

Anaphoric accessibility with flat update*

Lisa Hofmann

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Abstract This paper presents an intensional account of anaphoric accessibility, providing a unified analysis of anaphora with antecedents under negation and non-veridical operators. These include cases with double negation, bathroom-disjunctions, and modal subordination, which have previously received disparate analyses. The data raise two problems for classic dynamic approaches (Kamp 1981; Heim 1982; Groenendijk and Stokhof 1991): these miss the generalization that anaphora is possible whenever the antecedent referent exists in the local context, and they encounter a lookahead problem. While no previous account addresses both issues, this work resolves them in a unified way.

Building on analyses of modal subordination (Stone 1999; Brasoveanu 2006), the paper introduces a flat-update dynamic semantics, where expressions with the potential to introduce a discourse referent do so globally, regardless of their embedding context. Constraints on anaphora are derived based on discourse consistency and a pronominal existential inference interpreted relative to possible worlds in the local context. The analysis uses intensional representations of discourse referents, which store information about their embedding context, and its relationship to speaker commitments, explaining constraints imposed by negation and other non-veridical operators in terms of presuppositions about discourse referents.

1 Introduction

Negation constrains anaphora. While indefinites can usually provide anaphoric antecedents (1a), unspecific negated indefinites often cannot (1b) (Karttunen 1976).¹

- (1) a. Mary owns [a car]^{v1}. It_{v1} is red.
b. Mary doesn't own [a car]^{v2}. #It_{v2} is red.

Contrasts like (1) raise the question of how to characterize anaphoric accessibility — the conditions where a dependency between an anaphor and a potential antecedent is possible. Classic dynamic frameworks (Kamp 1981; Heim 1982; Groenendijk and

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¹Anaphora is indicated by co-indexation. Subscripts indicate anaphoric reference, and superscripts indicate introduction of a discourse referent.

Stokhof 1991, *et seq.*) have addressed the question by employing what we might call a *nested update* approach, capturing constraints imposed by non-veridical operators through nested discourse representations and a mechanism ensuring that anaphors find an antecedent on the same level of embedding or higher. This predicts that indefinites in the scope of negation cannot provide antecedents for anaphors outside that scope, providing an explanation for (1).

Various counterexamples to this prediction have been noted, including: (2a) doubly negated antecedents (Karttunen 1976; Krahmer and Muskens 1995); (2b) ‘bathroom-disjunctions’, where the antecedent is in the first disjunct and the anaphor in the second (Evans 1977; Roberts 1987, 1989); (2c) speakers disagreeing with each other (Hofmann 2019, 2022); and (2d) modal subordination with negated antecedents (Roberts, 1987, 1989; Frank, 1996).

- (2)
- a. Doubly negated antecedents (Krahmer and Muskens 1995: 5)
It’s not true that John didn’t bring [an umbrella]^{v1}.
It_{v1} was purple and stood in the hallway.
 - b. Bathroom-disjunctions (Roberts 1989: 12)
Either there isn’t [a bathroom]^{v2} in this house or it_{v2}’s in a funny place.
 - c. Disagreement (Hofmann 2019: 2d)
A: There isn’t [a bathroom]^{v3} in this house.
B: (What are you talking about?) It_{v3} is just in a weird place.
 - d. Modal subordination (Frank 1996: 8a)
Fred didn’t buy [a microwave oven]^{v4}.
He wouldn’t know what to do with it_{v4}.

No formally explicit account has captured all cases in (1) and (2). Here, I show that the data in (2) pose two distinct problems for nested update approaches: these miss a generalization regarding speaker commitments about discourse referents and they face a lookahead problem. While previous analyses do not address both issues, this work offers a unified resolution, representing a considerable improvement.

1.1 Problems for previous analyses

In dynamic theories of anaphora, utterances update a discourse context that stores *discourse referents* (drefs). A dref is a mapping from a variable name to a model object at a given point in discourse. Subsequent anaphors are interpreted as variables relative to a discourse state with the same mapping. This is illustrated in the Discourse Representation Structures (DRSs) in (3), using the meta-language syntax of Discourse Representation Theory (DRT, Kamp, 1981; Kamp and Reyle, 1993).

$$(3) \quad \text{Mary}^x \text{ has [a car]}^y. + \text{It}_y \text{ is red.} \rightsquigarrow$$

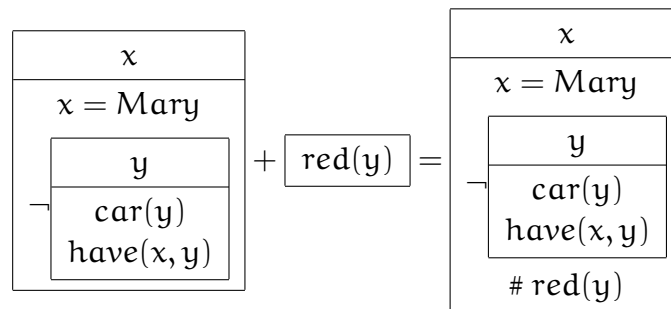
x, y
x = Mary car(y) have(x, y)

$$+ \text{red}(y) =$$

x, y
x = Mary car(y) have(x, y) red(y)

Sentential operators are treated as operations over updates, so that drefs invoked under non-veridical operators do not update the global discourse context. For example, negation specifies a counterfactual local update and imposes a condition that the global context does not allow this update. In DRT, this is implemented by adding drefs in subordinate DRSs. DRT enforces constraints on anaphora through a structural accessibility relation, constraining possible semantic representations: Variables can only be linked to a dref at the same level of embedding or higher. This explains the unacceptability of (1b), illustrated in (4).

$$(4) \quad \text{Mary}^x \text{ does not have [a car]}^y. + \# \text{It}_y \text{ is red.} \rightsquigarrow$$



The other classic dynamic systems operate similarly: File Change Semantics (Heim, 1982, 1983a) achieves the same by introducing drefs/files in local vs. global contexts, and Dynamic Predicate Logic (Groenendijk and Stokhof, 1991) distinguishes between externally static vs. dynamic operators. In all these systems, material in the scope of non-veridical operators affects truth-conditions but cannot introduce global drefs. An unwelcome prediction is that the unspecific indefinites under negation in (2) cannot provide antecedents for anaphora.

1.1.1 Commitment and veridicality

Assuming that non-veridical operators preclude anaphora misses a broader generalization: Roberts (1989) suggests that anaphora hinges on “the speaker’s commitment to the truth of a sentence” (p. 686). I adopt the assumption that the key distinction underlying anaphoric accessibility relates to the speaker’s epistemic stance — distinguishing between *veridical content*, where the speaker is committed to its truth, and

hypothetical or *counterfactual content*, where they are not. This allows for characterizing the contrast in (1), repeated here, at a denotational level.

- (1) a. Mary owns [a car]^{v₁}. It_{v₁} is red.
 b. Mary doesn't own [a car]^{v₂}. #It_{v₂} is red.

(1a) entails that the referent (*Mary's car*) exists but (1b) entails that it does not. Considering inferences about drefs, we can capture facts about accessibility that representational constraints cannot. For instance, indefinites in veridical contexts behave like the affirmative antecedent in (1a), even when embedded under negation. Examples include indefinites under double negation (2a), in complements of negated factives (5a), or in negated prejacent of negative implicatives (5b).

- (2a) It's not true that John didn't bring [an umbrella]^{v₁}.
 It_{v₁} was purple and stood in the hallway.

- (5) Adapting Karttunen (1976): (16, 14b)
 a. Bill didn't realize that he had [a dime]^{v₂}. It_{v₂} was in his pocket.
 b. John forgot not to bring [an umbrella]^{v₃}, but we had no room for it_{v₃}.

This data shows that relying solely on representational constraints is insufficient for making generalizations about accessibility. For instance, Krahmer and Muskens 1995 predict that indefinites introduce drefs only when embedded under an even number of negations, leaving the single negation cases in (5) unaddressed. Instead, we need a generalization over semantic values that considers speaker commitments. This is reflected in analyses of doubly negated antecedents in Gotham 2019, Elliott 2020, 2022, and Mandelkern 2022, where indefinites introduce drefs when existence of a referent is entailed. The analyses mentioned here are also applied to bathroom-disjunctions (2b) by interpreting the second disjunct in the context of the negation of the first. However, they rule out anaphora when the existence of a referent is denied, as in disagreement cases (2c), and modal subordination (2d), discussed below.

1.1.2 Lookahead problem

Nested update determines discourse-level accessibility based on the antecedent context alone: unspecific indefinites under non-veridical operators never introduce global drefs. However, the contrast in (6) highlights the need to consider the embedding context of the anaphor as well.

- (6) Mary doesn't have [a car]^{v₁}
 a. #It_{v₁} is parked outside.
 b. It_{v₁} would be parked outside.
 c. even though Cole said that it_{v₁}'s red.

Anaphora in non-veridical contexts (6b+c) can have negated antecedents, even when the speaker is committed that no referent exists. We call these cases *counterfactual anaphora*, in contrast to *veridical anaphora*, where the speaker commits to the referent's existence. Additionally, disagreement cases (2c) show that anaphora to negated indefinites is possible when the speaker using the anaphor (B) commits that a referent exists.

- (2c) A: There isn't [a bathroom]^{v4} in this house.
B: (What are you talking about?) It_{v4} is just in a weird place.

Nested update systems cannot address this data because they determine whether an indefinite is a potential antecedent when interpreting the discourse segment containing that indefinite. They cannot consider information about the embedding context of the anaphor, which becomes available later in discourse. The failure to incorporate this relevant information constitutes a lookahead problem.

The observation that the embedding context of a pronoun can enable anaphora to otherwise inaccessible antecedents is reflected in analyses of modal subordination (e.g., Roberts 1987, 1989; Frank 1996; Geurts 1999; van Rooij 2000). Here, variable mappings temporarily contributed by antecedents in non-veridical contexts are reintroduced in a local context for the pronoun. This is typically tied to propositional operators (modals or attitude reports) and thus cannot readily capture unembedded pronouns disagreement cases (2c) or veridical anaphora like (2a) or (5).

The counterexamples to nested update in (2) have received disparate analyses. Accounts of doubly negated antecedents and bathroom sentences (e.g., Krahmer and Muskens 1995; Gotham 2019; Elliott 2020; Mandelkern 2022) cannot address counterfactual anaphora or disagreement cases. Meanwhile, analyses of modal subordination (e.g., Roberts 1987, 1989; Frank 1996; Geurts 1999; van Rooij 2000) cannot handle unembedded veridical anaphora with unspecific antecedents.

1.2 Flat update and intensional drefs

This paper proposes a unified analysis of the cases in (1) and (2) in the spirit of flat-update analyses of modal subordination in Stone (1999), Stone and Hardt (1999), and Brasoveanu (2006, 2010a). Here, embedded drefs are admitted to the global level, while constraints on accessibility are explained by assuming (7).

- (7) A pronoun can be interpreted anaphorically, if
- a. its semantic value is provided by an overt antecedent expression, and
 - b. the presupposition that its referent exists is met in its local context (under some consistent interpretation of the discourse).

(7a) states a precondition on linguistic form (i.e., Heim’s, 1982, ‘formal link condition’) while (7b) adds constraints at the level of semantic values. This differs from nested update approaches, where accessibility constraints are applied to semantic representations. The analytical intuition behind the account is this: pronouns presuppose that their referent exists and indefinites entail it. These existential inferences can be evaluated globally, relative to speaker commitments, or locally, relative to a set of worlds that the speaker may consider an (im)possibility.

1.2.1 Flat update

The analysis involves introducing and interpreting individual drefs relative to possible worlds in the *local intensional context* (Heim 1982, 1983a; Stone 1999; Stone and Hardt 1999; Brasoveanu 2006, 2010a; Hofmann 2019, 2022). A local intensional context is a set of worlds where some embedded propositional content is true. Stone (1999) and Brasoveanu (2006) address the contrast in (8) by assuming that the indefinite *a wolf* globally introduces a dref for a hypothetical wolf, which exists in all worlds where the modal prejacent is true.

- (8) [A wolf]^v might walk in. (Roberts, 1989: 11)
- a. #It_v is gray.
 - b. It_v would eat you first.

The modalized indefinite introduces a *hypothetical dref* v , for a wolf, whose existence the speaker is not committed to. Subsequent anaphora in a veridical context (8a) is impossible because the existential presupposition is not met. In contrast, a pronoun under *would* (8b) can be interpreted locally, relative to the hypothetical prejacent. This explains the contrast in (8) by interpreting drefs relative to possible worlds and the unacceptability of (1b) (repeated here), as unspecific indefinites under negation come with a global inference that the referent does not exist.

- (1b) Mary doesn’t have [a car]^{v₂}. #It_{v₂} is red.

In (1b), the speaker introduces a *counterfactual dref* v_2 for Mary’s car and is committed to its non-existence. If the same speaker later uses a pronoun in a veridical context, that presupposed the existence of a referent, causing a contradiction. This rules out inaccessible anaphora based on discourse consistency.

1.2.2 Antecedents under negation

This work extends an analysis along those lines to the anaphora with negated antecedents in (2), developing a principled account of veridicality-based constraints on accessibility. Based on the veridicality of the embedding context of anaphors and

their antecedents, we can identify three cases where anaphora to negated indefinites is possible: (i) when the local context of the antecedent is veridical despite negation (double negation); (ii) when the antecedent is hypothetical or counterfactual, and the anaphor is also in a non-veridical context (modal subordination, bathroom-disjunctions); and (iii) when antecedent and anaphor are in a non-veridical discourse relation, in the sense of [Asher and Lascarides \(2003\)](#) (disagreement). The explanation for double negation (9) works as follows:

(9) It's not true that there isn't [a bathroom]^v in this house. It_v's upstairs.

The indefinite is in a veridical context, committing the speaker to the existence of a bathroom. It therefore enables a subsequent anaphor in a veridical context, as the existential presupposition is met globally. In contrast, that presupposition is interpreted locally for bathroom-sentences (10) and modal subordination cases (11).

(10) Either there isn't [a bathroom]^{v1} in this house or it_{v1}'s in a weird place.

(11) There isn't [a bathroom]^{v2} in this house. It_{v2} would be easier to find.

The indefinites in (10) and (11) introduce a dref for a hypothetical bathroom without the speaker committing to its existence. This dref can provide a value for the pronoun if its referent exists in all worlds in the pronoun's local context under some consistent interpretation of the discourse. Such an interpretation is available when the existential presupposition can be locally accommodated without contradiction, explaining why indefinites in non-veridical contexts can be antecedents for anaphors that are also in non-veridical contexts.

A third case of accessible anaphora to negated indefinites is cross-speaker anaphora where two speakers *A* and *B* disagree about whether the referent exists:

- (12) a. *A*: There isn't [a bathroom]^{v1} in this house.
 b. *B*: (What are you talking about?) It_{v1}'s upstairs.

These utterances are contradictory, but because the speakers disagree, that is consistent on a discourse level. Contradictory utterances can occur in disjunctive, or non-veridical, discourse relations. Conversely, when discourse segments containing anaphor and antecedent are interpreted in conjunction (e.g., as consecutive assertions by a single speaker), a veridical anaphor requires a veridical antecedent.

Veridicality constrains anaphora, and the condition (7) explains why: to interpret anaphora, we determine whether the referent of the antecedent exists in the local context of the anaphor. This depends on its epistemic status – whether the speaker believes it exists, might hypothetically exist, or does not exist.

1.2.3 Building on previous literature

The presented analysis extends [Stone \(1999\)](#) and [Brasoveanu \(2006\)](#) in two key ways: (i) implementing modal subordination without modal anaphora, and (ii) addressing counterfactual anaphora and negation. Stone and Brasoveanu analyze modal subordination as simultaneous individual and propositional anaphora: modals like English *would* retrieve a modal base anaphorically, providing a local context for pronouns in their prejacent.

To analyze other cases of hypothetical and counterfactual anaphora, I assume that non-veridical local contexts are not necessarily provided anaphorically. Instead, anaphora is possible whenever the referent can be assumed to exist in the local context without contradiction.

A key result from considering counterfactual anaphora will be that local contexts should be able to include (im)possibilities distinct from the global context, while many approaches limit local contexts to be subsets of the global context (e.g., [Heim 1982, et seq.](#); [Brasoveanu 2006](#); [Gotham 2019](#); [Elliott 2022](#); [Mandelkern 2022](#)). The flat-update systems in [Stone \(1999\)](#); [Brasoveanu \(2006\)](#) offer tools for handling dependencies in counterfactual contexts in principle, but neither explicitly addresses anaphora to negated content or the discourse effect of negation.

To address anaphora with negated antecedents, I adapt [Stone and Hardt's \(1999\)](#) dynamic negation, and assume flexible accommodation of existential presuppositions, applying the flat-update approach to a broader range of cases. This application, informed by careful empirical study of how veridicality affects anaphora, facilitates a comprehensive account of anaphoric accessibility.

1.3 Overview of the paper

In the following, [Section 2](#) illustrates how intensional and epistemic information about drefs is stored, how individual-denoting expressions are interpreted in their local context, and how that addresses anaphoric accessibility. [Section 3](#) introduces Intensional CDRT, the formal system used for the analysis. It provides definitions, assumptions for modeling language meaning, and illustrates the mechanism for anaphora. [Section 4](#) presents the analysis of anaphora to negated content, providing detailed derivations for (2a–d). [Section 5](#) compares this analysis to previous accounts of anaphora to hypothetical and counterfactual content. [Section 6](#) concludes.

2 Speaker commitments about discourse referents

This section illustrates the relationship between the global context set, local contexts, and intensional representations of individual drefs, using the example in (13).

- (13) S: Mary doesn't own [a car]^v
- even though Cole said that it_v's red.
 - #even though Cole knows that it_v's red.

The dynamic approach adopted here assumes that utterances update a discourse state, which stores information about drefs invoked in discourse. In Section 3, discourse states are formally defined as variable assignments, following Groenendijk and Stokhof 1991 and Muskens 1996.

2.1 Propositional operators and local contexts

I assume that sentential operators specify relations over *propositions* (sets of worlds), and propositional content invoked explicitly, by the preajcent of a propositional operator or a matrix assertion, contributes propositional drefs (Stone 1999; Bittner 2001, 2007; Brasoveanu 2006, 2010a; Murray 2014; Snider 2017).

For illustration, consider a toy universe with six worlds: in w_1 and w_2 , Mary doesn't own a car, and in w_3 – w_6 , she does. When someone says *Mary doesn't own a car*, two propositional drefs are introduced: ϕ_2 , for the negative preajcent, contains worlds where Mary owns a car (w_3 – w_6); and ϕ_1 , for the main assertion, is the negation of ϕ_2 and contains worlds not in ϕ_2 (w_1 – w_2), as shown in Figure 1.

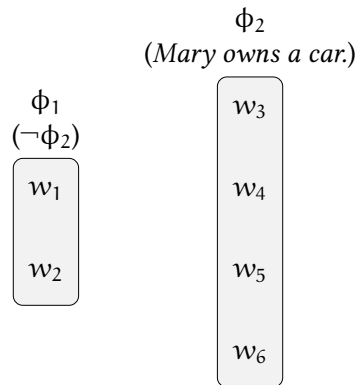


Figure 1: Propositional drefs contributed by update with *Mary doesn't own a car*.

Such propositional variables are assumed for matrix and embedded content throughout, independently motivated by two factors: first, by Snider's (2017) generalization that arguments of propositional operators systematically provide propositional antecedents; second, analogously to intensional systems that use variables for worlds of evaluation rather than interpretation function parameters (e.g., the version of Montague's 1973 Intensional Logic in Dowty et al. 1981). Here, intensional points of evaluation are sets of worlds, as in Heim 1983b, 1992. Propositional drefs, thus, integrate

Stalnakerian/Heimian context sets into a dynamic semantics with variable assignments, as in [AnderBois et al. 2015](#) for appositive relative clauses.

