Higher-order plurality: To what degree?[1]

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Abstract. Recent work by Grimau (2020) and Buccola et al. (2021) has rekindled the debate on the extent to which natural language allows for the construction of *higher-order* (or *structured*) pluralities (HOPs) — that is, pluralities of pluralities (Link, 1983; Landman, 1989; Schwarzschild, 1996). Over the decades, research on HOP has focused on ordinary entity-denoting plurals, yet the typology of semantic entities is generally assumed to be diverse (Rett, 2022), arguably including events, worlds, times, degrees, and more. At least some of these domains arguably have the same or similar mereology as the ordinary entity domain, most famously events (Bach, 1986), and more recently degrees (Dotlačil and Nouwen, 2015). A natural question that arises is: do any of these other semantic entities allow for HOP? I argue that the answer is *yes*, using reciprocal degree constructions as a case study (cf. Schwarz, 2007; Hsieh, 2021).

Keywords: degrees, mereology, plurality, semantic ontology, reciprocity

1. Introduction

Conjunctions of plural nominals, as in *the cats and the dogs*, appear to provide access to *higher-order*, or *structured*, pluralities — that is, pluralities of pluralities (Grimau, 2020; Buccola et al., 2021). Over the decades, research on higher-order plurality has focused on ordinary entity-denoting plurals (Link, 1984; Landman, 1989; Schwarzschild, 1996). Yet the typology of semantic entities is generally assumed to be diverse, arguably including events, worlds, times, degrees, and more (Rett, 2022). Some of these domains arguably share the same or similar mereology as the entity domain, most famously events (Bach, 1986), and more recently degrees (Dotlačil and Nouwen, 2015). This paper addresses the following question: do any other domains allow for the construction of higher-order pluralities? I argue that the answer is *yes*, based on evidence from reciprocal degree constructions (cf. Schwarz, 2007; Hsieh, 2021).

I begin by rehearsing the elements of some basic theories of plurality (§2). I then discuss previous arguments and diagnostics for higher-order plurality (§3). Next, I present an extension to degree-denoting nominals (§4). Finally, I move to non-degree nominals (reciprocal equatives) (§5) before concluding (§6).

2. Two theories of plurality

2.1. Background

A plurality of individuals is their mereological sum (Link, 1983). Individuals and individual sums are type e. The relation between them is represented by algebraic structure in the ontology: a is a *part of* the sum $a \sqcup b$ (that is, $a \leq a \sqcup b$). For example, the denotations of *student*, *students*,

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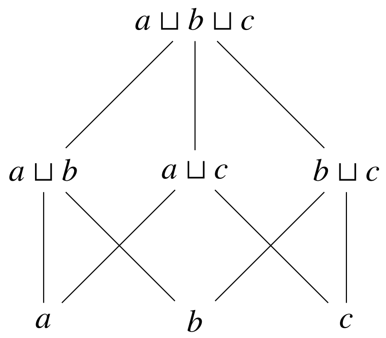


Figure 1: Hasse diagram representing ⟦students⟧ for ⟦student⟧ = $\{a, b, c\}$.

and *the students* may look as follows (see also Figure 1):

$$[\![\text{student}]\!] = \{a, b, c\} \qquad [\![\text{students}]\!] = \{a, b, c, a \sqcup b, a \sqcup c, b \sqcup c, a \sqcup b \sqcup c\}$$
$$[\![\text{the students}]\!] = \text{the maximal (top) element in } [\![\text{students}]\!] = a \sqcup b \sqcup c$$

While sets can be nested, giving rise to sets of sets, mereological sums cannot, as codified in the assumed properties of the summation operation, ⊔ (Champollion and Krifka, 2016):

- $(a \sqcup b) \sqcup c = a \sqcup (b \sqcup c) = a \sqcup b \sqcup c$ (associativity)
- $a \sqcup b = b \sqcup a$ (commutativity)
- $a \sqcup a = a$ (idempotence)

Mereological sums are thus flat, unstructured.

2.2. Higher-order plurality

A number of natural language constructions seem to provide access to higher-order pluralities (HOPs) — that is, pluralities of pluralities (Link, 1984; Landman, 1989). For instance, the conjunctive nominal *the French students and the Italian students* may intuitively refer to a plurality consisting of the sub-plurality of French students and the sub-plurality of Italian students. To see this more clearly, notice that a sentence like (1) can be true in a ‘symmetric’ context in which the French students hit the Italian students, and the Italian students hit the French students (Winter and Scha, 2015; Grimau, 2020), suggesting that we intuitively access the two sub-pluralities of French and Italian students.

