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On the role of causation in sufficiency and excess

Abstract: This paper concerns different conceptualizations for degree constructions of sufficiency and excess, such as with *enough* and *too* in English. According to many prior works, a sufficiency conveys that a measured degree meets or exceeds a minimum degree for some purpose (*meeting-the-minimum*), whereas an excessive conveys that a measured degree strictly exceeds a maximum degree for some purpose (*exceeding-the-maximum*). I will instead advocate for a causation-based conceptualization for sufficiency and excess, building on Schwarzschild 2008 and Grano 2022. Together with the monotonicity property of gradable predicates, I show that the causation-based descriptions can derive much of their respective truth conditions without stipulating the connection between sufficiency and meeting-the-minimum and between excess and exceeding-the-maximum as in previous accounts. I then present the facts from certain edge cases where the proposals diverge in their predictions; these prove to be problematic for the classic descriptions for these constructions, but are unproblematic for the causation-based formulation. Finally, I also discuss the relevance of my proposal to cross-linguistic variation in the expression of sufficiency and excess.

Keywords: sufficiency constructions, excessive constructions, causation, causal sufficiency, degrees, monotonicity

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To appear in Edoardo Cavirani and Anastasiia Vyshnevska (eds.), *Generative Perspectives on Degrees: The Semantics and Morphosyntax of Scalarity*. Preprint of January 2025.

1 Introduction

It is a truism in formal semantics that the meaning of a particular linguistic expression can be described in different ways. Consider for instance the description of the simple comparative in (1). Given standard assumptions for gradable predicates as relations between individuals and degrees, we might formulate the truth conditions for (1) as in (2a) or (2b). In prose, (2a) claims that there exists some degree d such that "Amy is d-tall" is true but "Bob is d-tall" is false, whereas (2b) first identifies the maximal degrees that make "Amy is d-tall" and "Bob is d-tall" true and claims that Amy's maximal tallness (her height) exceeds Bob's.

(1) Amy is taller than Bob.

(2) a. $\exists d\,[\text{Amy is } d\text{-tall} \wedge \neg(\text{Bob is } d\text{-tall})]$

b. $\max(\{d : \text{Amy is } d\text{-tall}\}) > \max(\{d : \text{Bob is } d\text{-tall}\})$

The truth conditional descriptions in (2a) and (2b) are in fact provably equivalent, again given suitable assumptions. But we may nonetheless have reason to prefer one mode of description over another as more plausible or appropriate for a particular purpose. For instance, it is possible to adapt the description in (2a) to a framework that describes gradable predicates such as *tall* without reference to degrees (see e.g. Klein, 1980; Bochnak, 2015), but not with (2b). The description of the form in (2a) also appears to transparently reflect the expression of comparison in some languages that instantiate what Stassen (1985) calls a "conjoined comparative," as in (3):

(3) Hayung ga bawiq (cikah), (ru) Yuraw ga rroq (cikah).
Hayung TOP tall a little CONJ Yuraw TOP short a little
'Hayung is taller than Yuraw.' [Squliq Atayal[1]]

1 This example comes from my fieldwork in Nan'ao, Taiwan, in 2007–2008. Notably, such conjoined comparatives in Atayal are not norm-related and support crisp judgments, similar to what has been reported by Deal and Hohaus (2019) for Nez Perce. I thank my teacher Hayung Yuraw for discussion.

On the other hand, as von Stechow (1984: 50) notes, if we wish to extend our inquiry to comparatives with differentials of the form in (40), we can do so with ease based on a description for comparison as in (2b), but not with that in (2a).

(4) Amy is 10cm taller than Bob.

And finally — to foreshadow some discussion later in this paper — the description in (2b) requires that both sets as described indeed have maximum values, which is true in this case, whereas a description of the form in (2a) may be used even in cases where the maximal element assumption does not hold.

In summary, the two descriptions in (2a,b) are logically equivalent and hence, in one sense, equally suitable for the truth conditions of (1). And yet, depending on our exact purposes and broader considerations, we may have reason to prefer one formulation over another.[2]

It is in this spirit that, in this paper, I discuss the description of the meanings expressed by sufficiency and excessive constructions, concentrating on English *enough* and *too* constructions as in (5) below. The most widely adopted approach to their truth conditions come from Nelson 1980 and Meier 2003, whereby sufficiency expresses that the measured degree *meets or exceeds* the *minimum* for the stated or implicit consequence (here: being able to drive), whereas excessive constructions express that the measured degree *exceeds* the *maximum* for the consequence. See (6a,b) below for informal descriptions of this form for examples (5a,b).

(5) a. Fred is old **enough** to (be able to) drive.
  b. Fred is **too** old to (be able to) drive.

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2 Put another way, adopting familiar terms from an adjacent literature (Chomsky, 1964 *et seq*), two semantic descriptions could be (nearly) equivalent in their *descriptive adequacy*, but differ in their *explanatory adequacy*. Another example, again within the domain of degree constructions, can be found in the literature around superlatives. See Kotek et al. 2011, 2015 for discussion of different conceptualizations for superlative expressions with English *most*, and the consideration of online verification procedures in experimental settings as potential evidence for or against particular conceptualizations.

(6) a. <u>sufficiency = meeting the minimum:</u>
Fred's age *meets or exceeds* ($\geq$) the *minimum* age for being able to drive.

b. <u>excessive = exceeding the maximum:</u>
Fred's age *exceeds* ($>$) the *maximum* age for being able to drive.

In contrast, Schwarzschild (2008) offers a different approach to the truth conditions of the excessive (with its extension to the sufficiency construction discussed in Grano 2022) that brings front and center the *causal* link between the measured degree and the consequence obtaining or not obtaining, here through the use of *because*.

(7) a. <u>sufficiency = the measured degree makes the consequence obtain:</u>
There is an age d such that (i) because Fred's age meets or exceeds d, Fred *is* able to drive and (ii) Fred's age meets or exceeds d

b. <u>excess = the measured degree makes the consequence *not* obtain:</u>
There is an age d such that (i) because Fred's age meets or exceeds d, Fred *is not* able to drive and (ii) Fred's age meets or exceeds d

The goal of this paper is to discuss the relationship between the Meier-style formulations in (6) and Schwarzschild's causation-based formulations in (7) and advocate for a version of the latter. Following some background in section 2, in section 3 I offer a revised and more detailed formalization based on Schwarzschild's intuition, incorporating insights from Nadathur and Lauer 2020 and Grano 2022, among others. I show that this causation-centric description can derive much of the Meier-style descriptions as in (6) which otherwise must be stipulated. Then in section 4, I present some examples that are problematic for formulations that specifically assume that sufficiency expresses an equative-like, *meeting or exceeding* ($\geq$) relation and that excess expresses a comparative-like, *exceeding* ($>$) relation, including the most widely adopted prior approaches. I conclude with discussion of some open questions in section 5.

2 Describing sufficiency and excess

I begin with some theoretical background and a brief overview of prior descriptions of sufficiency and excessive constructions, concentrating on the description and analysis of English *enough* and *too*.

2.1 Framework

I will adopt a standard framework for degree semantics in the tradition of Cresswell 1976 and von Stechow 1984. Degrees denote measures on a scale and are of type d, with commensurable degrees on a scale forming a total order $\leq$. Gradable predicates such as *tall* denote relations between degrees and individuals, of type $\langle d, et \rangle$, as in (8).[3]

(8) $[\![\text{tall}]\!]^w = \lambda d \, . \, \lambda x \, . \, \textsc{height}_w(x) \geq d$ (type $\langle d, et \rangle$)

In the general case, we can treat degree scales as (isomorphic to) the non-negative real numbers, $\mathbb{R}_{\geq 0}$; I then discuss particular practical contexts that may motivate departing from this assumption for scale structure in section 4. In some cases, I will indicate units on degrees.

Following the analyses in Heim 2000, Meier 2003, and others, I will assume simplified LFs for these constructions of the form in (9) below. The degree morphemes *enough* and *too* form a constituent with a (possibly implicit) clausal complement, which I will refer to as a *consequence*.[4] Such LFs may be thought of as the result of movement of an *enough/too*-headed degree phrase from the the gradable predi-

3 See e.g. Beck 2011, Morzycki 2015: ch. 3, and Hohaus and Bochnak 2020 for more recent introductions. Here for ease of presentation, I concentrate on examples with positive adjectives; i.e. *tall* as opposed to *short*, although my discussion extends to cases with negative polar adjectives, given suitable assumptions (see e.g. Kennedy, 2001; Meier, 2003).

cate's degree argument position. Predicate abstraction over the degree trace yields a degree description.

