Truthmaker semantics and natural language semantics*

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Abstract

Truthmaker semantics is a non-classical logical framework that has recently garnered significant interest in philosophy, logic, and natural language semantics. It redefines the propositional connectives and gives rise to more fine-grained entailment relations than classical logic. In its model theory, truth is not determined with respect to possible worlds, but with respect to truthmakers, such as states or events. Unlike possible worlds, these truthmakers may be partial; they may be either coherent or incoherent; and they are understood to be exactly or wholly relevant to the truth of the sentences they verify. Truthmaker semantics generalizes collective, fusion-based theories of conjunction; alternative-based theories of disjunction; and nonstandard negation semantics. This article provides a gentle introduction to truthmaker semantics aimed at linguists; describes applications to various natural language phenomena such as imperatives, ignorance implicatures, and negative events; and discusses its similarities and differences to related frameworks such as event semantics, situation semantics, alternative semantics, and inquisitive semantics.

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1 Introduction

Traditional semantic approaches often rely on possible worlds to explain meaning. For example, the sentence *It is raining* denotes the set of possible worlds where it is raining and is true if, and only if, the actual world is among them. Entailment is modeled as truth preservation at all possible worlds. For example, *It is raining and snowing* entails *It is raining* because in all the worlds where the first sentence is true, so is the second. But this view on entailment can be overly coarse-grained: all necessary sentences (such as *It is raining or not raining*, *It is snowing or not snowing*, etc.) have the same denotation and entail every sentence whatsoever; and any sentence entails any disjunction of which it is a disjunct (so that *It is raining* entails *It is raining or snowing*).

Truthmaker semantics offers a more nuanced perspective and a way out of these problems. It is a non-classical logical framework that redefines the semantics of conjunction, disjunction, and negation. Instead of evaluating sentences or propositions as true at certain possible worlds, truthmaker semantics takes their truth to depend on certain truthmakers—roughly, states or events whose occurrence is wholly relevant to the truth of the sentence. Truthmaker semantics is formally similar to situation semantics, inquisitive semantics, and other alternatives to possible-world semantics which have gained considerable attention in the linguistic literature. Compared with classical propositional logic and possible-world semantics, truthmaker semantics harmonizes better with certain intuitions about the meanings of connectives and provides a framework for formulating finer-grained theories of entailment and of propositions. If Davidsonian events are understood as truthmakers, it can also be seen as providing a way to provide a logic for event semantics within which propositional connectives can be given appropriate denotations that operate on sets of events.

This article provides a gentle introduction to truthmaker semantics for the working natural language semanticist, explains its relevance to linguistics, and gives an overview of some relevant recent research along with pointers to the literature. It is meant to complement Fine (2017c), which is primarily written for philosophers. Topics that are prominently discussed in Fine (2017c) but less so in this article include subject matter (known to linguists as *topics*), partial content, and scalar implicature. Topics that this article covers in more detail include the relation of truthmaker semantics to frameworks such as event semantics (with a focus on negative events and negative perception reports), alternative and inquisitive semantics, and situation semantics. I will only be touching relatively lightly on many issues pertaining to philosophy and logic outside of their applications to linguistics; the interested reader may consult Jago (2024) for an encyclopedia entry that goes into more depth on these areas. On the logical side, van Benthem (2019) provides a useful translation from truthmaker semantics into modal information logic (van Benthem, 1996).

The structure of this paper is as follows. Section 2 introduces the basic ideas behind truthmaker semantics, including the notions of truthmakers as exact verifiers and the semantics for the propositional connectives. Section 3 discusses how truthmaker semantics gives rise to more fine-grained entailment relations than classical logic. Section 4 compares truthmaker semantics with situation semantics, and Section 5 with alternative and inquisitive semantics. The following three sections describe case studies from the recent literature. Section 6 sketches a truthmaker-based approach to negative events, focusing primarily on negative perception reports but also discussing negative quantifiers and causation. Section 7 briefly explores extensions of truthmaker semantics beyond propositional logic, touching on predicate logic, modal logic, and other areas. Section 8 concludes.

2 Varieties of truthmaker semantics

At its core, truthmaker semantics is a family of related approaches to the semantics of propositional connectives built around the notion of *exact verification*. Exact verification means that sentences (i.e., well-formed formulas), the *truthbearers*, are evaluated at points, the *truthmakers*, that are wholly relevant to the sentences to which they assign True. These points are ordered by mereological parthood (written $\sqsubseteq$), and the semantics for conjunction is formulated in terms of pointwise mereological fusion (written $\sqcup$) or some equivalent operation. Intuitively, the fusion (also called sum) of some truthmakers is that truthmaker which you get when you put them together without adding anything extra; the fusion of any truthmaker with itself is that truthmaker (for background, see Champollion and Krifka 2016 or Cotnoir and Varzi 2021). "Pointwise", in this context, means that each conjunct denotes a set of truthmakers and the conjunction denotes the set of all fusions of a truthmaker for the first conjunct with a truthmaker for the second. For example, the pointwise fusion of the sets $\{s_1, s_2\}$ and $\{s_2, s_3, s_4\}$ is the set $\{s_1 \sqcup s_2, s_1 \sqcup s_3, s_1 \sqcup s_4, s_2, s_2 \sqcup s_3, s_2 \sqcup s_4\}$. Understood in this way, truthmaker semantics was independently discovered by scholars in a variety of disciplines, beginning with van Fraassen (1969). In its current incarnation, truthmaker semantics figures prominently in recent work by Fine (e.g. Fine, 2012a,b, 2014a,b, 2016, 2017a,b,c, 2018a,b,c, 2021) and Yablo (Yablo, 2014, 2016; Rothschild and Yablo, 2023); see also the recent collection of papers on Fine's work in Faroldi and Putte (2023). There are too many publications in philosophy and logic to review here; a comprehensive bibliography is maintained by Johannes Korbmacher and Mark Jago at `https://truthmakersemantics.github.`

io/. Within natural language semantics, approaches based on truthmaker semantics have been adopted or discussed only occasionally (e.g. Fernando, 2015; Moltmann, 2007, 2021; van Rooij, 2000, 2017).