(1) The French students and the Italian students hit each other.

But if conjunction of two pluralities generates a flat plurality, how does the compositional semantics gain access to the two sub-pluralities that hit each other?

$$[\![\text{the French students}]\!] = a \sqcup b \qquad [\![\text{the Italian students}]\!] = c \sqcup d$$
$$[\![\text{the French students and the Italian students}]\!] \stackrel{?}{=} a \sqcup b \sqcup c \sqcup d$$

There have been two prominent accounts in the literature. First, according to Landman (1989), a group-forming operator ($\uparrow$) may apply to plural nominals. Thus, $\uparrow$[*the French students*] *and* $\uparrow$[*the Italian students*] denotes a plurality of two atomic groups, where pluralities and groups are ontologically different, thus yielding higher-order structure:

$$\uparrow(a \sqcup b) \sqcup \uparrow(c \sqcup d)$$

Importantly, group formation is restricted by syntactic structure: the $\uparrow$ operator may only apply to nominals.

Second, according to Schwarzschild (1996), plural nominals are interpreted relative to a cover of their denotations — roughly, a way of dividing up the denotation into a *set* of pluralities. Thus, *the French students and the Italian students* just denotes the flat plurality $a \sqcup b \sqcup c \sqcup d$. One possible cover of this plurality divides them according to nationality: $\{a \sqcup b, c \sqcup d\}$. Covers are recoverable from context. Every possible cover corresponds to a distinct reading of the sentence.

Schwarzschild, arguing for covers, points out that sentences like (2) and (3) do not have a relevant node in the logical form for $\uparrow$ to apply to, and yet they are also true in the 'symmetric' context.

(2) The students from the two countries hit each other.

(3) The students hit each other.

But do sentences (1), (2), and (3) all have genuine symmetric readings?

3. Detecting symmetric readings

The two theories above make claims about what *readings* are available for a given sentence, but the fact that a sentence can be judged true in a particular situation does not mean that this situation corresponds to a distinct reading of the sentence. Rather, we must distinguish genuine ambiguity from underspecification (Gillon, 1990, 2004). If a sentence is genuinely ambiguous (between two or more readings), then the situations in which it is true under one reading differ from those in which it is true under another reading. For example, (4) is ambiguous between one reading that is true only in situations in which Sue has a baseball bat, and another reading that is true only in situations in which Sue has a flying mammal commonly known as a bat.

(4) Sue has a bat.

By contrast, if a sentence is underspecified in meaning, then it has a single reading, which can be true in different situations. For example, (5) is true in any situation where Sue has a sibling, regardless of the sibling's gender; that is, it is not ambiguous between one reading that is true only if Sue has a sister, another reading that is true only if Sue has a brother, and so on.

(5) Sue has a sibling.

Sometimes, one reading may be strictly stronger than another. For example, (6) is ambiguous between one reading that is true only if there is some particular book that every student read (the inverse scope reading), and another reading that is true only if every student read some (possibly different) book or other (the surface scope reading). The former reading entails the latter. Therefore, looking solely at the contexts in which the sentence may be judged true is not sufficient to determine whether the inverse scope reading exists.

(6) Every student read some book.

In the case at hand (... *hit each other*), reciprocal sentences have been claimed to allow a very weak reading based on an 'inclusive alternative ordering' (Dalrymple et al., 1998). Under this reading, sentence (3) could be paraphrased as 'Each student either hit or was hit by some other student'. The sentence would then be *true* in the symmetric situation, but this would not indicate that the sentence has a distinct *symmetric reading*.

To detect ambiguity, we therefore must consider not only when the sentence is true, but also when it is false. Sentences with ellipsis and negation help bring out the relevant intuitions.

(7) *Context: Sue owns a flying mammal but no sports equipment, while Jo owns a baseball bat but no animals.*

Sue owns a bat, but Jo doesn't. ⇝ **can** be judged true ✓

Here, we see that *Sue owns a bat* can be judged either *true* or *false*, depending on the reading, and the same for *Jo owns a bat*. With respect to a single reading, then, *Sue owns a bat, but Jo doesn't* can be judged true.