(9) LF: [[enough/too [$_Q$ PRO to (be able to) drive]] [$_D$ λd . Fred is d-old]]

The result is that *enough* and *too* take two arguments: the consequence proposition Q, of type $\langle s, t\rangle$, and the degree description D, intesionalized to type $\langle s, dt\rangle$. My illustration in (9) makes explicit a modal component within the consequence clause, which I will discuss further below.

2.2 Meeting-the-minimum and exceeding-the-maximum

The most prominent and well adopted description for the meaning of sufficiency and excessive constructions at this point is that in Meier 2003, also reflecting the earlier but less formalized description in Nelson 1980.[5] The truth conditions for both constructions relate the measured degree of D to a particular threshold computed based on the degree description D and the consequence Q; specifically, the latter considers the set of degrees d that satisfy a conditional description of the form "if $[\lambda w' . D(w')(d)]$ is true, Q is true."[6] As briefly introduced above, Meier claims (also echoing Nelson) that sufficiencies express meeting or exceeding ($\geq$; an equative) the minimum degree that makes the consequence true, whereas excessives express

4 In referring to a "(potential) consequence" in the general case, rather than a "goal" or "purpose" which suggest agent intention, I follow Fortuin 2013.

5 It appears that Meier (2003) was unaware of Nelson's work. I thank Lloyd Humberstone for providing me with a copy of Nelson 1980.

6 Formally, the denotation of the consequence clause in Meier 2003 differs so that it is what she calls an "incomplete conditional," allowing the degree description $[\lambda w' . D(w')(d)]$ to restrict the modal base of a modal in the consequence, following the Lewis/Kratzer approach to *if*-conditional interpretation. Here I will simply keep the informal "if" in the sketch of Meier's account in (10), but I note that this conditional relation foreshadows the reference to causation in the discussion below.

exceeding (>; a comparative) the maximum degree that makes the consequence true.[7] Points of contrast below are highlighted.

(10) ***Enough* and *too* in the style of Meier 2003:**

a. sufficiency = meeting the minimum:

$$[\![\text{enough}]\!]^w = \lambda Q_{\langle s,t\rangle} \,.\, \lambda D_{\langle s,dt\rangle} \,.\, \max(D(w)) \geq \min\ (\lambda d \,.\, \text{if } [\lambda w' \,.\, D(w')(d)] \text{ is true, } Q \text{ is true})$$

b. excessive = exceeding the maximum:

$$[\![\text{too}]\!]^w = \lambda Q_{\langle s,t\rangle} \,.\, \lambda D_{\langle s,dt\rangle} \,.\, \max(D(w)) > \max\ (\lambda d \,.\, \text{if } [\lambda w' \,.\, D(w')(d)] \text{ is true, } Q \text{ is true})$$

I will refer to Meier's mode of description for the truth conditions of sufficiency and excess as *meeting-the-minimum* and *exceeding-the-maximum*, respectively. On Meier's account, then, there are two points of variation in these descriptions — meeting (≥; equative-like) versus exceeding (>; comparative-like) and minimum versus maximum — which must covary.[8] But there is no deeper reason offered for why it is these combinations — meeting-the-minimum and exceeding-the-maximum, rather than for instance meeting-the-maximum or exceeding-the-minimum — that are expressed.

It's also worth highlighting the place of modality in Meier's account. For Meier 2003, there is no modal component encoded as part of the operators *enough* and *too* (10); instead, when a nonfinite consequence clause does not include an overt modal,

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7 The partial functions min and max can be defined as in (i) below. Technically, Meier's semantics makes use of "extents" in place of degrees, but this difference is not important here.

(i) a. $\min A = \iota x \,.\, x \in A \wedge \forall a \in A[x \leq a]$

b. $\max A = \iota x \,.\, x \in A \wedge \forall a \in A[a \leq x]$

8 As Meier (2003: 92) writes: "Constructions with *too* differ from constructions with *enough* in only two respects. First the comparison relation 'greater than or equal to' is replaced by 'greater than'. And second, the actual extent that an object has is not compared to the minimal extent that satisfies the corresponding conditional, but to the maximal extent."

an unpronounced possibility modal is posited. (I discuss finite consequence clauses below.) This explains the intuitive (near) equivalence of the examples in (25) above with and without *be able to*.[9] Although some analysts may be put off by the postulation of a covert modal, Grano (2022) notes that this may fall under the umbrella of a broader generalization, which holds true at least for English, that nonfinite clauses in a range of different constructions are quite generally interpreted with some sort of modal character, even without an overt modal.

von Stechow, Krasikova, and Penka (2004) take issue with both the need under Meier's account "to stipulate the distribution of the minimal-maximal operators in the construction" (p. 16) and for the postulation of a covert modal in nonfinite consequence clauses, two features of Meier's account that I've highlighted above. Instead, following discussion in Heim 2000 on scopal interactions between degree constructions and modals, von Stechow et al. (2004) argue that sufficiency expresses meeting or exceeding ($\geq$) the degree that *must* be met where the consequence holds (i.e. *meeting-the-necessary*) and excess expresses exceeding ($>$) the degree that *can* be met where the consequence holds (*exceeding-the-possible*). These modal meanings are built into *enough* and *too*, and no covert modal is posited for overtly modal-less consequences. Although von Stechow et al.'s account appears to eliminate one point of complexity in comparison with Meier 2003 — eliminating the need for reference to a minimum or maximum degree — we still have two points of variation between the meaning schemas for the two constructions: meeting ($\geq$) versus exceeding ($>$) and necessity versus possibility, which must covary.

Concentrating on sufficiency constructions, Grano (2022) refers to the two approaches to the syntax/semantics of modality exemplified by Meier 2003 and von Stechow et al. 2004 as the "modal-clause" approach and the "modal-*enough* approach," respectively; let us more generally call the latter the "modal-operator approach." Grano (2022) shows that sufficiency constructions with finite consequence

9 Expanding on discussion in Meier 2003, Grano (2022: 126) shows that the covert possibility modal in nonfinite consequences that is hypothesized under this approach is more flexible in its choice of accessibility relation than any particular overt modal counterpart in English.

clauses containing overt modals, as in (11) below, can be interpreted straightforwardly on the modal-clause approach (p. 129), such as with the semantics for *enough* in (10b) above, but are challenging for the modal-operator approach (pp. 135–138), where we assume *enough* to include its own necessity modal, which here takes an additional modal in its complement.

(11) **Sufficiency with finite consequence clause, with different overt modals:**
Pat is tall enough that she might/must be the thief. (Grano, 2022)

All of these works surveyed so far, however, agree on the basic description — going back at least to Nelson 1980 (see pp. 108–109) — that sufficiencies convey that the measured degree *meets or exceeds* ($\geq$) a particular threshold and excessives convey that the measured degree *exceeds* ($>$) a particular threshold, where the threshold is determined in some fashion based on the consequence. This reflects the overwhelming success of this approach to the truth conditions of these constructions. And yet, however, I will in fact challenge this shared aspect of these prior descriptions in section 4 below.

2.3 The *because* aspect

An important component of the meaning of these constructions that appears to be lacking thus far is what Humberstone (2006) calls “the *because* aspect” of these constructions. That is, the idea that these constructions seem to convey that the degree to which D holds somehow makes the consequence obtain or not obtain. Schwarzschild (2008) offers a semi-formal account for the meaning of excessive *too* which puts “the *because* aspect” front and center, reflected in (12b) below, making use of a BECAUSE operator as in (13). Although Schwarzschild (2008) only describes the truth conditions for the excessive, Grano (2022: 131–134) illustrates its potential extension to sufficiency constructions, leading to a description as in (12a), differing only in the absence of negation, highlighted below.

(12) ***Enough* and *too* in the style of Schwarzschild 2008:**

a. $[\![\text{enough}]\!]^w = \lambda Q_{\langle s,t\rangle} \,.\, \lambda D_{\langle s,dt\rangle} \,.\, \exists d \left[\text{BECAUSE}_w(\lambda w' \,.\, D(w')(d))(Q)\right]$

b. $[\![\text{too}]\!]^w = \lambda Q_{\langle s,t\rangle} \,.\, \lambda D_{\langle s,dt\rangle} \,.\, \exists d \left[\text{BECAUSE}_w(\lambda w' \,.\, D(w')(d))(\neg Q)\right]$

(13) **BECAUSE from Schwarzschild 2008: 325:**

$\text{BECAUSE}_w(p)(q)$ is true iff (i) p is a reason for q and (ii) p and q are true in w.