What makes truthmaker semantics challenging to describe is that the framework provides many design choices without an established standard, and that much of it is open for interpretation. For example, there is variation in the semantics of propositional connectives or in the underlying notion of propositional content. In this section I go through conjunction, disjunction, and negation, and explain the basic features of truthmaker semantics as well as some of the relevant points of variation. More complete formal descriptions of specific versions of truthmaker semantics are found in the survey article Fine (2017c) and in Fine (2017a,b) or Champollion and Bernard (2024). The latter paper explicitly aims at being accessible to linguists.

2.1 Basic ideas

Recall that model-theoretic logics based on classical possible-world semantics map each propositional letter P to its denotation or intension, a set which is understood intuitively as containing just those possible worlds at which P is true. A set of recursive Tarski-style semantic clauses then extends this mapping from propositional letters to arbitrary sentences. Formally, each model supplies an interpretation I that maps each propositional letter to a subset $I(P)$ of the set W of possible worlds. If one wishes to define truth in the model, one member of W is designated as the actual world; a sentence is considered true just in case its denotation contains this world. For example, the sentence *It is raining* denotes the set of all possible worlds in which it is raining, and it is true just in case the actual world is one of them. The Tarski-style clauses are formulated in terms of a $[\![\cdot]\!]$ function which is the same across all models except in the case of propositional letters, where it is defined in terms of I. This function encodes the meaning of conjunction, disjunction, and negation in terms of intersection, union, and complement:

(1) **Possible-world semantics**

a. $[\![P]\!] = I(P)$ if P is a propositional letter
b. $[\![\phi \wedge \psi]\!] = [\![\phi]\!] \cap [\![\psi]\!]$
c. $[\![\phi \vee \psi]\!] = [\![\phi]\!] \cup [\![\psi]\!]$
d. $[\![\neg\phi]\!] = W \setminus [\![\phi]\!]$

In model-theoretic treatments of truthmaker semantics, a model likewise maps each propositional letter P to a set $I(P)$; as in the classical treatment, a set of recursive Tarski-style semantic clauses then extends this mapping to arbitrary sentences.

The sets $I(P)$ are subsets of a set S, the "state space", equipped with an algebraic structure that provides a mereological fusion operation $\sqcup$ and a parthood relation $\sqsubseteq$. Following Fine (2017c), I will refer to the members of S interchangeably as "truthmakers" or "states". These are mere labels and not meant to imply anything about the nature of these entities. As we will see, truthmakers can be understood in various ways; but to get an intuitive grip on them, it is helpful to think of P initially as standing for an atomic sentence like *It is raining* or *It is sunny* and to think of its truthmakers, the members of $I(P)$, as raining events or sunny states in the sense of Davidson (1967) and Parsons (1990) (but see Moltmann 2021), and of S as the set of all eventualities including their parts and fusions. Davidsonian event semantics associates typical sentences with more than one event, i.e., more than one truthmaker; and different varieties of truthmaker semantics emerge depending on whether I is constrained so as to map each atomic sentence P to a set containing exactly one truthmaker, or to a set containing at least one truthmaker, or left unconstrained so that some propositional letters may lack truthmakers. The assumption that each propositional letter has exactly one truthmaker is in effect made in van Fraassen (1969), and in any system where propositional letters have themselves as truthmakers or are isomorphic with them; but it is absent from most contemporary systems (e.g. Fine 2017c).

The algebraic structure of a state space will be in the general vicinity of complete lattices or semilattices. Linguists who are familiar with mereology (for an overview see Champollion and Krifka 2016) may find it useful to think of classical extensional mereology, whose models are complete Boolean algebras with the bottom element removed. Once the bottom element is put back in, complete Boolean algebras are special cases of complete lattices.[1] To the mereological structure of truthmaker semantics, one can add modal structure by defining a nonempty subset $S^\diamond \subseteq S$ to be the set of possible states (Fine, 2017a). It is then assumed that any part of any possible state is itself possible. As we follow the partial order $\sqsubseteq$ upwards within $S^\diamond$, going from ordinary possible states to larger and larger states, at some point we may find maximal possible states (which can also be characterized as those possible states s that contain every state whose fusion with s is possible); but we may also encounter ever larger possible states that are bounded only by impossible states. Maximal possible states correspond to possible worlds and provide a natural connection to possible-world semantics, but they are not integral to truthmaker semantics.

[1] Complete Boolean algebras are precisely those complete lattices which are complemented and distributive. Champollion and Bernard (2024) argue that distributivity is a desirable property in state spaces for truthmaker semantics, and Fine (2017a) frequently assumes distributivity. I am not aware of any discussions of complementedness in the literature on truthmaker semantics.

Any truthmaker for a sentence should be wholly or exactly relevant to the truth or falsity of that sentence and therefore devoid of irrelevant material; this is why one speaks of truthmaking as an exact rather than inexact relation between truthmakers and sentences. For example, if we identify the truthmakers for *It is raining* with raining events as above, the fusion of a raining event with a snowing event is not a truthmaker for *It is raining*; though as noted below, it will be a truthmaker for the conjunction *It is raining and snowing.* One may call this fusion an "inexact" truthmaker for *it is raining* and an "exact" truthmaker for *It is raining and snowing.* In general, an inexact truthmaker for a sentence is any truthmaker among whose parts is an exact truthmaker. Exact truthmakers relate to inexact truthmakers as truthmaker semantics relates to situation semantics; see Section 4 for a discussion. Most of the time, a possible world is not an exact truthmaker but at best an inexact truthmaker for ordinary sentences because it contains extraneous material that does not bear on them. In what follows, when I simply speak of "truthmakers" I will always refer to exact rather than inexact truthmakers.

Defining truth in a model is useful for linguistic applications and can be done in different ways. One can define truth at a world in terms of being made true by part of that world. This strategy can also be used to define absolute truth by designating some world to be the actual world. Alternatively, one can regard the members of a designated set of possible states that is closed under fusion and parthood to be actual (i.e., to obtain or occur). Their fusion may be thought of as corresponding to the actual world. One then considers a sentence to be true just in case it has at least one truthmaker that is actual.

If one assumes that all possible truthmakers are parts of worlds, a set-of-truthmakers proposition can be turned into a set-of-worlds proposition by replacing every one of its truthmakers by the set of all worlds of which that truthmaker is a part. But this mapping is not one-to-one; for example, truthmaker semantics will typically assign different sets of truthmakers to the necessary sentence *It is raining or not raining* and to the likewise necessary sentence *It is snowing or not snowing* even though the sentences are true in the same possible worlds (namely in all of them). This illustrates that truthmaker semantics is hyperintensional, as it can distinguish between sentences with the same intension (Fox and Lappin, 2005).