Modeling local contexts as propositional drefs offers flexibility due to the random, non-deterministic nature of dref introduction ([Groenendijk and Stokhof 1991](#)), allowing new drefs to point to any object of the appropriate type, provided it satisfies the conditions imposed by information given about the referent. The local context variable ϕ_2 can, in principle, represent any set of worlds where the embedded content is true, if it also satisfies the relationship introduced by the propositional operator and the pronoun’s accessibility conditions.

One may expect that incorporating Heimian local context sets allows for combining a flat-update semantics for individual drefs with a Heimian account of projection and satisfaction of the pronominal existential presupposition.² However, that account relies on local contexts being subsets of the global context. Consequently, local contexts under negation cannot include (im)possibilities distinct from the global context, preventing the account from handling counterfactual anaphora. To see why, consider the Heimian negation in (14) and its predictions for anaphora between two negative environments (15).

(14) Negation, [Heim 1983b](#): (15)

$$c' = c + \neg p = c - c_1$$

$$c_1 = c + p$$

(15) [Kibble 1994](#): (31), attributed to Paul Dekker

John doesn’t have [a car]^{v₁}, so he doesn’t have to wash it_{v₁}.

According to (14), update with the first clause in (15) yields an output context c' containing all c -worlds where John’s car does not exist. The negated pronoun in the second clause presupposes that it does. Since this presupposition is not met in the new input context c' , Heim predicts accommodation. But because c' already contradicts the presupposition, both global and local accommodation would create an inconsistent discourse, predicting (15) to be unacceptable.³

Heim’s approach analyzes projection of embedded presuppositions by testing whether they can be admitted to the global context. Consequently, any local context under negation will be a subset of the global context, causing the contradiction in (15). To handle anaphora with counterfactual antecedents, hypothetical local contexts, like ϕ_2 above, must point more freely to a set of worlds where the embedded content is true without requiring overlap with the global context.

This adjustment, while necessary, sacrifices Heim’s account of presupposition satisfaction and projection, which relies on the connection between local and global con-

²Many thanks to an anonymous reviewer who pushed me to make this point explicit.

³A similar point can be made for a presuppositional variant, e.g., [Kadmon 2001](#) Ch. 8, (5):
A: *I don’t have a dog.*; B: *So at least you don’t have to walk your dog.*

texts. Notably, Heim’s (1992) analysis of non-veridical doxastic attitudes (like English *believe*) also dissociates local from global contexts. Here, embedded presuppositions are evaluated against the attitude-holder’s doxastic state, predicting no projection — an issue not resolved there. Reconciling an analysis of projection with counterfactual anaphora is deferred to future research. For now, the anaphora to negated antecedents discussed here can be addressed by interpreting the existential presupposition locally. Below, I therefore treat this presupposition as locally accommodated, on par with asserted content, by essentially deriving it as an entailment.

2.2 Interpreting individuals relative to worlds

We admit all individual drefs to the global discourse state, even counterfactual ones, while storing information about the worlds where they refer. This is done by interpreting individual drefs intensionally, relative to the possible worlds in the local context of the expression invoking them. Variable mappings can vary across worlds, and variables may have a referent in some worlds, but not others (Carnap 1947; Stone 1999; Brasoveanu 2006).

For our universe, assume three individuals: the person Mary, and two cars *a* and *b*. Mary owns *a* in w_3 and w_4 , and Mary owns *b* in w_5 and w_6 . Figure 2 illustrates individual and propositional drefs contributed by an utterance of *Mary doesn’t own [a car]^v*.

The indefinite *a car* introduces a dref *v* relative to the worlds in its local context: For each ϕ_2 -world, *v* points to a car that Mary owns in that world. For worlds outside ϕ_2 , *v* has no referent, modeled by mapping it to a dummy individual \star . Following Brasoveanu 2006, 2010a,b, \star is the universal falsifier — a semantic object of which any lexical relation is false.

2.3 Commitment and veridicality

Assertion is modeled as intersective update of a global context set (Stalnaker 1978, 2002; Heim 1982, 1983b). Here, we assume that the global context is a speaker commitment set (following Gunlogson 2001),⁴ and a designated propositional variable ϕ_{DC_S} stores the current commitment set of a speaker *S* (following AnderBois et al. 2015). For illustration, assume that *S* enters into the discourse with some commitments and ϕ_{DC_S} initially points to $\{w_1, w_2, w_3, w_5\}$. Their assertion of *Mary doesn’t own a car* narrows ϕ_{DC_S} to contain only ϕ_1 -worlds, illustrated in Figure 3.

⁴Treating the global context as representing speaker commitments, rather than a common ground, enables a straightforward analysis of disagreement cases (Section 4.3). The common ground between interlocutors *A* and *B* is the intersection of their commitment sets (i.e., $CG_{A,B} = DC_A \cap DC_B$, Gunlogson 2001: pp. 42–43).

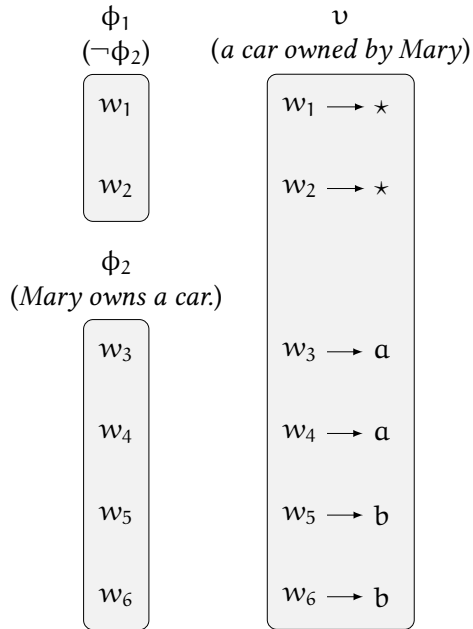


Figure 2: Drefs contributed by an utterance of *Mary doesn't own a car*.

The discourse state tracks S's epistemic stance about individual drefs by their relation to the global context set. We operationalize speaker commitments about drefs by distinguishing veridical, hypothetical, and counterfactual drefs:

- (16) Given a speaker S and a discourse state,
- a. a propositional dref ϕ is *veridical* iff it points to a proposition whose truth S is committed to ($\phi_{DC_S} \subseteq \phi$). ϕ is *hypothetical* iff S is not committed to its truth ($\phi_{DC_S} \not\subseteq \phi$), and *counterfactual* iff S is committed that it's false ($\phi_{DC_S} \cap \phi = \emptyset$).
 - b. an individual dref v is *veridical* iff for all worlds in ϕ_{DC_S} , v has a referent (other than *). v is *hypothetical* iff that is not the case, and *counterfactual* iff it points to * for all worlds in ϕ_{DC_S} .

Accordingly, in our example, ϕ_1 is a veridical dref, whereas ϕ_2 and v are hypothetical, more specifically, counterfactual drefs.

2.4 Anaphoric pronouns in their local contexts

When S asserts *Cole said that it_v's red*, the pronoun is interpreted as a variable v with an existential presupposition. The question is whether this inference can be

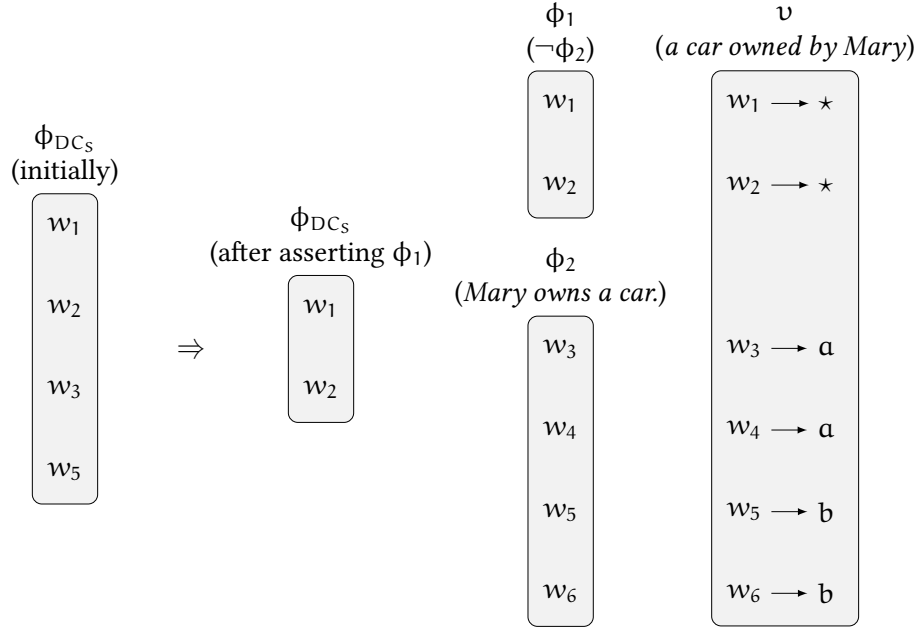


Figure 3: Discourse update where S asserts *Mary doesn't own a car*.

consistently locally accommodated. In this account, *local accommodation* means that the existential inference affects the local context ϕ_2 in the same way as the entailed content in the scope of negation does – by imposing a condition that it is true in all ϕ_2 -worlds.

For illustration, assume that a is red in all worlds, b is not, and *Cole said that it_v's red* in worlds w_1, w_3, w_5 . The update, illustrated in Figure 4, adds a propositional dref ϕ_3 for the matrix content (w_1, w_3, w_5) and narrows ϕ_{DC_S} by eliminating w_2 , where it is false. It also adds ϕ_4 for the embedded content, providing a local context for interpreting the pronoun. According to (7), the dref v can provide its value only if (i) the referent of v exists in all ϕ_4 -worlds, and (ii) this is consistent with other discourse information, particularly the veridicality of the embedding context. We call a discourse *consistent* iff it characterizes a discourse state where the commitment set ϕ_{DC_S} is non-empty.

(i) Existential presupposition. If the pronominal presupposition is locally accommodated in ϕ_4 , v must exist in all ϕ_4 -worlds. v was introduced counterfactually, relative to ϕ_2 , and has a referent in all and only the ϕ_2 -worlds (w_3 – w_6). Consequently, ϕ_4 can only contain worlds from the counterfactual antecedent context ($\phi_4 \subseteq \phi_2$),

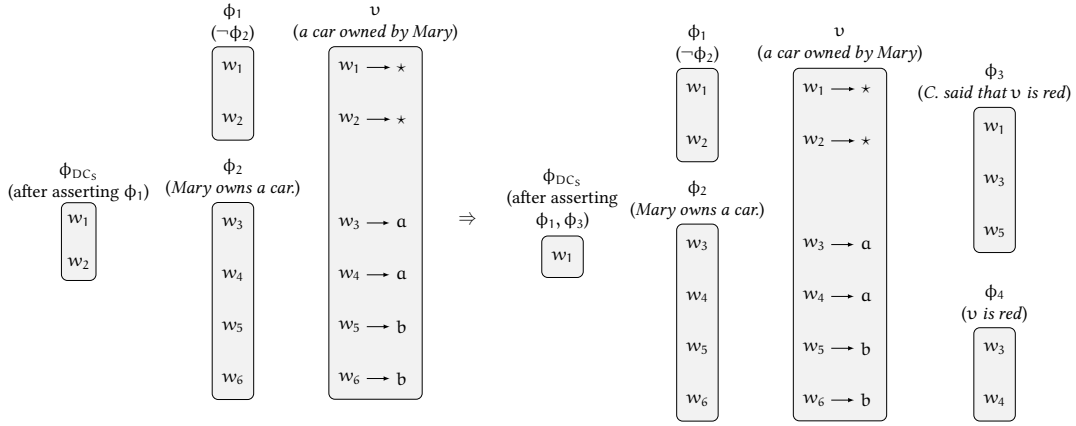


Figure 4: Discourse update when S utters *Cole said that it_v's red*.

and thus cannot overlap with ϕ_{DC_S} . Therefore, ϕ_4 is also interpreted counterfactually, due to the anaphoric dependency.

(ii) Consistency. Accommodating the existential presupposition in ϕ_4 works only if this is consistent with other discourse information. ϕ_4 is further constrained by the embedded content and the veridicality of the embedding context. Instead of containing all worlds where v has a referent, ϕ_4 is limited to worlds where the content of the prejacent is true and v maps to the red car a (w_3, w_4). Additionally, ϕ_4 is introduced by the complement of *Cole said*, establishing the relation that Cole said ϕ_4 in all ϕ_3 -worlds. Since this is a non-veridical relation, ϕ_4 does not need to be entailed by ϕ_3 or ϕ_{DC_S} . Therefore, accommodating v 's existence in ϕ_4 avoids contradictory commitments for S, and anaphora is predicted to be possible.

Conversely, if ϕ_4 were introduced veridically, as in (13b): *Cole knows that it's red*, accommodating the presupposition in ϕ_4 would conflict with the veridicality of the embedding. The veridical relation requires that ϕ_4 is entailed by the main assertion ϕ_3 , and, in turn, by ϕ_{DC_S} ($\phi_{DC_S} \subseteq \phi_4$). This would, further, require that v were interpreted veridically, in ϕ_4 . But since v is counterfactual, this would create contradictory commitments for S, making this discourse inconsistent.

2.5 Interim conclusion

In a flat-update dynamic system, all drefs are introduced globally into the discourse state (e.g., Figure 4), and individual drefs store information about the circumstances where they refer. Here, accessibility constraints are stated relative to propositional information available in discourse and systematically affected by truth-functional

properties of sentential operators, particularly their veridicality. This section illustrated how these constraints operate at a denotational level by constraining relationships between the denoted objects.

To account for accessibility based on existential inferences and discourse consistency, drefs must be interpreted intensionally. While propositional drefs are already intensional, individual drefs must be relativized to possible worlds, allowing the existential inference to be evaluated relative to those worlds. The next section illustrates the formal implementation.

3 Intensional CDRT

The analysis is implemented in an intensional version of Compositional DRT (CDRT, [Muskens, 1996](#)), which emulates dynamic update in a static logic. I most closely follow [Brasoveanu’s \(2006\)](#) implementation of CDRT (Dynamic Ty₂), adding intensionality and propositional drefs.

The system uses four basic types: *t* (truth-values), *e* (entities), *w* (worlds), and *s* (variable assignments). Object-language utterances are translated into a dynamic DRS-metalanguage, representing updates as defined in (17), comprising a list of new drefs ($\delta_1, \dots, \delta_n$), and a series of output conditions (C_1, \dots, C_n). Updates are interpreted as static relations between an input assignment i_s and output j_s .

$$(17) \quad \boxed{\begin{array}{c} \delta_1, \dots, \delta_n \\ C_1, \\ \dots \\ C_n \end{array}} := \lambda i_s. \lambda j_s. i[\delta_1, \dots, \delta_n]j \wedge C_1(j) \wedge \dots \wedge C_n(j)$$

ICDRT differs from classic nested-update systems in three key ways (see also [Stone, 1999](#); [Brasoveanu, 2006, 2010a](#); [Hofmann, 2022](#)): (i) flat update: all drefs are introduced globally; (ii) static operators: the truth-functional contribution of sentential operators is expressed as static relations over sets of worlds; (iii) intensional drefs: variable-to-entity mappings are relativized to possible worlds.

To achieve this, I adopt an approach of encapsulated quantification over possible worlds ([Stone 1997, 1999](#); [Bittner 2001, 2007, 2011](#)), where dependencies of individuals on modal quantification are stored in complex static objects referred to by drefs.⁵

⁵Other dynamic approaches to intensionalizing individual drefs include representing discourse contexts as sets of world-assignment pairs ([Heim 1982, et seq.](#)) or as plural information states in a system with selectively distributive update ([Brasoveanu 2006, 2010a](#)). Analyses using world-assignment pairs ([van Rooij 2000](#); [Elliott 2020, 2022](#); [Mandelkern 2022](#)) face empirical limitations discussed in Section 5. Brasoveanu’s plural system provides a comprehensive framework for anaphora to quantificational dependencies. While this paper focuses on the effect of intensional and epistemic infor-

Specifically, individual drefs store *individual concepts*, i.e., functions from worlds to entities (Carnap 1947). To model cases where a variable has no referent in certain worlds, we include the universal falsifier \star_e as a dummy value in the domain of entities (following Brasoveanu 2006).

This section introduces the formal system by illustrating how it captures the contrast that a veridical pronoun (18c) is acceptable with a veridical antecedent (18a) but not with a counterfactual one (18b).

- (18) a. [There is [a bathroom]^v here.]^{ϕ₁}
 b. [There isn't [[a bathroom]^v here.]^{ϕ₂}]^{ϕ₁}
 c. [It_v is upstairs.]^{ϕ₃}

Sections 3.1 and 3.2 illustrate the interpretation of the antecedent utterances. Section 3.1 introduces propositional drefs, conditions on their interpretation, and the contribution of propositional operators. While both (18a) and (18b) introduce a veridical propositional dref ϕ_1 , for the main assertion, only (18b) also invokes a counterfactual proposition ϕ_2 for the negative prejacent. Section 3.2 outlines how indefinites, individual drefs, and predicative conditions are interpreted relative to their local intensional context. In (18a), the dref v for *a bathroom* is introduced veridically (relative to ϕ_1), whereas in (18b), it is introduced counterfactually (relative to ϕ_2). Section 3.3 explains how the pronoun in a veridical context (18c) is licensed by the veridical antecedent in (18a) and provides definitions for assertion, commitment sets, and discourse consistency. Section 3.4 shows how interpretation fails for the veridical anaphor (18c) with a counterfactual antecedent (18b). A full list of ICDRT definitions and meta-language translations for a fragment of English are provided in the appendix.

3.1 Drefs, conditions, and propositional operators

Representations for the two (simplified) antecedent utterances are given in (19).

(19) a. $\text{DEC}_S^{\phi_1}(\textit{there is } a^v \textit{ bathroom}) \quad \rightsquigarrow$

$\phi_1, \phi_1 : v$
$\phi_{\text{DC}_S} \in \phi_1$
$\textit{bathroom}_{\phi_1}(v)$

mation on (singular) anaphora, choosing a simpler system, the analysis could also be expressed in Brasoveanu's system (with appropriate assumptions about counterfactual contexts).

$$\begin{array}{l}
\text{b. } \text{DEC}_S^{\phi_1}(\text{NOT}^{\phi_2}(\text{there is } a^v \text{ bathroom})) \quad \rightsquigarrow \\
\begin{array}{|l}
\hline
\phi_1, \phi_2, \phi_2 : v \\
\hline
\phi_{\text{DC}_S} \in \phi_1 \\
\phi_1 = \overline{\overline{\phi_2}} \\
\text{bathroom}_{\phi_2}(v) \\
\hline
\end{array}
\end{array}$$

Assumptions about the contribution of linguistic expressions and formal definitions for the components of (19) will be presented throughout this section, starting with propositional drefs and conditions on their interpretation.

Introducing drefs. A dref δ is a function from assignments to model objects.⁶ For example, the propositional dref ϕ_1 in (19a) is a function of type $s(\text{wt})$, from assignments to sets of worlds in which there is a bathroom. Drefs are introduced to a discourse state by *variable update*, which is random assignment of values to a variable (Groenendijk and Stokhof 1991). The relation $i[\delta]j$ holds between assignments i_s and j_s iff they differ at most in the value assigned to δ . Because variable update is non-deterministic, any model object of matching type is a possible referent. Subsequent discourse updates successively constrain possible values by imposing conditions on the output state.