It is worth stepping back and appreciating how different these semantic entries in (12) look from that from Meier's approach as in (10). In particular, I will highlight that the Schwarzschild-style formulation offers the simplest — and as a matter of taste, in my own opinion, the conceptually most attractive — understanding for the difference between *enough* and *too*: the measured degree is such that it is a reason for Q to obtain (in sufficiencies), or for $\neg Q$ to obtain (in excessives). This difference in the presence or absence of negation immediately explains their contrasts in the licensing of NPIs, such as *give a damn* in (14):

(14) **Excessives license NPIs but sufficiencies do not:** (Linebarger, 1987: 328)

John is {too tired / *tired enough} to give a damn.

Grano (2022: 130–134) also highlights a further conceptual advantage of Schwarzschild's account, again related to the availability of finite consequences with *enough*. Grano notes that sufficiencies with modal-less finite consequences as in (15c) lead to non-cancellable actuality inferences: in this case, that Bo hit the target. This is in contrast to the minimally contrasting examples with a nonfinite consequence in (15a) and with a finite consequence with an overt possibility modal in (15b).

(15) **Actuality inferences with different consequence types:** (Grano, 2022: 127)

a. Bo was fast enough to hit the target, but actually he didn't hit the target.

b. Bo was fast enough that he could hit the target,
but actually he didn't hit the target.

c. Bo was fast enough that he hit the target,
(#but actually he didn't hit the target).

Under Meier's account, all consequences must have a modal, whether overt or covert, as the mode of composing the degree description and the consequence clause involves something akin to constructing and evaluating a conditional, as a modal restrictor in the Lewis/Kratzer style. We would therefore have to posit another covert modal in (15a), this time for finite clauses: a circumstantial necessity modal. In contrast, as Grano notes, the pattern in (15) is explained quite naturally on a Schwarzschild-type account, with minimal modification to make it a modal-clause approach: "the entailment patterns in [(15)] follow straightaway from the fact that the BECAUSE operator, like its object language counterpart *because*, is veridical on both of its propositional arguments" (p. 132). (15a,b) entail Q, that he could hit the target with a possibility modal (covert in (15a)), whereas (15c) simply directly entails Q, with no modalization.

However, despite this advantage in offering a straightforward account for some patterns of actuality inferences, Grano does not ultimately commit to nor specifically advocate for Schwarzschild's formulation. He expresses one reason for his ambivalence in a footnote: the approach in Schwarzschild 2008 is, "as it stands, incomplete, because it relies on a meta-language operator BECAUSE that is not formally defined. A reduction of BECAUSE to more familiar theoretical terms, perhaps using possible worlds, might enable a more careful investigation…" (p. 133, note 19). My own proposal and discussion below will do just this.

3 Proposal

I now present and advocate for my own proposal for the sufficiency and excessive constructions. My proposal takes what Humberstone (2006) calls "the *because* aspect" to play a central role, following Schwarzschild 2008 in relating the satisfaction of the degree description by a particular degree with the consequence obtaining or not obtaining. I improve upon Schwarzschild 2008 by unpacking his BECAUSE relation and clarifying the modal nature of the consequence clause, building especially on discussions in Grano 2022, Nadathur and Lauer 2020, and Nadathur 2019, 2023a.

My basic proposal for the semantics of sufficiency and excessive morphemes is given in (16). For presentational purposes, I continue to describe these meanings as associated with the English expressions *enough* and *too*, although I intend for my core proposal to extend to other languages as well.

(16) **Proposal (compact, to be expanded):**

a. $[\![\text{enough}]\!]^w = \lambda Q_{\langle s,t\rangle} \, . \, \lambda D_{\langle s,dt\rangle} \, . \, \exists d \, [(\lambda w' \, . \, D(w')(d)) \rhd_w Q \, \wedge \, D(w)(d)]$

b. $[\![\text{too}]\!]^w = \lambda Q_{\langle s,t\rangle} \, . \, \lambda D_{\langle s,dt\rangle} \, . \, \exists d \, [(\lambda w' \, . \, D(w')(d)) \rhd_w \neg Q \, \wedge \, D(w)(d)]$

where $A \rhd B$ indicates that A is *causally sufficient* for B.

After composition with its arguments Q and D, these constructions will claim the existence of a degree d such that $(\lambda w' \, . \, D(w')(d))$ is causally sufficient ($\rhd$) for Q or $\neg Q$ and D indeed attains this degree d in the evaluation world w. The one point of difference between *enough* and *too* is highlighted in (16).

This section is organized as follows. I first discuss the notion of causal sufficiency and motivate the use of this notion (over causal necessity) in section 3.1 below. I then expand upon the denotations in (16) by restating the causal sufficiency relation $\rhd$ in more familiar modal semantic terms. In section 3.2, I briefly comment on examples involving non-causal explanatory relations. In section 3.3, I will show that my proposal derives truth conditions for sufficiency and excessive constructions that capture much of — but are not exactly equivalent to — the truth conditions as described in classic descriptions such as Nelson 1980, Meier 2003, and von Stechow et al. 2004, due to the logical properties of degree descriptions. I then present new data that serves to distinguish between the two types of accounts in section 4.

3.1 Formalizing *because* as causal sufficiency

My proposal follows the core insight of Humberstone (2006) and Schwarzschild (2008) that a relation we might call BECAUSE plays a key role in the semantics of excessives and sufficiencies. The question is how exactly this intuitive notion should be modeled. Here, I will first concentrate on modeling causal senses of BECAUSE and briefly comment on non-causal senses in section 3.2 below.

Nadathur and Lauer (2020) argue for incorporating notions of reasoning based on situational causal dynamics into our formal semantic toolkit.[10] In particular, they argue that we, as linguists, must distinguish between two different causal relations, *causal necessity* (◁) and *causal sufficiency* (▷),[11] which theories of causal reasoning allow us to do. Informally, causal necessity refers to a relationship where "a cause is taken to render its effect possible" whereas causal sufficiency refers to a relationship where "a cause renders its effect not only possible, but inevitable" (Nadathur and Lauer, 2020: 2). I refer readers to Nadathur and Lauer 2020, Bar-Asher Siegel and Boneh 2019, 2020, and Glass 2023 for discussion of linguistic phenomena that highlight the need for distinguishing between these two different notions of causation. See also Nadathur and Lauer 2020 and Nadathur's subsequent work for discussion of the formal modeling of these two causal notions using causal models (in particular, so-called *structural equation models*; see Pearl 2000 and Schulz 2011).

10 Many prior works have argued that notions of causation, based on our intuitions thereof, cannot be reduced to basic notions of logical dependency or counterfactual reasoning. Here I follow the perspective that causal claims specifically reflect (arguably non-linguistic) reasoning based on the causal dynamics relevant to a particular situation: "a combination of situation-specific information, past experience, and generally accepted truths about the world (e.g., physical constants and laws)" (Nadathur, 2023a: 263). See also for example Copley and Wolff 2015, Nadathur and Lauer 2020 (especially page 10 and footnote 8), McHugh 2023, and citations there for further discussion.

11 I follow the notation in Nadathur's more recent work (Nadathur 2023a: 61, 2023b: 327), where ▷ and ◁ specifically indicate causal sufficiency and causal necessity, respectively. Her earlier dissertation (see Nadathur, 2019: 211) instead uses ► and ◄ for these notions, with ▷ and ◁ instead indicating more general notions of sufficiency and necessity.

Note that there are thus two senses of the word "sufficiency" here, which are important to keep apart, as von Fintel (2024: 151) also emphasizes. I will attempt to be very systematic about referring to the relevant causal relation as "causal sufficiency" throughout (which contrasts with "causal necessity"), in order to avoid any confusion with the *enough*-type construction under investigation, which I will refer to as (bare) "sufficiency" (which contrasts with "excessive").

With this brief background on causal reasoning and notions of causation in place, let us return to the issue of causation in the semantics of sufficiency and excessive constructions. As mentioned above in my proposal (16), I argue that the relevant notion here, corresponding to Schwarzschild's BECAUSE in (12–13), is *causal sufficiency*. These constructions then require that there is some degree d such that (i) meeting or exceeding some degree d is *causally sufficient* for the consequence Q to (not) obtain and (ii) the measured degree indeed meets or exceeds this degree d.

Due to the nature of causal sufficiency, we predict that a true sufficiency entails that the consequence Q is true while a true excessive entails that the consequence Q is false.[12] Here I reiterate that I follow Meier 2003 and Grano 2022 in adopting the so-called modal-clause approach to modality in these constructions: nonfinite consequences are necessarily modal, using an unpronounced possibility modal where there is no overt modal. This explains the (near) equivalence of examples with a nonfinite consequence and a finite consequence with a possibility modal as in (17a,b) below, as well as the non-modalized consequence entailment with the finite consequence in (17c). See evidence from the attempted cancellation of hitting the target in (15) above.[13]

12 This echoes Schwarzschild's (2008) definition for $\text{BECAUSE}_w(p)(q)$, reproduced in (13) above, which specifically requires p and q to be true in w.