Though I have identified truthmakers with events and truthbearers with ordinary declarative sentences, anything that can be fused together and have parts can in principle serve as a truthmaker; and anything that can have truth or satisfaction conditions can in principle serve as a truthbearer. Thus, commands can be truthbearers made true by courses of action (Fine, 2018a,b); the products of claims and desires ("attitudinal objects") can be truthbearers made true by situations, actions, etc. (Moltmann, 2020, 2021, in prep); and noun phrases can be truthbearers made true by ordinary, pluralized, and negative individuals (Fine, 2017a; Bledin, 2024) or variable objects (Moltmann, 2020). Here, though, I focus on truthmaker semantics for a simple propositional language with conjunction, disjunction, and negation. These operations are central to logical systems and are essential for understanding the core principles of truthmaker semantics.

2.2 Conjunction

Natural language semantics analyzes conjunction in terms of either intersection or collective formation (Champollion, 2016, 2019). The intersective approach is adopted in possible worlds semantics, but also within the treatment of conjunction of generalized quantifiers (Barwise and Cooper, 1981; Winter, 2001); the collective approach figures prominently within the treatment of conjunction of proper names and represents a conjunction as denoting the collective individual that results from mereologically fusing the two referents (Krifka, 1990; Heycock and Zamparelli, 2005; Schmitt, 2019, 2021). Truthmaker conjunction is collective, and expresses the pointwise mereological fusion of two sets of truthmakers:

(2) **Semantic Rule for Truthmaker Conjunction**
$[\![\phi \wedge \psi]\!] = \{s_1 \sqcup s_2 \mid s_1 \in [\![\phi]\!], s_2 \in [\![\psi]\!]\}$
(The conjunction of ϕ and ψ denotes the set of all fusions of a truthmaker for ϕ with a truthmaker for ψ.)

To illustrate, assume that P has exactly one truthmaker, symbolized as p, and similarly for Q and q. Then $[\![P \wedge Q]\!] = \{p \sqcup q\}$ and $[\![P \wedge P]\!] = \{p \sqcup p\} = \{p\} = [\![P]\!]$. In models where P has more than one truthmaker, we do not in general have $[\![P \wedge P]\!] = [\![P]\!]$ unless some additional assumptions are made. For example, Fine (2017a) ensures that $[\![P \wedge P]\!] = [\![P]\!]$ by ensuring that all propositions are closed under fusion (see below).

Some versions of truthmaker semantics allow for impossible truthmakers; in these versions, if ϕ and ψ are jointly impossible, then even if they are possible individually, $\phi \wedge \psi$ will have only impossible truthmakers. Other versions of truthmaker semantics disallow impossible truthmakers and restrict the semantic rule above accordingly.

2.3 Disjunction

Another point of variation within truthmaker semantics is the disjunction clause, defined either as union, or as union with additional collective formation. Fine (2017c) calls these two possibilities non-inclusive and inclusive semantics:

(3) **Semantic rule for Truthmaker Disjunction (non-inclusive version)**
$[\![\phi \vee \psi]\!] = [\![\phi]\!] \cup [\![\psi]\!]$
(The disjunction of ϕ and ψ denotes the set that contains any truthmaker for ϕ or for ψ.)

(4) **Semantic rule for Truthmaker Disjunction (inclusive version)**
$[\![\phi \vee \psi]\!] = [\![\phi]\!] \cup [\![\psi]\!] \cup [\![\phi \wedge \psi]\!]$
(The disjunction of ϕ and ψ denotes the set that contains any truthmaker for ϕ, for ψ, or for their conjunction.)

If each of the disjuncts is closed under fusion, the inclusive semantics preserves that property for the disjunction. While inclusive and non-inclusive semantics may result in different truthmakers, they both result in the same classical truth conditions (i.e., the conditions under which the sentence has at least one actual truthmaker do not change).

To illustrate the non-inclusive semantics, let us assume that P, Q, and R have p, q, and r respectively as their sole truthmakers. Then we have $[\![P \wedge Q]\!] = \{p \sqcup q\}$, $[\![P \vee Q]\!] = \{p, q\}$, $[\![P \wedge (P \vee Q)]\!] = [\![P \vee (P \wedge Q)]\!] = \{p, p \sqcup q\}$, $[\![P \wedge (Q \vee R)]\!] = [\![(P \wedge Q) \vee (P \wedge R)]\!] = \{p \sqcup q, p \sqcup r\}$, $[\![P \vee (Q \wedge R)]\!] = \{p, q \sqcup r\}$, $[\![(P \vee Q) \wedge (P \vee R)]\!] = \{p, p \sqcup q, p \sqcup r, q \sqcup r\}$. This also illustrates that in models where propositional letters have exactly one truthmaker each there is a close correspondence between the denotations of sentences and their disjunctive normal forms (DNF). Not all of these DNF are minimal: for example, $P \vee (P \wedge Q)$ is already a DNF but has P as an equivalent, and minimal, DNF; and as we have seen, p is not the only truthmaker for $P \vee (P \wedge Q)$. Even if one allows propositional letters to have more than one truthmaker, it is still possible to achieve this close correspondence, and to strengthen the logic that arises from the system, by constraining the class of models. For example, Fine (2017a) entertains "regular" state spaces, i.e., spaces in which propositions are closed under fusion (as described above) and where the models and semantic clauses are furthermore amended to make all propositions convex with respect to the parthood relation (i.e., if s_{small} and s_{large} are members of a proposition then so is any s_{medium} such that $s_{small} \sqsubseteq s_{medium} \sqsubseteq s_{large}$). On the one hand, such amendments lead to various desirable properties of the logic of truthmaker semantics; on the other hand, they sometimes produce undesirable truthmakers. For example, suppose (plausibly) that the truthmakers for the sentence *One or all of the children ate the cookie* are s_1, s_2, s_3 in which each of three children ate the cookie by itself, and their sum $s_1 \sqcup s_2 \sqcup s_3$ where they ate it together (Brast-McKie, 2021). Convexity will enforce that $s_1 \sqcup s_2$ will also be a truthmaker for the sentence. Likewise, closure under fusion will enforce that if s_1 and s_2 make *Exactly one of the children ate the cookie* true then so does $s_1 \sqcup s_2$.