DRS-conditions are properties of discourse states. Given a state i_s dynamic predication over drefs is interpreted by checking whether the corresponding static relation holds of the referents of the arguments at i . For instance, the assertions in (19) impose the condition (20a), requiring that ϕ_{DC_S} entails ϕ_1 at the output, based on dynamic inclusion (20b).

$$\begin{array}{ll}
(20) \quad \text{a. Assertion:} & \phi_{\text{DC}_S} \in \phi_1 = \lambda i_s. \phi_{\text{DC}_S}(i) \subseteq \phi_1(i) \\
\quad \quad \text{b. Dynamic inclusion:} & \phi_1 \in \phi_2 := \lambda i_s. \phi_1(i) \subseteq \phi_2(i)
\end{array}$$

In (19b), negation contributes the output condition (21a), requiring that ϕ_1 and ϕ_2 are complements, based on definitions in (21b+c).

$$\begin{array}{ll}
(21) \quad \text{a. Negation:} & \phi_1 = \overline{\overline{\phi_2}} = \lambda i_s. \phi_1(i) = \overline{\phi(i)} \\
\quad \quad \text{b. Dynamic identity:} & \alpha \equiv \beta := \lambda i_s. \alpha(i) = \beta(i) \\
\quad \quad \text{c. Dynamic complementation:} & \overline{\overline{\phi}} := \lambda i_s. \overline{\phi(i)}
\end{array}$$

The truth-functional contribution of sentential operators is captured as static relations over sets of worlds, allowing for a classical boolean treatment. This distin-

⁶Following Landman (1986), Muskens (1996), and Brasoveanu (2006), I treat assignments as atomic objects, and drefs as type-lifted variables functioning as projection functions over assignments.

guishes flat update from nested update systems, which treat sentential operators as operations over updates.

Declarative assertion. Both utterances in (21) are assertions of a proposition ϕ_1 by a speaker S . I assume that a declarative sentential mood operator $\text{DEC}_S^{\phi_1}$ combines with the propositional content of an utterance (Wittgenstein 1953; Stenius 1967) and introduces ϕ_1 as a dref storing the content of its prejacent (Bittner 2001, 2007, 2011; Murray 2014). The pragmatic effect of assertion as intersective update of the context set is captured by the condition $\phi_{\text{DC}_S} \in \phi_1$.

This condition narrows permissible values of ϕ_{DC_S} to subsets of ϕ_1 by indexically invoking the dref ϕ_{DC_S} and stating that ϕ_1 is true in these worlds. The compositional contribution of the declarative operator is given in (22), where $;$ is dynamic conjunction (i.e. relation composition).

$$(22) \quad \text{DEC}_S^{\phi} \rightsquigarrow \lambda \mathcal{P}_{s(\text{wt}),s(\text{st})}.[\phi \mid \phi_{\text{DC}_S} \in \phi]; \mathcal{P}(\phi)$$

Negation. I assume that propositional operators combine with the propositional content of their prejacent and introduce a dref for that content (following Stone and Hardt 1999; Brasoveanu 2006; Snider 2017). In (19b), negation contributes the dref ϕ_2 and the condition stating that ϕ_1 and ϕ_2 are contradictory opposites ($\phi_1 = \overline{\overline{\phi_2}}$). The compositional translation of sentential negation is shown in (23).

$$(23) \quad \text{NOT}^{\phi'} \rightsquigarrow \lambda \mathcal{P}_{s(\text{wt}),s(\text{st})}.\lambda \phi.[\phi' \mid \phi \equiv \overline{\overline{\phi'}}]; \mathcal{P}(\phi')$$

3.2 Interpreting individual drefs in their local context

Indefinites introduce drefs relative to a local context. For illustration, the representation (19) is repeated here:

(19) a.	$\text{DEC}_S^{\phi_1}(\textit{there is } a^v \textit{ bathroom})$	\rightsquigarrow	$\phi_1, \phi_1 : v$
			$\phi_{\text{DC}_S} \in \phi_1$ $\text{bathroom}_{\phi_1}(v)$
b.	$\text{DEC}_S^{\phi_1}(\text{NOT}^{\phi_2}(\textit{there is } a^v \textit{ bathroom}))$	\rightsquigarrow	$\phi_1, \phi_2, \phi_2 : v$
			$\phi_{\text{DC}_S} \in \phi_1$ $\phi_1 = \overline{\overline{\phi_2}}$ $\text{bathroom}_{\phi_2}(v)$

In (19b), the term $[\phi_2 : v]$, contributed by the indefinite, introduces an individual dref v relative to the local context under negation ϕ_2 (based on Stone 1999; Stone and Hardt 1999). The condition contributed by the predicate ($\text{bathroom}_{\phi_2}\{v\}$) ensures that v is a bathroom in all ϕ_2 -worlds. In (19a), without negation, this condition and the introduction of v are interpreted relative to the matrix proposition ϕ_1 .

The representation for the dynamic proposition contributed by the clause *there is a^v bathroom* is given in (24),⁷ illustrating that the local context for interpreting propositional content is compositionally provided as a propositional variable.

$$(24) \quad \lambda\phi.[\phi : v \mid \text{bathroom}_{\phi}(v)]$$

3.2.1 Introducing individual drefs

An *individual dref* v is a function of type $s(\text{we})$ mapping assignments and worlds to individuals. Values of individual drefs can therefore differ based on assignments and worlds. To model cases where an individual dref does not have a referent, assume that individual concepts can map to a dummy element \star_e , s.t. any lexical relation yields falsity whenever \star is one of its arguments.⁸

Relative Variable Update. An indefinite α^v introduces an individual dref v along with information about the worlds in which it has a referent. This is done by interpreting variable update relative to possible worlds in the local context ϕ (based on Stone 1999; Stone and Hardt 1999).

$$(25) \quad \text{Variable Update with } v \text{ relative to } \phi \\ i[\phi : v]j \text{ abbreviates the conjunction of:} \\ \text{(i) } i[v]j \\ \text{(ii) } \forall w_w. (\phi(j)(w) \leftrightarrow v(j)(w) \neq \star)$$

(25i) states that j is an update of i that differs at most wrt the value assigned to v . (25ii) states that j maps v and ϕ in a way that, for any world w , $v(j)(w)$ maps to something other than \star (i.e., it has a referent) iff $\phi(j)(w)$ is true, and to \star otherwise. This definition of relative variable update encapsulates universal quantification over possible worlds in the interpretation of indefinites, and ensures that v has a referent in all and only the ϕ -worlds.

Stone’s (1999) formulation of relative variable update involves unidirectional implication in condition (25ii), stating that v exists in all ϕ -worlds, while allowing that it

⁷This simplified representation for existential sentences is based on Francez’s (2009) assumption that the existential “pivot” (here: *a bathroom*) is a generalized quantifier. Francez suggests that a contextual domain implicitly provides the nuclear scope argument, which is omitted here for simplicity.

⁸To guarantee this, we axiomatically state that for any n -ary lexical relation R : $[[R]]^M \subseteq (D_e^M \setminus \{\star\})^n \times D_w^M$, based on Brasoveanu 2010a, fn. 5.

may exist in non- ϕ -worlds. However, to address unspecific indefinites under negation, it is crucial to assume that their referents does not exist outside of the local context. For instance, (26) is ruled out because the first utterance is inconsistent with existence of a referent on a global level.

(26) #There isn't a bathroom. It is upstairs.

For our example (19), relative variable update guarantees that v has a referent in all and only ϕ_1 -worlds for the positive case, and the ϕ_2 -worlds in the negative case.

Predication. Conditions involving predicates (like $\text{bathroom}_{\phi_1}(v)$) are interpreted relative to a propositional variable ϕ (indicated using subscripts on predicates) and abbreviate output conditions as in (27).

(27) Dynamic predication
 $R_{\phi}(v) := \lambda i_s. \forall w_w. (\phi(i)(w) \rightarrow R(v(i)(w))(w))$
 where $R_{e(wt)}$ is a unary lexical relation

(27) ensures that ϕ serves as a local context for interpreting the relation invoked by the condition: given a state i , for each world w in $\phi(i)$, R is true of the individual stored in v at i and w . This definition encapsulates universal quantification over the worlds in the local context in the interpretation of predicates. In (19), the predicative condition (i.e., v is a bathroom) is interpreted relative to the veridical context ϕ_1 in the affirmative case and the counterfactual ϕ_2 in the negated case.

3.2.2 Local contextual entailment

From the definitions of relative variable update and predication, it follows that individual variables locally entail the existence of a referent. While this inference is part of the presupposed content for pronouns, I will show here only how it follows from the at-issue contribution of both indefinites and the interpretation of individual variables in predicative conditions (for reasons discussed in Section 2.1).

(28) defines that an individual variable is entailed in its local context, iff it has a referent throughout the local context at a given discourse state.

(28) v is entailed in the context of ϕ at i , iff:
 For each world in $\phi(i)$, $v(i)$ has an existing referent, i.e.:
 $\forall w_w. (\phi(i)(w) \rightarrow v(i)(w) \neq \star_e)$

For indefinites, local contextual entailment in the local context ϕ at output j follows from the condition on relative variable update (25ii). For interpreting an individual variable v in a condition $R_{\phi}(v)$, it follows from the assumption that \star_e is a falsifier. For any world w , it will be the case that: (29a) any assignment mapping v to \star at w

it will make R false at w ; and, by contraposition, (29b) any assignment mapping v to an individual making R true will not map it to \star .

- (29) For any unary relation R , individual v , assignment i , and world w ,
- $v(i)(w) = \star \rightarrow \neg R(v(i)(w))(w)$.
 - $R(v(i)(w))(w) \rightarrow v(i)(w) \neq \star$.

Accordingly, indefinites or individual variables in predicative conditions introduce an existential inference to their local context.

3.3 Veridical drefs, speaker commitments, and consistency

We can now show how (19a), repeated here, provides an antecedent for a veridical anaphor in (30).

(19a)	$\text{DEC}_S^{\phi_1}(\textit{there is a}^v \textit{ bathroom})$	\rightsquigarrow	$\phi_1, \phi_1 : v$
			$\phi_{\text{DC}_S} \in \phi_1$ $\textit{bathroom}_{\phi_1}(v)$
(30)	$\text{DEC}_S^{\phi_3}(\textit{it}_v \textit{ is upstairs})$	\rightsquigarrow	ϕ_3
			$\phi_{\text{DC}_S} \in \phi_3$ $\textit{upstairs}_{\phi_3}(v)$

I first explicate assumptions about assertion and speaker commitments, in order to define discourse consistency and illustrate how the dref v in (19a) is interpreted as veridical. Then, I show how interpreting the pronoun in (30) veridically yields a consistent discourse.

3.3.1 Commitment and consistency

Discourse contexts. We interpret updates in a discourse context relative to a model providing the semantic interpretation for the underlying static logic, along with information about the interlocutors and current discourse state. A *discourse context* C is a tuple $\langle M, \text{INT}, i \rangle$, where:

- M is a model,
- $\text{INT} \subseteq D_e^M$ is the set of interlocutors, and
- $i_s \in D_s^M$ is the current discourse state.

A *discourse state* i is an assignment that, for each $x \in \text{INT}^C$, stores a propositional dref ϕ_{DC_x} to represent x 's commitments, which can be referred to indexically.

Discourse consistency is enforced by axiomatically assuming that commitment sets are non-empty for any discourse state i and interlocutor x :

$$(31) \quad \text{Consistency:} \quad \forall i_s, x_e (x \in \text{INT} \rightarrow \phi_{DC_x}(i) \neq \emptyset)$$

We assume idealized initial discourse states, which contain no information except that about the conversational context. These are derived from a *null assignment* (32) holding no information for discourse variables.

$$(32) \quad \text{An null assignment } i_\emptyset \text{ is an assignment, s.t.:} \\ \forall v_{s(\text{we})} (v(i_\emptyset) = \lambda w. \star), \text{ and } \forall \phi_{s(\text{wt})} (\phi(i_\emptyset) = \lambda w. 1)$$

An *initial state* (33) is one resulting from implicitly introducing a dref for each interlocutor's commitment set to a null assignment:

$$(33) \quad \text{An initial state } i_0^C \text{ in context } C \text{ is s.t. for all } A, B, \dots \in \text{INT}^C: \\ i_\emptyset[\phi_{DC_A}, \phi_{DC_B}, \dots]i_0^C$$

Propositional drefs associated with speaker commitments are introduced implicitly and accessed indexically via the declarative operator. Non-deterministically introducing these variables and successively placing conditions on them models the pragmatics of intersective assertion (as in [AnderBois et al. 2015](#)).

Discourse update is defined in (34):

$$(34) \quad \text{An update with } D_{s(\text{st})} \text{ in context } C = \langle M, \text{INT}, i \rangle \text{ is successful iff there exists} \\ \text{some output context } C' = \langle M, \text{INT}, j \rangle, \text{ s.t.} \\ \llbracket D(i)(j) \rrbracket^M = 1.$$

We assume that an update is acceptable iff it is successful and leaves all interlocutors with non-empty commitment sets (as required by discourse consistency).

3.3.2 Assertion and veridical drefs

Assertion of (19a) by speaker S constrains possible values for ϕ_{DC_S} .

$$(19a) \quad \text{DEC}_S^{\phi_1}(\textit{there is } a^v \textit{ bathroom}) \quad \rightsquigarrow \quad \begin{array}{|c|} \hline \phi_1, \phi_1 : v_1 \\ \hline \phi_{DC_S} \in \phi_1 \\ \text{bathroom}_{\phi_1}(v) \\ \hline \end{array}$$

For illustration, assume a context $C_1 = \langle M_1, \{S\}, i \rangle$, where $\{S\}$ is a singleton set of interlocutors, i is an initial state, and M_1 is a model, such that:

- $D_w^{M_1} = \{w_{bu}, w_b, w_u, w_0\}$,
- $D_e^{M_1} = \{S, b, \star\}$,
- b is a bathroom in w_{bu} and w_b , and
- b is upstairs in w_{bu} and w_u .

At the initial state i , any non-empty subset of D_w is a possible value for ϕ_{DC_S} , reflecting the absence of conversational commitments. Updating C_1 with (19a) yields an output context $C'_1 = \langle M_1, \{S\}, j \rangle$, where possible values of $\phi_{DC_S}(j)$ are propositions where there is a bathroom.

Table 1: Assertion eliminating possible values of ϕ_{DC_S}

$\phi_{DC_S}(i)$	$\phi_{DC_S}(j)$
$\{w_{bu}, w_b, w_u, w_0\}$	
$\{w_{bu}, w_b, w_u\}$	
$\{w_{bu}, w_b, w_0\}$	
$\{w_{bu}, w_u, w_0\}$	
$\{w_b, w_u, w_0\}$	
$\{w_{bu}, w_b\}$	$\{w_{bu}, w_b\}$
$\{w_{bu}, w_u\}$	
$\{w_{bu}, w_0\}$	
$\{w_b, w_u\}$	
$\{w_b, w_0\}$	
$\{w_u, w_0\}$	
$\{w_{bu}\}$	$\{w_{bu}\}$
$\{w_b\}$	$\{w_b\}$
$\{w_u\}$	
$\{w_0\}$	

(19a) affects S 's commitments as illustrated in Table 1 by introducing a propositional dref ϕ_1 , placing conditions on its interpretation and then using ϕ_1 to constrain ϕ_{DC_S} . First, v is introduced relative to ϕ_1 , which, due to local contextual entailment, has existential import: v has a referent in the ϕ_1 -worlds. Second, the condition $\text{bathroom}_{\phi_1}(v)$ ensures that ϕ_1 only contains worlds where v is a bathroom. Therefore, ϕ_1 can only point to a set of worlds with a bathroom, like $\{w_{bu}, w_b\}$. Finally, the condition $\phi_{DC_S} \in \phi_1$ limits possible values of ϕ_{DC_S} to subsets of ϕ_1 .

Table 2: Assertion eliminating possible values of ϕ_{DC_S}

$\phi_{DC_S}(j)$	$\phi_{DC_S}(k)$
$\{w_{bu}, w_b\}$	
$\{w_{bu}\}$	$\{w_{bu}\}$
$\{w_b\}$	

Informativeness and defeasible maximization. After the assertion, any non-empty set of worlds where there is a bathroom is a possible value for $\phi_{DC_S}(j)$. The discourse state stores the current context set and all its non-empty subsets (as in [Gunlogson 2001](#); [AnderBois et al. 2015](#)). However, only the largest one ($\{w_{bu}, w_b\}$) accurately reflects S 's commitments, since they have not committed about whether 'b is upstairs', which is true in w_{bu} , but false in w_b . To resolve this, we assume a pragmatic maximization mechanism for commitment sets: (35) defines an output state j as pragmatically privileged when no other possible assignment maps a superset to the drefs for commitment sets in the context.

- (35) Pragmatic maximization for commitment sets:
 When updating context C with D , those possible outputs j most accurately reflect the available information, s.t.:
- $\llbracket D(i_C)(j) \rrbracket^{M_C} = 1$, and
 - $\llbracket \forall h_s, x_e ((D(i_C)(h) \wedge x \in \text{INT}_C) \rightarrow \neg(\phi_{DC_x}(j) \subset \phi_{DC_x}(h))) \rrbracket^{M_C} = 1$

(35) can be motivated by Gricean quantity (if the speaker intended to commit to something more informative, they would have done so; [Grice 1975](#)), or the information-theoretic principle of maximum entropy (the knowledge state that best represents available information is the one with the most uncertainty; [Jaynes 1957](#)).

Importantly, pragmatic maximization for context sets must be defeasible.⁹ Smaller sets remain available to enable further assertions. For example, asserting $\text{DEC}_S^{\phi_3}(it_u)$ (is upstairs) will restrict ϕ_{DC_S} to $\{w_{bu}\}$, where *there is a bathroom* and *the bathroom is upstairs* (see Table 2). Smaller sets, therefore, cannot be eliminated from consideration, but remain live options for continuing the discourse. However, they will be omitted in the following exposition.

⁹The need to explicitly state defeasible maximization over context sets arises from modeling Heimian context sets as propositional variables in a referential dynamic system, but the mechanism itself is not new. In Heim's update semantics, context set maximization is implicit in the set builder notation, and defeasible in later updates.

Veridical drefs. The dref v for *a bathroom* is introduced relative to its local context ϕ_1 , which itself is entailed by speaker commitments. Consequently, ϕ_1 and v are veridical drefs, according to definitions in Section 2, stated formally here:

(36) Veridical drefs:

A dref δ is *veridical* relative to $\chi \in \text{INT}$ at a discourse state i , iff δ is contextually entailed in $\phi_{\text{DC}_\chi}(i)$, and *non-veridical* otherwise.

- a. For individual drefs: $\forall w(w \in \phi_{\text{DC}_\chi}(i) \rightarrow \delta(i)(w) \neq \star)$
- b. For propositional drefs: $\phi_{\text{DC}_\chi}(i) \subseteq \delta(i)$

(37) Counterfactual drefs:

A dref δ is *counterfactual* relative to $\chi \in \text{INT}$ at a discourse state i , iff its non-existence or falsity is contextually entailed in $\phi_{\text{DC}_\chi}(i)$.

- a. For individual drefs: $\forall w(w \in \phi_{\text{DC}_\chi}(i) \rightarrow \delta(i)(w) = \star)$
- b. For propositional drefs: $\phi_{\text{DC}_\chi}(i) \cap \delta(i) = \emptyset$

Accordingly, v is veridical for S in C'_1 because it has a referent in each ϕ_{DC_S} -world. ϕ_1 is veridical for S in C'_1 because it is entailed by S 's commitments. Figure 5 illustrates these relationships, where the shaded area indicates worlds compatible with speaker commitments, *viz.* w_{bu}, w_b where v points to the bathroom b .