13 Grano's discussion concentrates on sufficiency constructions, which allow for finite consequences, but the predicted entailment of $\neg Q$ by excessive *too* constructions with nonfinite consequences — often with covert possibility modals parallel to (17a) — cannot be verified. Excessives in English do not allow for finite consequences, which appears to be a syntactic quirk without deeper, semantic motivation. On this point, see discussion in Grano 2022: 141–143.

(17) **Sufficiencies entail their consequences:** (Grano, 2022: 132–133)

a. Bo was fast enough [$_Q$ to hit the target].
$\Rightarrow Q \approx$ Bo could hit the target

b. Bo was fast enough [$_Q$ that he could hit the target].
$\Rightarrow Q =$ Bo could hit the target

c. Bo was fast enough [$_Q$ that he hit the target]
$\Rightarrow Q =$ Bo hit the target

Note that this general pattern of consequence entailment in sufficiencies and their denial in excessives is due to the causal sufficiency $\triangleright$ relation in their semantics in (16) in the position corresponding to the imprecise BECAUSE in Schwarzschild 2008. We do not predict these entailments if the relevant notion were instead causal necessity. I illustrate this point for the case of sufficiency constructions: if meeting or exceeding a particular threshold degree were instead causally necessary for a consequence Q to be true, we would be claiming that the only way Q comes about as an effect of the causal dynamics is if the measured degree meets or exceeds some threshold; we then assert that this threshold is in fact met. This however does not entail the consequence Q. (The same point holds for excessives as well, with their entailment of $\neg Q$.)

In addition to the derivation of these consequence entailment facts, I will discuss a few examples that more directly illustrate the role of causal sufficiency in these constructions. First, consider the sufficiency constructions in (18).

(18) **Causal sufficiency in sufficiency constructions:**

a. The sun was bright enough that I sneezed. (Grano, 2022: 128)

b. They were {lucky enough / smart enough / rich enough / famous enough} to get away with murder.

Grano (2022: 128) writes that example (18a) with a finite consequence clause "gives the impression that the sun's brightness caused me to sneeze (not an unusual occurrence for those of us who have the photic sneeze reflex)." But there can also be other reasons why one might sneeze. The sun being bright to some particular degree is, for some individuals, causally sufficient for them to sneeze, not causally necessary. Next, I note that each of the variants of example (18b) is intuitively natural: some high degree of luck makes it possible to get away with murder, but so does a high degree of intelligence, or fortune, or fame. There are many different ways to get away with murder,[14] but crucially, no particular one of these qualities is required: one could for instance get away with murder by their intelligence alone, or by their fame alone, etc.

The same point can be made for the semantics of excessives:

(19) **Causal sufficiency in excessive constructions:**

a. The sun is too hot to touch. (Schwarzschild, 2008: 317)

b. [Pat has a strong alibi. And besides...]
she is too tall to be the thief. (based on Grano, 2022)

Schwarzschild (2008) describes (19a) as "saying that the sun's temperature makes it impossible for us to touch it," as an informal demonstration of his causation-based semantics. But interestingly, he follows this with, "In fact, other factors prevent us from touching the sun, such as its distance from the earth. These do not detract from the claim in [(19a)]... the various causes depend on different physical properties (temperature, distance) and each is a sufficient cause" (p. 317). As this passage suggests, the sun's temperature is causally sufficient for one to not be able to touch it, but the matter of its temperature is not causally necessary for one to not be able to touch it. Similarly, in (19b), we clearly establish a relevant factor — a strong alibi — which in and of itself suggests that Pat could not be the thief. But in addition, we note that she exceeds some relevant threshold of height, which is also already causally sufficient to disqualify her as a suspect. Neither factor is causally necessary for the effect of Pat not possibly being the thief.[15]

14 Or so I've been told.

15 I return to this example in section 3.2 below, to discuss whether the claim here is best described as "causal" or not.

These examples clearly demonstrate that, in both sufficiencies and excessives, meeting or exceeding some threshold degree is *causally sufficient* for the consequence Q to be true in sufficiencies and false in excessives. In either case, the threshold degree does not have to be a causal necessity for $Q/\neg Q$.

I also acknowledge that there are certain examples that, at first glance, invite a description where meeting or exceeding a particular degree is causally *necessary*, rather than causally sufficient, for the consequence to (not) obtain. For instance, consider example (20) below. Meier (2003: 89) notes that "It is possible, for example, to utter a sentence like [(20)] in a situation where we know that Bertha meets the age requirement but not the height requirement to get a certain part in a play."

(20) Bertha is old enough to get the part (although she is not tall enough).
(Meier, 2003: 89)

Here, I emphasize again that I adopt the modal-clause approach, where the relevant consequence Q in (20) includes a covert possibility modal. In particular, as Meier (2003: 89–90) and Grano (2022: 125–127) discuss, this covert possibility modal in non-finite consequence clauses allows for counterfactual interpretations. This allows us to "abstract away from Bertha's other actual properties" (Meier, 2003: 90) that are independent of the age claim at issue.

Taking a page from Grano 2022 (specifically, page 126), evidence for this understanding — that the covert modal within Q holds the key to this interpretation in (20) — comes from the behavior of corresponding examples with finite consequence clauses. Consider the examples in (21) below.

(21) [Context from Meier 2003: 89: "Assume that Bertha is 18 years old and 5 feet tall and that she must be at least 16 years old and at least 6 feet tall in order to qualify for the role."]
Although she is not tall enough...
a. # Bertha is old enough [that she *can* get the part].
b. Bertha is old enough [that she *could* get the part].

c. # Bertha is old enough [that she gets/got the part].

To be felicitous in the context in question, the finite consequence clause must use a modal such as *could* with so-called X-marking (von Fintel and Iatridou, 2023) to indicate counterfactuality, as in (21b). The variants with non-X-marked *can* (21a) or without a modal (21c) are judged as infelicitous in the context. The counterfactuality in (21b) allows us to consider worlds where some other salient factor is different — namely, where she has greater height — then commenting only on the causally sufficient nature of her age within that counterfactual context.

Having established the correct notion of causation that corresponds to Schwarzschild's BECAUSE in the denotation of sufficiency and excessive constructions, I return to the issue of its formal implementation. Nadathur and Lauer (2020) argue for the idea that linguistic expressions related to causation make use of a more general cognitive capacity for causal reasoning, which may be formalized using causal models. The relations of causal sufficiency ($\rhd$) and causal necessity ($\lhd$) were defined there in those terms. However, Kaufmann (2013) develops a technique for encapsulating the causal dynamics encoded in such models into *causal premises*, which can be used as an ordering source in a more familiar, Kratzer-style modal semantics. The result is that we can now define causal sufficiency and causal necessity in familiar quantificational terms. Nadathur (2019, 2023a) does just that, offering definitions as in (22) below.[16]

(22) **Causal sufficiency and causal necessity in causal premise semantics:**

(based on Nadathur 2019: 305, 2023a: 180)

Let CAUS(w) be the contextually-determined set of causally optimal worlds accessible from world w, which do not predetermine the truth or falsity of propositions p and q.

a. p is *causally sufficient* for q in w ($p \rhd_w q$) iff

$$\forall w' \in \textsc{Caus}(w)[p(w') \rightarrow q(w')]$$

b. p is *causally necessary* for q in w ($p \lhd_w q$) iff

$$\forall w' \in \textsc{Caus}(w)[\neg p(w') \rightarrow \neg q(w')]$$

Using this conception of causal sufficiency, then, I now restate my proposal for the semantics of *enough* and *too* from (16) above as in (23) below. Again, I highlight the negation in *too* below to emphasize the one point of difference between the two denotations.

(23) **Proposal (expanded):**

a. $[\![\text{enough}]\!]^w = \lambda Q_{\langle s,t\rangle} \, . \, \lambda D_{\langle s,dt\rangle} \, .$
$$\exists d \, [\forall w' \in \textsc{Caus}(w) \, [D(w')(d) \rightarrow Q(w')] \; \wedge \; D(w)(d)]$$

b. $[\![\text{too}]\!]^w = \lambda Q_{\langle s,t\rangle} \, . \, \lambda D_{\langle s,dt\rangle} \, .$
$$\exists d \, [\forall w' \in \textsc{Caus}(w) \, [D(w')(d) \rightarrow \neg Q(w')] \; \wedge \; D(w)(d)]$$

where $\textsc{Caus}(w)$ is the contextually-determined set of causally optimal worlds accessible from world w.

Unpacking the causal relation in this way will be useful for understanding the relationship between the meanings expressed by sufficiency and excessive constructions on my proposal versus earlier accounts. I turn to this comparison in section 3.3 below.