Fine (2017c) exploits the non-inclusive semantics to sketch a truthmaker semantic account of scalar implicatures of disjunctive sentences like the following (see also van Rooij 2013, 2017 for a similar account):

(5) a. John had toast or cereal for breakfast.
b. John had toast or cereal or toast and cereal for breakfast.

Sentence (5a) gives rise to the scalar implicature that John did not have both toast and cereal, while (5b) lacks it. Toast is an exact truthmaker for both sentences, and so is cereal; on the non-inclusive semantics, the fusion of toast and cereal is only an inexact truthmaker for (5a) but an exact truthmaker for (5b). For Fine, scalar implicatures arise from the presumption that a true statement has among its exact truthmakers what he calls the "relevant situation" (here, the fusion of all those things that John actually ate for breakfast). If John ate both toast and cereal, this presumption is satisfied for (5b) but not for (5a), so the difference in implicatures is predicted.

2.4 Negation

Different versions of truthmaker semantics also arise depending on how negation is treated. Here the two relevant versions are called unilateral and bilateral. I discuss each of them in turn.

2.4.1 Unilateral negation

It is possible, as a first approximation, to think of unilateral negation as relating sentences to their antonymic counterparts, so that the truthmakers for "Mary stayed" are the paradigm cases of truthmakers for "Mary did not leave". For details on antonymic event negation and a development of this idea in terms of negative events, see Higginbotham (1983) and Bernard and Champollion (2024). Formally, unilateral semantics takes states to stand in a binary exclusion relation $\perp$ that is added to the system as a primitive (i.e., not defined in terms of anything else); intuitively, $s \perp s'$ is intended to express that s refutes, precludes, prevents, knocks out, contradicts, or contravenes s'. Truthmakers for negated sentences can then be derived from those for the corresponding nonnegated sentences in terms of this exclusion relation. Possible states can be defined in terms of the exclusion relation if desired, but not vice versa.[2] To this effect, one first defines an inexact counterpart of the exclusion relation as follows; s_1 *conflicts with* s_2 just in case some part of s_1 excludes some part of s_2. Then a possible state can be defined as any state which does not conflict with itself. Exclusion is required to satisfy various formal constraints whose effect is to ensure at the model-theoretic level that exclusion expresses what it is intended to, and to ensure at the semantic level that exactly one of any sentence and its negation is true (Fine, 2017a; Champollion and Bernard, 2024; Plebani, Rosella, and Saitta, 2022).[3]

One way to give a unilateral semantics for negation is to take a truthmaker for $\neg\phi$ to be any fusion of a set of truthmakers that, between them, exclude some part of every truthmaker for ϕ:

(6) **Semantic Rule for Truthmaker Negation (unilateral version)**
$[\![\neg\phi]\!] = \{\bigsqcup S \mid [\forall s \in S\ \exists s' \in [\![\phi]\!].\ s \perp s'] \wedge [\forall s' \in [\![\phi]\!]\ \exists s \in S.\ s \perp s']\}$
(The negation of ϕ denotes the set of all fusions of sets of truthmakers that each exclude some part of a truthmaker for ϕ, with no truthmaker for ϕ left out.)

Other variants can also be used. For example, Champollion and Bernard (2024) use a definition that relates to (6) in the same way the noninclusive definition for disjunction relates to the inclusive definition. Fine (2017a, p. 659) analyzes both kinds of definitions for negation.

2.4.2 Bilateral negation

Instead of exclusion, bilateral truthmaker semantics introduces a primitive falsity-making relation. A sentence is not only mapped to a set of truthmakers (or verifiers), which are used in the definition of truth, but also to a set of falsitymakers (or falsifiers), which are used in the definition of falsity. The clause for negation flips truthmakers and falsitymakers.

More formally, while unilateral models contain an interpretation function I which

[2] In order for two states to stand in the exclusion relation, it is not enough for their fusion to be impossible. Otherwise, every impossible state would exclude every state whatsoever, leading to problems with the semantic rule for negation below.

[3] With slight variations, these authors adopt or entertain the same constraints. In Champollion and Bernard (2024), for example, these constraints are the following: Exclusion is symmetric; no part of any world-state excludes any self-conflicting state; any two possible states whose sum is impossible conflict with one another; every possible world conflicts with every impossible state. Further constraints are also plausible, e.g., making exclusion cumulative, prohibiting exclusion of the null state, requiring nonempty states to have excluders, etc.

assigns propositional letters to sets of verifiers, bilateral models contain two functions I^+ and I^- which assign propositional letters to sets of verifiers and sets of falsifiers respectively; the semantic clauses are similarly extended. In order to ensure (if desired) that every sentence is true or false but not both, models are constrained so that that no verifier is compatible with a falsifier for the same sentence ("exclusivity"), and that any possible state is compatible with a verifier or with a falsifier of any given sentence ("exhaustivity", Fine 2017a). Here, two states are compatible just in case their fusion is a possible state. The semantic rules of the connectives extend exclusivity and exhaustivity to arbitrary sentences. The falsitymaking side of the conjunction clause is typically fashioned to be dual to the truthmaking side of the disjunction clause, and vice versa. As before, one can choose between inclusive and noninclusive variants, and it is also possible to mix and match. For concreteness, the following summarizing clauses use the inclusive style throughout.