Things are different for *sibling*.

(8) *Context: Sue has a brother but no sister, while Jo has a sister but no brother.*

Sue has a sibling, but Jo doesn't. ⇝ **cannot** be judged true ✗

Here, *Sue has a sibling* and *Jo has a sibling* are both just plain *true*, and hence *Sue has a sibling, but Jo doesn't* is just plain *false*.

Returning now to (1), (2), and (3), Buccola et al. (2021) report the following judgments based on an experiment that was conducted and statistically analyzed:

(9) *Context: This class has only French and Italian students. On Monday, a fight broke out: the French students hit the Italian students, and the Italian students hit the French students. On Tuesday, another fight broke out, but this time within the two groups: the French students hit one another, and the Italian students hit one another.*

a. On Monday, the Fr. students and the It. students hit each other, but not on Tuesday. ⇝ **can** be judged *true* ✓

b. On Monday, the students from the two countries hit each other, but not on Tuesday. ⇝ **can** be judged *true* ✓

c. On Monday, the students hit each other, but not on Tuesday. ⇝ **cannot** be judged *true* ✗

Thus, (1) and (2), but not (3), share a reading — the symmetric reading — that is synonymous with (10), which states that members of each group (plurality) hit members of the other.

(10) The Fr. students hit the It. students, and the It. students hit the Fr. students.

The upshot is that genuine symmetric readings, hence HOPs, are available, depending on the form of the nominal — sometimes even *without* conjunction.[2]

4. Extension to degrees

Do the domains of other semantic entities (beyond type e) provide access to HOP? Below we explore the domain of degrees (type d).

4.1. Forming and conjoining (atomic) degree nominals

Plausibly, a nominal like *the weight of this apple* denotes a degree, such as 150 g,[3] so that (11a) means (11b), where μ_w is a measure function that maps this apple, a, to its weight.

(11) a. The weight of this apple is 150 g.

b. $\mu_w(a) = 150\,\text{g}$

We may conjoin two such nominals, as in *the weight of this apple and the weight of that plum*, to form a plural nominal. To this plural nominal, we may apply a predicate like *are 150 g and 100 g*, as in (12a).

(12) a. The weight of this apple and the weight of that plum are 150 g and 100 g.

b. $\mu_w(a) \sqcup \mu_w(p) = 150\,\text{g} \sqcup 100\,\text{g}$

Dotlačil and Nouwen (2015) argue that such sentences have cumulative readings that provide prima facie evidence for *degree pluralities*. Thus, (12a) means (12b), where $\sqcup$ is the (degree-based version of) the summation operation: the degree sum consisting of the apple's weight and the plum's weight is identical to the degree sum consisting of the degree 150 g and the degree 100 g.[4] These truth conditions are equivalent to saying that the apple weighs one of 150 g or

[2] A theoretical side question is: how do we model HOPs in such a way that they are available for sentences like (1) and (2), but not (3)? Buccola et al. (2021) propose two ideas, one an enrichment of Landman's groups with scope-taking, the other a revision of Schwarzschild's covers with dynamic semantics. This paper does not address this question — I assume that either theoretical avenue would work equally well for the data to come — but see Buccola et al. 2021 if interested.

[3] I say "plausibly", but this assumption/claim may not be uncontroversial. For example, one may alternatively say that such nominals denote tropes, rather than degrees, in the sense of Moltmann 2009. I will have to leave a detailed comparison of degree- vs. trope-based analyses (and predictions) to future work, but see §6 for some very brief remarks. The main point for now is that I assume such nominals denote something other than ordinary entities.

[4] Just to be extra clear, $\sqcup$ simply sums up two degrees, just like it sums up two entities, to yield a sum (plurality) — it has nothing to do with *adding* degrees in any way.

100 g, and the plum has the other weight, which is the cumulative reading.

This analysis of (12a) mirrors how we could analyze a similar sentence with ordinary entities, such as *The guest with purple hair and the guest wearing sandals are Sue and Jo*:

$$[\![\text{the guest with purple hair}]\!] \sqcup [\![\text{the guest wearing sandals}]\!] = \text{Sue} \sqcup \text{Jo}$$

Of course, in both cases there is also an order inference: (12a) intuitively implies that the apple's weight is 150 g and that the plum's weight is 100 g, not the other way around; and we likewise infer that Sue has purple hair and that Jo is wearing sandals, not the other way around. But these inferences are plausibly implicatures, as they can be reinforced with *respectively*, suspended with *but I don't know which one has which weight* (*. . . which guest is which*), or canceled with *but not in that order*.

