdot \lambda I_{\text{STDD}} \cdot \lambda x_e \cdot G\text{-DIMENSION}(x) \subseteq \iota I [I - I_{\text{STDD}} = I_{\text{DIFF}}]$$

$$(5) \quad \llbracket \text{COMPARE tall} \rrbracket_{\langle dt, \langle dt, et \rangle \rangle} \\ = \lambda I_{\text{DIFF}} \cdot \lambda I_{\text{STDD}} \cdot \lambda x_e \cdot \text{HEIGHT}_{\langle e, dt \rangle}(x) \subseteq \iota I [I - I_{\text{STDD}} = I_{\text{DIFF}}]$$

The meaning of gradable adjectives (Zhang and Zhang 2024)



The meaning of *tall*



The meaning of *short*

$$(3) \quad \llbracket \text{tall} \rrbracket \stackrel{\text{def}}{=} \lambda I_{\text{DIFF}}. \lambda I_{\text{STDD}}. \lambda x. \underbrace{I_{\text{DIFF}} \subseteq [0, +\infty)}_{\text{non-negative presup.}}. \text{HEIGHT}(x) \subseteq \iota I [I - I_{\text{STDD}} = I_{\text{DIFF}}]$$

(i.e., the height of x **reaches** the comparison standard, I_{STDD} .)

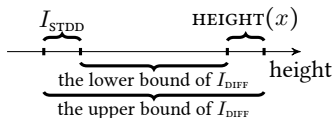
\rightsquigarrow the difference between them, I_{DIFF} , is **non-negative**)

$$(6) \quad \llbracket \text{short} \rrbracket \stackrel{\text{def}}{=} \lambda I_{\text{DIFF}}. \lambda I_{\text{STDD}}. \lambda x. \underbrace{I_{\text{DIFF}} \subseteq [0, +\infty)}_{\text{non-negative presup.}}. \text{HEIGHT}(x) \subseteq \iota I [I_{\text{STDD}} - I = I_{\text{DIFF}}]$$

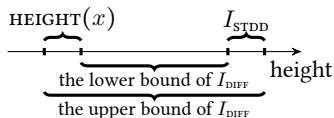
(i.e., the height of x **does not exceed** the comparison standard, I_{STDD} .)

\rightsquigarrow the difference between them, I_{DIFF} , is **non-negative**)

Major uses of gradable adjectives: Positive use



The meaning of *tall*

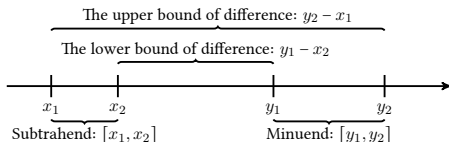


The meaning of *short*

- (7) [[Lucy is POS tall]]
 $\Leftrightarrow \text{HEIGHT}(\text{Lucy}) \subseteq \iota I [I - \underbrace{[d_{\text{POS}}^c, d_{\text{POS}}^c]}_{I_{\text{STDD}}} = \underbrace{[0, +\infty)}_{I_{\text{DIFF}}}]$
- $\Leftrightarrow \text{HEIGHT}(\text{Lucy}) \subseteq [d_{\text{POS}}^c, +\infty)$
- (8) [[Lucy is POS short]]
 $\Leftrightarrow \text{HEIGHT}(\text{Lucy}) \subseteq \iota I [\underbrace{[d_{\text{POS}}^c, d_{\text{POS}}^c]}_{I_{\text{STDD}}} - I = \underbrace{[0, +\infty)}_{I_{\text{DIFF}}}]$
- $\Leftrightarrow \text{HEIGHT}(\text{Lucy}) \subseteq (-\infty, d_{\text{POS}}^c]$

(See Zhang and Zhang 2024)

Subtraction between intervals



$$(9) \quad \underbrace{[y_1, y_2]}_{\text{minuend: position}} - \underbrace{[x_1, x_2]}_{\text{subtrahend: position}} = \underbrace{[y_1 - x_2, y_2 - x_1]}_{\text{difference: distance between positions}}$$