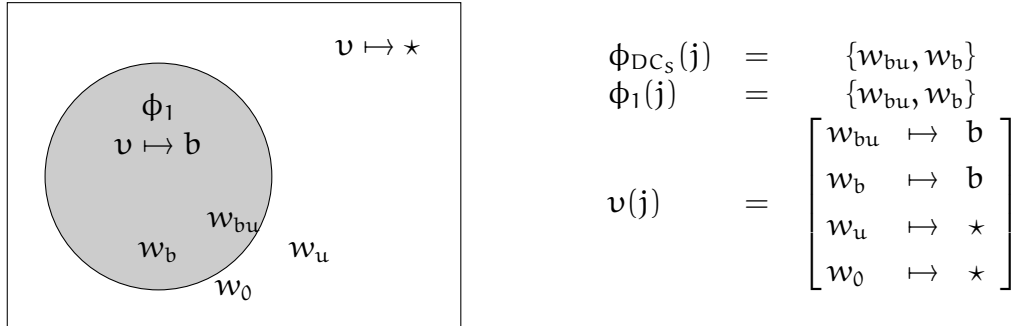


Figure 5: Drefs after updating with ‘*There is [a bathroom]*’ in C_1

3.3.3 Veridical anaphor

Because v was introduced veridically, it can be referenced by a pronoun in a veridical context. Anaphoric pronouns are represented as variables with the same name as their antecedent. Co-indexation is assumed as part of the syntactic representation, while semantic conditions on accessibility filter possible syntactic parses. The representation of the update for the veridical anaphor is repeated here:

$$(30) \quad \text{DEC}_S^{\phi_3}(\textit{it}_v \textit{ is upstairs})$$

\rightsquigarrow

ϕ_3
$\phi_{\text{DC}_S} \in \phi_3$
$\textit{upstairs}_{\phi_3}(v)$

The variable is interpreted in the condition $\textit{upstairs}_{\phi_3}(v)$, stating that v is upstairs in ϕ_3 . Therefore, local contextual entailment requires that v has a referent in the local context ϕ_3 . Because ϕ_{DC_S} entails ϕ_3 , this is a veridical context, and the update commits the speaker that the referent exists.

When updating C'_1 with (30) (see Figure 6), the discourse will remain consistent iff ϕ_{DC_S} is a subset of the non-empty intersection of the two assertions ϕ_1 and ϕ_3 . Because v is locally entailed in both, the local existence requirement does not create contradictory commitments for S , predicting that anaphora is possible.

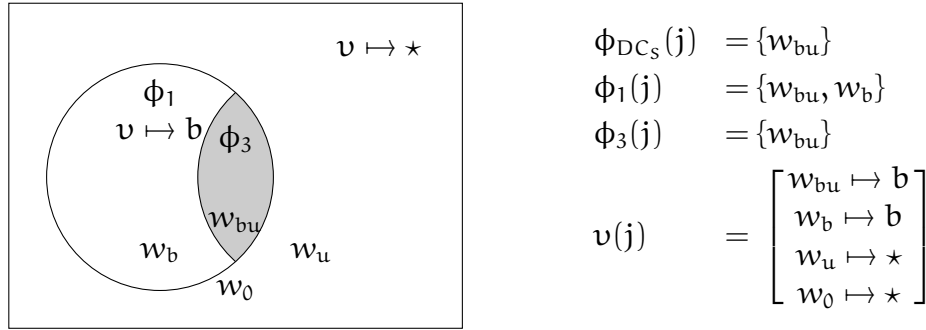


Figure 6: Drefs after updating with ‘*There is [a bathroom]^v*’; ‘*It_v is upstairs.*’

Constraints on individual anaphora. The condition on interpreting individual anaphora in (7) requires that the antecedent referent α^v exists in all worlds in the local context of the anaphor β_v under some consistent interpretation of the discourse. We can now derive this, which we do in two steps:

First, we define which drefs are accessible for a pronoun based on local contextual entailment and consistency: a pronoun β_v can be interpreted at a discourse state i , only if its referent $v(i)$ maps to an individual (other than \star) in each world of its local context $\phi_1(i)$ without causing a contradiction.

(38) Accessibility:

A dref v is accessible for reference by an (object-language) pronoun β_v at a state i , only if v is entailed in the local context ϕ_1 of β_v and the discourse is consistent at i .

By relative variable update, an indefinite α^v introduces a dref v relative to its own local context ϕ_2 , and v is locally entailed in all and only the ϕ_2 -worlds. Therefore, v will be entailed in the anaphor's local context ϕ_1 , only if ϕ_1 is a subset of ϕ_2 . This leads to the *subset requirement*:

- (39) Subset requirement:
 An indefinite α^v can be the antecedent for β_v at i , only if the local context of β_v ϕ_1 is a subset of the local context of α^v ϕ_2 at i .

This derives the condition in (7), requiring that the antecedent referent exists in all worlds in the local context of the anaphor.

3.4 Negation, counterfactual drefs, inaccessible antecedent

To see how the condition on accessibility can fail, consider a veridical anaphor (19b) with a counterfactual antecedent (30):

(19b)	$\text{DEC}_S(\text{NOT}(\textit{there is a bathroom}))$	\rightsquigarrow	$\phi_1, \phi_2, \phi_2 : v$ $\phi_{\text{DC}_S} \subseteq \phi_1$ $\phi_1 = \overline{\phi_2}$ $\textit{bathroom}_{\phi_2}(v)$
(30)	$\text{DEC}_S^{\phi_3}(\textit{it}_v \textit{ is upstairs})$	\rightsquigarrow	ϕ_3 $\phi_{\text{DC}_S} \subseteq \phi_3$ $\textit{upstairs}_{\phi_3}(v)$

3.4.1 Counterfactual antecedent

In (19b), the indefinite is interpreted unspecifically in the scope of negation, introducing v relative to the counterfactual ϕ_2 -worlds. Updating C_1 with (19b) yields an output C_1'' where the discourse state j maps drefs as shown in Figure 7.

At j , ϕ_2 includes worlds where there is a bathroom ($\{w_{bu}, w_b\}$). v maps to the bathroom b for each world in ϕ_2 , and to \star for all other worlds. ϕ_1 is the complement of ϕ_2 , containing all worlds without a bathroom ($\{w_u, w_0\}$), where v maps to \star . Since ϕ_1 represents the asserted proposition, ϕ_{DC_S} is narrowed to entail ϕ_1 .

Propositional maximization. Based on the definitions above, the output C_1'' in Figure 7 is just one of several possible outputs. Again, we must exclude non-maximal values for propositional variables, in this case, for local contexts. Specifically, we select the output j mapping ϕ_2 to the largest set of worlds with a bathroom ($\{w_{bu}, w_b\}$).

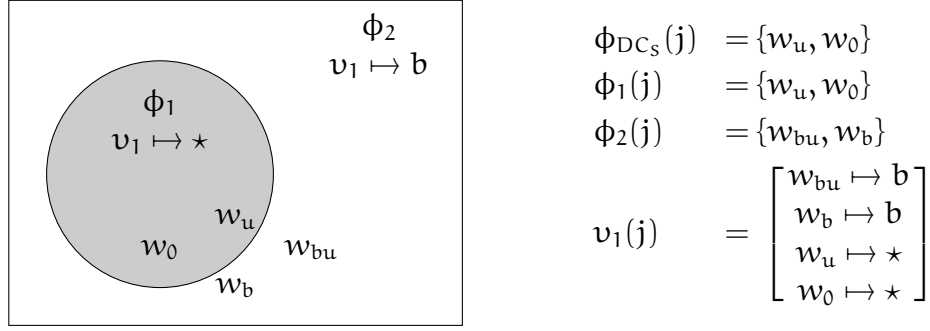


Figure 7: Drefs after updating with $\text{DEC}_S(\text{NEG}(\text{there is a bathroom}))$ in C_1

This is essential for deriving accurate truth-conditions for negation and other operators, and, unlike pragmatic maximization for global contexts, should not be defeasible. To see why, consider alternative values for j shown in Table 3.

Table 3: Drefs after updating C_1 with $\text{DEC}_S(\text{NEG}(\text{there is a bathroom}))$.

	$\phi_{\text{DC}_S}(j)$	$\phi_1(j)$	$\phi_2(j)$	$v(j)$
1	$\{w_u, w_0\}$	$\{w_u, w_0\}$	$\{w_{bu}, w_b\}$	$\begin{bmatrix} w_{bu} \mapsto b \\ w_b \mapsto b \\ w_u \mapsto \star \\ w_0 \mapsto \star \end{bmatrix}$
2	$\{w_b, w_u, w_0\}$	$\{w_b, w_u, w_0\}$	$\{w_{bu}\}$	$\begin{bmatrix} w_{bu} \mapsto b \\ w_b \mapsto \star \\ w_u \mapsto \star \\ w_0 \mapsto \star \end{bmatrix}$
3	$\{w_{bu}, w_u, w_0\}$	$\{w_{bu}, w_u, w_0\}$	$\{w_b\}$	$\begin{bmatrix} w_{bu} \mapsto \star \\ w_b \mapsto b \\ w_u \mapsto \star \\ w_0 \mapsto \star \end{bmatrix}$
4	$\{w_{bu}, w_b, w_u, w_0\}$	$\{w_{bu}, w_b, w_u, w_0\}$	\emptyset	$\begin{bmatrix} w_{bu} \mapsto \star \\ w_b \mapsto \star \\ w_u \mapsto \star \\ w_0 \mapsto \star \end{bmatrix}$

(19b) states that ϕ_2 contains only worlds where *there is a bathroom*. This holds for $\{w_{bu}, w_b\}$ (row #1), the maximal set where that's the case, but also for any (possibly empty) subset. The semantics defined above allows ϕ_1 and ϕ_{DC_S} to map to sets containing worlds where there is a bathroom (e.g., rows #2–4), deriving incorrect truth conditions for negation. To avoid this, ϕ_2 must pick out the largest set where *there is a bathroom*, so that ϕ_1 , contains all and only worlds where there isn't. We achieve

this by defining an operator that maximizes a propositional dref ϕ given an update D , restricting output states to those assigning maximal sets to ϕ (following [Brasoveanu 2006](#)).

$$(40) \quad \mathbf{max}_{\phi}(D) := \lambda i. \lambda j. D(i)(j) \wedge \forall k (D(i)(k) \rightarrow \neg(\phi(j) \subset \phi(k)))$$

Adding maximization to propositional operators, we represent $\text{DEC}_S(\text{NEG}(\textit{there is a bathroom}))$ as (41):

$$(41) \quad \text{DEC}_S(\text{NOT}(\textit{there is a bathroom})) \rightsquigarrow \left(\begin{array}{|c|} \hline \phi_1 \\ \hline \phi_{\text{DC}_S} \subseteq \phi_1 \\ \hline \end{array} ; \mathbf{max}_{\phi_1} \left(\begin{array}{|c|} \hline \phi_2 \\ \hline \phi_1 \equiv \overline{\phi_2} \\ \hline \end{array} ; \mathbf{max}_{\phi_2} \left(\begin{array}{|c|} \hline \phi_2 : v \\ \hline \text{bathroom}_{\phi_2}(v) \\ \hline \end{array} \right) \right) \right)$$

As a result, the drefs behave as intended: ϕ_2 includes all and only worlds with a bathroom, while its negation ϕ_1 , holds all other worlds. Consequently, none of the worlds in S 's commitment set are ϕ_2 -worlds where v is defined, making ϕ_2 and v are counterfactual drefs for S .

3.4.2 Veridical anaphor fails

Further updating C_1'' with (30) fails because the pronominal existence requirement creates an inconsistent discourse. The pronominal variable is interpreted in the condition $\text{upstairs}_{\phi_3}(v)$, requiring that v is locally contextually entailed in ϕ_3 . Since v was introduced relative to ϕ_2 , the local context of the anaphor ϕ_3 must be a subset of ϕ_2 , the counterfactual antecedent context.s

Because both utterances are assertions by the same speaker, consistency requires that ϕ_{DC_S} be a subset of the non-empty intersection of ϕ_1 and ϕ_3 . This creates a contradiction: if consistency is satisfied, ϕ_3 overlaps with the veridical ϕ_1 , meaning there are worlds in ϕ_3 where v_1 has no referent, so the interpretation fails. If local contextual entailment is satisfied, ϕ_3 is a subset of the counterfactual ϕ_2 , and the intersection of ϕ_1 and ϕ_3 is empty, committing the speaker to a contradiction. Thus, there is no successful update with both (19b) and (30).

3.5 Interim conclusion

ICDRT uses [Stone's \(1999\)](#) encapsulated quantification over possible worlds and drefs for individual concepts to interpret indefinites and pronouns relative to their local intensional context. This section showed how these mechanisms explain the constraints on anaphoric accessibility imposed by non-veridical operators. The analysis addresses the basic contrast (1) that motivated classic nested update approaches through local contextual entailment and discourse consistency. Moreover, it's flex-

ible enough to analyze the anaphora to negated content in (2) as regular cases of discourse anaphora, as shown in the next section.

4 Anaphora to negated content

This section illustrates how the analysis enables us to understand why anaphora to negated content is possible in the cases in (2), simplified here as (42).

- (42) a. Double negation:
It's not the case that there isn't a bathroom. It is upstairs.
- b. Disjunction:
Either there isn't a bathroom, or it is upstairs.
- c. Disagreement:
A: There isn't a bathroom.
B: It is upstairs.
- d. Modal subordination:
There isn't a bathroom. It would be upstairs.

4.1 An actual bathroom: double negation

The acceptability of veridical anaphora with doubly negated antecedents (42a) follows from the analysis of veridical anaphora in Section 3. The doubly negated indefinite is interpreted veridically, making it an accessible antecedent for the anaphor.

(42a) It's not the case that there isn't a bathroom. It is upstairs.

4.1.1 Analysis of double negation

The representation of the doubly negated antecedent utterance is given in (43).

$$(43) \quad \text{DEC}_S(\text{NOT}(\text{NOT}(\textit{there is [a bathroom]}^v))) \rightsquigarrow$$

$$\frac{\phi_1}{\phi_{\text{DC}_S} \subseteq \phi_1} ; \max_{\phi_1} \left(\frac{\phi_2}{\phi_1 \equiv \overline{\phi_2}} ; \max_{\phi_2} \left(\frac{\phi_3}{\phi_2 \equiv \overline{\phi_3}} ; \max_{\phi_3} \left(\frac{\phi_3 : v}{\text{bathroom}_{\phi_3}(v)} \right) \right) \right)$$

(43) introduces the drefs ϕ_1 , ϕ_2 , ϕ_3 , and v , and places conditions on their interpretation. The effect of this update in context C_1 (from Section 3.3.2) is illustrated in Figure 8. At the output j , ϕ_3 is the maximal set of worlds where v is a bathroom, i.e., $\{w_{bu}, w_b\}$. ϕ_2 is the complement of ϕ_3 , i.e., $\{w_u, w_0\}$, where there is no bathroom. ϕ_1 , for the main assertion, in turn, points to the complement of ϕ_2 .

Double negation, treated as double complementation over propositional drefs, results in $\phi_1(j) = \phi_3(j)$. Because the dref for *a bathroom* v is introduced relative to

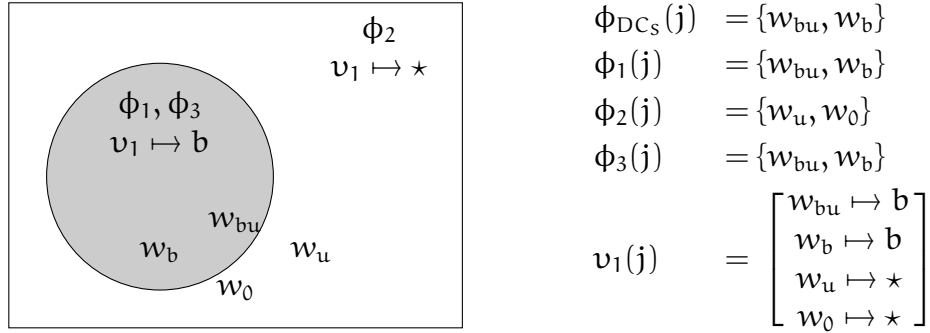


Figure 8: Drefs after update with ‘It’s not the case that there isn’t a bathroom’

ϕ_3 , it also exists in all ϕ_1 worlds, and by assertion in all ϕ_{DC_s} -worlds. It’s a veridical dref. As such, it can provide the interpretation for a veridical anaphor, analogously to the veridical antecedent in Section 3.3.3.

4.1.2 Further applications: Negated factives and negative implicatives

Because the analysis of double negation relies on the veridicality of the antecedent context, it extends to negated indefinites under negated factives and negative implicatives, repeated here.

- (5) a. Bill didn’t realize that he had [a dime]^{v2}. It_{v2} was in his pocket
based on Karttunen (1976): (16)
- b. John forgot not to bring [an umbrella]^{v3}, although we had no room for it_{v3}.
based on Karttunen (1976): (14b)

While deriving a projective inference of (5a) or veridicality in (5b) is beyond this paper’s scope, we can see that once we do, we get an explanation of why anaphora is possible here.

4.2 An optional bathroom: disjunction

In bathroom-disjunctions (42b), anaphor and antecedent occur in separate disjuncts:

(42b) Either there isn’t a bathroom, or it is upstairs.

The unacceptable case of veridical anaphora with negated antecedents in Section 3.4 is ruled out because anaphor and antecedent are contributed in assertions by the same speaker, requiring a non-empty intersection between the asserted propositions. In

contrast, disjunction does not require a non-empty intersection between its operands, which is why anaphora is possible here.

4.2.1 Analysis of bathroom-disjunctions

(42b) is represented as (44), where the disjunctive condition $\phi_1 \equiv \phi_2 \uplus \phi_3$ states that the matrix proposition ϕ_1 is the union of the propositions corresponding to the disjuncts ϕ_2 and ϕ_3 .¹⁰

(44) $\text{DEC}_S((\text{either}) \text{ there isn't a bathroom, or it is upstairs}) \rightsquigarrow$

$$\begin{array}{c}
 \begin{array}{|c|} \hline \phi_1 \\ \hline \phi_{\text{DC}_S} \in \phi_1 \\ \hline \end{array} ; \mathbf{max}_{\phi_1} \left(\begin{array}{|c|} \hline \phi_2, \phi_3 \\ \hline \phi_1 \equiv \phi_2 \uplus \phi_3 \\ \hline \end{array} ; \right. \\
 \mathbf{max}_{\phi_2} \left(\begin{array}{|c|} \hline \phi_4 \\ \hline \phi_2 \equiv \phi_4 \\ \hline \end{array} ; \mathbf{max}_{\phi_4} \left(\begin{array}{|c|} \hline \phi_4 : \nu_1 \\ \hline \text{bathroom}_{\phi_4}(\nu_1) \\ \hline \end{array} \right) \right) ; \\
 \left. \mathbf{max}_{\phi_3} \left(\begin{array}{|c|} \hline \text{upstairs}_{\phi_3}(\nu_1) \\ \hline \end{array} \right) \right)
 \end{array}$$

Figure 9 shows the result of updating C_1 with (44). At the output j , ϕ_1 is the union of ϕ_2 and ϕ_3 . The dref ϕ_4 , representing the negative prejacent in the first disjunct, is the maximal proposition where a bathroom exists ($\{\omega_{bu}, \omega_b\}$). ϕ_4 is the local context of the indefinite and thus ν points to the bathroom b in all and only the ϕ_4 -worlds. ϕ_2 , representing the first disjunct, is the complement of ϕ_4 ($\{\omega_u, \omega_0\}$), where ν does not exist. ϕ_3 , for the second disjunct, is the maximal proposition where ν is upstairs. This update is successful because the output j assigns a value to ϕ_3 , where ν is locally entailed without violating consistency.

Local contextual entailment: The variable ν is interpreted relative to ϕ_3 . Therefore, its referent exists in every ϕ_3 -world, and ϕ_3 is a subset of the antecedent context $\phi_4 = \{\omega_{bu}, \omega_b\}$. Specifically, it contains the worlds $\omega \in \phi_4$ where $\nu(j)(\omega)$ is upstairs, i.e. $\{\omega_{bu}\}$. Thus, ϕ_3 does not overlap with ϕ_2 , where ν maps to \star , effectively yielding an exclusive interpretation of the disjunction.

Consistency: Disjunction does not require overlap between the disjunct propositions (ϕ_2, ϕ_3). Therefore, this update is compatible with the output state shown in Figure 9 where ν exists in all ϕ_3 -worlds, and the speaker commits to the (non-empty) union of the disjuncts.