3.2 A note on *because* beyond causation

I take a moment here to discuss the possibility of excessives and sufficiencies expressing claims that are not strictly speaking about “causation” per se. In some cases, the relationship between the claim of meeting or exceeding a particular degree and the consequence obtaining or not, which I analyze as the causal sufficiency relation $\triangleright$ above, may be a non-causal relation instead.

16 The definitions here are simplified for presentational purposes. In particular, Nadathur describes these relations in terms of a consistent set of propositions c. The causally optimal worlds are determined using a set of premises (for instance, determined from a circumstantial modal base) and an ordering source determined by the causal dynamics, following Kaufmann 2013. I collapse these components together in my reference to the context-dependent $\textsc{Caus}(w)$. I refer interested readers to the discussions in Nadathur 2019: 296–306, 2023a: 175–181.

Humberstone (2006) includes a warning to this effect: “The ‘because’ here is not specifically a causal ‘because.’ For instance, a geometrical figure with eleven sides has too many sides to be a decagon” (p. 267). Indeed, we might hesitate to say there is a “causal” relationship between having more than 11 sides (the degree claim) and the negated consequence of not being a decagon. The relationship here is instead a constitutive one, reflecting what it means to be a decagon.

Dean McHugh points out that there are related linguistic contrasts between *because* and expressions such as *cause* as well. In particular, he observes the following difference in the felicity of paraphrases for Grano’s *too tall to be the thief* example in (19b) above:

(24) **Paraphrases for (19b) with *cause* vs *become*:**

a. Pat is not able to be the thief *because* she is so/too tall.

b. ?? Pat being so/too tall *caused* her to be unable to be the thief.

The *because* variant in (24a) is much more natural than the *cause* variant in (24b). This contrast appears to reflect a known difference between the two expressions (see e.g. McHugh 2023: 180–181 and note 5 there) such that *because* but not *cause* can be used to describe the epistemic basis for particular claims. Dean McHugh (p.c.) writes, “This suggests that Humberstone and Schwarzschild were onto something when they described the causal component of sufficiency and excessive constructions using *because* rather than *cause*.”

As one reviewer helpfully suggests, the alternative notion relevant in such cases might be that of “grounding,” which can be thought of as a relation of non-causal explanation (see e.g. Fine 2012). I will not pursue an in-depth discussion of this notion and its relation to causation here.[17] As the reviewer notes, for current purposes, we may be heartened by the fact that some scholars suggest that grounding relations can also be productively modeled using Pearl-style structural equation models (see

17 I thank Bob Beddor and Helen Beebee for helping me find my bearings in the grounding literature.

e.g. Schaffer, 2016; Emmerson, 2023), which would allow for the modal semantic restatement for *enough* and *too* as in (23) above.

I will leave a full investigation into sufficiencies and excessives involving non-causal dependencies for another occasion. In this paper, I will simply naively refer to the relevant relation underlying "the *because* aspect" as causal sufficiency throughout, despite Humberstone's cautionary note.

3.3 Towards the classical truth conditions

I now turn to a comparison of the claims expressed by sufficiency and excessive constructions under my proposal versus earlier accounts. I will show in particular that my causation-based proposal derives truth conditions that are similar — although not equivalent to — that of the classic descriptions such as in Nelson 1980, Meier 2003, and von Stechow et al. 2004.

Concretely, let us consider the truth conditions for the sufficiency and excessive constructions in (25) below, repeated from (5) in the introduction. In each case, the intensionalized degree description D and consequence Q are as in (26) below.[18] Following the modal-clause approach as in Meier 2003 and Grano 2022, I assume that there is a covert possibility modal — here, with a deontic accessibility relation, ACC — within the consequence Q.

(25) a. Fred is old enough to (be able to) drive.
LF: [enough [$_Q$ CAN [PRO$_{\text{Fred}}$ to drive]]] [$_D$ λd [Fred is d-old]]

b. Fred is too old to (be able to) drive.
LF: [too [$_Q$ CAN [PRO$_{\text{Fred}}$ to drive]]] [$_D$ λd [Fred is d-old]]

(26) a. $[\![\text{old}]\!]^w = \lambda d \,.\, \lambda x \,.\, \text{AGE}_w(x) \geq d$

b. $D = \lambda w \,.\, \lambda d \,.\, \text{AGE}_w(\text{Fred}) \geq d$

18 Intensionalization of D is not reflected in the LF as abstraction over a world variable. I instead assume the use of a rule of composition such as Intensional Functional Application here; see Heim and Kratzer 1998: 308ff and von Fintel and Heim 1997–2021.

c. $Q = \lambda w \,.\, \exists w' \in \text{Acc}(w)\, [\text{drive}_{w'}(\text{Fred})]$

It will be important here to emphasize one formal property of degree predicates. Gradable predicate denotations such as for *old* in (26a) are *downward-scalar*, in the sense that, if some individual is d-old for a particular degree d, they will also be d'-old for all lower degrees, $d' < d$. I define this notion in (27), following Heim 2000.[19]

(27) A function G of type $\langle d, et \rangle$ is *downward-scalar* iff
$\forall x\, \forall d\, \forall d'\, [(G(d)(x) \wedge d' < d) \rightarrow G(d')(x)]$

Because degree descriptions D in a sufficiency or excessive construction are built by abstracting over the degree variable of a gradable predicate, they too will be downward-scalar in the basic case.[20] In our simple examples in (25) above, the degree description D in (26a) is downward-scalar, which can be verified using a suitably adapted definition for the term. This downward-scalar property of the degree descriptions will be important in my discussion below.

The truth conditions for these degree constructions can be described in terms of the set of degrees $\mathcal{D}$ for the gradable predicate in D — that is, here, a set of ages for Fred — where the construction will be true. Consider the sets $\mathcal{D}_{enough}$ and $\mathcal{D}_{too}$ for the sufficiency and excessive constructions, respectively. Their membership de-

19 Although this definition is based on Heim 2000: 216, Heim as well as Nouwen (2011) refer to the property simply as monotonicity on the degree argument. I choose to follow authors such as Abrusán and Spector (2011: 110) and Beck (2012: 238, 2013: 6) in using the more descriptive term "downward-scalar," introduced as a formal property of predicates of degrees or numbers of type $\langle d, t \rangle$ by Beck and Rullmann (1999: 257).

20 If there is an entailment-reversing operator in the description, such as negation, the degree descriptions will be upward-scalar instead. (Note that we need not be worried about non-monotonic quantifiers intervening, due to an observed restriction on quantifier scope-taking in degree descriptions, called the Heim-Kennedy Constraint; see Kennedy 1997, Heim 2000: 223, a.o.) My basic claim here, that my proposal yields truth conditions similar to that of the classic accounts (with the exception of the boundaries; see section 4 below) also extends to such cases, but here for presentational purposes, I will simply consider the basic case where D is downward-scalar, such as with (26).

scriptions echo the causal requirement for degree d in the denotations for *enough* and *too* in (23).

(28) a. $\mathcal{D}_{enough} = \{d \mid \forall w' \in \text{CAUS}(w)\,[D(w')(d) \rightarrow Q(w')]\}$

b. $\mathcal{D}_{too} = \{d \mid \forall w' \in \text{CAUS}(w)\,[D(w')(d) \rightarrow \neg Q(w')]\}$

These $\mathcal{D}$ sets (if non-empty) are intervals with a lower bound but no upper bound, which we can prove using the downward-scalar property of D. First, we can show that if $d \in \mathcal{D}_{enough}$, any higher degree $d' > d$ will also be in $\mathcal{D}_{enough}$. Consider the membership description for $\mathcal{D}_{enough}$: for any world $w' \in \text{CAUS}(w)$, $D(w')(d) \rightarrow Q(w')$ (because $d \in \mathcal{D}_{enough}$) and $D(w')(d') \rightarrow D(w')(d)$ (because D is downward-scalar), and therefore $D(w')(d') \rightarrow Q(w')$. This guarantees $\forall w' \in \text{CAUS}(w)\,[D(w')(d') \rightarrow Q(w')]$, and so the higher degree $d' \in \mathcal{D}_{enough}$. Notice that this reasoning does not apply in the other direction: given $d \in \mathcal{D}_{enough}$, we cannot guarantee that a lower degree ($d' < d$) will also be in $\mathcal{D}_{enough}$. This argument in this paragraph applies equally to $\mathcal{D}_{too}$ as well.

It then follows that both sets in (28) are intervals with lower bounds but no upper bound. That is, they will either be of the form $(\theta, \infty) = \{x \in S \mid \theta < x\}$ (lower-open) or $[\theta, \infty) = \{x \in S \mid \theta \leq x\}$ (lower-closed), where S is the relevant scale and θ is the greatest lower bound of $\mathcal{D}$. Recall that I assume $S = \mathbb{R}_{\geq 0}$ in the general case, although other scale assumptions will become relevant in section 4 below.