(7) **Unilateral Truthmaker Semantics (inclusive clauses)**

a. $[\![P]\!] = I(P)$ if P is a propositional letter
b. $[\![\phi \wedge \psi]\!] = \{s_1 \sqcup s_2 \mid s_1 \in [\![\phi]\!], s_2 \in [\![\psi]\!]\}$
c. $[\![\phi \vee \psi]\!] = [\![\phi]\!] \cup [\![\psi]\!] \cup [\![\phi \vee \psi]\!]$
d. $[\![\neg\phi]\!] = \{\bigsqcup S \mid [\forall s \in S\ \exists s' \in [\![\phi]\!].\ s \perp s'] \wedge [\forall s' \in [\![\phi]\!]\ \exists s \in S.\ s \perp s']\}$

(8) **Bilateral Truthmaker Semantics (inclusive clauses)**

a. $[\![p]\!]^+ = I^+(P)$ if P is a propositional letter
b. $[\![p]\!]^- = I^-(P)$ if P is a propositional letter
c. $[\![\phi \wedge \psi]\!]^+ = \{s_1 \sqcup s_2 \mid s_1 \in [\![\phi]\!]^+, s_2 \in [\![\psi]\!]^+\}$
d. $[\![\phi \wedge \psi]\!]^- = [\![\phi]\!]^- \cup [\![\psi]\!]^- \cup [\![\phi \vee \psi]\!]^-$
e. $[\![\phi \vee \psi]\!]^+ = [\![\phi]\!]^+ \cup [\![\psi]\!]^+ \cup [\![\phi \wedge \psi]\!]^+$
f. $[\![\phi \vee \psi]\!]^- = \{s_1 \sqcup s_2 \mid s_1 \in [\![\phi]\!]^-, s_2 \in [\![\psi]\!]^-\}$
g. $[\![\neg\phi]\!]^+ = [\![\phi]\!]^-$
h. $[\![\neg\phi]\!]^- = [\![\phi]\!]^+$

In the bilateral system, where negation acts as a flip-flop, any sentence ϕ and its doubly negated sentence $\neg\neg\phi$ have the same truthmakers; in unilateral semantics, ϕ and $\neg\neg\phi$ will be true in the same models but may differ in their truthmakers. Similarly, a bilateral system which does not mix and match inclusive and noninclusive styles will assign the same verifiers to $\neg\phi \vee \neg\psi$ and to $\neg(\phi \wedge \psi)$; in a unilateral system, these sentences will be true in the same models but $\neg(\phi \wedge \psi)$ may have additional verifiers. In linguistics, this kind of asymmetry is arguably desirable (Ciardelli, Zhang, and Champollion, 2018; Champollion and Bernard, 2024), despite philosophical and logical motivations against it (Fine, 2017a; Brast-McKie, 2021).

Outside of the truthmaker semantics literature, unilateral and bilateral semantics are sometimes also called *Australian Plan* and *American Plan* respectively (Meyer and Martin, 1986).

3 Fine-grained entailment relations

Truthmaker semantics provides a variety of fine-grained ways to formalize notions like propositions and to distinguish between entailment relations that are not distinguishable in classical propositional logic (though they are distinguishable in many partial logics, Blamey 2002). This section discusses some of these relations and linguistic applications. While only model theory is discussed, proof-theoretic treatments are also available (Fine, 2016; Fine and Jago, 2019; Korbmacher, 2022). My focus on model theory is guided by the prevalence of model-theoretic semantics in linguistics, but proof-theoretic applications are well worth exploring. Thus, Koralus and Mascarenhas (2013) use a proof-theoretic truthmaker semantics to formalize the mental model theory of deductive inference (Johnson-Laird, 1983). See also Knudstorp (2023) for a model-theoretic study of various logics arising from truthmaker semantics.

While one can define classical entailment both in possible-world semantics and in truthmaker semantics, the latter framework also provides various nonclassical entailment relations. For example, *conjunctive entailment* or *analytic containment* holds from $P \wedge Q$ to P but not from P to $P \vee R$; *disjunctive entailment* or *exact consequence* holds from P to $P \vee R$ but not from $P \wedge Q$ to P. Classical entailment, of course, holds in both these cases. Despite appearances, conjunctive and disjunctive entailment can be defined purely semantically (I give only the clauses for verifiers here). The definitions are taken from Fine (2017a).

(9) **Conjunctive entailment**
ϕ conjunctively entails ψ just in case $[\forall s_1 \in [\![\phi]\!].\ \exists s_2 \in [\![\psi]\!].\ s_2 \sqsubseteq s_1] \wedge [\forall s_2 \in [\![\psi]\!].\ \exists s_1 \in [\![\phi]\!].\ s_2 \sqsubseteq s_1]$.
(Any truthmaker of ϕ has a part that is a truthmaker of ψ, and any truthmaker of ψ is part of a truthmaker of ϕ.)[4]

(10) **Disjunctive entailment**
ϕ disjunctively entails ψ just in case $[\![\phi]\!] \subseteq [\![\psi]\!]$.

[4]Conjunctive entailment in (9), which Fine calls containment, is only defined for propositions with at least one verifier but does not depend on whether they are closed under fusion. For an alternative definition which requires closure under fusion but which also works for propositions without verifiers, see Brast-McKie (2021, p. 1489) under *Essence*.

(Any truthmaker of ϕ is also a truthmaker of ψ.)

A close relation between conjunctive and disjunctive entailment can be observed in the bilateral semantics, where ϕ conjunctively (resp. disjunctively) entails ψ just in case $\neg\phi$ disjunctively (resp. conjunctively) entails $\neg\psi$, assuming that ϕ and ψ each have at least one verifier (Fine, 2017a).

3.1 Conjunctive entailment

Conjunctive entailment can model reasoning in which the conclusion does not go beyond the subject matter (the topic) of the premise. The philosophical literature has linked a variety of intuitive concepts to subject matter preservation. One of these concepts is aboutness, i.e., the relation between a sentence and its subject matter (Yablo, 2014; Hawke, 2018; Fine, 2020; Brast-McKie, 2021). As described in (Plebani and Spolaore, 2021), the notion of subject matter in the philosophical literature is closely related to the Question under Discussion, i.e., the implicit or explicit question that the utterance of a sentence is addressing at any given time (Roberts, 2012). Linguists may think of the subject matter of a sentence as related to its topic-value (von Stechow, 1981), background (Krifka, 2001), or focus semantic value (Rooth, 2016).

Concepts that are related to aboutness involve agreement between interlocutors, partial truth, verisimilitude, and others (Yablo 2014, pp. 12–14; Fine 2020). One linguistic application is Ross's paradox of imperatives (Ross, 1944):

(11) a. Stamp this letter and post it!
b. Post this letter!
c. Post this letter or burn it!

There is an intuitive sense in which (11b) follows from (11a) but (11c), which introduces the additional subject matter of burning the letter, does not follow from (11b).

While classical logic would lead us to expect no such asymmetry, truthmaker semantics provides the basis for an explanation: (11a) entails (11b) only conjunctively while (11b) entails (11c) only disjunctively. The idea is to treat imperatives as sets of alternatives, conceived as actions (or inactions) in compliance with these imperatives, and to take the operative entailment relation to be conjunctive entailment. A semantics along these lines is developed in Fine (2018a); see also Fine (2014a, 2017c). Similar systems and proposals are also found, among others, in Stelzner (1992), van Rooij (2000), Aloni (2007), and Aloni and Ciardelli (2013).