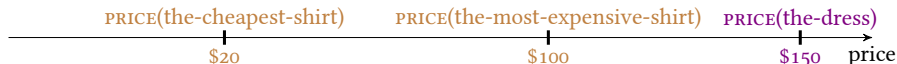
$$(10) \quad \text{Given the subtrahend position } [a, b] \text{ and the difference } [c, d], \\ \text{Minuend position} = [b + c, a + d] \quad (\text{defined when } b + c \leq a + d) \\ \text{HEIGHT(Lucy)} \subseteq \iota I [I - [d_{\text{pos}}^c, d_{\text{pos}}^c] = [0, +\infty)] \Leftrightarrow \text{HEIGHT(Lucy)} \subseteq [d_{\text{pos}}^c, +\infty)$$

$$(11) \quad \text{Given the minuend position } [a, b] \text{ and the difference } [c, d], \\ \text{Subtrahend position} = [b - d, a - c] \quad (\text{defined when } b - d \leq a - c) \\ \text{HEIGHT(Lucy)} \subseteq \iota I [[d_{\text{pos}}^c, d_{\text{pos}}^c] - I = [0, +\infty)] \Leftrightarrow \text{HEIGHT(Lucy)} \subseteq (-\infty, d_{\text{pos}}^c]$$

(See Moore 1979)

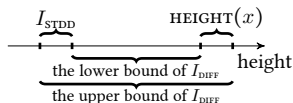
Interlude

- (10) Given the subtrahend position $[a, b]$ and the difference $[c, d]$,
Minuend position = $[b + c, a + d]$ (defined when $b + c \leq a + d$)

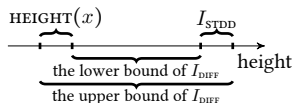


- (12) \llbracket The dress is up to \$60 more expensive than every shirt is \rrbracket
 $\Leftrightarrow \text{PRICE}(\text{the-dress}) \subseteq$
 $\iota I[I - [\text{price of the cheapest shirt}, \text{price of the most expensive shirt}] = (0, \$60)]$
The definedness condition for the minuend: the price difference between the most expensive and the cheapest shirt is no more than \$60. Under the above context: undefined!!
- (13) The giraffe is exactly 5 inches taller than every tree is.
 \rightsquigarrow We have the inference that every tree is of the same height. Why?
 $\text{HEIGHT}(\text{the-giraffe}) \subseteq \iota I[I - I_{\text{STDD}} = [5'', 5'']]$, thus the upper and lower bound of I_{STDD} needs to be the same to meet the definedness requirement

Major uses of gradable adjectives: Measurement sentence



The meaning of *tall*



The meaning of *short*

- (14) \llbracket Lucy is 6 feet tall \rrbracket ‘at least’ reading and ‘exactly’ reading
 (I_{STDD} is the zero point, the starting point for having a height)

a. $HEIGHT(Lucy) \subseteq \iota I [I - \underbrace{[0, 0]}_{I_{STDD}} = \underbrace{[6', +\infty)}_{I_{DIFF}}] \Leftrightarrow HEIGHT(Lucy) \subseteq [6', +\infty)$

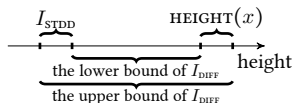
b. $HEIGHT(Lucy) \subseteq \iota I [I - \underbrace{[0, 0]}_{I_{STDD}} = \underbrace{[6', 6']}_{I_{DIFF}}] \Leftrightarrow HEIGHT(Lucy) \subseteq [6', 6']$

- (15) \llbracket Lucy is 5 feet short \rrbracket **Ungrammatical!** (\llbracket short \rrbracket has no starting point)

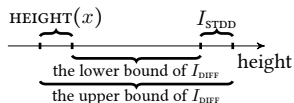
\llbracket short $\rrbracket \stackrel{\text{def}}{=} \lambda I_{DIFF}. \lambda I_{STDD}. \lambda x. \underbrace{I_{DIFF} \subseteq [0, +\infty)}_{\text{non-negative presup.}}. HGHT(x) \subseteq \iota I [I_{STDD} - I = I_{DIFF}]$

\rightsquigarrow If Lucy's height is at the position $[5', 5']$, compared with I_{STDD} that is $[0, 0]$, the non-negative presupposition of I_{DIFF} is violated.

Major uses of gradable adjectives: Degree question



The meaning of *tall*



The meaning of *short*

(16) [[How tall is Lucy]]

a. $\lambda I_{\text{DIFF}}.\text{HEIGHT}(\text{Lucy}) \subseteq \iota I [I - \underbrace{[0, 0]}_{I_{\text{STDD}}} = I_{\text{DIFF}}]$ No evaluativity!