¹⁰Dynamic union: $\phi_1 \uplus \phi_2 := \lambda i_s. \phi_1(i) \cup \phi_2(i)$

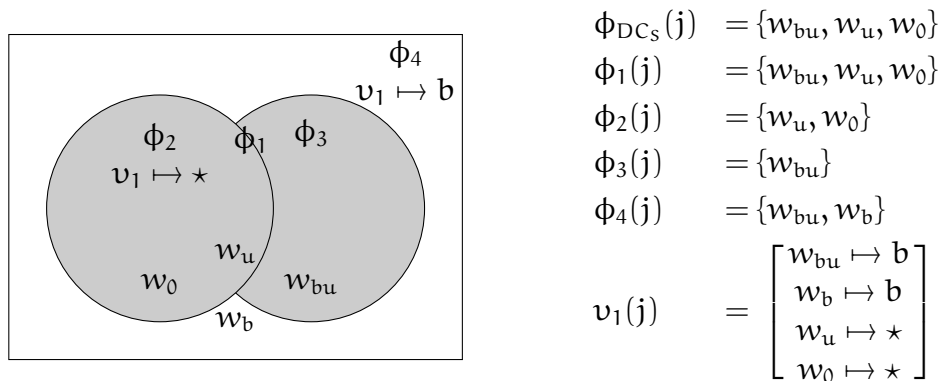


Figure 9: Drefs after update with ‘Either there isn’t a bathroom or it’s upstairs’

4.2.2 Discussion: Exclusive or inclusive disjunction

Dynamic analyses of anaphora or presupposition under disjunction often assume that the local context of the second disjunct is derived by locally updating with the negation of the first, enabling bathroom-anaphora or presupposition filtering (e.g., [Karttunen 1974](#); [Roberts 1989](#); [Krahmer and Muskens 1995](#)). This is typically linked to an exclusive semantics for disjunction. In contrast, in the example above, the entailment relation between the local contexts of the anaphor (ϕ_3) and its antecedent (ϕ_4) results from the anaphoric link, not the disjunction itself.

We may question whether this is desirable. Our boolean semantics for disjunction permits inclusive interpretations, including the unacceptable (45), where the second disjunct contextually entails the first:

(45) #Either there is a bathroom in this house, or it’s upstairs.

[Elliott \(2020\)](#) suggests that such cases are independently ruled out by Hurford’s constraint—the observation that disjunctions are infelicitous when one disjunct entails the other ([Hurford 1974](#)). While its status and derivation are debated, [Gazdar \(1979\)](#) showed that the constraint cannot be tied to the literal semantics of disjunction, using (46), where the second disjunct entails the literal content of first:

(46) Some or all of them were there. [Gazdar 1979](#): (65)

Here, the implicature of the first disjunct (not all of them were there) is sufficient for introducing an exclusive interpretation. Thus, Hurford’s constraint might be viewed as a pragmatic filter on semantic representations, so that (45) is derivable from the semantics of disjunction, but is unavailable pragmatically.

4.3 A contested bathroom: disagreement

Anaphora in disagreement (42c) is possible, because different speakers may contradict each other without creating an inconsistent discourse.

- (42c) A: There isn't a bathroom.
B: It is upstairs.

The updates in (42c) are the same as the counterfactual antecedent and veridical anaphor in Section 3.4, but they update commitments of A and B, respectively.

$$(47) \quad \text{DEC}_A(\text{NEG}(\text{there is [a bathroom]}^v)) \rightsquigarrow$$

$$\left(\frac{\phi_1}{\phi_{\text{DC}_A} \in \phi_1} \right); \mathbf{max}_{\phi_1} \left(\left(\frac{\phi_2}{\phi_1 \equiv \phi_2} \right); \mathbf{max}_{\phi_2} \left(\left(\frac{\phi_2 : v}{\text{bathroom}_{\phi_2}(v)} \right) \right) \right)$$

(47) introduces ϕ_1 veridically, and introduces ϕ_2 and v (for a bathroom in ϕ_2) counterfactually, relative to A's commitments.

$$(48) \quad \text{DEC}_B(\text{it}_v \text{ is upstairs}) \rightsquigarrow \left(\frac{\phi_3}{\phi_{\text{DC}_B} \in \phi_3} \right); \mathbf{max}_{\phi_3} \left(\left(\text{upstairs}_{\phi_3}(v) \right) \right)$$

(48) introduces ϕ_3 , for B's assertion veridically for B, which also serves as the local context for interpreting the pronominal variable in the condition $\text{upstairs}_{\phi_3}(v)$.

For illustration, consider a context $C_2 = \langle M_2, \text{INT}, i \rangle$, with two interlocutors $\text{INT} = \{A, B\}$, where i is an initial state. M_2 is a model where $D_e^{M_2} = \{A, B, b, \star\}$, $D_w^{M_2} = \{w_{bu}, w_b, w_u, w_0\}$, b is a bathroom in $\{w_{bu}, w_b\}$, and b is upstairs in $\{w_{bu}, w_u\}$. An update with (47) and (48) in C_2 yields an output state j , assigning drefs as shown in Figure 10. Worlds compatible with A's commitments are shaded in gray, and those compatible with B's commitments have a gray hatch.

This update is successful because j assigns a value to ϕ_3 , where v is locally entailed without violating consistency. Since v was introduced relative to ϕ_2 , ϕ_3 must be a subset of ϕ_2 , which is counterfactual for A. The anaphoric link, therefore, derives an interpretation where B's assertion contradicts A's assertion. However, because their commitments are separate, they can contradict each other without creating empty commitment sets, thereby satisfying *consistency*.¹¹

¹¹More generally, anaphora with negated antecedents can be enabled by enabling contradictory interpretations of two discourse segments in non-veridical discourse relations (in the sense of Segmented DRT, SDRT, Asher and Lascarides 2003). Another example is (self-)correction:

- (49) There isn't a bathroom in this house. In fact, I realized it's just in a weird place.

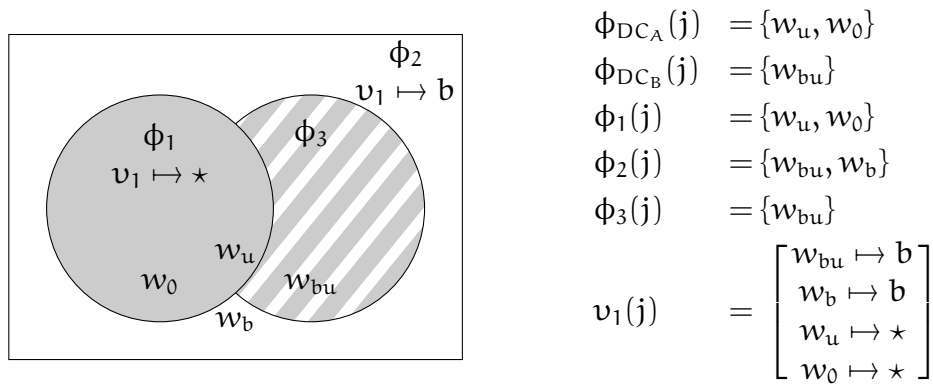


Figure 10: Drefs after updating C_2 with $DEC_A(\text{NEG}(\text{there is [a bathroom]}^p))$; $DEC_B(\text{it}_v \text{ is upstairs})$

4.4 A hypothetical bathroom: modal subordination

Modal subordination with counterfactual antecedents is possible, because the anaphor itself is in a non-veridical context.

(42d) There isn't a bathroom. It would be upstairs.

Stone (1999) and Brasoveanu (2006, 2010a) analyze modal subordination as simultaneous anaphora to propositions and individuals. They treat *would* as a Kratzerian epistemic necessity modal (Kratzer 1981) that anaphorically retrieves a hypothetical proposition for its modal base, providing a local context for interpreting the pronoun. While this captures the context-dependence of epistemic modals (e.g., Kratzer 1981; Roberts 1987; Frank 1996; Frank and Kamp 1997; Geurts 1999; Anand and Hacquard 2013), this paper argues that anaphora with hypothetical and counterfactual antecedents can also occur in other non-veridical contexts like under non-veridical propositional attitudes:

(50) There isn't a bathroom in this house, but Sue still {hopes/believes} it's upstairs.

Like modals, the non-veridical attitude *hope* has been argued to depend on a doxastic information state, providing the restrictor for the modal quantification (Asher 1987; Maier 2015). Doxastic attitudes like *believe* introduce an information state that can serve as a modal base for epistemic modals (Kratzer 1981; Anand and Hacquard 2013) or restrictor for *hope* (Asher 1987; Maier 2015). This aligns with a standard semantics for doxastic attitudes (Heim 1992), shown in (51).

- (51) a. $\llbracket \text{believe}(x_e)(p_{wt})(w_w) \rrbracket^{M,g} = \llbracket \text{Dox}_x(w) \subseteq p \rrbracket^{M,g}$
b. Doxastic Belief states: $\llbracket \text{Dox}_x(w) \rrbracket^{M,g} = \{w' \in D_w : w' \text{ conforms to what } \llbracket x \rrbracket^{M,g} \text{ believes in } \llbracket w \rrbracket^{M,g}\}$

To illustrate the generality of the pattern for anaphora in non-veridical contexts, and for simplicity of exposition, this section derives the dependency for a hypothetical anaphor with a counterfactual antecedent using the example in (52).

- (52) a. S: There isn't a bathroom.
b. S: Sue believes it's upstairs.

4.4.1 Semantic representations

(52a) is represented as (41) (see Section 3.4), repeated here:

$$(41) \quad \text{DEC}_S(\text{NOT}(\textit{there is a bathroom})) \rightsquigarrow$$

$$\left(\begin{array}{c} \phi_1 \\ \hline \phi_{\text{DC}_S} \in \phi_1 \end{array} \right); \max_{\phi_1} \left(\begin{array}{c} \phi_2 \\ \hline \phi_1 \equiv \phi_2 \end{array} \right); \max_{\phi_2} \left(\begin{array}{c} \phi_2 : v \\ \hline \text{bathroom}_{\phi_2}(v) \end{array} \right)$$

Importantly, v , the dref for *a bathroom* is introduced counterfactually in ϕ_2 .

(52b) is represented as (53):

$$(53) \quad \text{DEC}_S(\textit{Sue believes(it}_v \textit{ is upstairs)}) \rightsquigarrow$$

$$\left(\begin{array}{c} \phi_3 \\ \hline \phi_{\text{DC}_S} \in \phi_3 \end{array} \right); \max_{\phi_3} \left(\begin{array}{c} v_2 \\ \hline v_2 \equiv \textit{Sue} \\ \text{believe}_{\phi_3}(v_2, \phi_4) \end{array} \right); \max_{\phi_4} \left(\text{upstairs}_{\phi_4}(v_1) \right)$$

(53) contributes the non-veridical relation that ϕ_4 is a proposition which v_2 (i.e. Sue)¹² believes in all ϕ_3 -worlds.¹³

4.4.2 The effect of the update

Assume a context $C_3 = \langle M_3, \text{INT}, i \rangle$, where $\text{INT} = \{S\}$ and i is an initial state. M_3 includes $D_e = \{b, \textit{sue}, S, \star\}$, $D_w = \{w_{bus}, w_{bu}, w_{bs}, w_b, w_{us}, w_u, w_s, w_0\}$, and the interpretation of the predicates *bathroom*, *upstairs*, and *believe* characterizes the propositions in Table 4.

After updating C_3 with (41); (53), the output j assigns drefs as shown in Figure 11. Here, ϕ_4 is the local context of the pronoun, so *local contextual entailment* requires

¹²Proper names contribute an individual dref, and a condition equating the new dref with a discourse constant associated with the proper name. The constant $\textit{Sue} := \lambda i_s. \lambda w_w. \textit{sue}_e$ points to *sue* across all assignments and worlds.

¹³ $\text{believe}_{\phi_3}(v_2, \phi_4) = \lambda i. \forall w(w \in \phi_3(i) \rightarrow \text{believe}(v_2(i))(\phi_4(i))(w) = \lambda i. \forall w(w \in \phi_3(i) \rightarrow \text{Dox}_{v_2(i)}(w) \subseteq \phi_4(i))$

Table 4: Relevant propositions in \mathcal{M}_3

	$\lambda w.bathroom(b)(w)$	$\lambda w.upstairs(b)(w)$	$\lambda w.believe(sue)(\{w_{bus}, w_{bu}\})(w)$
w_{bus}	1	1	1
w_{bu}	1	1	0
w_{bs}	1	0	1
w_b	1	0	0
w_{us}	0	1	1
w_u	0	1	0
w_s	0	0	1
w_0	0	0	0

that v have a referent in all ϕ_4 -worlds and ϕ_4 be a subset of the antecedent context ϕ_2 (i.e., $\phi_4(j) \subseteq \{w_{bus}, w_{bu}, w_{bs}, w_b\}$), specifically, those ϕ_2 -worlds where v is upstairs: $\{w_{bus}, w_{bu}\}$.

To maintain *consistency* ϕ_{DC_S} must be a subset of the non-empty intersection of ϕ_1 and ϕ_3 , which are associated with S 's consecutive assertions. ϕ_1 contains worlds without a bathroom ($\{w_{us}, w_u, w_s, w_0\}$), and ϕ_3 contains worlds where *Sue believes* ϕ_4 ($\{w_{bus}, w_{bs}, w_{us}, w_s\}$). Due to the non-veridical embedding, no entailment is required between ϕ_3 and ϕ_4 , and ϕ_4 need not overlap with the speaker's commitment set.

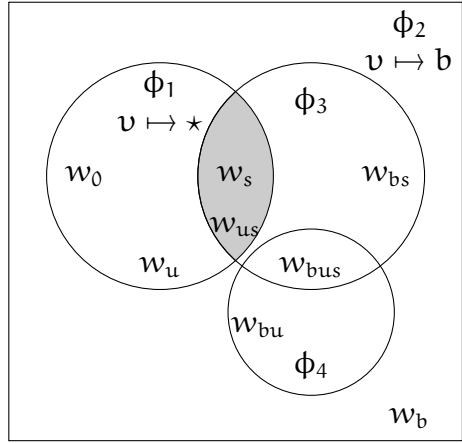
Consequently, the output j characterizes a *consistent* discourse where the hypothetical local context of the anaphor is a subset of the counterfactual antecedent context ϕ_2 . Because this relationship results from the anaphoric link, the analysis does not rely on anaphorically provided local contexts, or simultaneous anaphora to individuals and propositions. This allows us to extend a flat-update approach beyond modal subordination.

4.4.3 Further applications: Hypothetical anaphora

Since this analysis achieves modal subordination without modal anaphora, it applies similarly to other cases of hypothetical anaphora. These include anaphora between (15) negative environments (Kibble 1994), (54) modalized contexts (Elliott 2022), and (55) attitude contexts (Karttunen 1976). In such cases, the speaker is not committed to the existence of a referent, but anaphora is possible because the pronoun doesn't evoke such a commitment either.

(15) Kibble 1994: (31)

John doesn't have [a car]^{v1}, so he doesn't have to wash it_{v1}.



$$\begin{aligned}
\phi_{DC_s}(j) &= \{w_{us}, w_s\} \\
\phi_1(j) &= \{w_{us}, w_u, w_s, w_0\} \\
\phi_2(j) &= \{w_{bus}, w_{bu}, w_{bs}, w_b\} \\
v(j) &= \begin{bmatrix} w_{bus} \mapsto b \\ w_{bu} \mapsto b \\ w_{bs} \mapsto b \\ w_b \mapsto b \\ w_{us} \mapsto * \\ w_u \mapsto * \\ w_s \mapsto * \\ w_0 \mapsto * \end{bmatrix} \\
\phi_3(j) &= \{w_{bus}, w_{bs}, w_{us}, w_s\} \\
\phi_4(j) &= \{w_{bus}, w_{bu}\} \\
v_2(j) &= \lambda w.sue
\end{aligned}$$

Figure 11: Drefs after update with *There isn't [a bathroom]; 'Sue believes it's upstairs*

- (54) Elliott (2022): (2)
 Maybe there is [no bathroom]^{v1} in this house,
 and maybe it_{v1}'s in a funny place.
- (55) Adapted from Karttunen 1976: (19)
 Bill says he saw [a lion]^{v2} on the street. He claims that it_{v2} had escaped from
 the zoo.

Because these anaphors are interpreted hypothetically, they can have hypothetical antecedents.

4.4.4 Discussion: Accommodation

Assuming local accommodation throughout forces a reevaluation of the modal subordination contrast in (8). Assumption accommodation, local accommodation is predicted to be possible in the absence of contradictory information in both (8a+b).

- (8) [A wolf]^v might walk in.
 a. #It_v is gray.
 b. It_v would eat you first.

To understand the unavailability of (8a), we draw on independent reasons for why the veridical anaphor is inconsistent with an antecedent under 'might': possibility claims carry a scalar implicature negating a necessity claim, which is often so strong that

cancellation needs to be signaled explicitly (see, e.g., [Gazdar 1979](#); [Sauerland 2004](#); [Fox 2014](#)). When doing that, veridical anaphora with antecedents under *might* becomes possible:

- (56) There might be a bathroom in this house. In fact, I just remembered, it's upstairs.

This observation aligns with an approach centering discourse consistency rather than representational constraints. Because various inferences, including implicatures, affect speaker commitments, discourse consistency depends on both literal truth-conditions and pragmatic inference.

The newly gained flexibility also has some welcome consequences, predicting veridical anaphora to semantically hypothetical antecedents, as in (57):

- (57) a. I heard that there is a bathroom in this house.
b. It's upstairs.

While various pragmatic factors influence how committed to embedded propositions listeners take a speaker to be,¹⁴ (57a) does not commit the speaker to the existence of a bathroom based on the literal content. However, assuming accommodation of the pronominal existence inference in (57b) predicts that the speaker commits that the referent exists, which seems accurate.¹⁵

4.5 Discussion: Truth conditions and weak/strong readings

While this paper focuses on accessibility conditions, I briefly discuss whether weak (existential) or strong (universal) truth conditions (in the sense of [Chierchia 1990](#)) are, or should be, predicted for the cases in (42), repeated here:

¹⁴E.g., (non-)at-issueness of embedded propositions (e.g., [Abrusán 2011](#); [Simons et al. 2017](#); [Beaver et al. 2017](#); [Tonhauser et al. 2018](#)), interlocutor beliefs ([Degen and Tonhauser 2021](#)), and the credibility of the source of speech reports ([Rieh 2010](#); [de Marneffe et al. 2012](#)).

¹⁵Assuming accommodation also predicts anaphora in cases like (58a), due to an anonymous reviewer, and [Rothschild's 2017](#) (58b), where the propositional inferences available in discourse affect anaphoric possibilities.

- (58) a. If Ms President declares bankruptcy, there will be a motion to censor and it will likely pass. If she does not and prints money instead to pay for our debt, it won't pass, because our MPs won't get so panicky.
b. [Rothschild 2017](#): (11)
Either it's a holiday or a customer will come in. And if it's not a holiday, he'll want to be served.

- (42) a. Double negation:
It's not the case that there isn't a bathroom. It is upstairs.
- b. Disjunction:
Either there isn't a bathroom, or it is upstairs.
- c. Disagreement:
A: There isn't a bathroom.
B: It is upstairs.
- d. Modal subordination:
There isn't a bathroom. It would be upstairs.

The present analysis derives weak readings throughout, because individual concepts highlight at most one individual per world. For instance, (42a) is predicted to be true iff some bathroom is upstairs, regardless of whether others are downstairs; and the same applies to B's response in (42c). However, (42a) is commonly judged to be false when some bathroom in the domain is downstairs, suggesting a strong reading. Yet, a doubly negated antecedent can support existential truth-conditions: (59a) can be true even if Logan has another credit card that is not on the table. Relatedly, Elliott 2020 observes that this case supports maximal plural anaphora (59b).

- (59) It's not true that Logan has no credit card.
- a. It's on the table.
- b. They're on the table. (Elliott 2020: 82)

For disjunction, the predicted weak reading is that (42b) is true iff there are no bathrooms in the domain, or some bathroom is upstairs, regardless of whether others are downstairs. This contrasts with the common intuition that (42b) is false when there is some bathroom that is not upstairs, suggesting a universal reading. However, Elliott (2020, 2024) shows that existential readings for bathroom-disjunctions are possible, and Chatain and Spector's (forthcoming) experimental data indicates that existential readings are generally available, while certain contexts also support universal readings.