Conjunctive entailment is also relevant for the analysis of judgments concerning inferences between ordinary declarative statements. Compared with classical entailment, it is arguably closer to the intuitive idea that the content of the premises of a deductive inference contains that of the conclusion. Within linguistics, discrepancies between classical entailment and intuitive judgments about correct reasoning are typically seen as resulting from pragmatic factors. For example, the intuition that from *It is raining and snowing* we can infer *It is raining* is much clearer than the intuition that from *It is raining* we can infer *It is raining or the sun is shining* (Geurts, 2005; Zimmermann, 2000). An important question about the semantics-pragmatics divide concerns the status of this difference. The traditional picture starts from the observation that by uttering $\phi \vee \psi$, a speaker typically signals that they believe that exactly one of the disjuncts is true but do not know which one (Tarski, 1941, p. 21). This *ignorance implicature* is traditionally seen as resulting either from the assumption that a cooperative and opinionated speaker would have uttered either $\phi \wedge \psi$ or one of its disjuncts (Grice, 1975), or alternatively, from the weaker assumption that a disjunction requires the speaker's beliefs to be compatible with $\phi \wedge \neg\psi$ and with $\neg\phi \wedge \psi$ (Stalnaker, 1975). On this traditional approach, one can maintain that ϕ semantically entails $\phi \vee \psi$ but that this is masked by the fact that no cooperative speaker who utters ϕ is in a position to felicitously assert $\phi \vee \psi$. In effect, the traditional approach treats preservation of subject matter as belonging to pragmatics. In contrast, the truthmaker-based approach sketched in Fine (2017c) in effect views preservation of subject matter as belonging to semantic entailment and, accordingly, denies that ϕ semantically entails $\phi \vee \psi$ (in the relevant, i.e., conjunctive sense) in the first place.

This intriguing possibility raises the question whether the line indicating the division of labor between semantics and pragmatics might have to be redrawn. One advantage of the truthmaker-based approach is its immediate applicability within embedded contexts. For example, part of the problem of logical omniscience consists in explaining why one may know something without knowing all of its consequences, even though knowledge distributes over conjunction (*John knows it is raining and snowing; therefore he knows it is raining and he knows it is snowing*, Barwise and Perry 1983, p. 205). A truthmaker-based approach to this problem is found in Hawke, Özgün, and Berto (2019).

3.2 Disjunctive entailment

Disjunctive entailment has applications, for example, in the analysis of counterfactuals. Generally, removing one disjunct from the disjunctive antecedent of a counterfactual is truth-preserving. This principle is known as Simplification of Disjunctive Antecedents (SDA):

(12) a. If you had posted or hand-carried the letter, it would have arrived.
b. If you had posted the letter, it would have arrived.

Because classical logic treats the entailment from ϕ to $\phi \vee \psi$ and the entailment from $\phi \wedge \chi$ to ϕ as equally valid, any compositional logic of conditionals that builds on propositional logic and validates SDA will predict that adding a conjunct to a counterfactual antecedent should likewise be truth-preserving (Fine, 1975; Ellis, Jackson, and Pargetter, 1977). But (12b) does not entail (13b) (Goodman, 1947; Sobel, 1970):

(13) a. If you had stamped the letter, it would have arrived.
b. If you had stamped and burned the letter, it would have arrived.

Truthmaker-based semantics for counterfactuals that account for these observations are found in Fine (2012a,b) and Santorio (2018) (for discussion see also Fine 2017c; Ciardelli, Zhang, and Champollion 2018; McHugh (2022, 2023); and Güngör 2022). The general idea is to apply the semantics of the counterfactual pointwise to each disjunct, and in fact to each truthmaker, of the antecedent (see Alonso-Ovalle 2009 for a similar intuition). This derives SDA and, more generally, has the effect that the relevant notion is disjunctive rather than conjunctive entailment. But in cases where an individual disjunct has more than one truthmaker, distribution over truthmakers may be too strong if these truthmakers are considered to be events in the Davidsonian sense. For example, a Lewis (1973)-style counterfactual semantics ranks scenarios described by the antecedent for similarity to the actual world in order to exclude scenarios involving gratuitous deviations from actuality. Applying the counterfactual semantics pointwise ensures that neither disjunct is excluded wholesale even when it is more deviant than the other; ranking should take place only within disjuncts. This poses a compositionality problem (Embry, 2014): Both on classical and truthmaker semantics, the denotation of a disjunction fails to expose its disjuncts. For example, if $[\![\phi]\!] = \{s_1, s_2\}$ and $[\![\psi]\!] = \{s_3\}$ then $[\![\phi \vee \psi]\!] = \{s_1, s_2, s_3\}$, from which neither $[\![\phi]\!]$ nor $[\![\psi]\!]$ can be recovered. So if a counterfactual semantics for *If* $\phi \vee \psi$ *then* ... is applied pointwise to $[\![\phi \vee \psi]\!]$, the points will be the three truthmakers s_1, s_2, and s_3 and not the two disjuncts, and the counterfactual will not be able to carry out separate rankings for $\{s_1, s_2\}$ and for $\{s_3\}$. One approach to this problem is to accept and embrace the failure of SDA (Lewis, 1973). McHugh (2022, 2023) develops this approach in a truthmaker-based framework and also gives a truthmaker-based account of causation and *because*-clauses.

3.3 Inexact entailment

Truthmaker semantics is related to relevance logics (Mares, 2024). In particular, relevant first degree entailment (Belnap, 1977; Dunn, 1976) can be formalized in the following way (recall that an inexact truthmaker for a sentence ϕ is any state among whose parts is an exact truthmaker for ϕ):

(14) **Inexact entailment**
ϕ inexactly entails ψ just in case for all s, if $[\![\phi]\!]$ contains some part of s then $[\![\psi]\!]$ contains some (perhaps different) part of s.
I.e., iff $\{s_1 \mid \exists s_2 \sqsubseteq s_1.\ s_2 \in [\![\phi]\!]\} \subseteq \{s_1 \mid \exists s_3 \sqsubseteq s_1.\ s_3 \in [\![\psi]\!]\}$.
(Any inexact truthmaker for ϕ is also an inexact truthmaker for ψ.)