\rightsquigarrow How far Lucy's height measurement is from / above the zero point

b. $\lambda I_{\text{DIFF}}.\text{HEIGHT}(\text{Lucy}) \subseteq \iota I [I - \underbrace{[d_{\text{POS}}^c, d_{\text{POS}}^c]}_{I_{\text{STDD}}} = I_{\text{DIFF}}]$ Evaluativity!

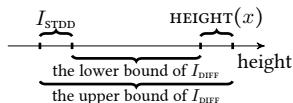
\rightsquigarrow How far Lucy's height is from / above the contextual threshold of being tall

(17) [[How short is Lucy]]

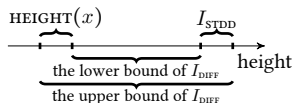
$\lambda I_{\text{DIFF}}.\text{HEIGHT}(\text{Lucy}) \subseteq \iota I [\underbrace{[d_{\text{POS}}^c, d_{\text{POS}}^c]}_{I_{\text{STDD}}} - I = I_{\text{DIFF}}]$ Evaluativity!

\rightsquigarrow How far Lucy's height is from / below the contextual threshold of being short

Major uses of gradable adjectives: Degree question



The meaning of *tall*



The meaning of *short*

$$(16a) \quad \llbracket \text{How tall is Lucy} \rrbracket = \lambda I_{\text{DIFF}}. \text{HEIGHT}(\text{Lucy}) \subseteq \iota I [I - \underbrace{[0, 0]}_{I_{\text{STDD}}} = I_{\text{DIFF}}]$$

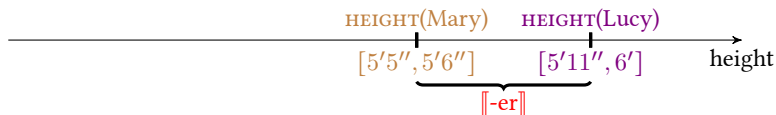
$$(17) \quad \llbracket \text{How short is Lucy} \rrbracket = \lambda I_{\text{DIFF}}. \text{HEIGHT}(\text{Lucy}) \subseteq \iota I [\underbrace{[d_{\text{POS}}^c, d_{\text{POS}}^c]}_{I_{\text{STDD}}} - I = I_{\text{DIFF}}]$$

(18) An answerhood operator Ans_{DIFF} (which returns a maximally informative true answer) is defined for a set of intervals p s.t.
 $\exists I [p(I) \wedge \forall I' [[p(I') \wedge I' \neq I] \rightarrow I \subset I']]$ (this maximally informative I exists)
 When defined, $\text{Ans}_{\text{DIFF}} \stackrel{\text{def}}{=} \lambda p_{\langle dt, t \rangle}. \iota I [p(I) \wedge \forall I' [[p(I') \wedge I' \neq I] \rightarrow I \subset I']]$

$$(19) \quad \text{Position-M} \stackrel{\text{def}}{=} \lambda I_{\text{DIFF}}. \iota I [I - I_{\text{STDD}} = I_{\text{DIFF}}] \quad \text{Minuend position}$$

$$(20) \quad \text{Position-S} \stackrel{\text{def}}{=} \lambda I_{\text{DIFF}}. \iota I [I_{\text{STDD}} - I = I_{\text{DIFF}}] \quad \text{Subtrahend position}$$

Major uses of gradable adjectives: Clausal comparative



(21) $\llbracket \text{Lucy is taller than Mary is tall} \rrbracket$
 $\text{HEIGHT}(\text{Lucy}) \subseteq_{\text{I}} \text{I} \left[\underbrace{\text{I} - \llbracket \text{than Mary is tall} \rrbracket}_{I_{\text{STDD}}} \right] = \underbrace{\llbracket \text{er} \rrbracket}_{I_{\text{DIFF}}}$

- a. $\llbracket \text{than Mary is tall} \rrbracket = \mathbf{Position-M}[\mathbf{Ans}_{\text{DIFF}}[\text{how tall Mary is}]]$
 $= \text{HEIGHT}(\text{Mary}) = [5'5'', 5'6'']$ under the above context
- b. $\llbracket \text{er} \rrbracket \stackrel{\text{def}}{=} (0, +\infty) \rightsquigarrow$ extending the value $\llbracket \text{than Mary is tall} \rrbracket$ in addressing the Current Question ‘how tall Lucy is’
- c. $\text{HT}(\text{Lucy}) \subseteq_{\text{I}} \text{I} \left[\text{I} - [5'5'', 5'6''] = (0, +\infty) \right] \Leftrightarrow \text{HT}(\text{Lucy}) \subseteq (5'6'', +\infty)$