4.2. Forming higher-order degree pluralities

To construct a *higher-order degree plurality* (HODP), we may try conjoining two plural degree nominals, such as *the weights of these apples and the weights of those plums*. However, to test for HODP, we also need a suitable predicate to apply to it, so that the resulting sentence is grammatical and has a reading that necessitates access to a HODP. By analogy with the symmetric readings discussed above, I suggest that *(be) equal* is one suitable candidate.

(13) The weights of these apples and the weights of those plums are equal.

If a genuine symmetric reading is available for (13), then it should be synonymous with (14), which does not involve any HODP, and which has a transitive use of *equal*.

(14) The weights of these apples are equal to the weights of those plums.

Prima facie judgments from informants (collected informally) indicate that this synonymy *does* hold. But what exactly do (13) and (14) mean? (Note, crucially, that the plural *weights* indicates that we are *not* talking about the *total* weight of the apples or of the plums.) To answer this question, note first that when *equal* applies to two atomic degrees, as in (15a) or (15b), the answer is clear: *equal* says that the two degrees are the same.

(15) a. The weight of this apple and the weight of that plum are equal.

b. The weight of this apple is equal to the weight of that plum.

c. $\mu_w(a) = \mu_w(p)$

Generalizing this idea to (13) and (14), the prediction is that *equal* says that the two degree pluralities (or groups), that of the apple weights and that of the plum weights, are the same.

(16) $\mu_w(a_1) \sqcup \cdots \sqcup \mu_w(a_i) = \mu_w(p_1) \sqcup \cdots \sqcup \mu_w(p_j)$

4.3. Testing the prediction

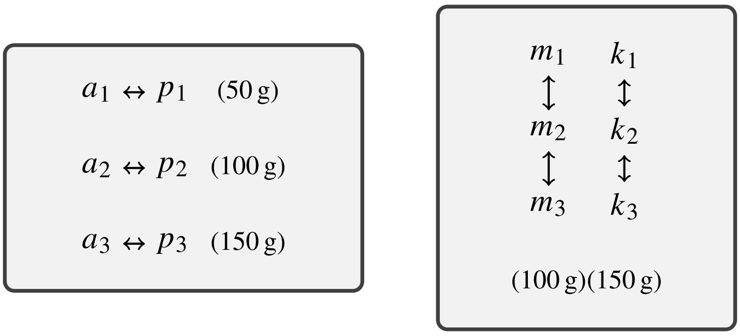
Does the equality in (16) correspond to a reading of (13)/(14)? To be sure, the most natural context that comes to mind may be one where all the plums and apples have the same weight, say 125 g each. The equality in (16) correctly comes out true in this context, given that $125\,\text{g} \sqcup \cdots \sqcup 125\,\text{g} = 125\,\text{g}$ by idempotence, and hence both sides of the equality are 125 g.[5]

But the equality holds in many other contexts as well: in general, (16) holds if and only if each apple weighs the same as some plum, and each plum weighs the same as some apple. Note that this weak reading is reminiscent of weak readings of reciprocal constructions more generally, as mentioned above (Dalrymple et al., 1998).

Informants report that (13) and (14) can both have this reading, although contextual support may be necessary for certain verifying cases (like with reciprocal constructions more generally).

Following the methodology of Buccola et al. (2021), we may construct a context that verifies the truth conditions for one case (apples and plums), but not for another (mandarins and kiwis).

(17) *Context: A scientist is conducting an experiment to test whether a drug she created causes fruit to grow to an exact weight, depending on the dosage of the drug, and regardless of the type of fruit. She first tests three apples and three plums, using three different dosages: the smallest dosage causes the first apple-plum pair to each weigh 50 g; the medium dosage causes the second apple-plum pair to each weigh 100 g; and the largest dosage causes the third apple-plum pair to each weigh 150 g. She then tests three mandarins and three kiwis: all three dosages cause all three mandarins to weigh 100 g each and all three kiwis to weigh 150 g each.*



a. The weights of the apples and the weights of the plums are equal. ⇝ **can** be *true*

b. The weights of the mandarins and the weights of the kiwis are equal. ⇝ **can** be *false*

c. The weights of the apples and the weights of the plums are equal, but the weights of the mandarins and the weights of the kiwis are not. ⇝ **can** be *true* ✓

Relative to this context, informants report that (17a) can be judged true, while (17b) can be judged false, hence also that (17c) can be judged true.