For modal subordination, assessing weak versus strong readings is more challenging due to interactions with the modal quantifier and because it is not obvious how we reason about counterfactual quantities. Disentangling "every non-existent bathroom would be upstairs" from "some non-existent bathroom would be upstairs" requires additional work.

The literature preliminarily indicates that both weak and strong readings are generally available for the configurations in (42), and pragmatic factors determine which interpretation is chosen, as suggested for donkey sentences (e.g., Brasoveanu 2006; Champollion et al. 2019). To derive strong readings, we could retain the current semantics and assume a pragmatic strengthening mechanism (Elliott 2024). Alterna-

tively, the system could be upgraded to handle anaphora to universal quantification or plural DPs, introducing a systematic weak/strong ambiguity as in Brasoveanu’s (2006; 2010a; 2010b) flat-update plural system.

4.6 Interim conclusion

The analysis provides unified account of various cases of anaphora to negated content that have previously received disparate analyses. By incorporating speaker commitments about drefs, it derives constraints on accessibility from two core principles: (i) local contextual entailment of a referent for individual-denoting expressions, and (ii) discourse consistency.

The analysis addresses the data in (1) and (2) while avoiding the empirical challenges identified for nested update in Section 1: It captures the generalization that indefinites in veridical contexts provide antecedents for veridical anaphora, even when non-veridical operators are present (as with double negation); and it explains how indefinites in hypothetical and counterfactual contexts can provide antecedents based on the anaphor’s embedding context, avoiding the lookahead problem (for modal subordination, inter-speaker disagreement).

Beyond empirical advantages, the analysis improves on a conceptual problem of nested update highlighted by Elliott (2020) (based on Soames 1989, where the point is made for presupposition projection): Because nested update treats sentential operators as operations over updates, it must define both their truth-functional contribution and their effect on the anaphoric potential of content in their scope. This often leads to multiple possible definitions for the same operator, which vary in their anaphoric possibilities but maintain identical truth-conditions, making the choice between them more descriptive than explanatory.

This analysis partially solves Soames’s explanatory challenge, regarding the question of whether material within an operator’s scope is accessible for anaphora outside it (see also discussion in Elliott 2020).¹⁶ For example, the nested update semantics for disjunction usually permits dependencies between disjuncts but not beyond them—it is internally dynamic but externally static, in Groenendijk and Stokhof’s (1991) terms. An truth-conditionally equivalent formulation, program disjunction (*ibid.*), is internally static but externally dynamic.

In ICDRT, propositional operators are always conditionally dynamic (both internally and externally), while their discourse effect is derived from their truth-conditional contribution. Encapsulating quantification over sets of worlds with relativized drefs allows us to interpret individual drefs relative to the truth-conditional information

¹⁶Soames’s explanatory challenge also raises the question of why operators like disjunction allow for cataphora while conjunction does not, which is not addressed here. One might explore a dynamic explanation related to representational simplicity, because dynamic logic offers a basic asymmetric conjunction operator, while connectives like *or*, *but*, *if* require more involved representations.

stored in propositional drefs. Sentential operators are defined using boolean set-relations, ensuring compatibility with inferential rules from static propositional logic. This setup allows for truth-conditional inferences to influence the availability of individual drefs in various contexts.

5 Comparison to other accounts

This section compares the proposed flat-update dynamic account to other analyses of anaphora with hypothetical or negated antecedents, focusing on their ability address the data in (1) and (2). Section 5.1 examines accounts where indefinites introduce drefs only under certain conditions, primarily developed for double negation and bathroom-disjunctions (Krahmer and Muskens, 1995; Gotham, 2019; Elliott, 2020; Mandelkern, 2022), and argues that they cannot handle counterfactual anaphora. Section 5.2 considers approaches where referential information, rather than just propositional content, is accommodated in certain contexts (e.g Roberts, 1989; Frank and Kamp, 1997; Geurts, 1999; Lewis, 2021). These were designed mainly for modal subordination and face limitations for unembedded veridical anaphora, like double-negation and disagreement cases. Whereas the proposed analysis derives the cases in (2) as regular instances of anaphora, integrating them into a general pattern of veridicality-based constraints, no previous account addresses all cases in (2).¹⁷

5.1 Conditional variable update

In analyses involving conditional variable update, indefinites introduce drefs only when they have an existing referent, offering a principled explanation for why indefinites under double negation and in bathroom-disjunctions can be anaphoric antecedents. However, these accounts rule out counterfactual anaphora, partially reproducing the lookahead-problem, and preventing a unified analysis of the cases in (1) and (2). Here, I discuss implementations in bilateral systems (Krahmer and Muskens 1995; Elliott 2020, 2022), and via a projective inference associated with negated indefinites (Krahmer and Muskens 1995; Elliott 2020, 2022).

5.1.1 Bilateral dynamics

Bilateralism is a way of specifying a partial logic, stating truth- and falsity-conditions, where formulas that fail to meet both are undefined. Krahmer and Muskens's

¹⁷Another notable account addressing double negation, bathroom-disjunction, and hypothetical modal subordination is Qian's (2014) analysis in Type-Theoretic Dynamic Logic. I don't discuss it in detail here for two reasons: Qian acknowledges that it does not extend to counterfactual anaphora (pp. 239f), and for reasons of space, as the formalism is less related to the one used here.

(1995) Double Negation DRT (DN-DRT) and Elliott’s (2022) Bilateral Update Semantics (BUS) define an interpretation function with positive and negative interpretations and a ‘flip-flop’ negation operator, which applies to an update and returns the interpretation of opposite polarity (60).

- (60) Bilateral negation Krahmer and Muskens (1995), p. 367
- a. $\llbracket \neg D \rrbracket^+ = \llbracket D \rrbracket^-$
 - b. $\llbracket \neg D \rrbracket^- = \llbracket D \rrbracket^+$

In DN-DRT, the positive interpretation of an update is a set of input-output pairs making it true, while the negative interpretation falsifies it. Existentially quantified updates introduce a dref only under their positive interpretation, whereas the negative interpretation implements the condition that the positive update is impossible.

- (61) a. $\llbracket [x \mid \text{bathroom}(x)] \rrbracket^+ = \{\langle i, j \rangle \mid i[x]j \wedge j(x) \in I(\text{bathroom})\}$
 b. $\llbracket [x \mid \text{bathroom}(x)] \rrbracket^- = \{\langle i, i \rangle \mid \neg \exists j (i[x]j \wedge j(x) \in I(\text{bathroom}))\}$

Double negation flip-flops twice, returning to the positive interpretation of the pre-jacent. Unlike standard dynamic negation, bilateral negation validates double negation elimination (DNE), whereby (62b) is interpreted like (62a), predicting that both can be followed by (62c).

- (62) a. There is a bathroom. \rightsquigarrow $[x \mid \text{bathroom}(x)]$
 b. It’s not the case that there isn’t a bathroom. \rightsquigarrow $\neg\neg[x \mid \text{bathroom}(x)]$
 c. It’s upstairs. \rightsquigarrow $[\text{upstairs}(x)]$

DN-DRT also addresses bathroom-disjunctions by interpreting disjunctions of the form $D_1 \vee D_2$ like conditionals of the form $\neg D_1 \rightarrow D_2$. While these formulas are statically equivalent, the bathroom-disjunction (63a) is assumed to also be dynamically equivalent to the conditional in (63b) and by DNE to the one in (63c).

- (63) a. Either there isn’t a bathroom, or it is upstairs.
 $\rightsquigarrow \neg\neg[x \mid \text{bathroom}(x)] \rightarrow [\text{upstairs}(x)]$
 b. If it’s not true that there isn’t a bathroom in this house, it is upstairs.
 $\rightsquigarrow \neg\neg[x \mid \text{bathroom}(x)] \rightarrow [\text{upstairs}(x)]$
 c. If there is a bathroom, it is upstairs.
 $\rightsquigarrow [x \mid \text{bathroom}(x)] \rightarrow [\text{upstairs}(x)]$

(63a) is interpreted by analogizing disjunction to a standard dynamic interpretation for conditionals. Because this analysis relies on the particular semantics for negation and disjunction, it doesn’t systematically extend to other operators, leaving modal subordination unaddressed. Further, indefinites under single negation never intro-

duce drefs, incorrectly ruling out counterfactual anaphora.

Elliott’s (2022) BUS more systematically derives the anaphoric potential of embedded indefinites from truth-functional properties of sentential operators and incorporates intensionality, which allows for modeling epistemic uncertainty and modal semantics. Formulas update a Heimian information state s , i.e., a set of possibilities represented as world-assignment pairs (Heim 1982). As in DN-DRT, existentially quantified updates introduce a dref only under the positive interpretation:

$$(64) \quad \begin{aligned} \text{a.} \quad & \llbracket [x \mid \text{bathroom}(x)] \rrbracket^+ = \\ & \lambda s. \{ \langle w, h \rangle \mid \langle w, g \rangle \in s \wedge g[x]h \wedge h(x) \in I(\text{bathroom})(w) \} \\ \text{b.} \quad & \llbracket [x \mid \text{bathroom}(x)] \rrbracket^- = \\ & \lambda s. \{ \langle w, g \rangle \in s \mid \neg \exists h (g[x]h \wedge h(x) \in I(\text{bathroom})(w)) \} \end{aligned}$$

Using bilateral negation, BUS addresses double negation like DN-DRT does. Bathroom-disjunctions (65a) are analyzed interpreting the positive update as the union of updates corresponding to alternative ways of verifying the disjunction. The output derived in (65b) may include elements where the first disjunct holds (there isn’t a bathroom) and the second disjunct does not need to be verified and can remain undefined. It may also include elements falsifying the first disjunct (i.e., it’s not true that there isn’t a bathroom). Since bilateral negation validates DNE, these elements include a dref for *a bathroom*, licensing the pronoun in the second disjunct (65c).

$$(65) \quad \begin{aligned} \text{Elliott (2022): (26–30)} \\ \text{a.} \quad & \llbracket \neg [x \mid \text{bathroom}(x)] \vee [\text{upstairs}(x)] \rrbracket^+(s) = \\ \text{b.} \quad & \llbracket \neg [x \mid \text{bathroom}(x)] \rrbracket^+(s) \cup \\ & (\llbracket \neg [x \mid \text{bathroom}(x)] \rrbracket^-; \llbracket [\text{upstairs}(x)] \rrbracket^+)(s) = \\ \text{c.} \quad & \llbracket [x \mid \text{bathroom}(x)] \rrbracket^-(s) \cup \llbracket [x \mid \text{bathroom}(x)]; [\text{upstairs}(x)] \rrbracket^+(s) \end{aligned}$$

Since BUS information states include multiple possibilities, hypothetical indefinites, like in bathroom-disjunctions, lead to output states where a dref is made *partially familiar*, i.e., defined in some but not all possibilities. Similarly, Elliott accounts for cases like (66) by assuming the indefinite introduces a dref in only possibilities where a bathroom exists, and the pronoun is evaluated relative to a subset of the discourse state.

(66) Maybe there is a bathroom in this house. Maybe it’s upstairs.

A similar approach for pronouns under *would* could account for hypothetical modal subordination. However, this cannot be extended to counterfactual anaphora: an update like *There isn’t a bathroom* will not make a dref (even partially) familiar, so the output context involves no possibilities where a dref was introduced. This is-

sue, related to the problem discussed for Heimian update semantics in Section 2.1, arises because pronouns are interpreted relative to (subsets of) the global context set, which only contains live possibilities, while counterfactual drefs are not stored in discourse. Analyzing counterfactual anaphora would require substantial changes. Consequently, bilateral accounts cannot address modal subordination with negated antecedents or inter-speaker disagreement, and miss important aspects of how double negation elimination and bathroom-disjunctions fit within the broader picture of veridicality-based constraints on accessibility.

5.1.2 Introducing drefs via projective content

Gotham’s (2019) DPL with ‘unique excluded middle’ (UEM) and Mandelkern’s (2022) bounded theory address double negation and bathroom-disjunctions by associating indefinites under negation (67a+b) with a projective inference like (67c).

- (67) a. There isn’t [a bathroom]_x.
 b. It’s not true that there isn’t [a bathroom]_x.
 c. If there is a bathroom, x is a bathroom.

(67c) introduces x as a dref for *a bathroom*, only if the indefinite has an existing referent (67b), but not in (67a).

Gotham’s (2019) DPL with UEM is a nested-update system where discourse states are sets of variable assignments (here in CDRT-notation for consistency). The at-issue content of (67b) doesn’t introduce a dref for *a bathroom* but only imposes a condition that such an update is possible (68a). The innovation is an excluded-middle (EM) inference (68b), which outputs the union of updating input assignments with either the prejacent (introducing x for *a bathroom*) or its negation (imposing the condition that the positive update is impossible).

- (68) a. $\llbracket \neg\neg[x \mid \text{bathroom}(x)] \rrbracket = \{\langle i, i \rangle \mid \exists j (i[x]j \wedge j(x) \in I(\text{bathroom}))\}$
 b. $\llbracket [x \mid \text{bathroom}(x)] \cup \neg[x \mid \text{bathroom}(x)] \rrbracket = \llbracket [x \mid \text{bathroom}(x)] \rrbracket \cup \llbracket \neg[x \mid \text{bathroom}(x)] \rrbracket$

Update with (68a) retains all possibilities where some bathroom x can be found and (68b) ensures that x is introduced as a dref in these possibilities, deriving that doubly negated indefinites introduce global drefs.

Bathroom-disjunctions are also interpreted with the EM-inference and can be approached by considering different ways of verifying the disjunction. The at-issue content receives a standard dynamic interpretation (69), stating that update with at least one of the disjuncts is possible.

- (69) $\llbracket \neg[x \mid \text{bathroom}(x)] \vee [\text{upstairs}(x)] \rrbracket = \{\langle i, i \rangle \mid \neg\exists j (\llbracket [x \mid \text{bathroom}(x)] \rrbracket (i)(j) \vee \llbracket [\text{upstairs}(x)] \rrbracket (i)(j))\}$

The EM-inference introduces a dref for negated indefinites iff the referent exists. In (69), either the first disjunct is true (there is no bathroom), or it's false, in which case a dref x is introduced, providing an antecedent for the pronoun in the second disjunct. While this extensional system captures double negation and bathroom-disjunctions, it cannot address epistemic uncertainty or modal subordination.

A minimal extension of this system introducing intensionality (e.g., using FCS-style context sets of world-assignment pairs) would allow indefinites to introduce drefs for all possibilities where their referent exists, as in Elliott's (2022) BUS. However, since these discourse states track epistemic possibilities (like Heim's update semantics discussed in Section 2.1), they can't store counterfactual drefs, ruling out anaphora with counterfactual antecedents.

That is essentially what is done in Mandelkern's (2022) bounded theory, which uses projective inferences to introduce drefs and context sets of world-assignment pairs. It assumes a two-dimensional semantics: truth-conditional content is interpreted statically, while existential inferences of indefinites and pronouns are evaluated on a separate dynamic dimension of presupposition-like *bounds*. Context update with a formula p restricts a Heimian information state to contain elements where p is statically true and *satt*, i.e., its bounds are satisfied.

(70) Simplified from Mandelkern (2022), p. 17

$$c[p] = \{\langle w, g \rangle \mid \llbracket p \rrbracket^{w,g} = 1, \text{ and } p \text{ is satt at } \langle c, w, g \rangle\}$$

Bounds can impose conditions on the output context distributively (checking each element $\langle w, g \rangle$), or collectively for the input context c .

An update with an indefinite (*there is [a bathroom]_x*) has a static existential semantics on its truth-conditional dimension (71a), imposing the condition that for each $\langle w, g \rangle$ in the output, g can be modified so that x points to a bathroom in w . The indefinite also has a witness bound (71b), requiring that if there is a bathroom at $\langle w, g \rangle$, then $g(x)$ is a bathroom.

(71) a. $\llbracket \exists x(\text{bathroom}(x)) \rrbracket^{w,g} = 1$, iff
 $\exists d \in D_e : \llbracket \text{bathroom}(x) \rrbracket^{g[x \rightarrow d], w} = 1$
 b. Witness bound for indefinites:
 $\exists x(\text{bathroom}(x))$ is satt at $\langle c, w, g \rangle$, only if:
 If $\exists d \in D_e : \llbracket \text{bathroom}(x) \rrbracket^{g[x \rightarrow d], w} = 1$,
 then $g(x) \in I(\text{bathroom})(w)$.

(71b) introduces a dref x in each possibility where there is a bathroom, without affecting possibilities where there isn't. In case of double negation, this analysis predicts that a dref is introduced for all output possibilities.

An update with a pronoun (*it_x is upstairs*) statically imposes the condition that $g(x)$ is upstairs in w by (72a). Anaphoric pronouns also have a familiarity bound (72b), requiring that x is familiar throughout the input context c .

- (72) a. $\llbracket \iota x(\text{upstairs}(x)) \rrbracket^{w,g} = 1$, iff $\llbracket \text{upstairs}(x) \rrbracket^{w,g} = 1$
 b. Familiarity bound for anaphoric pronouns:
 $\iota x(\text{upstairs}(x))$ is satt at $\langle c, w, g \rangle$, only if:
 $\forall \langle w', g' \rangle \in c : g'(x)$ is defined

Bathroom-disjunctions (73) are addressed by interpreting the familiarity-bound of the pronoun in the second disjunct in a local context that results from updating with the negation of the first:

- (73) a. $\llbracket \neg \exists x(\text{bathroom}(x)) \vee \text{upstairs}(x) \rrbracket^{w,g} = 1$, iff
 $\llbracket \neg \exists x(\text{bathroom}(x)) \rrbracket^{w,g} = 1$, or $\llbracket \text{upstairs}(x) \rrbracket^{w,g} = 1$
 b. $\neg \exists x(\text{bathroom}(x)) \vee \text{upstairs}(x)$ is satt at $\langle c, w, g \rangle$, iff:
 (i) $\neg \exists x(\text{bathroom}(x))$ is satt at $\langle c, g, w \rangle$, and
 (ii) $\text{upstairs}(x)$ is satt at $\langle c[\neg \exists x(\text{bathroom}(x))], g, w \rangle$

The familiarity-bound in the second disjunct (73bii) requires that x is familiar throughout the local context derived by updating c with the negation of the first disjunct. Since this local update is a doubly negative update, this correctly derives the possibility of anaphora here.

Further, the witness-bound (73bi) introduces a dref x for *a bathroom* at output possibilities where a bathroom exists. Like BUS, the bounded theory makes hypothetical drefs partially familiar and could be extended to address modal subordination with hypothetical antecedents. However, it also faces the same limitation: because discourse states reflect epistemic possibilities, counterfactual indefinites introduce no referential information, failing to account for counterfactual anaphora.

5.1.3 Interim conclusion

Conditional update accounts capture double negation and bathroom-disjunction but face limitations for counterfactual antecedents in modal subordination and disagreement cases. Systems incorporating possible worlds in their discourse states (Elliott 2022; Mandelkern 2022) can analyze anaphora with hypothetical antecedents (bathroom-disjunction, modal subordination) by assuming partially familiar drefs.

Allowing discourse states to store partially familiar (i.e., hypothetical) drefs partly addresses the lookahead-problem of nested update approaches: discourse states contain enough information to predict that subsequent pronouns are accessible whenever their referent exists in the local context. However, the assumed global contexts include only possibilities that are still epistemically viable (cf. discussion for

Heim’s update semantics Section 2.1), and modeling discourse states as sets of world-assignment-pairs conflates referential and epistemic information in one single context set. Therefore, the discourse states cannot store counterfactual referential information, reproducing the lookahead-problem for anaphora with counterfactual antecedents, which are predicted to be impossible.