It may also be useful to visualize these $\mathcal{D}$ sets on number lines. Supposing that the constructions make non-trivial claims — that is, that $\mathcal{D}$ is neither the empty set nor the entire scale S — these intervals are as indicated in (29) below.

(29) **Degrees ensuring felicity and truth, $\mathcal{D}$:**

a. $\underline{\mathcal{D}_{enough}:}$ b. $\underline{\mathcal{D}_{too}:}$

For the general case in (29), I use ◒ in these number lines to indicate the intervals' greatest lower bounds, staying agnostic between open (○) and closed (●) variants. Whether the greatest lower bounds are included in these intervals or not will be the topic of section 4.

I return now to the concrete case of our basic examples in (25) above. First, suppose that people are allowed to drive in the relevant context from age 18. The ages that make *Fred is old enough to drive* true, $\mathcal{D}_{enough}$, is the set of ages such that Fred being that age is causally sufficient for him to be able to drive; in this context, this will be the closed, lower-bounded interval $[18, \infty)$. Second, consider a context where people are allowed to drive up to the age of 70 but not greater than that. The ages that make *Fred is too old to drive* true, $\mathcal{D}_{too}$, is the set of ages such that Fred being that age is causally sufficient for him to *not* be able to drive; this will be the open, lower-bounded interval $(70, \infty)$.[21] In each case, we thereby derive overall truth conditions that are equivalent to that predicted by the classic accounts such as Meier 2003 and von Stechow et al. 2004: sufficiency has the equative-like truth conditions that require meeting-the-minimum (or meeting-the-necessary) age for driving whereas the excessive has comparative-like truth conditions that require exceeding-the-maximum (or exceeding-the-possible) age for driving.

21 The minimum and maximum age requirements can also be imposed simultaneously, for instance in a context where people are allowed to drive from age 18 to 70, but not outside of this range. In such cases, technically $\mathcal{D}_{enough}$ as defined in (28) is empty: Fred being d-old for some $d \in [18, 70]$ is not causally sufficient for Fred being able to drive, as Fred might also be d'-old for some $d' > 70$, and hence, in fact too old to drive.

Following Rullmann 1995 and Heim 2006, among others, Beck (2013) discusses the semantics of comparatives with *than*-clauses that may potentially denote a minimum or maximum point of a scalar interval. There, Beck suggests two possible interpretational strategies in such situations, which we can also adopt here. The first is to simply limit our attention to a particular subset of the scale, over which the causal sufficiency between the degree claim and the consequence still holds. Concretely, for instance, suppose we know we are discussing a young adult, so we concentrate on $S' \approx [16, 30)$, a subset of the full scale $S = \mathbb{R}_{\geq 0}$. Limiting our attention to S', $\mathcal{D}_{enough} = [18, 30)$ which is equal to $[18, \infty)$ over S'.

The second repair strategy Beck suggests is to exceptionally interpret the degree predicate as exact and hence non-scalar. In this particular example, the modified gradable predicate would be $D = (\lambda w\ .\ \lambda d\ .\ \text{AGE}_w(\text{Fred}) = d)$ (cf (26b)). This modification also arrives at the correct truth conditions for the *enough* example in (25a) in this context, requiring that Fred be exactly d-old with $d \in \mathcal{D}_{enough} = [18, 70]$.

In this way, my causation-centric account can derive overall truth conditions for examples such as in (25) that are equivalent to that of the classic accounts, without having to stipulate any link between sufficiency and *meeting-the-minimum* and between excess and *exceeding-the-maximum*. Given the overall successes of the truth conditional descriptions of the classic accounts, the fact that my account is able to derive the same effect from the nature of the causal link, without stipulating the two covarying parameters of meeting/exceeding and minima/maxima (or necessity/possibility), is a welcome result of my account and a conceptual advantage.

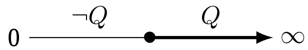
4 Comparative-like sufficiencies and equative-like excessives

I have now presented my causation-based proposal and shown that it derives similar — and in many basic examples identical — truth conditions for sufficiency and excessive constructions to those of the classic descriptions of Nelson 1980, Meier 2003, von Stechow et al. 2004, and others. However, their predictions are not identical. In this section, I will present novel data that shows that the causation-based approach is empirically superior and that these previous (and currently mainstream) approaches cannot be maintained.

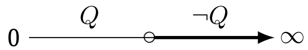
Recall that the classic accounts describe sufficiencies as having "equative-like" truth conditions that are true if and only if the measured degree *meets or exceeds* ($\geq$) a particular threshold degree, while excessives have "comparative-like" truth conditions that are true if and only if the measured degree *exceeds* ($>$) a particular threshold. We can restate this in terms of the shape of the set of degrees $\mathcal{D}$, which are degrees for the gradable predicate that make the construction true. A sufficiency construction, having equative-like truth conditions, is predicted to have $\mathcal{D}_{enough} = [\theta, \infty)$, with a closed lower bound. An excessive construction, having comparative-like truth conditions, is predicted to have $\mathcal{D}_{too} = (\theta, \infty)$. These $\mathcal{D}$ structures are illustrated in the number lines in (30) below:

(30) **Degrees ensuring felicity and truth, $\mathcal{D}$, on the classic accounts:**

a. $\underline{\mathcal{D}_{enough}:}$



b. $\underline{\mathcal{D}_{too}:}$



On these classic descriptions, as reflected in (30), the threshold degree θ *does* make a sufficiency true but does *not* make an excessive true. One way that we might restate and understand this is to say that the threshold degree θ is generally associated with the truth of the stated consequence Q, according to these descriptions. I give this generalization in (31).

(31) **The status of the threshold degree, according to the classical proposals:**
Let θ be the greatest lower bound of $\mathcal{D}$, the set of degrees that make a sufficiency or excessive construction true. According to the classic proposals such as Meier 2003, θ is associated with the truth of Q. Specifically:

a. In sufficiencies, *meeting-the-minimum* means θ is a degree for D that makes Q true.

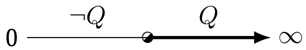
b. In excessives, *exceeding-the-maximum* means θ is a degree for D that does *not* make Q *false*, i.e. it is possible that Q is true.

I comment on this generalization in (31) again below.

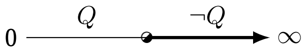
In contrast, I will also repeat the illustration of the shape of $\mathcal{D}_{enough}$ and $\mathcal{D}_{too}$ predicted by my account, in (32) below. As I showed in the preceding section, $\mathcal{D}_{enough}$ and $\mathcal{D}_{too}$ (if non-empty) are both lower bounded intervals with no upper bound, but the nature of that lower bound — as open or closed — is underspecified by my proposal, which I indicate with ◑. In other words, if the classic descriptions are correct, this is one aspect of the meaning of these constructions which my own proposal fails to derive.

(32) **Degrees ensuring felicity and truth, $\mathcal{D}$, on my account:** =(29)

a. $\mathcal{D}_{enough}$:



b. $\mathcal{D}_{too}$:



The questions we must ask in order to adjudicate between these two descriptions are then the following:

(33) a. Is it possible for a sufficiency construction to have truth conditions that are comparative-like ($> \theta$), where the threshold degree does *not* make the construction true, but any higher value does?
I.e. $\mathcal{D}_{enough} = (\theta, \infty)$, with an open lower bound

b. Is it possible for an excessive construction to have truth conditions that are equative-like ($\geq \theta$), where the threshold degree θ *does* make the construction true, in addition to any higher value?
I.e. $\mathcal{D}_{too} = [\theta, \infty)$, with a closed lower bound

These questions zoom in on the point of difference between the predictions of the classic accounts, in (30), and the predictions of my causation-based proposal, in (32).

I contend that the answer to both questions is *yes*, that uses with such interpretations are attested, contrary to the predictions of the classic descriptions. I present such examples in (34–35) below. In both cases, the (a) example is a (perhaps somewhat artificial) example about a mathematical claim, whereas the (b) example is based on an attested set of circumstances in the actual world. I also give the sets $\mathcal{D}$ of degrees which make the target sentences true.

(34) **Sufficiencies that have no minimum true degree:**

a. [*r* is a positive real number.]
The value of r is large enough that the function $y = r^x$ is increasing.
$\mathcal{D} = (1, \infty)$

b. [The Lufthansa baggage policy specifies that, "a piece of baggage is considered to be excess baggage when it weighs more than 23 kg," and that excess baggage incurs an additional fee.[22] We do not know the precision of the scale used to weigh the luggage.]