[5]An interpretation that entails the same weight for all fruit pieces may well correspond to a distinct reading, derived from the flat plurality of all apple and plum weights, but I leave this potential reading aside in this paper.

Here is another example along a different dimension, for good measure.

(18) *Context: Various basketball teams take part in a tournament. For reasons of fairness, when two teams play, it is important that each player from one team can be paired with a player from the other team of equal height, and vice versa. Team A and Team B mutually satisfy this requirement, but Team C and Team D do not.*

a. The heights of Team A's players and the heights of Team B's players are equal.
$\rightsquigarrow$ **can** be *true*

b. The heights of Team C's players and the heights of Team D's players are equal.
$\rightsquigarrow$ **can** be *false*

c. The heights of Team A's players and the heights of Team B's players are equal, but the heights of Team C's players and the heights of Team's D's players are not.
$\rightsquigarrow$ **can** be *true* ✓

Again, informants report that (18a) can be judged true in this context, while (18b) can be judged false, hence also that (18c) can be judged true.

In summary, the existence of symmetric readings of sentences with conjunctions of plural degree nominals provides support for HODPs: the conjunctions denote HODPs whose degree sub-pluralities (or subgroups) are required to be identical to one another, which in turn means that each degree atom of one must be a degree atom of the other.[6]

4.4. What about *unequal*?

Another predicate worth considering is *(be) unequal*. A natural position to take is that *unequal* is just the negation of equal. Intuitions about singular degree-denoting nominals support this idea: (19a) and (20a) have opposite truth conditions, given in (19b) and (20b), respectively.

(19) a. The weight of this apple and the weight of that plum are equal.

b. $\mu_w(a) = \mu_w(p)$

(20) a. The weight of this apple and the weight of that plum are unequal.

b. $\mu_w(a) \neq \mu_w(p)$

Following this line, we predict that (21a) and (22a) likewise have opposite truth conditions.

(21) a. The weights of these apples and the weights of those plums are equal.

b. $\mu_w(a_1) \sqcup \cdots \sqcup \mu_w(a_i) = \mu_w(p_1) \sqcup \cdots \sqcup \mu_w(p_j)$

[6]Along the lines of Buccola et al. 2021, one may wonder whether conjunction is necessary for such readings, by considering nominals like *the weights of the two fruits*, in a context with two salient fruits (apples and plums). I suspect that the facts are the same as with ordinary entities, namely that they allow access to the relevant sub-pluralities of apple weights and plum weights, but I will have to leave confirmation to future work.

(22) a. The weights of these apples and the weights of those plums are unequal.

b. $\mu_w(a_1) \sqcup \cdots \sqcup \mu_w(a_i) \neq \mu_w(p_1) \sqcup \cdots \sqcup \mu_w(p_j)$

Is this prediction correct? First, note that for the context in (17) above, where each apple is paired with a plum of equal weight, and vice versa, informants report that (22a) can be judged *false*, as predicted.

But in which contexts can such a sentence be judged *true*? Similar to before, the most natural context that comes to mind may be one where *no* apple has the same weight as *any* plum (and vice versa) (cf. Schwarz, 2007, which we will return to). This is the mandarin-kiwi case from (17), and indeed in that context, the *unequal* variant of (17b), (23a) below, can be judged true, as predicted by the inequality in (23b), while the *unequal* variant of (17a), viz. (22a), can be judged false, as predicted by the inequality in (22b).

(23) a. The weights of the mandarins and the weights of the kiwis are unequal.

b. $\mu_w(m_1) \sqcup \cdots \sqcup \mu_w(m_i) \neq \mu_w(k_1) \sqcup \cdots \sqcup \mu_w(k_j)$

But in general, (22b) holds the moment there is an apple whose weight is not shared by any plum, or there is a plum whose weight is not shared by any apple. So, imagine the same context as (17), slightly modified.