5.2 Accommodating referential information

Accommodation-based accounts were developed to address modal subordination (Roberts 1987, 1989; Geurts 1999; Frank 1996) or the interaction of anaphora and negation more generally (Lewis 2021). They differ from the present account, because they involve accommodation, not just of a propositional existence-presupposition, but also of referential information about variable mappings.

5.2.1 Recycling local contexts

Accounts in Roberts (1987, 1989), Geurts (1999), Frank (1996), and Frank and Kamp (1997) allow previously subordinated discourse contexts to be reintroduced as the local context of interpretation for an anaphor under modal subordination.

Roberts’s (1989) DRS-copying mechanism reuses previously subordinated modal sub-DRSs through accommodation in contexts where (i) a pronoun occurs in a hypothetical context; (ii) there is a plausible hypothetical antecedent in the preceding discourse; and (iii) the antecedent has been explicitly introduced. This mechanism derives modal subordination and bathroom sentences. Later refinements (Geurts 1999; Frank 1996; Frank and Kamp 1997) assuming that dynamic local contexts are retrieved anaphorically and therefore subject to constraints on anaphora.

Because these accounts are designed to address modal subordination and the lookahead problem, but they limit accommodation to hypothetical contexts, like embedded under *would*. This limitation prevents an extension to unembedded veridical anaphora in cases of double negation or disagreement.¹⁸

5.2.2 Accommodation as replanning in dynamic pragmatics

Lewis’s (2021) dynamic pragmatic approach combines a static d-type semantics with pragmatic discourse update, where constraints on anaphora are pragmatic. Speakers introduce drefs by signaling a plan to talk about the referent again later. Using an overt DP to assert its existence is the default strategy to signal this, which explains anaphora with veridical antecedents, including double negation cases.

This approach, based on the idea that “plans [...] constrain the possible moves one can make” (p. 1420), also allows for *replanning* when intentions change or were

¹⁸For further empirical challenges, see Stone (1999) and Brasoveanu (2006).

initially unclear to the hearer. Lewis views dref accommodation as a form of replanning, permitting pronouns without explicit antecedents when speakers act “as though a discourse referent has already been introduced” (p. 1424). Therefore, any individual-denoting phrase can potentially participate in anaphoric dependencies, subject to certain constraints, like in flat-update dynamic semantics.

Because the constraints on planning and replanning are not spelled out explicitly, the account lacks a predictive formalization. However, Lewis identifies three cases where replanning enables anaphora: (i) the utterance containing the pronoun explains why the first utterance was issued, (ii) both utterances address the same QUD, or (iii) the antecedent utterance negates a property of an object rather than its existence. (i) and (ii) can apply to modal subordination, where the utterances containing antecedent and anaphor can be interpreted in an explanation-relation or as addressing the same QUD.

While Lewis addresses double-negation and offers an explanation for modal subordination, she does not discuss bathroom-disjunction or disagreement cases. A discourse-structural explanation could extend to bathroom-disjunctions, if the disjuncts are construed as addressing the same QUD. However, Lewis suggests that ‘[a]ccommodation [...] does not occur whenever a speaker’s intentions are recognizable’ (p. 1425), ruling out the disagreement-cases, as they cannot be explained by changes in the first speaker’s intentions or by the second speaker adapting their understanding of what those intentions might be.

A reviewer notes that alternative conceptions of replanning are possible, which could better explain disagreement, and that an accommodation-based approach would be more successful than flat update in handling antecedentless anaphora, like:

- (74) Based on [Lewis 2021](#) (38)
- A: I’m getting married.
B: Oh yeah? Who is she?

While an explicit formalization dref accommodation could offer new insights and grounds for a more principled theory-comparison, Lewis’s account is not explicit enough to provide this basis (though see [Chatain](#), forthcoming, for a formal analysis allowing for certain antecedentless cases). Importantly, such an analysis should be constrained enough to address the marble-contrast in (75).

- (75) [Roberts \(1987\)](#), attributed to Barbara Partee:
- a. I dropped ten marbles and found all but [one of them]^{v1}. It_{v1}’s probably under the couch.
 - b. I dropped ten marbles and found nine of them. # It_v’s probably under the couch.

The tension between (74) and (75) raises the question of whether anaphora generally require antecedents, in which case apparently antecedentless occurrences like (74) require special explanations (e.g., via bridging from *married* to one of its arguments *she*). Or possibly, explicit antecedents are not always necessary, and cases like (75), which seem to require them, need alternative explanations (e.g., by operationalizing a notion of salience). Further research is needed to clarify the kind of data required to decisively support either approach.

5.2.3 Interim conclusion

Accommodation-based accounts handle modal subordination and bathroom-disjunctions but have limitations. Lewis's pragmatic approach extends to double negation, but her conception of accommodation as replanning does not address anaphora with counterfactual antecedents in inter-speaker disagreement. More explicit constraints on accommodation would be essential for a predictive, testable account. Dynamic accounts that recycle local contexts only do so in embedded cases, thereby precluding a unified analysis of the data considered here.

6 Conclusion

Negation constrains anaphora, because anaphora is sensitive to speaker commitments about discourse referents. This conclusion arises from careful empirical study of how anaphoric accessibility interacts with the veridicality of the embedding contexts of both anaphors and their antecedents. It reveals a view of anaphoric polarity-sensitivity as part of a broader pattern of sensitivity to veridicality and local contextual entailments.

This paper argued that a flat-update dynamic semantics is best suited to analyze the counterexamples to classic dynamic approaches in (2) and the presented generalization about anaphora and veridicality. To model constraints on anaphora, indefinites and pronouns must be interpreted relative to their local intensional context, Discourse states must store information about worlds in which individual referents exist and how these worlds relate to the speaker's conversational commitments. This approach addresses the constraints imposed by non-veridical operators by tracking speaker commitments about discourse referents.

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A Formal appendix: Intensional CDRT

Based on [Muskens's 1996](#) CDRT, and implementation in [Brasoveanu 2006](#), Ch. 3 (CDRT / Dynamic Ty₂). Adding possible worlds, intensionality, and interpreting individual-denoting expressions relative to encapsulated quantification over worlds, based on [Stone 1999](#).

A.1 Static logic definitions

- (1) Types
 - a. Basic static types
 $\mathbf{BasSTyp} := \{t, w, e\}$ (truth values, worlds, individuals)
 - b. Static types
 \mathbf{STyp} : the smallest set including $\mathbf{BasSTyp}$ and all elements (σ, τ) , s.t. $\sigma, \tau \in \mathbf{STyp}$
 - c. Basic dynamic types
 $\mathbf{BasDynTyp} := \{s\}$ (variable assignments)
 - d. Dref types
 \mathbf{DRTyp} : the smallest set s.t. if $\tau \in \mathbf{STyp}$, then $(s\tau) \in \mathbf{DRTyp}$
 - e. Basic types
 $\mathbf{BasTyp} := \mathbf{BasSTyp} \cup \mathbf{BasDynTyp}$
 - f. Types
 \mathbf{Typ} : the smallest set including \mathbf{BasTyp} and all elements $(\sigma\tau)$, s.t. $\sigma, \tau \in \mathbf{Typ}$
- (2) Basic expressions
For every type $\tau \in \mathbf{Typ}$, there is a denumerable set of τ -constants \mathbf{Con}_τ and a denumerably infinite set of τ -variables $\mathbf{Var}_\tau = \{v_{\tau 0}, v_{\tau 1}, \dots\}$. e.g.:
 - a. $\mathbf{Con}_e = \{\text{mary}_e, \text{sue}_e, \dots\}$
 - b. $\mathbf{Con}_s = \{i, i', \dots, j, j', \dots, k, k', \dots\}$
 - c. Lexical relations: An n -ary lexical relation is a constant of type $e^n(wt)$, where $e^n(wt)$ is defined as follows:
 $e^0(wt) := wt$ and $e^{m+1}(wt) := e(e^m(wt))$; e.g.:
 - (i) $\mathbf{Con}_{e(wt)} = \{\text{bathroom}_{e(wt)}, \text{upstairs}_{e(wt)}, \dots\}$
 - (ii) $\mathbf{Con}_{e(e(wt))} = \{\text{own}_{e(e(wt))}, \dots\}$

(3) Terms

a. For any type $\tau \in \mathbf{Typ}$, the set of τ -terms is the smallest set, s.t.:

(i) $\mathbf{Con}_\tau \cup \mathbf{Var}_\tau \subseteq \mathbf{Term}_\tau$

(ii) $\alpha(\beta) \in \mathbf{Term}_\tau$,

for any $\alpha \in \mathbf{Term}_{(\sigma,\tau)}$, $\beta \in \mathbf{Term}_\sigma$, $\sigma \in \mathbf{Typ}$

(iii) $(\lambda v.\alpha) \in \mathbf{Term}_\tau$,

for any $\tau = (\sigma, \rho)$, $v \in \mathbf{Var}_\sigma$, $\alpha \in \mathbf{Term}_\rho$, $\sigma, \rho \in \mathbf{Typ}$

b. For formulas (terms of type t), the following apply in addition:

(i) $(\alpha = \beta) \in \mathbf{Term}_t$,

for any $\alpha, \beta \in \mathbf{Term}_\sigma$, $\sigma \in \mathbf{Typ}$

(ii) $(i[\delta]j) \in \mathbf{Term}_t$

for any $i, j \in \mathbf{Term}_s$, $\delta \in \mathbf{Term}_\sigma$, $\sigma \in \mathbf{DRTyp}$

(iii) for any $\alpha, \beta \in \mathbf{Term}_t$:

$(\neg\alpha) \in \mathbf{Term}_t$

$(\alpha \wedge \beta) \in \mathbf{Term}_t$

$(\alpha \vee \beta) \in \mathbf{Term}_t$

$(\alpha \rightarrow \beta) \in \mathbf{Term}_t$

$(\alpha \leftrightarrow \beta) \in \mathbf{Term}_t$

(iv) for any $v \in \mathbf{Var}_\sigma$, $\sigma \in \mathbf{Typ}$, $\alpha \in \mathbf{Term}_t$:

$\exists v(\alpha) \in \mathbf{Term}_t$

$\forall v(\alpha) \in \mathbf{Term}_t$

c. Abbreviations; for any $\alpha, \beta \in \mathbf{Term}_{(\sigma,t)}$, $\gamma \in \mathbf{Term}_\sigma$, $\sigma \in \mathbf{Typ}$:

(i) $(\gamma \in \beta) := \beta(\gamma)$

(ii) $(\alpha \subseteq \beta) := \forall v_\sigma(\alpha(v) \rightarrow \beta(v))$

(iii) $\bar{\alpha} := \lambda v_\sigma.\neg\alpha(v)$

(iv) $(\alpha \cap \beta) := \lambda v_\sigma.\alpha(v) \wedge \beta(v)$

(v) $(\alpha \cup \beta) := \lambda v_\sigma.\alpha(v) \vee \beta(v)$

(4) Frames, models, assignments

a. A *standard frame* F is a set $\{D_\tau : \tau \in \mathbf{Typ}\}$ s.t.:

(i) D_e, D_t, D_w , and D_s are disjoint sets,

(ii) $D_{\sigma\tau} = \{f : f \text{ is a total function from } D_\sigma \text{ to } D_\tau\}$, for any $\sigma, \tau \in \mathbf{Typ}$,

(iii) $D_t = \{0, 1\}$, and

(iv) D_e contains a dummy value \star_e .

- b. A *model* M is a pair $\langle F^M, \llbracket \cdot \rrbracket^M \rangle$, s.t.:
- (i) F^M is a standard frame
 - (ii) Non-logical constants: $\llbracket \cdot \rrbracket^M$ assigns an object $\llbracket \alpha \rrbracket^M \in D_\tau^M$ to each $\alpha \in \mathbf{Con}_\tau$ for any $\tau \in \mathbf{Typ}$
 - \star_e is the universal falsifier, i.e., for any n-ary lexical relation R : $\llbracket R \rrbracket^M \subseteq (D_e^M \setminus \{\star\})^n \times D_w^M$
 - (iii) M satisfies the following axioms (to make objects of type s behave as variable assignments, see [Muskens 1996](#), p. 156)
 - Identity of assignments:** $\forall i_s, j_s. (i \llbracket j \leftrightarrow i = j)$
 - Discourse variables:** $\mathbf{dVar}(\delta)$, for any dref name δ of any type $\tau \in \mathbf{DRType}$, \mathbf{dVar} is a non-logical predicate constant identifying the non-constant functions of type $s\tau$, for any $\tau \in \mathbf{STyp}$.
 - Enough assignments:** $\forall i_s, \delta_{s\tau}, f_\tau (\mathbf{dVar}(\delta) \rightarrow \exists j_s. (i \llbracket \delta j \wedge \delta(j) = f))$, for any type $\tau \in \mathbf{STyp}$
 - Unique dref names:** $\mathbf{dVar}(\delta_\tau) \wedge \mathbf{dVar}(\delta'_\tau) \rightarrow \delta \neq \delta'$, for any distinct dref names δ, δ' and type $\tau \in \mathbf{DRType}$
- c. An M -assignment function g assigns to each variable $v \in \mathbf{Var}_\tau$ an element $g(v) \in D_\tau^M$ for any type $\tau \in \mathbf{Typ}$. $g[v \mapsto d]$ is the M -assignment identical to g except that it assigns v to d (for $v \in \mathbf{Var}_\tau, d \in D_\tau^M$).

(5) The interpretation function $\llbracket \cdot \rrbracket^{M,g}$ is defined as follows:

- a. Constants: For $\alpha \in \mathbf{Con}_\tau, \tau \in \mathbf{Typ}$:
 - $\llbracket \alpha \rrbracket^{M,g} = \llbracket \alpha \rrbracket^M$;
- b. Variables: For $v \in \mathbf{Var}_\tau, \tau \in \mathbf{Typ}$:
 - $\llbracket v \rrbracket^{M,g} = g(v)$;
- c. Function application: For $\alpha \in \mathbf{Term}_{(\tau,\sigma)}, \beta \in \mathbf{Term}_\tau, \tau, \sigma \in \mathbf{Typ}$:
 - $\llbracket \alpha(\beta) \rrbracket^{M,g} = \llbracket \alpha \rrbracket^{M,g}(\llbracket \beta \rrbracket^{M,g})$
- d. Lambda-abstraction: For $\alpha \in \mathbf{Term}_\tau, v \in \mathbf{Var}_\sigma, \sigma, \tau \in \mathbf{Typ}$:
 - $\llbracket \lambda v. \alpha \rrbracket^{M,g} = \{ \langle d, \llbracket \alpha \rrbracket^{M,g(v \mapsto d)} \rangle \mid d \in D_\tau^M \}$;
- e. Identity: For $\alpha, \beta \in \mathbf{Term}_\tau, \tau \in \mathbf{Typ}$:
 - $\llbracket \alpha = \beta \rrbracket^{M,g} = 1$, iff $\llbracket \alpha \rrbracket^{M,g} = \llbracket \beta \rrbracket^{M,g}$;
- f. Variable update: For $\delta \in \mathbf{Term}_\sigma, \sigma \in \mathbf{DRType}, i, j \in \mathbf{Term}_s$:
 - $\llbracket i \llbracket \delta j \rrbracket^{M,g} = 1$, iff
 - $\llbracket \forall \delta'_\sigma [\mathbf{dVar}(\delta') \wedge \delta' \neq \delta] (\delta'(i) = \delta'(j)) \rrbracket^{M,g} = 1$ and
 - $\llbracket \forall \delta'_\tau [\mathbf{dVar}(\delta')] (\delta'(i) = \delta'(j)) \rrbracket^{M,g} = 1$, for all $\tau \neq \sigma, \tau \in \mathbf{DRType}$;

- g. Unary connective(s): For $\alpha \in \mathbf{Term}_t$:
- Negation: $\llbracket \neg \alpha \rrbracket^{M,g} = 1$, iff $\llbracket \alpha \rrbracket^{M,g} = 0$;
- h. Binary connectives: For $\alpha, \beta \in \mathbf{Term}_t$:
- $\llbracket \alpha \wedge \beta \rrbracket^{M,g} = 1$, iff $\llbracket \alpha \rrbracket^{M,g} = 1$ and $\llbracket \beta \rrbracket^{M,g} = 1$
 - $\llbracket \alpha \vee \beta \rrbracket^{M,g} = 1$, iff $\llbracket \alpha \rrbracket^{M,g} = 1$ or $\llbracket \beta \rrbracket^{M,g} = 1$;
 - $\llbracket \alpha \rightarrow \beta \rrbracket^{M,g} = 1$, iff $\llbracket \alpha \rrbracket^{M,g} = 0$ or $\llbracket \beta \rrbracket^{M,g} = 1$;
 - $\llbracket \alpha \leftrightarrow \beta \rrbracket^{M,g} = 1$, iff $\llbracket \alpha \rrbracket^{M,g} = \llbracket \beta \rrbracket^{M,g}$;
- i. Quantifiers: For $\alpha \in \mathbf{Term}_t, \nu \in \mathbf{Var}_\tau, \tau \in \mathbf{Typ}$:
- $\llbracket \exists \nu(\alpha) \rrbracket^{M,g} = 1$, iff $\langle d, 1 \rangle \in \llbracket \lambda \nu. \alpha \rrbracket^{M,g}$, for some $d \in D_\tau^M$;
 - $\llbracket \forall \nu(\alpha) \rrbracket^{M,g} = 1$, iff $\langle d, 1 \rangle \in \llbracket \lambda \nu. \alpha \rrbracket^{M,g}$, for all $d \in D_\tau^M$;
- j. Propositional attitudes
- Doxastic information state:
 $\llbracket \text{DOX}_x(w) \rrbracket^{M,g} = \{w' \in D_w : w' \text{ conforms to what } \llbracket x \rrbracket^{M,g} \text{ believes in } \llbracket w \rrbracket^{M,g}\}$
 - Assertive information state:
 $\llbracket \text{ASSERTIVE}_x(w) \rrbracket^{M,g} = \{w' \in D_w : w' \text{ conforms to the communicative commitments of } \llbracket x \rrbracket^{M,g} \text{ in } \llbracket w \rrbracket^{M,g}\}$
 - true* as vacuous (veridical) propositional embedder:
 $\llbracket \text{true}(p_{wt})(w_w) \rrbracket^{M,g} = \llbracket p(w) \rrbracket^{M,g}$
 - believe* as non-veridical doxastic attitude:
 $\llbracket \text{believe}(x_e)(p_{wt})(w_w) \rrbracket^{M,g} = 1$, iff $\llbracket \text{DOX}_x(w) \rrbracket^{M,g} \subseteq \llbracket p \rrbracket^{M,g}$
 - prove* as veridical assertive attitude:
 $\llbracket \text{prove}(x_e)(p_{wt})(w_w) \rrbracket^{M,g} = 1$, iff
 $\llbracket \text{ASSERTIVE}_x(w) \rrbracket^{M,g} \subseteq \llbracket p \rrbracket^{M,g}$ and $\llbracket p(w) \rrbracket^{M,g} = 1$
 - mistaken* as anti-veridical assertive attitude:
 $\llbracket \text{mistaken}(x_e)(p_{wt})(w_w) \rrbracket^{M,g} = 1$, iff
 $\llbracket \text{ASSERTIVE}_x(w) \rrbracket^{M,g} \subseteq \llbracket p \rrbracket^{M,g}$ and $\llbracket p(w) \rrbracket^{M,g} = 0$

(6) Truth: A formula $\varphi \in \mathbf{Term}_t$ is *true*...

- wrt M, g iff $\llbracket \varphi \rrbracket^{M,g} = 1$.
- in M iff it is true wrt any assignment g .