Your box is heavy enough to incur a fee.

$\mathcal{D} = (23\text{kg}, \infty)$

(35) **Excessives that have a minimum true degree:**

a. [*r* is a positive real number.]

The value of r is too large for the geometric series $\sum_{n=0}^{\infty} r^n$ to converge.

$\mathcal{D} = [1, \infty)$[23]

b. [The rules of the Paws ’n Play dog park in Lansing, Michigan specify that “Large dogs (26 lbs and above) are not allowed in the small run area. Small dogs (under 26 lbs) are not allowed in the large dog run area.”[24] We do not know the precision of the tools they would use to determine a dog’s weight in any context of potential enforcement action.]

Your dog is too heavy to be in the small run area.

$\mathcal{D} = [26\text{ lbs}, \infty)$

It is worth emphasizing that the classic accounts, which describe sufficiencies as *meeting or exceeding* some threshold ($\geq \theta$) and excessives as *exceeding* some threshold ($> \theta$) *cannot* account for the behavior of the examples in (34) and (35), and are therefore not descriptively adequate. These include the descriptions in Nelson 1980: 108, Meier 2003: 87, 92, and von Stechow et al. 2004: 20. I am not aware of any prior work discussing the existence of such examples as a challenge to these proposals.

I note that the exact way in which these preceding accounts fall short in the face of such data differs by account to account. For Meier 2003, sufficiencies make ref-

22 From https://www.lufthansa.com/us/en/excess-baggage, accessed May 14, 2024.

23 For any $|r| < 1$, the series converges to $1/(1-r)$. For any $|r| \geq 1$, the series diverges.

24 From https://www.lanoakparkdistrict.org/paws-n-play/paws-n-play-rules/, accessed May 14, 2024.

erence to *the* minimal degree (p. 87) and excessives make reference to *the* maximal degree (p. 92), from a set of degrees that — paraphrasing slightly, closer to my own account — make the consequence true. (See (10) and note 6 above.) The relevant set of degrees for example (34b) does not have a unique minimum degree, predicting presupposition failure by ι in min. (See note 7 above for definitions of min and max.)

We might also consider a variant of Meier's approach, making reference to greatest lower bound (*infimum*) and least upper bound (*supremum*) in place of min and max, respectively, as in (36) below.[25] In this case, as the set of degrees for D that make the consequence true is lower bounded in a sufficiency and upper bounded in an excessive, we will necessarily be able to refer to its infimum or supremum, respectively, and so no presupposition failure will arise. However, this modification does not suffice. Even though we will not encounter presupposition failure, the hard-coded $\geq$ relation in *enough* as meeting-the-infimum and $>$ in *too* as exceeding-the-supremum (see (10) above) will still make the incorrect prediction that sufficiencies are equative-like and excessives are comparative-like in their truth conditions.

(36) **Meier-style denotations (10) using infima and suprema (also incorrect):**

a. sufficiency = meeting the infimum:

$$[\![\text{enough}]\!]^w = \lambda Q_{\langle s,t\rangle} \,.\, \lambda D_{\langle s,dt\rangle} \,.\, \text{max}(D(w)) \geq \text{inf}\ (\lambda d \,.\, \text{if } [\lambda w' \,.\, D(w')(d)] \text{ is true, } Q \text{ is true})$$

25 The operations inf (greatest lower bound) and sup (least upper bound) can be defined in terms of min and max (note 7) as follows:

(i) a. $\text{inf}\, A = \text{max}\,\{x : x \in S \wedge \forall a \in A[x \leq a]\}$

b. $\text{sup}\, A = \text{min}\,\{x : x \in S \wedge \forall a \in A[a \leq x]\}$

Where a set has a unique minimum or maximum value, $\text{inf}\, A = \text{min}\, A$ and $\text{sup}\, A = \text{max}\, A$, respectively. For instance, for a lower-closed interval $A = [\theta, \infty)$, $\text{inf}\, A = \text{min}\, A = \theta$. However, inf and min (as well as sup and max) come apart where there is an open boundary: for instance, for a lower-open interval $A = (\theta, \infty)$, $\text{inf}\, A = \theta$ but $\text{min}\, A$ is undefined.

b. excessive = exceeding the supremum:

$$[\![\text{too}]\!]^w = \lambda Q_{\langle s,t\rangle} \,.\, \lambda D_{\langle s,dt\rangle} \,.\, \max(D(w)) > \text{sup}\ (\lambda d \,.\, \text{if } [\lambda w' \,.\, D(w')(d)] \text{ is true, } Q \text{ is true})$$

The proposal in von Stechow et al. 2004 appears at first glance to be more resilient, as their formalization expresses a set-containment relation ($\subseteq$ or $\subset$) between two sets of degrees, without any intermediate step of calculating their maximum or minimum values. However, consider the truth conditions that von Stechow et al. (2004) would predict for the sufficiency example (34b) in (37) below, modeled after their example (76). (H^* is a contextually-determined modal base, "fine" is the proposition that a fine is incurred, and "box" stands in for the referent of *your box*.)

(37) $$\{d \mid \forall w\,[(w \in H^* \wedge \text{fine}_w(\text{box})) \rightarrow \textsc{weight}_w(\text{box}) \geq d]\} \subseteq \{d \mid \textsc{weight}_{w^*}(\text{box}) \geq d\}$$

Now consider the left hand set, $\{d \mid \forall w\,[(w \in H^* \wedge \text{fine}_w(\text{box})) \rightarrow \textsc{weight}_w(\text{box}) \geq d]\}$. In the context in (34b) above, this will be the interval $[0, 23\text{kg}]$, because all fine-incurring boxes weigh at least 23kg, as well as any lower degrees, by downward-scalarity. The set however does not include any degrees higher than 23kg. Suppose for contradiction that $d >$ 23kg is in this set. Because the relevant scale ($\mathbb{R}_{\geq 0}$) is dense, there will always be a degree d' between 23kg and d: 23kg $< d' < d$. There is then a possible world in H^* where the box weighs exactly d' and incurs a fine. Therefore d cannot be in the set. The left hand set will therefore be exactly $[0, 23\text{kg}]$. The von Stechow et al. 2004 style account then predicts the sentence in example (34b) to be true when the measured degree is exactly 23kg, contrary to fact, because the left hand set in (37) is $[0, 23\text{kg}]$. In effect, this mode of description is unable to distinguish between the meanings of sufficiencies and excessives with and without minimum true degrees.

Given the existence of examples such as in (34) and (35), which pose an empirical challenge to the truth conditional descriptions in many influential prior accounts, we might wonder why those classical truth conditional descriptions seem to be correct for the vast majority of examples. I believe there are at least two factors at play here.

The first factor is the nature of the scale that is assumed for interpretation, i.e. the set of values that degrees can range over. Here I have followed a standard mode of presentation by treating the scale of degrees as (isomorphic to) the non-negative real numbers, $\mathbb{R}_{\geq 0}$. An important property of the real numbers is that it is *dense*:

(38) An ordered set S is *dense* iff for any two values $a, b \in S$ where $a < b$, there is another value $c \in S$ such that $a < c < b$.

However, in practice, most real life contexts have a relevant level of (im)precision, identifiable from the context or else reasonably inferred (see e.g. Lasersohn, 1999; Klecha, 2018). Taking this level of precision into account, we are licensed to consider a scale that is coarser, and correspondingly, in a technical sense, *non-dense* (cf 38), i.e. *discrete* (*pace* Fox and Hackl, 2006). When the scale is discrete, a sufficiency with no logical minimum true degree, of the form in (34), can be restated as a sufficiency *with* a minimum true degree.

As an illustration of this effect, consider the interpretation of (39) below:

(39) **Sufficiency that invites a precise, minimum degree:**
[The current world record for the largest lake trout by weight is 32.65kg.[26] These records are based on weights rounded to two decimal places.]
This lake trout is heavy enough to set a new world record!
$\mathcal{D} = [32.66\text{kg}, \infty)$

Hypothetically, if such world records cared about differences of arbitrary precision, we would expect any lake trout larger than the current world record to set a new world record: $\mathcal{D} = (32.65, \infty)$. But because we practically know that all measurements in this domain are rounded to the nearest hundredth of a kilogram, a descriptively equative-like interpretation as $\mathcal{D} = [32.66, \infty)$ also counts as a faithful

26 From https://igfa.org/member-services/world-record/common-name/Trout,%20lake, accessed May 14, 2024.

translation here. Formally, this is because, if the scale S does not include any values that fall between 32.65 and 32.66, then the intervals $(32.65, \infty)$ and $[32.66, \infty)$ are equivalent over S.