(24) *Context: The scientist tests three mandarins and three kiwis: the smallest dosage causes the first mandarin-kiwi pair to each weigh 50 g; the medium dosage causes the second mandarin-kiwi pair to each weigh 100 g; and the largest dosage causes the third apple to weigh 150 g and the third kiwi to weigh 200 g.*

m_1	$\leftrightarrow$	k_1	(50 g)
m_2	$\leftrightarrow$	k_2	(100 g)
m_3			(150 g)
		k_3	(200 g)

Informants report that (23a) can indeed be judged *true* in this context, as predicted by the analysis advocated for here: the conjunctive nominal denotes a HODP whose degree sub-pluralities (or subgroups) are required to be distinct from one another.

In summary, to the extent that the judgments reported above stand, we have evidence for HODP from symmetric (equative) readings of sentences with conjunctions of plural, degree-plurality-denoting nominals.

That being said, searching for HODP from (conjunctions of) definite descriptions may not be the most convincing route, as Rett (2022) warns us:

> "There is no limit on the sort of entity that can be referred to with a name or definite description, given the innovativeness of natural language." (Rett, 2022)

While Rett was referring specifically to putative *atomic* ("basic") semantic entities (such as degrees), her point is still well taken.

As it turns out, the data presented here are very related to the *reciprocal equative* construction first discussed by Schwarz (2007), which do not involve definite descriptions of degrees.

5. Reciprocal equatives

The reciprocal equative construction discussed by Schwarz (2007) is exemplified by (25a) and its German equivalent, (25b).

(25) a. The apples and the plums are equally heavy.

b. Die Äpfel und die Pflaumen sind gleich schwer.

In such cases, according to Schwarz, we form an ordinary, flat plurality of entities ($a_1 \sqcup \cdots \sqcup a_i \sqcup p_1 \sqcup \cdots \sqcup p_j$), to which the predicate *gleich schwer* (*equally heavy*) applies. Schwarz proposes that *gleich* (*equally*) is a degree operator that relates the adjective *schwer* (*heavy*) and this flat plurality Z of apples and plums, with respect to a contextually given set C. The meaning thus derived is in (26).

(26) If $x, y \leq Z$ and $x, y \in C$, then $\{d \mid x \text{ is } d\text{-heavy}\} = \{d \mid y \text{ is } d\text{-heavy}\}$.

For $C = \{\text{the apples, the plums}\}$, we derive that the *total* weight of apples is equal to the *total* weight of plums: $\{d \mid \text{the apples are } d\text{-heavy}\} = \{d \mid \text{the plums are } d\text{-heavy}\}$.

For $C = \{a_1, \ldots, a_i, p_1, \ldots, p_j\}$, we derive that *all* of the fruit pieces have the same weight: $\{d \mid a_1 \text{ is } d\text{-heavy}\} = \cdots = \{d \mid a_i \text{ is } d\text{-heavy}\} = \{d \mid p_1 \text{ is } d\text{-heavy}\} = \cdots = \{d \mid p_j \text{ is } d\text{-heavy}\}$.

Importantly, however, it is not possible under this analysis to derive the sort of symmetric reading described above. This semantics for *gleich* (*equally*) hardcodes universal reciprocity over members of C. No choice of C will help derive anything weaker.

Yet judgments are clear (at least for English) that (13) and (25a) have synonymous readings (ignoring any evaluative inference that may arise from the use of *heavy*). Thus, if the claims made here are accurate — that (13) and hence (25a) have genuine symmetric readings — then an alternative account that associates them with (16), repeated below, may be desirable, and perhaps necessary.[7]

(16) $\mu_w(a_1) \sqcup \cdots \sqcup \mu_w(a_i) = \mu_w(p_1) \sqcup \cdots \sqcup \mu_w(p_j)$

Finally, Schwarz notes a problem for his account involving *unterschiedlich* 'unequally'.

[7] Hsieh (2021) provides an important and interesting alternative account in terms of degree pluralities (not HODPs), covers, and distributivity. This account likewise forces universal quantification (albeit in a different way, through distributivity), hence is open to the same criticism.

(27) Die Äpfel und die Pflaumen sind unterschiedlich schwer.

The word-by-word counterpart in English, *The apples and the plums are unequally (differently) heavy*, is awkward at best, hence one reason why Schwarz focuses on German, but *The apples and the plums are unequal in weight* seems to come close.