- Without **Position-M**: comparing 2 distances away from a certain reference position: Lucy is farther away from the reference than Mary is
- With **Position-M**: comparing 2 positions along a height scale: Lucy’s position involves an increase compared to Mary’s position

Comparatives with *than*-clause internal quantifiers

$$(22) \quad \llbracket \text{The dress is more expensive than every shirt is expensive} \rrbracket$$

$$\text{PRICE}(\text{the-dress}) \subseteq \underbrace{\iota I [I - \llbracket \text{than every shirt is expensive} \rrbracket]}_{I_{\text{STDD}}} = \underbrace{\llbracket \text{more} \rrbracket}_{I_{\text{DIFF}}}$$

- a. $\llbracket \text{than every shirt is expensive} \rrbracket =$
Position-M $[\text{Ans}_{\text{DIFF}} \llbracket \text{how expensive every shirt is} \rrbracket] =$
Position-M $[\text{Ans}_{\text{DIFF}} [\lambda I_{\text{DIFF}}. \forall x [\text{shirt}(x) \rightarrow \text{PRICE}(x) \subseteq$
 $\iota I [I - I_{\text{STDD}} = I_{\text{DIFF}}]]]]$, which is
 [price of the cheapest shirt, price of the most expensive shirt]
- b. $\llbracket \text{more} \rrbracket \stackrel{\text{def}}{=} (0, +\infty)$
- c. $\text{PRICE}(\text{the-dress}) \subseteq \iota I [I -$
 [price of the cheapest shirt, price of the most expensive shirt] =
 $(0, +\infty)]$
 $\Leftrightarrow \text{PRICE}(\text{the-dress}) \subseteq (\text{price of the most expensive shirt}, +\infty)$

Comparatives with *than*-clause internal quantifiers and numerical differentials

$$(23) \quad \llbracket \text{The dress is up to \$60 more expensive than every shirt is expensive} \rrbracket$$

$$\text{PRICE}(\text{the-dress}) \subseteq \iota I [I - \underbrace{\llbracket \text{than every shirt is expensive} \rrbracket}_{I_{\text{STDD}}} = \underbrace{\llbracket \text{up to \$60 more} \rrbracket}_{I_{\text{DIFF}}}]$$

a. $\llbracket \text{than every shirt is expensive} \rrbracket =$
Position-M[**Ans**_{DIFF} [**how expensive every shirt is**]] =
Position-M[**Ans**_{DIFF} [$\lambda I_{\text{DIFF}}. \forall x [\text{shirt}(x) \rightarrow \text{PRICE}(x) \subseteq \iota I [I - I_{\text{STDD}} = I_{\text{DIFF}}]]]]$],
 which is [price of the cheapest shirt, price of the most expensive shirt]

b. $\llbracket \text{up to \$60 more} \rrbracket = (0, +\infty) \cap (-\infty, \$60] = (0, \$60]$

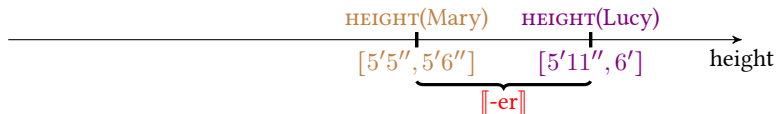
c. $\text{PRICE}(\text{the-dress}) \subseteq \iota I [I -$
 [price of the cheapest shirt, price of the most expensive shirt] = $(0, \$60]$]

\Leftrightarrow

$\text{PRICE}(\text{the-dress}) \subseteq (\text{price of the most expensive shirt, price of the cheapest shirt} + \$60]$

(defined when the most expensive shirt is not more than \$60 more expensive than the cheapest shirt is)

Less



(19) $\llbracket \text{Lucy is taller than Mary is tall} \rrbracket$
 $\text{HEIGHT(Lucy)} \subseteq_{\iota I} [I - \underbrace{\llbracket \text{than Mary is tall} \rrbracket}_{I_{\text{STDD}}}] = \underbrace{\llbracket \text{er} \rrbracket}_{I_{\text{DIFF}}}]$