A.2 Dynamic metalanguage definitions

- (7) Conditions (type st):
- Predication:

$$R_\phi\{v_1, \dots, v_n\} := \lambda i_s. \forall w_w. (\phi(i)(w) \rightarrow R(v_1(i)(w), \dots, v_n(i)(w))(w))$$
for $R \in \mathbf{Term}_{e^n(wt)}$, $v \in \mathbf{Term}_{s(we)}$, $\phi \in \mathbf{Term}_{s(wt)}$
 - Identity:

$$\alpha \equiv \beta := \lambda i_s. \alpha(i) = \beta(i),$$
for $\alpha, \beta \in \mathbf{Term}_\tau$, $\tau \in \mathbf{DrTyp}$
 - Inclusion:

$$\phi_1 \subseteq \phi_2 := \lambda i_s. \phi_1(i) \subseteq \phi_2(i),$$
for $\phi_1, \phi_2 \in \mathbf{Term}_{s(wt)}$
- (8) Set operations (for $\phi_1, \phi_2 \in \mathbf{Term}_{s(wt)}$):
- Complementation:

$$\overline{\phi_1} := \lambda i_s. \overline{\phi_1(i)}$$
 - Intersection:

$$\phi_1 \cap \phi_2 := \lambda i_s. \phi_1(i) \cap \phi_2(i)$$
 - Union:

$$\phi_1 \cup \phi_2 := \lambda i_s. \phi_1(i) \cup \phi_2(i)$$
- (9) Updates (DRSs, type s(st)):
- Variable update:

$$[\delta] := \lambda i_s. \lambda j_s. i[\delta]j,$$
for $\delta \in \mathbf{Term}_{st}$, $\tau \in \mathbf{STyp}$
 - Relative variable update:

$$[\phi : v] := \lambda i_s. \lambda j_s. i[v]j \wedge \forall w_w. (\phi(j)(w) \leftrightarrow v(j)(w) \neq \star),$$
for $\phi \in \mathbf{Term}_{s(wt)}$, $v \in \mathbf{Term}_{s(we)}$
 - DRSs with conditions:

$$[C] := \lambda i_s. \lambda j_s. i = j \wedge C(j),$$
for $C \in \mathbf{Term}_{st}$
- (10) Operations over updates (for $D, D_1, D_2 \in \mathbf{Term}_{s(st)}$):
- Dynamic conjunction:

$$D_1; D_2 := \lambda i_s. \lambda j_s. \exists h_s. (D_1(i)(h) \wedge D_2(h)(j))$$
 - Maximization over propositional drefs:

$$\mathbf{max}_\phi(D) := \lambda i. \lambda j. D(i)(j) \wedge \forall k. (D(i)(k) \rightarrow \neg(\phi(j) \subset \phi(k)))$$
- (11) Multiple drefs and conditions:
- $[C_1, \dots, C_n] := \lambda i_s. \lambda j_s. i = j \wedge C_1(j) \wedge \dots \wedge C_n(j),$
for $C_1, \dots, C_n \in \mathbf{Term}_{st}$
 - $[\delta_1, \dots, \delta_n] := [\delta_1]; \dots; [\delta_n],$
for $\delta \in \mathbf{Term}_\sigma$, $\sigma \in \mathbf{DRTyp}$

- (12) DRS notation:
 $[\delta_1, \dots, \delta_n \mid C_1, \dots, C_n] := \lambda i_s. \lambda j_s. ([\delta_1, \dots, \delta_n]; [C_1, \dots, C_n])(i)(j)$
 for $\delta \in \mathbf{Term}_\sigma$, $\sigma \in \mathbf{DRTyp}$, $C_1, \dots, C_n \in \mathbf{Term}_{st}$

A.3 Translations for a fragment of English

- (13) Defining dynamic “meta-types” for an intensional dynamic system
- e abbreviates $s(w_e)$ (individual drefs)
 - w abbreviates $s(wt)$ (propositional drefs)
 - t abbreviates $s(st)$ (updates / dynamic formulas)
- (14) Proper Names, e.g.: $Mary \rightsquigarrow \lambda P_{e(wt)}. \lambda \phi_w. [v \mid v \equiv Mary_e]; P(v)(\phi)$
 (where $Mary = \lambda i_s. \lambda w_w. mary_e$)
- (15) Unary Predicates, e.g.: $bathroom \rightsquigarrow \lambda v_e. \lambda \phi_w. [bathroom_\phi(v)]$
- (16) Indefinite Determiner: $a^v \rightsquigarrow \lambda P_{e(wt)}. \lambda P'_{e(wt)}. \lambda \phi_w. [\phi : v]; P(v)(\phi); P'(v)(\phi)$
- (17) Pronouns, e.g.: $it_v \rightsquigarrow \lambda P_{e(wt)}. \lambda \phi_w. P(v)(\phi)$
- (18) Sentential operators:
- $\mathbf{NOT}^{\phi'} \rightsquigarrow \lambda \mathcal{P}_{wt}. \lambda \phi_w. [\phi' \mid \phi \equiv \overline{\phi'}]; \mathbf{max}_{\phi'}(\mathcal{P}(\phi'))$
 - $\mathbf{OR}^{\phi', \phi''} \rightsquigarrow \lambda \mathcal{P}'_{wt}. \lambda \mathcal{P}''_{wt}. \lambda \phi_w. [\phi', \phi'' \mid \phi \equiv \phi' \uplus \phi''];$
 $\mathbf{max}_{\phi'}(\mathcal{P}'(\phi')); \mathbf{max}_{\phi''}(\mathcal{P}''(\phi''))$
 - $\mathbf{IF}^{\phi', \phi''} \rightsquigarrow \lambda \mathcal{P}'_{wt}. \lambda \mathcal{P}''_{wt}. \lambda \phi_w. [\phi', \phi'' \mid \phi \equiv \overline{\phi'} \uplus \phi''];$
 $\mathbf{max}_{\phi_1}(\mathcal{P}'(\phi')); \mathbf{max}_{\phi_2}(\mathcal{P}''(\phi''))$
 - $\mathbf{AND}^{\phi', \phi''} \rightsquigarrow \lambda \mathcal{P}'_{wt}. \lambda \mathcal{P}''_{wt}. \lambda \phi_w. [\phi' \mid \phi \in \phi]; \mathbf{max}_{\phi_1}(\mathcal{P}'(\phi'));$
 $[\phi'' \mid \phi \in \phi'']; \mathbf{max}_{\phi_2}(\mathcal{P}''(\phi''))$
 - $\mathbf{believes} \rightsquigarrow \lambda \mathcal{P}_{wt}. \lambda v_e. \lambda \phi_w. [\phi'_w \mid \mathbf{believe}_\phi(v, \phi')]; \mathbf{max}_{\phi'}(\mathcal{P}(\phi'))$
- (19) Declarative mood: $\mathbf{DEC}_S^\phi \rightsquigarrow \lambda \mathcal{P}_{wt}. [\phi \mid \phi_{DC_S} \in \phi]; \mathbf{max}_\phi(\mathcal{P}(\phi))$
- (20) “Glue”
- $there \rightsquigarrow \lambda Q_{(e(wt), wt)}. Q$
 - $be \rightsquigarrow \lambda Q_{(e(wt), wt)}. Q$
 - $be \rightsquigarrow \lambda P_{e(wt)}. P$

A.4 Discourse update, truth

- (21) Contexts, states, updates
- a. A *discourse context* C is a tuple $\langle M, \text{INT}, i \rangle$ s.t.:
 - (i) M is a model.
 - (ii) $\text{INT} \subseteq D_e^M$ is the set of interlocutors.
 - (iii) $i_s \in D_s^M$ is a discourse state.
 - b. A *discourse state* i in C is an assignment which, for each $x \in \text{INT}^C$, stores a propositional dref ϕ_{DC_x} , s.t. x 's commitments hold in $\phi_{\text{DC}_x}(i)$.
 - c. A *null assignment* i_\emptyset is an assignment that holds no information for discourse variables, i.e.:

$$\forall v_{s(\text{we})}(\mathbf{dVar}(v) \rightarrow v(i_\emptyset) = \lambda w. \star), \text{ and}$$

$$\forall \phi_{s(\text{wt})}(\mathbf{dVar}(\phi) \rightarrow \phi(i_\emptyset) = \lambda w. 1)$$
 - d. An *initial state* i_0^C in C is s.t. for all $A, B, \dots \in \text{INT}^C$:

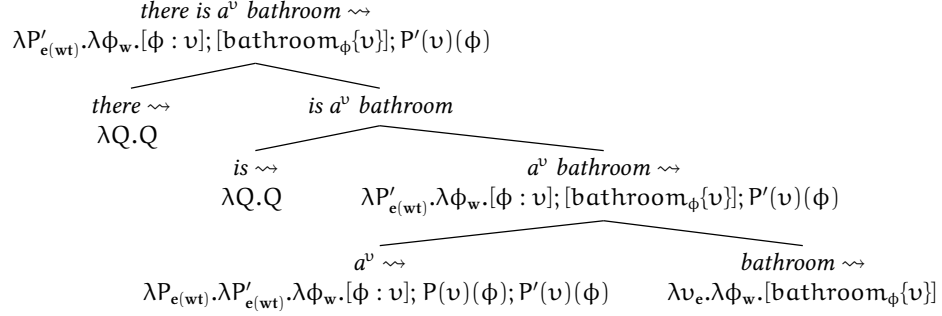
$$i_\emptyset[\phi_{\text{DC}_A}, \phi_{\text{DC}_B}, \dots]i_0^C$$
 - e. Commitment sets satisfy consistency: The set associated with the commitments of an interlocutor x is non-empty, for any assignment i :

$$\forall x_e, i_s(x \in \text{INT} \rightarrow \phi_{\text{DC}_x}(i) \neq \emptyset)$$
 - f. A *discourse update* with $D \in \mathbf{Term}_{s(\text{st})}$ in $C = \langle M, \text{INT}, i \rangle$ is successful iff there is some output context $C' = \langle M, \text{INT}, j \rangle$, s.t. $\llbracket D(i)(j) \rrbracket^M = 1$.
- (22) Discourse-level truth (relative to communicative agents)
- For context C , interlocutor $x \in \text{INT}_C$, dynamic propositions $\mathcal{P}, \mathcal{P}' \in \mathbf{Term}_{\text{wt}}$:
- a. \mathcal{P} is true for x in C , iff
 - x can assert \mathcal{P} in C without change in commitments, i.e.:
 - Any successful update with $([\phi \mid \phi_{\text{DC}_x} \in \phi]; \mathcal{P}(\phi))$ in C is s.t. $\phi_{\text{DC}_x}(i) = \phi_{\text{DC}_x}(j)$ where i/j are input/output discourse state respectively.
 - b. \mathcal{P} is consistent for x in C , iff
 - x can assert \mathcal{P} in C without violating consistency, i.e.:
 - There is a successful update with $([\phi \mid \phi_{\text{DC}_x} \in \phi]; \mathcal{P}(\phi))$ in C , s.t. $\phi_{\text{DC}_x}(j) \neq \emptyset$ at output state j .
 - c. \mathcal{P} is false for x in C , iff \mathcal{P} is inconsistent for x in C . smallskip
 - d. \mathcal{P} contextually entails \mathcal{P}' for x in C , iff
 - After x asserts \mathcal{P} in C , \mathcal{P}' is globally true, i.e.:
 - Any successful update with $([\phi \mid \phi_{\text{DC}_x} \in \phi]; \mathcal{P}(\phi))$ in C has an output context C' s.t. \mathcal{P}' is true for x in C' .
 - e. \mathcal{P} literally entails \mathcal{P}' , iff \mathcal{P} contextually entails \mathcal{P}' for any context C and any $x \in \text{Int}_C$.

A.5 Compositional derivations

(23) *there is a^v bathroom* \rightsquigarrow $\lambda\phi_w. [\phi : v]; [\text{bathroom}_\phi\{v\}]$

a. Uncontextualized meaning (based on Francez 2009: 13–15)



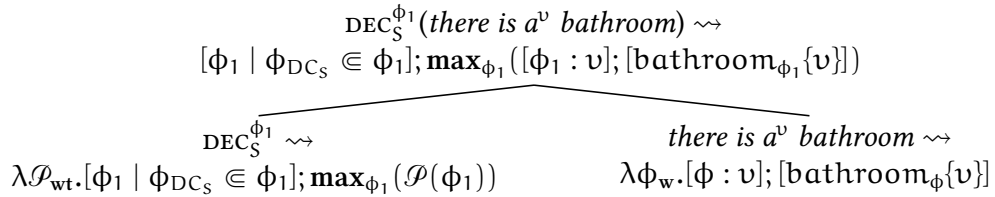
b. Contextualization (Francez 2009: 16) provides a contextual domain C_{et} as implicit argument for the nuclear scope of the “pivot” quantifier in existential constructions. For an intensional version, we use a predicate $C_{e(\text{wt})}$, which is true of individuals iff they are in the contextual domain at a given world. We assume a corresponding dynamic predicate $C_\phi\{v\}$. Therefore:

there is a^v bathroom (contextualized) \rightsquigarrow
 $\lambda\phi_w. [\phi : v]; [\text{bathroom}_\phi\{v\}]; [C_\phi\{v\}]$

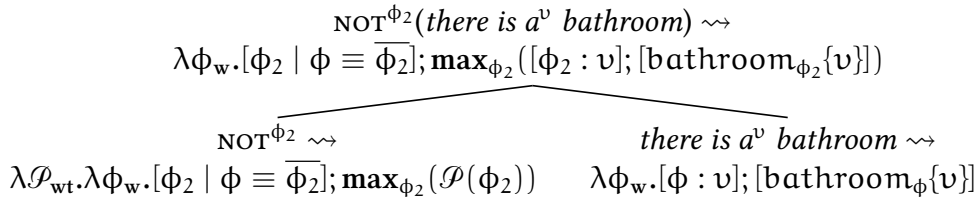
c. For simplicity, we ignore the implicit contextual domain argument, and use:

there is a^v bathroom \rightsquigarrow $\lambda\phi_w. [\phi : v]; [\text{bathroom}_\phi\{v\}]$

(24) $\text{DEC}_S^{\phi_1}(\text{there is } a^v \text{ bathroom}) \rightsquigarrow$
 $[\phi_1 \mid \phi_{\text{DC}_S} \in \phi_1]; \mathbf{max}_{\phi_1}([\phi_1 : v]; [\text{bathroom}_{\phi_1}\{v\}])$



(25) $\text{NOT}^{\phi_2}(\text{there is } a^v \text{ bathroom}) \rightsquigarrow$
 $\lambda\phi_w. ([\phi_2 \mid \phi \equiv \overline{\phi_2}]; \mathbf{max}_{\phi_2}([\phi_2 : v]; [\text{bathroom}_{\phi_2}\{v\}]))$



$$(26) \quad \text{NOT}^{\phi_2}(\text{NOT}^{\phi_3}(\text{there is a}^v \text{ bathroom})) \rightsquigarrow \\ \lambda\phi_w.[\phi_2 \mid \phi \equiv \overline{\phi_2}]; \mathbf{max}_{\phi_2}([\phi_3 \mid \phi_2 \equiv \overline{\phi_3}]; \mathbf{max}_{\phi_3}([\phi_3 : v]; [\text{bathroom}_{\phi_3}\{v\}])]$$

$$\lambda\phi_w.[\phi_2 \mid \phi \equiv \overline{\phi_2}]; \mathbf{max}_{\phi_2}([\phi_3 \mid \phi_2 \equiv \overline{\phi_3}]; \mathbf{max}_{\phi_3}([\phi_3 : v]; [\text{bathroom}_{\phi_3}\{v\}])) \\ \begin{array}{c} \text{NOT}^{\phi_2} \rightsquigarrow \qquad \qquad \qquad \text{NOT}^{\phi_3}(\text{there is a}^v \text{ bathroom}) \rightsquigarrow \\ \lambda\mathcal{P}_{wt}.\lambda\phi_w.[\phi_2 \mid \phi \equiv \overline{\phi_2}]; \mathbf{max}_{\phi_2}(\mathcal{P}(\phi_2)) \qquad \lambda\phi_w.[\phi_3 \mid \phi \equiv \overline{\phi_3}]; \mathbf{max}_{\phi_3}([\phi_3 : v]; [\text{bathroom}_{\phi_3}\{v\}]) \end{array}$$

$$(27) \quad \text{it}_v \text{ is upstairs} \rightsquigarrow \lambda\phi_w.[\text{upstairs}_{\phi}\{v\}]$$

$$\begin{array}{c} \text{it}_v \text{ is upstairs} \rightsquigarrow \\ \lambda\phi_w.[\text{upstairs}_{\phi}\{v\}] \\ \begin{array}{c} \text{it}_v \rightsquigarrow \qquad \qquad \qquad \text{is upstairs} \rightsquigarrow \\ \lambda\mathcal{P}_{e(wt)}.\lambda\phi_w.P(v)(\phi) \qquad \lambda v_e.\lambda\phi_w.[\text{upstairs}_{\phi}\{v\}] \\ \begin{array}{c} \text{is} \rightsquigarrow \qquad \qquad \qquad \text{upstairs} \rightsquigarrow \\ \lambda\mathcal{P}_{e(wt)}.P \qquad \lambda v_e.\lambda\phi_w.[\text{upstairs}_{\phi}\{v\}] \end{array} \end{array} \end{array}$$

$$(28) \quad \text{NOT}^{\phi_4}(\text{there is a}^v \text{ bathroom}) \text{OR}^{\phi_2, \phi_3} \text{it}_v \text{'s upstairs} \rightsquigarrow \\ \lambda\phi_w.[\phi_2, \phi_3 \mid \phi \equiv \phi_2 \uplus \phi_3]; \\ \mathbf{max}_{\phi_2}([\phi_4 \mid \phi_2 \equiv \overline{\phi_4}]; \mathbf{max}_{\phi_4}([\phi_4 : v]; [\text{bathroom}_{\phi_4}\{v\}])); \\ \mathbf{max}_{\phi_3}([\text{upstairs}_{\phi_3}\{v\}])$$

$$\begin{array}{c} \text{NOT}^{\phi_4}(\text{there is a}^v \text{ bathroom}) \text{OR}^{\phi_2, \phi_3} \text{it}_v \text{'s upstairs} \rightsquigarrow \\ \lambda\phi_w.[\phi_2, \phi_3 \mid \phi \equiv \phi_2 \uplus \phi_3]; \\ \mathbf{max}_{\phi_2}([\phi_4 \mid \phi_2 \equiv \overline{\phi_4}]; \mathbf{max}_{\phi_4}([\phi_4 : v]; [\text{bathroom}_{\phi_4}\{v\}])); \\ \mathbf{max}_{\phi_3}([\text{upstairs}_{\phi_3}\{v\}]) \\ \begin{array}{c} \text{NOT}^{\phi_4}(\text{there is a}^v \text{ bathroom}) \rightsquigarrow \qquad \qquad \qquad \text{OR}^{\phi_2, \phi_3} \text{it}_v \text{'s upstairs} \rightsquigarrow \\ \lambda\phi_w.[\phi_4 \mid \phi \equiv \overline{\phi_4}]; \mathbf{max}_{\phi_4}([\phi_4 : v]; [\text{bathroom}_{\phi_4}\{v\}]) \qquad \lambda\mathcal{P}'_{wt}.\lambda\phi_w.[\phi_2, \phi_3 \mid \phi \equiv \phi_2 \uplus \phi_3]; \\ \mathbf{max}_{\phi_2}(\mathcal{P}'(\phi_2)); \mathbf{max}_{\phi_3}([\text{upstairs}_{\phi_3}\{v\}]) \\ \begin{array}{c} \text{OR}^{\phi_2, \phi_3} \rightsquigarrow \qquad \qquad \qquad \text{it}_v \text{'s upstairs} \rightsquigarrow \\ \lambda\mathcal{P}''_{wt}.\lambda\mathcal{P}'_{wt}.\lambda\phi_w.[\phi_2, \phi_3 \mid \phi \equiv \phi_2 \uplus \phi_3]; \qquad \lambda\phi_w.[\text{upstairs}_{\phi}\{v\}] \\ \mathbf{max}_{\phi_2}(\mathcal{P}'(\phi_2)); \mathbf{max}_{\phi_3}(\mathcal{P}''(\phi_3)) \end{array} \end{array} \end{array}$$