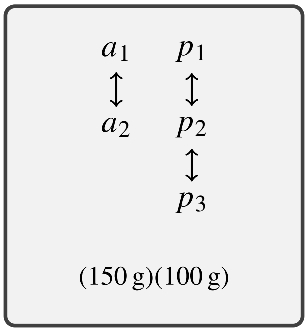
Schwarz assumes that *unterschiedlich*, like *gleich*, involves universal quantification:

(28) If $x, y \leq Z$ and $x, y \in C$ (and $x \neq y$), then $\{d \mid x$ is d-heavy$\} \neq \{d \mid y$ is d-heavy$\}$.

For $C = \{$the apples, the plums$\}$, we derive that the *total* weight of apples is different from the *total* weight of plums: $\{d \mid$ the apples are d-heavy$\} \neq \{d \mid$ the plums are d-heavy$\}$.

For $C = \{a_1, \ldots, a_i, p_1, \ldots, p_j\}$, we derive that *none* the pieces of fruit have the same weight: $\{d \mid a_1$ is d-heavy$\} \neq \cdots \neq \{d \mid a_i$ is d-heavy$\} \neq \{d \mid p_1$ is d-heavy$\} \neq \cdots \neq \{d \mid p_j$ is d-heavy$\}$.

Now, Schwarz asks us to consider a scenario where a_1 and a_2 each weigh 150 g, while p_1, p_2, and p_3 each weigh 100 g.



Neither choice of C above yields a true reading of (27). And yet (27) can truthfully describe this context. Schwarz writes, "Apparently the sentence can be read as conveying that each of the apples has a different weight than each of the plums." This paraphrase is essentially a *symmetric* one, albeit stronger than the one I have advocated for here, namely: 'not every apple has the same weight as some plum, or not every plum has the same weight as some apple'.

Schwarz concludes:

> "I suspect that this problem indicates that building reciprocity into the semantics of reciprocal degree operators in the way the lexical entries [given here] do is ultimately incorrect."

The problem can be overcome if we assume that degree pluralities are directly compared in the symmetric way advocated for here. For *unterschiedlich*, the two degree pluralities must simply be unequal (which still allows for *some* apples and plums to have the same weight, as in the modified mandarin-kiwi context in (24) above).

6. Conclusion

The main result of this paper is tentative evidence for the construction of higher-order pluralities of degrees. Naturally, a number of open issues remain, among which are the following.

First, the judgments reported here were informally collected from a handful of informants, and although the judgments were broadly confirmed by audience members where this work was presented, it may still be worth running a larger-scale experiment to test their accuracy and robustness (for example, using the methodology of Buccola et al. 2021).

Second, I reported that Schwarz's (2007) adjectival sentences are synonymous with the degree-denoting-nominal variants, at least in English, but this claim should be checked, including for German. Assuming they are synonymous, should we analyze both constructions in the same way, and if so, how do we do so compositionally? Or is it possible that they have different compositional analyses, but with the same end result (Chris Kennedy, p.c.)?

Third, in English, *equal* can be further modified with an approximative like *roughly*. How do we deal with that, on an analysis where *equal* has been assumed to encode the strictly binary mathematical relation = (that is, either $x = y$, or not)? Presumably, we start by analyzing what it means for two degree *atoms* to be 'roughly equal' (*The weight of this apple and the weight of that plum are roughly equal*), and then generalize that to degree *pluralities* that are 'roughly equal'.

Fourth, how do we deal with a sentence like *The weights of these apples (together) amount to (add up to) 300 g* (thanks to a Sinn und Bedeutung reviewer for this example)? The plural nominal cannot just denote a degree plurality, since idempotence would predict the sentence to be true if each apple weighed 300 g. One solution may be to say that, even if two apples each weigh 300 g, *the weight of this apple* and *the weight of that apple* may well denote two distinct things — for instance, intensionalized (or structured) degrees, or perhaps tropes in the sense of Moltmann 2009. This move would still allow us to sum up such denotations and form higher-order pluralities, but would effectively eliminate idempotence.

Fifth, can the kind of investigation of degrees presented here extend to even further types of entities, such as events, worlds, times, and others whose domains may involve mereological structure? What sort of grammatical constructions, perhaps corresponding to the symmetric and reciprocal equative constructions discussed here, would we need to investigate?

Needless to say, I must leave these issues — and the many other remaining ones — open for future research.

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