(24) $\llbracket \text{Mary is less tall than Lucy is tall} \rrbracket$
 $\text{HEIGHT(Mary)} \subseteq_{\iota I} [I - \underbrace{\llbracket \text{than Lucy is tall} \rrbracket}_{I_{\text{STDD}}}] = \underbrace{\llbracket \text{less} \rrbracket}_{I_{\text{DIFF}}}]$

- (25) a. $\llbracket \text{er} \rrbracket \stackrel{\text{def}}{=} (0, +\infty)$ an increase based on a contextual salient base
 b. $\llbracket \text{less} \rrbracket \stackrel{\text{def}}{=} \text{LITTLE}[\llbracket \text{er} \rrbracket] = [0, 0] - (0, +\infty) = (-\infty, 0)$
 a negative increase: a decrease (to be revisited)

Discussion: What is a negative increase

- Additivity is a phenomenon of QUD-based anaphoricity, indicating an extension of a previous salient answer in addressing the QUD.
 - In the domain of scalar values, there is not necessarily entailment between a lower and a higher value along a scale.
- (26)
- a. Lucy is exactly 6 feet tall \neq Lucy is between 5'5 and 5'8" tall
 - b. Lucy is between 5'5 and 5'8" tall \neq Lucy is exactly 6 feet tall
- Thus along a scale, both $(0, +\infty)$ (which means moving a distance towards one direction of the scale) and $(-\infty, 0)$ (which means moving a distance towards the other direction of the scale) can be considered extensions of a previous salient answer in addressing the Current Question (i.e., about the measurement of the subject of a comparative).
 - However ...

Discussion: Not to negate the increase, but to change the comparison direction

$$(3) \quad \llbracket \text{tall} \rrbracket \stackrel{\text{def}}{=} \lambda I_{\text{DIFF}} \cdot \lambda I_{\text{STDD}} \cdot \lambda x \cdot \underbrace{I_{\text{DIFF}} \subseteq [0, +\infty)}_{\text{non-negative presup.}} \cdot \text{HEIGHT}(x) \subseteq \iota I [I - I_{\text{STDD}} = I_{\text{DIFF}}]$$

$$(6) \quad \llbracket \text{short} \rrbracket \stackrel{\text{def}}{=} \lambda I_{\text{DIFF}} \cdot \lambda I_{\text{STDD}} \cdot \lambda x \cdot \underbrace{I_{\text{DIFF}} \subseteq [0, +\infty)}_{\text{non-negative presup.}} \cdot \text{HGHT}(x) \subseteq \iota I [I_{\text{STDD}} - I = I_{\text{DIFF}}]$$

- Analyzing *less* as $(0, +\infty)$ is at odds with the non-negative presupposition of gradable adjectives.

- Remedy: decompose $\llbracket \text{less} \rrbracket$ into an operator **OPPOSITE** and $\llbracket \text{er} \rrbracket$, then **OPPOSITE** changes the direction of comparison, not the polarity of I_{DIFF}

$$(27) \quad \text{OPPOSITE}_{\langle \langle dt, \langle dt, et \rangle \rangle, \langle dt, \langle dt, et \rangle \rangle \rangle} \stackrel{\text{def}}{=} \lambda G_{\langle dt, \langle dt, et \rangle \rangle} \cdot \lambda I_{\text{DIFF}} \cdot \lambda I_{\text{STDD}} \cdot \lambda x \cdot G\text{-DIMENSION}(x) \subseteq \iota I [I - I_{\text{STDD}} = [0, 0] - I_{\text{DIFF}}]$$

$$(28) \quad \begin{array}{ll} \text{a.} & \text{OPPOSITE} \llbracket \text{tall} \rrbracket = \llbracket \text{short} \rrbracket \quad \rightsquigarrow \llbracket \text{less tall} \rrbracket = \llbracket \text{shorter} \rrbracket \\ \text{b.} & \text{OPPOSITE} \llbracket \text{short} \rrbracket = \llbracket \text{tall} \rrbracket \quad \rightsquigarrow \llbracket \text{less short} \rrbracket = \llbracket \text{taller} \rrbracket \end{array}$$

Interim summary

- We have developed a new analysis of gradable adjectives and comparatives based on
 - considering *-er/more* an additive particle like *another*
 - interval subtraction