Within natural language semantics, the closest relative to relevance logics is situation semantics, the first among a number of related frameworks I discuss next.

4 Relation to situation semantics

Similar frameworks to truthmaker semantics include situation semantics, alternative semantics, and inquisitive semantics. (One may also mention possibility semantics, though it has been less influential among linguists; see e.g. Holliday 2021 and references therein.) Here and in the next section, I outline the relationship of truthmaker semantics with these approaches.

Situation semantics was introduced into natural language semantics by Barwise and Perry (1981), where situations are defined as tuples consisting of relations, objects and locations (see Barwise and Perry 1983 for a book-length treatment; for overviews, see Devlin 2006, Seligman and Moss 2011, and Kratzer 2021). Many subsequent formulations, instead, treat situations as undefined primitives that form a mereological structure. Kratzer (1989, 2021) adopts a unilateral semantics. Muskens (1995, ch. 7), following Dunn (1976) and Blamey (1986), takes a four-valued approach which can be viewed as a bilateral system. This semantics relates to the classical (unilateral) semantics for the propositional connectives in the same way as the bilateral and unilateral clauses for truthmaker semantics relate to each other. Muskens's approach is related to data semantics, which is likewise bilateral (Veltman, 1985, part III).

The main difference between situation semantics and truthmaker semantics concerns the relationship between sentences and their verifiers (i.e., truthmakers or situations) in the Tarski clauses for truth. Recall that in truthmaker semantics, any verifier of a sentence must be wholly relevant to its truth. In situation semantics, by contrast, a situation in which a sentence is true may contain extraneous material (with exceptions such as the characterization relation in Schubert 2000, which is rather like exact verification). The set of verifiers for a propositional letter in situation semantics is assumed to be upward closed (also called "hereditary" or "persistent") under the parthood relation $\sqsubseteq$, in contrast to truthmaker semantics (and event semantics, see Eckardt 1998) but similarly to many other semantics. While truthmaker semantics defines inexact verification in terms of exact verification which is in turn primitive, in situation semantics it is inexact verification which is primitive. Because exact truthmakers need not be minimal, it is controversial whether exact verification can be recovered from inexact verification in situation semantics (Fine, 2017c; Deigan, 2019; Leitgeb, 2020; Kratzer, 2021).

Situation semantics models entailment as truth preservation over situations; this corresponds to inexact consequence as in (14) above. Situations are typically understood as parts of possible worlds, though they may also bear other relations to them. Partiality gives rise to truth value gaps, which can be propagated in various ways, though linguistic versions of situation semantics are often tentative or inexplicit about their treatment of negation and disjunction (Barwise and Perry 1983; Kratzer 1989; for discussion see also Kanazawa, Kaufmann, and Peters 2005; Kratzer 2012, p. 134, n. 17; and Mandelkern and Rothschild 2019).

5 Relation to alternative and inquisitive semantics

This section discusses the relation between truthmaker semantics and the related frameworks of alternative and inquisitive semantics. Alternative semantics uses notions from focus semantics (Rooth, 2016). It defines disjunction as an operation that forms a set whose members are the meanings of the disjuncts, and then propagates these elements pointwise through the derivation (Alonso-Ovalle, 2009; Rooth, 1985). Inquisitive semantics uses the classical semantics for conjunction and disjunction in terms of intersection and union, but treats propositions as sets of information states that are downward closed under the subset relation $\subseteq$, where an information state is defined as a set of possible worlds (Ciardelli, Groenendijk, and Roelofsen, 2018).

Comparing truthmaker semantics with alternative semantics is challenging because the latter is not a full-fledged logic; however, a logic in the spirit of alternative semantics can be obtained from inquisitive semantics by dropping the requirement that sets of information states be downward closed (Ciardelli, Roelofsen, and Theiler, 2017).

A key distinction between inquisitive and truthmaker semantics lies in how they model the elements of a proposition. In inquisitive semantics, these elements (the "alternatives") are sets of possible worlds; the more informative and specific such an alternative is, the smaller it is; when an alternative obtains, so does at least one of its parts (the actual world). Consequently, combining propositions involves shrinking their elements. This aligns with the intersective theory of conjunction and the Stalnakerian approach to discourse semantics, where new knowledge restricts the range of possibilities (e.g. Stalnaker, 1978). In truthmaker semantics, the elements of a proposition are states; the more informative and specific such a state is, the larger it is; when a state obtains, so do all of its substates (its parts). Consequently, combining propositions involves growing their elements. As we have seen in Section 2.2, it is therefore appropriate to represent the combination of information by fusing states, as in the collective theory of conjunction.

In keeping with classical logic and situation semantics (but in contrast with truthmaker semantics, alternative semantics, and most relevance logics), inquisitive semantics treats $\phi \vee (\phi \wedge \psi)$ as semantically equivalent to ϕ; in logical terms, inquisitive semantics validates the Absorption law $\phi \Leftrightarrow \phi \vee (\phi \wedge \psi)$, with "$\Leftrightarrow$" indicating identity of supporting information states. This is because the assumption in inquisitive semantics that sets of information states are downward closed has the consequence that any information state supporting $(\phi \wedge \psi)$ also supports ϕ, similarly to what has been observed above for situation semantics; truthmaker semantics has no corresponding assumption. Within linguistics, $\phi \vee (\phi \wedge \psi)$ is an example of a *Hurford disjunction*, a disjunction in which one disjunct entails the other (Hurford, 1974). Hurford disjunctions sound infelicitous in many cases (*#John is from Paris or from France*) but not always (*One or both of the children are on the left*). They provide a rich testing ground not only for distinguishing classical possible-worlds semantics from finer-grained semantic theories, but also for distinguishing theories based on whether they validate the above Absorption law. Thus Ciardelli and Roelofsen (2017) argue that infelicitous Hurford disjunctions challenge theories that invalidate the law (such as truthmaker semantics) because such theories cannot account for the infelicity in terms of redundancy; in contrast, Güngör (2022) argues that felicitous Hurford disjunctions challenge theories that validate the law (such as inquisitive semantics and classical possible-world semantics).