Compare this effect in the interpretation of the *lake trout* example (39) to the *baggage policy* example in (34b) above. In (34b), we may assume that enforcement of the stated rules are based on an exact interpretation of the stated measure (23 kilograms), where no difference is ignorable, if discernible. Importantly — and unlike in (39) — the context in (34b) specifies that we do not know what level of precision can be assumed. We therefore cannot treat the scale as something coarser than the default $\mathbb{R}_{\geq 0}$. The same goes for excessives with a minimum true degree such as the *dog park* example in (35b); if a level of precision were identifiable, the sentence would allow for an *exceeding-the-maximum* formulation, not possible in (35b) as stated. In addition, for the mathematical examples in (34a) and (35a), we explicitly specify that the relevant scale is the positive real numbers. In such mathematical discussions, no pragmatic slack is allowed (see e.g. Lasersohn, 1999: 524), and so we maintain the use of the dense scale.

The second factor which I believe is at play here is how we as speakers conceptualize the consequence Q and the associated threshold degree θ, the greatest lower bound of $\mathcal{D}$. Specifically, I hypothesize that there is a communicative preference for discussing consequences Q so that the threshold degree θ is associated with the truth of Q. For instance, if we are discussing ages associated with voting or heights associated with being the thief, etc., it is most natural to describe these acceptable values themselves by describing minimum and maximum bounds which are included in the corresponding degree extension. This in effect explains (by stipulation) the observation in (31) above.

What we observe with the examples in (34) and (35) is that this preference is not absolute. Concretely, concentrating on the non-mathematical contexts, this hypothesized preference would mean that, in the *baggage policy* context in (34b), we might be more likely to discuss the weights associated with a consequence of the form 'to (be able to) check the box for free,' which is compatible with its threshold degree of

23kg, or in the dog park context (35b), we might be more likely to discuss the weights associated with ‘to be in the large dog run area,’ which is compatible with its threshold degree of 26 pounds. However, if incurring a fine or being in the small run area are salient consequences in the current discourse, this overall preference can be overridden.

I hypothesize that these two pressures, while not in force in all contexts, together conspire to make it so that sufficiencies generally are compatible with “equative-like” ($\geq$) interpretations and excessives generally are compatible with “comparative-like” ($>$) interpretations, in accordance with the generalization in (31). Nonetheless, formulations for the semantics of sufficiency and excessive constructions that strictly assume or derive this generalization are incorrect.

5 Conclusion

As I noted in the introduction, a recurring challenge for the semantic analyst is to distinguish between two conceptually distinct modes of description for a particular linguistic expression. In the case of sufficiency and excessive constructions as with English *enough* and *too*, broadly two approaches are found in the literature: one that treats sufficiencies and excessives as truth-conditionally varieties of equatives and comparatives, respectively (due to Nelson, Meier, and others), and one that treats them as expressing some causal (or similar) relation between the measured degree and the consequence obtaining or not (due to Humberstone and Schwarzschild). I have referred to the former approach as the “classic” description, due to its relatively wide adoption in existing literature.

This paper makes multiple contributions to the study of English sufficiency and excessive constructions, which overall motivate the adoption of the causal approach over the classic descriptions. First, I argued that the causal relation in the Humberstone/Schwarzschild style approach should be modeled as causal sufficiency (or a similar notion of sufficient grounding). Given this, I showed that my denotations predict a subtle difference in truth conditions as opposed to the classic accounts, in terms of whether or not the set of degrees that satisfy the causal claim is predicted to have a minimal element. My causation-based proposal accurately models the behavior of sufficiency and excessive constructions in cases where these sets have a lower bound but not a minimum — that is, what I have called "comparative-like sufficiencies" and "equative-like excessives" — but the classic accounts and subsequent work with similar formulations fails to capture such uses.

I believe that the empirical superiority of the causal account is a welcome result also from the perspective of how we conceptualize the relationship between sufficiency and excess. Under my proposal, like the proposal in Schwarzschild 2008 that I build on, there is only one point of variation between the two constructions: the presence or absence of negation in the description of the relevant degree; see (16) and (23). There are not two covarying points of variation as with Meier's *meeting-the-minimum* versus *exceeding-the-maximum* and von Stechow et al.'s *meeting-the-necessary* versus *exceeding-the-possible* (discussed in §2.2). I would argue that the mode of description here for the relationship between the two constructions is much more illuminating.

I will conclude the paper with two open issues for the causation-based account. The first is that *too* excessives comfortably take differential measure phrases but *enough* sufficiencies do not:

(40) ***Too* allows for differentials but *enough* does not:**
   a. Silvio is {a bit / much / 5 cm} *too* tall to (be able to) walk through this door.
   b. * Silvio is {a bit / much / 5 cm} tall *enough* to (be able to) touch the ceiling.

As Chris Kennedy notes, this is one empirical point where the classical accounts appear to have an immediate advantage over the causation-based approach. Specifically, we note that comparatives take differential measure phrases but equatives do not (41), and so this contrast as in (40) immediately falls out from accounts that treat excessives and sufficiencies as sharing a semantic core with comparatives and equatives, respectively.

(41) **Comparatives allow for differentials but sufficiencies do not:**
Silvio is {a bit / much / 5 cm} {taller than / *as tall as} this door.

Under the causation-based accounts such as my proposal here, this effect as in (40) requires a separate explanation. I unfortunately do not have a solution to this issue and describe it here as an open problem.[27] However, I also note that the apparent analogy between excessives vs sufficiencies and comparatives vs equatives in this regard breaks down in cases of sufficiencies with no minimum true degree, i.e. in those with comparative-like truth conditions. In such cases, differentials are still unacceptable, at least to my ear:

(42) **Differentials still disallowed in comparative-like sufficiency:**
[Context repeated from (34b): The Lufthansa baggage policy specifies that, "a piece of baggage is considered to be excess baggage when it weighs more than 23 kg," and that excess baggage incurs an additional fee. We do not know the precision of the scale used to weigh the luggage.]
* Your box is {a bit / much / 2 kg} heavy enough to incur a fee.

The second issue concerns the derivation of actuality entailment behaviors from nonfinite consequence clauses under higher perfective aspect (see e.g. Bhatt 1999 and Hacquard 2020). Hacquard (2005) shows that sufficiency and excessive constructions in French, under higher perfective aspect, entail the non-modalized content of their nonfinite consequent clauses or its negation:

27 One might wonder if the contrast reflects a syntactic difference in whether the position to host the measure phrase is available (for instance, inspired by the discussion of differentials in Stateva 2003), correlating with the pre-predicate word order of *too* versus post-predicate *enough*. See Erlewine and Nguyen 2022, 2024 for discussion of Vietnamese degree constructions, where such differences in surface word order indeed reflect deeper structural differences between different degree morphemes, including two varieties of excessives. Unfortunately, even if such an account were successful for explaining the contrast between English *too* and *enough*, it would not explain the difference between *more* and *as*.

(43) **French actuality entailments under perfective aspect:** (Hacquard, 2005: 81)

a. Jean a été assez rapide pour s'enfuir (#mais il ne s'est pas enfui).
'Jean was.PFV quick enough to escape (#but he didn't escape).'

b. Jean a été trop lent pour s'enfuir (#mais il s'est enfui).
'Jean was.PFV too slow to escape (#but he still escaped).'

As a reviewer notes, my proposal here may complicate the derivation of such effects. Previous work on the derivation of such actuality entailment effects (as in Hacquard 2005 as well as Nadathur 2023a) build on what I called a "modal-operator" approach, treating nonfinite consequent clauses as not inherently modalized. However, my proposal here builds on the arguments in Grano 2022 (see (15) and (17) above) that nonfinite clauses are necessarily understood as containing a modal, even if covert. In this paper, I have concentrated on describing and explaining the basic truth conditions of sufficiencies and excessives as degree constructions, leaving the explanation for such (non)actuality inference patterns using my proposal here for future work.

Despite the challenges that these remaining issues may pose to the causation-based approach that I develop and argue for here, I will reiterate again the empirical inadequacy of the classic alternative accounts: *Enough* sufficiencies can have comparative-like truth conditions and *too* excessives can have equative-like truth conditions, which challenge the classic accounts but are as predicted by my causation-based account. I hope that the success of the proposal here and its motivation will then instead inform future work on related, open issues such as the semantics of differentials and actuality effects.

Acknowledgment: I thank Chris Kennedy and Dean McHugh for detailed comments on an earlier draft, as well as Bob Beddor, Helen Beebee, Tom Grano, Prerna Nadathur, and the audience at the 2024 Singapore Summer Meeting for related discussion. I also thank two anonymous reviewers who may already be named above. This work was supported by a fellowship at the Helsinki Collegium for Advanced Studies, which is gratefully acknowledged.

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