The new difference-based view	
Assumption	Interval scales
Comparison	Subtraction: $M_1 - M_2 = D$
Representations of ⊗ operations on scalar values	Intervals (i.e., set of degrees) ⊗ interval subtraction
The semantics of <i>-er/more</i>	Additivity a default positive difference: $(0, +\infty)$

$$(3) \quad \llbracket \text{tall} \rrbracket \stackrel{\text{def}}{=} \lambda I_{\text{DIFF}}. \lambda I_{\text{STDD}}. \lambda x. \underbrace{I_{\text{DIFF}} \subseteq [0, +\infty)}_{\text{non-negative presup.}}. \text{HEIGHT}(x) \subseteq \iota I [I - I_{\text{STDD}} = I_{\text{DIFF}}]$$

$$(6) \quad \llbracket \text{short} \rrbracket \stackrel{\text{def}}{=} \lambda I_{\text{DIFF}}. \lambda I_{\text{STDD}}. \lambda x. \underbrace{I_{\text{DIFF}} \subseteq [0, +\infty)}_{\text{non-negative presup.}}. \text{HGHT}(x) \subseteq \iota I [I_{\text{STDD}} - I = I_{\text{DIFF}}]$$

Outline

- 1 Formal analysis of comparatives
- 2 Comparatives in *-er*-less languages
(to be discussed on Day 5 along with Chinese cousins of English *even*)
- 3 Further discussion

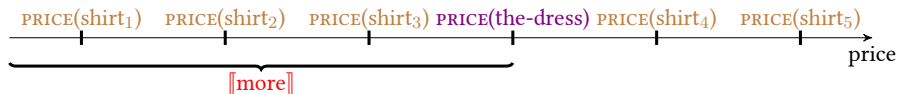
Outline

- 1 Formal analysis of comparatives
- 2 Comparatives in *-er-less* languages
(to be discussed on Day 5 along with Chinese cousins of English *even*)
- 3 Further discussion

How the current additivity/difference-based analysis of comparatives helps solve more puzzles or shed some light on them

- Comparatives with *than*-clause internal modified numerals
- Incomplete comparatives
- Comparison between differences that result from comparisons
- ...(NPI licensing of the *than*-clause, see [Zhang 2020a](#))

Comparatives with *than*-clause internal modified numerals



[[The dress is **more** expensive than exactly 3 shirts are expensive]]

(29) [[The dress is **more** expensive than exactly 3 shirts are expensive]]

$$\text{PRICE(the-dress)} \subseteq_{\iota} I [I - \underbrace{\text{[[than exactly 3 shirts are expensive]]}}_{I_{\text{STDD}}} = \underbrace{\text{[[more]]}}_{I_{\text{DIFF}}}]$$

- Zhang (2020b): A post-suppositional analysis à la Brasoveanu (2013)
 - ▶ The information of the minuend **PRICE(the-dress)** and the differential **[[more]]** is made use of to compute the subtrahend I_{STDD}
 - ▶ The cardinality of the maximal sum of shirts s.t., their price falls within I_{STDD} (computed from the step above) is checked (whether it's equal to 3) as post-suppositional requirement.

(See also Schwarzschild 2008)

Incomplete comparatives

- When there is an overt *than*-expression, a numerical measurement can play the role of comparison standard:

- (30) a. Lucy is taller than 6 feet. $\text{HEIGHT}(\text{Lucy}) \subseteq (6', +\infty)$
b. Mary is not 6^u feet tall. Lucy is taller than that_u.
 $\text{HEIGHT}(\text{Lucy}) \subseteq (6', +\infty)$

- However, in **incomplete comparatives** (which do not have an overt *than*-expression), it seems that numerical measurements cannot play the role of comparison standard (see Sheldon 1945, Schwarzschild 2010, Li 2023):

- (31) a. Mary is not 6 feet tall. Lucy is taller.
 $\sim \text{HEIGHT}(\text{Lucy}) \subseteq \iota I [I - \text{HEIGHT}(\text{Mary}) = (0, +\infty)]$
 $\not\sim \text{HEIGHT}(\text{Lucy}) \subseteq (6', +\infty)$
b. Mary is not POS tall. Lucy is taller.
 $\sim \text{HEIGHT}(\text{Lucy}) \subseteq \iota I [I - \text{HEIGHT}(\text{Mary}) = (0, +\infty)]$
 $\not\sim \text{HEIGHT}(\text{Lucy}) \subseteq (d_{\text{POS}}^c, +\infty)$

Incomplete comparatives (Zhang and Zhang 2024)

- Comparative morpheme *-er/more*, as an additive particle, extends a previous salient answer in addressing the Current Question.
 - A previous salient answer: a **position** along a relevant scale (here a height scale)