The formal relationship between inquisitive semantics and truthmaker semantics has been studied by Ciardelli (2013); this paragraph is based on that note. Inquisitive semantics can be seen as a special case of truthmaker semantics where there is only one incoherent state, corresponding to the empty set of possible worlds, and where a state is part of another one just in case it contains every world the second one contains. The correspondence between the two frameworks extends beyond conjunction and disjunction to the conditional, which is treated similarly in inquisitive semantics and in Fine's truthmaker semantics for intuitionistic logic (Fine, 2014b). One can obtain inquisitive semantics from truthmaker semantics by associating with each propositional letter p a single truthmaker, the set of all worlds at which p is true. Inquisitive semantics can then be understood as truthmaker semantics on models of this kind, with inexact instead of exact verification. Under certain natural assumptions, Ciardelli (2013) shows (roughly) that one version of truthmaker semantics can be obtained from inquisitive semantics if we drop the constraint that every propositional letter has a single truthmaker and assume that truthmakers are never part of one another.

6 Application to negative events and perception reports

Here and below, I offer case studies involving applications of truthmaker semantics to natural language. We have already seen that the relation that Davidsonian events bear to the sentences describing them is similar to the relation between truthmakers and the sentences that they exactly verify, and that truthmaker conjunction is analogous to the collective approach to conjunction, which has been applied to event semantics (e.g. Carlson, 1987; Moltmann, 1992; Lasersohn, 1995). This suggests that an analogue to the treatment of negation in truthmaker semantics is applicable to event semantics and can be brought to bear on the problem of negative events. This is the problem of deciding, for any event predicate ϕ, which events, if any, should be regarded as events of not ϕ-ing. Bernard and Champollion (2024) propose a solution that is conceptually related to the unilateral treatment of negation described in Section 2.4. They analyze negative causation (*I kept the child awake by not turning out the light*), negative quantifiers like *no student*, and negative perception reports like the following:

(15) a. John saw Mary not leave.
b. John saw the dog not bark.

Here, negative events serve as truthmakers for sentences and clauses that describe non-occurrence. This is particularly relevant for analyses involving a predicate or relation over events. For example, traditional approaches to perception reports analyze *John saw Mary stay* in terms of a relation between a seeing event and a staying event (Higginbotham, 1983; Parsons, 1990). Extending these approaches to negated perception reports like those in (15) is challenging because these sentences seem to imply the perception of the absence of an event of a certain kind. In some cases, these perceived absences can be thought of as ordinary events under a negative description; thus, for example, what is seen in (15a) can be seen as a staying event by Mary. But in most cases, no obvious nonnegative description is available, and additional events need to be introduced to play an analogous role to these staying events. Another challenge involves providing a compositional treatment of negation as a modifier of event predicates. Moreover, a treatment of negation along these lines needs to be suitably constrained to guarantee familiar logical properties such as excluded middle, noncontradiction, or downward-entailingness. Bernard and Champollion (2024) incorporate negative events into the semantic analysis in a way that meets these and other challenges by analyzing negation in terms of a function Neg from sets of events to sets of events. The familiar logical properties are ensured by requiring that for any event predicate P, there is an actual P event if and only if there is no actual "anti-P" event (i.e., no event in $Neg(P)$). If desired, this requirement can be derived from an exclusion-based semantics for negation as described in Section 2.4, so as to ensure that the "anti-P" events are exact rather than inexact verifiers (Champollion and Bernard, 2024). The non-finite perception report in (15), for example, can then be analyzed in terms of a relation between a seeing event and an anti-Mary-leaving event:

(16) $$\exists e.\ \mathit{actual}(e) \wedge \mathit{Exp}(e) = \mathit{John} \wedge \mathit{see}(e) \wedge \\ \mathit{Th}(e) \in \mathit{Neg}(\lambda e''.\ \mathit{Ag}(e'') = \mathit{Mary} \wedge \mathit{leave}(e''))$$

This says that there is an event of Mary not leaving that John actually saw. Assuming that John did not hallucinate, it follows that this event is actual, and hence, that no leaving by Mary is actual. This line of work underscores the versatility of truthmaker semantics. It demonstrates its potential to inspire and enrich theories formulated within existing frameworks of natural language semantics.

7 Beyond propositional logic

The following are several ways in which truthmaker semantics extends beyond the propositional connectives discussed earlier. The conditional $\phi \to \psi$ may be treated as a shorthand for $\neg\phi \vee \psi$ or $\neg(\phi \wedge \neg\psi)$, or alternatively, as a distinct primitive with its own logic. Some extensions of truthmaker semantics take the latter path, such as truthmaker semantics for intuitionistic logic (Fine, 2014b) and the truthmaker semantics for counterfactuals mentioned in Section 3.2 above.

A truthmaker predicate logic can be obtained by generalizing conjunction and disjunction to existential and universal quantifiers. Constants, variables and predicates can be treated in analogy with classical predicate logic, by enriching models with domains of individuals and by modeling n-ary predicates as denoting functions from n individuals to sets of truthmakers. Alternatively, one may introduce a special treatment for the universal quantifier by adding to the corresponding conjunction a special kind of "that's all" truthmaker representing the fact that the conjunction is exhaustive; this is called a "totality state" (Armstrong, 1997, 2004; Fine, 2017c; Moltmann, 2019). On this view, a truthmaker for the conjunction *Mercury, Venus, Earth, and Mars are rocky* is the fusion of four states (of Mercury's being rocky, etc.), while a truthmaker for *All the inner planets are rocky* is the fusion of four such states plus a totality state of Mercury, Venus, Earth and Mars being exactly the inner planets. Truthmaker semantics for predicate logic has not been worked out in much detail; and the same is true for extensions for modal logic, temporal logic, and generalized quantifiers (though see Bledin 2024 for a promising approach to the semantics of noun phrases based on negative individuals). Modal logics for truthmaker semantics are likewise a topic of active research (Anglberger, Faroldi, and Korbmacher, 2016; Moltmann, 2018, in prep; Kim, 2024).

8 Conclusion

While much progress has been made in developing truthmaker semantics for propositional and sentential logic, further work is needed to extend it to a full-fledged treatment of predicate logic, generalized quantifiers, and other natural language constructions. A compositional truthmaker semantics in the style of Montague (1974) has yet to be worked out in detail. Nonetheless, truthmaker semantics has already demonstrated its potential as a powerful tool for precise semantic analysis that is informed by both linguistic intuitions and philosophical logic.

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