- (44)
- a. Mary is not 6 feet tall. Lucy is taller.
 $\rightsquigarrow \text{HEIGHT}(\text{Lucy}) \subseteq \iota I [I - \text{HEIGHT}(\text{Mary}) = (0, +\infty)]$
 $\not\rightsquigarrow \text{HEIGHT}(\text{Lucy}) \subseteq (6', +\infty)$
- b. Mary is not POS tall. Lucy is taller.
 $\rightsquigarrow \text{HEIGHT}(\text{Lucy}) \subseteq \iota I [I - \text{HEIGHT}(\text{Mary}) = (0, +\infty)]$
 $\not\rightsquigarrow \text{HEIGHT}(\text{Lucy}) \subseteq (d_{\text{POS}}^c, +\infty)$

- Under the current analysis, in a measurement sentence, the numerical measurement plays the role of I_{DIFF} , meaning the distance away from the zero point. Thus this numerical measurement cannot be a salient position for the use of *-er/more*.
- Then the contextual threshold in the positive use is probably never a salient value in a discourse. Thus it cannot be the antecedent for

Comparison between differences

$$(19) \quad \llbracket \text{Lucy is taller than Mary is tall} \rrbracket \\ \text{HEIGHT}(\text{Lucy}) \subseteq {}_l I [I - \underbrace{\llbracket \text{than Mary is tall} \rrbracket}_{I_{\text{STDD}}} = \underbrace{\llbracket \text{er} \rrbracket}_{I_{\text{DIFF}}}]$$

$$\llbracket \text{than Mary is tall} \rrbracket = \text{Position-M}[\text{Ans}_{\text{DIFF}}[\llbracket \text{how tall Mary is} \rrbracket]]$$

(32) Mona is more happy than Jude is sad.

(Kennedy 1999, Zhang and Ling 2021)

a. **Comparison 1** – along a scale of happiness:

Mona's happiness vs. the threshold of happiness

\leadsto Mona is happy

b. **Comparison 2** – along a scale of sadness:

Jude's sadness vs. the threshold of sadness

\leadsto Jude is sad

c. **Comparison 3** – along a scale of deviation / difference size

difference from Comparison 1 vs. difference from Comparison 2

- The comparison between differences should be derived without the operator **Position-M**.

Comparison between differences

$$(45) \quad \llbracket \text{Mona is much+er happy than Jude is sad} \rrbracket$$

$$\text{HAPPINESS}(\text{Lucy}) \subseteq$$

$$\iota I [I - [d_{\text{POS-HAPPY}}^c, d_{\text{POS-HAPPY}}^c] = \iota I [I - \underbrace{\llbracket \text{than Jude is sad} \rrbracket}_{I_{\text{STDD}}} = \underbrace{\llbracket \text{er} \rrbracket}_{I_{\text{DIFF}}}]]$$

Here $\llbracket \text{than Jude is sad} \rrbracket = \text{Ans}_{\text{DIFF}} \llbracket \text{how sad Jude is} \rrbracket$

$$= \text{Ans}_{\text{DIFF}} [\lambda I_{\text{DIFF}} . \text{SADNESS}(\text{Jude}) \subseteq \iota I [I - [d_{\text{POS-SAD}}^c, d_{\text{POS-SAD}}^c] = I_{\text{DIFF}}]]$$

Today's take-home messages

- Day 2 (yesterday) and Day 3 (today): Comparatives and *-er/more*
 - How an additivity-based perspective improve our understanding of scalarity-related phenomena?
 - What is additivity?
- Today
 - Formal analysis of gradable adjectives, including
 - ★ antonyms
 - ★ *-er/more*
 - ★ *less*
 - ★ various uses of gradable adjectives
 - ★ *than*-clause internal quantifiers
 - ★ numerical differentials
 - Cross-linguistic phenomena: languages without morphemes like *-er/more*
 - etc.

Tomorrow

- Day 1: Basics of scales and degrees; how they are relevant to natural language
 - What are scales? What are their formal properties? What operators do they support?
- Day 2 and Day 3: Comparatives and *-er/more*
 - How an additivity-based perspective improve our understanding of scalarity-related phenomena?
 - What is additivity?
- Day 4 and Day 5: *Even* and its cross-linguistic siblings
 - How a scalarity-based perspective improve our understanding of additivity-related phenomena?

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